

Computer algebra independent integration tests

6-Hyperbolic-functions/6.2-Hyperbolic-cosine/6.2.2-e-x-^m-a+b-x^n-^p-cosh

Nasser M. Abbasi

July 24, 2021

Compiled on July 24, 2021 at 10:34pm

Contents

1	Introduction	7
1.1	Listing of CAS systems tested	7
1.2	Results	8
1.3	Performance	12
1.4	list of integrals that has no closed form antiderivative	13
1.5	list of integrals solved by CAS but has no known antiderivative	14
1.6	list of integrals solved by CAS but failed verification	14
1.7	Timing	15
1.8	Verification	15
1.9	Important notes about some of the results	15
1.9.1	Important note about Maxima results	15
1.9.2	Important note about FriCAS and Giac/XCAS results	16
1.9.3	Important note about finding leaf size of antiderivative	16
1.9.4	Important note about Mupad results	17
1.10	Design of the test system	18
2	detailed summary tables of results	19
2.1	List of integrals sorted by grade for each CAS	19
2.1.1	Rubi	19
2.1.2	Mathematica	19
2.1.3	Maple	20
2.1.4	Maxima	20

2.1.5	FriCAS	20
2.1.6	Sympy	20
2.1.7	Giac	21
2.1.8	Mupad	21
2.2	Detailed conclusion table per each integral for all CAS systems	22
2.3	Detailed conclusion table specific for Rubi results	44
3	Listing of integrals	51
3.1	$\int x^3(a + bx) \cosh(c + dx) dx$	51
3.2	$\int x^2(a + bx) \cosh(c + dx) dx$	55
3.3	$\int x(a + bx) \cosh(c + dx) dx$	59
3.4	$\int (a + bx) \cosh(c + dx) dx$	63
3.5	$\int \frac{(a+bx) \cosh(c+dx)}{x} dx$	66
3.6	$\int \frac{(a+bx) \cosh(c+dx)}{x^2} dx$	70
3.7	$\int \frac{(a+bx) \cosh(c+dx)}{x^3} dx$	74
3.8	$\int \frac{(a+bx) \cosh(c+dx)}{x^4} dx$	78
3.9	$\int \frac{(a+bx) \cosh(c+dx)}{x^5} dx$	82
3.10	$\int x^2(a + bx)^2 \cosh(c + dx) dx$	86
3.11	$\int x(a + bx)^2 \cosh(c + dx) dx$	90
3.12	$\int (a + bx)^2 \cosh(c + dx) dx$	94
3.13	$\int \frac{(a+bx)^2 \cosh(c+dx)}{x} dx$	97
3.14	$\int \frac{(a+bx)^2 \cosh(c+dx)}{x^2} dx$	101
3.15	$\int \frac{(a+bx)^2 \cosh(c+dx)}{x^3} dx$	105
3.16	$\int \frac{(a+bx)^2 \cosh(c+dx)}{x^4} dx$	109
3.17	$\int \frac{(a+bx)^2 \cosh(c+dx)}{x^5} dx$	113
3.18	$\int \frac{x^4 \cosh(c+dx)}{a+bx} dx$	118
3.19	$\int \frac{x^3 \cosh(c+dx)}{a+bx} dx$	123
3.20	$\int \frac{x^2 \cosh(c+dx)}{a+bx} dx$	128
3.21	$\int \frac{x \cosh(c+dx)}{a+bx} dx$	132
3.22	$\int \frac{\cosh(c+dx)}{a+bx} dx$	136
3.23	$\int \frac{\cosh(c+dx)}{x(a+bx)} dx$	139
3.24	$\int \frac{\cosh(c+dx)}{x^2(a+bx)} dx$	143
3.25	$\int \frac{\cosh(c+dx)}{x^3(a+bx)} dx$	147
3.26	$\int \frac{x^4 \cosh(c+dx)}{(a+bx)^2} dx$	152

3.27	$\int \frac{x^3 \cosh(c+dx)}{(a+bx)^2} dx$	159
3.28	$\int \frac{x^2 \cosh(c+dx)}{(a+bx)^2} dx$	165
3.29	$\int \frac{x \cosh(c+dx)}{(a+bx)^2} dx$	170
3.30	$\int \frac{\cosh(c+dx)}{(a+bx)^2} dx$	175
3.31	$\int \frac{\cosh(c+dx)}{x(a+bx)^2} dx$	179
3.32	$\int \frac{\cosh(c+dx)}{x^2(a+bx)^2} dx$	184
3.33	$\int \frac{x^3 \cosh(c+dx)}{(a+bx)^3} dx$	190
3.34	$\int \frac{x^2 \cosh(c+dx)}{(a+bx)^3} dx$	196
3.35	$\int \frac{x \cosh(c+dx)}{(a+bx)^3} dx$	201
3.36	$\int \frac{\cosh(c+dx)}{(a+bx)^3} dx$	206
3.37	$\int \frac{\cosh(c+dx)}{x(a+bx)^3} dx$	210
3.38	$\int \frac{\cosh(c+dx)}{x^2(a+bx)^3} dx$	215
3.39	$\int \frac{\cosh(c+dx)}{x^3(a+bx)^3} dx$	221
3.40	$\int x^3 (a + bx^2) \cosh(c + dx) dx$	227
3.41	$\int x^2 (a + bx^2) \cosh(c + dx) dx$	231
3.42	$\int x (a + bx^2) \cosh(c + dx) dx$	235
3.43	$\int (a + bx^2) \cosh(c + dx) dx$	239
3.44	$\int \frac{(a+bx^2) \cosh(c+dx)}{x} dx$	243
3.45	$\int \frac{(a+bx^2) \cosh(c+dx)}{x^2} dx$	247
3.46	$\int \frac{(a+bx^2) \cosh(c+dx)}{x^3} dx$	251
3.47	$\int \frac{(a+bx^2) \cosh(c+dx)}{x^4} dx$	255
3.48	$\int \frac{(a+bx^2) \cosh(c+dx)}{x^5} dx$	259
3.49	$\int x^2 (a + bx^2)^2 \cosh(c + dx) dx$	263
3.50	$\int x (a + bx^2)^2 \cosh(c + dx) dx$	268
3.51	$\int (a + bx^2)^2 \cosh(c + dx) dx$	272
3.52	$\int \frac{(a+bx^2)^2 \cosh(c+dx)}{x} dx$	276
3.53	$\int \frac{(a+bx^2)^2 \cosh(c+dx)}{x^2} dx$	280
3.54	$\int \frac{(a+bx^2)^2 \cosh(c+dx)}{x^3} dx$	284

3.55	$\int \frac{(a+bx^2)^2 \cosh(c+dx)}{x^4} dx$	288
3.56	$\int \frac{(a+bx^2)^2 \cosh(c+dx)}{x^5} dx$	292
3.57	$\int \frac{x^4 \cosh(c+dx)}{a+bx^2} dx$	296
3.58	$\int \frac{x^3 \cosh(c+dx)}{a+bx^2} dx$	301
3.59	$\int \frac{x^2 \cosh(c+dx)}{a+bx^2} dx$	306
3.60	$\int \frac{x \cosh(c+dx)}{a+bx^2} dx$	311
3.61	$\int \frac{\cosh(c+dx)}{a+bx^2} dx$	315
3.62	$\int \frac{\cosh(c+dx)}{x(a+bx^2)} dx$	319
3.63	$\int \frac{\cosh(c+dx)}{x^2(a+bx^2)} dx$	323
3.64	$\int \frac{\cosh(c+dx)}{x^3(a+bx^2)} dx$	328
3.65	$\int \frac{x^4 \cosh(c+dx)}{(a+bx^2)^2} dx$	333
3.66	$\int \frac{x^3 \cosh(c+dx)}{(a+bx^2)^2} dx$	339
3.67	$\int \frac{x^2 \cosh(c+dx)}{(a+bx^2)^2} dx$	345
3.68	$\int \frac{x \cosh(c+dx)}{(a+bx^2)^2} dx$	351
3.69	$\int \frac{\cosh(c+dx)}{(a+bx^2)^2} dx$	356
3.70	$\int \frac{\cosh(c+dx)}{x(a+bx^2)^2} dx$	362
3.71	$\int \frac{\cosh(c+dx)}{x^2(a+bx^2)^2} dx$	368
3.72	$\int \frac{x^3 \cosh(c+dx)}{(a+bx^2)^3} dx$	374
3.73	$\int \frac{x^2 \cosh(c+dx)}{(a+bx^2)^3} dx$	381
3.74	$\int \frac{x \cosh(c+dx)}{(a+bx^2)^3} dx$	389
3.75	$\int \frac{\cosh(c+dx)}{(a+bx^2)^3} dx$	395
3.76	$\int \frac{\cosh(c+dx)}{x(a+bx^2)^3} dx$	402
3.77	$\int \frac{\cosh(c+dx)}{x^2(a+bx^2)^3} dx$	410
3.78	$\int \frac{\cosh(c+dx)}{x^3(a+bx^2)^3} dx$	418

3.79	$\int x^3 (a + bx^3) \cosh(c + dx) dx$	426
3.80	$\int x^2 (a + bx^3) \cosh(c + dx) dx$	430
3.81	$\int x (a + bx^3) \cosh(c + dx) dx$	434
3.82	$\int (a + bx^3) \cosh(c + dx) dx$	438
3.83	$\int \frac{(a+bx^3) \cosh(c+dx)}{x} dx$	442
3.84	$\int \frac{(a+bx^3) \cosh(c+dx)}{x^2} dx$	446
3.85	$\int \frac{(a+bx^3) \cosh(c+dx)}{x^3} dx$	450
3.86	$\int \frac{(a+bx^3) \cosh(c+dx)}{x^4} dx$	454
3.87	$\int x (a + bx^3)^2 \cosh(c + dx) dx$	458
3.88	$\int (a + bx^3)^2 \cosh(c + dx) dx$	463
3.89	$\int \frac{(a+bx^3)^2 \cosh(c+dx)}{x} dx$	468
3.90	$\int \frac{(a+bx^3)^2 \cosh(c+dx)}{x^2} dx$	473
3.91	$\int \frac{(a+bx^3)^2 \cosh(c+dx)}{x^3} dx$	478
3.92	$\int \frac{(a+bx^3)^2 \cosh(c+dx)}{x^4} dx$	483
3.93	$\int \frac{(a+bx^3)^2 \cosh(c+dx)}{x^5} dx$	488
3.94	$\int \frac{x^4 \cosh(c+dx)}{a+bx^3} dx$	493
3.95	$\int \frac{x^3 \cosh(c+dx)}{a+bx^3} dx$	499
3.96	$\int \frac{x^2 \cosh(c+dx)}{a+bx^3} dx$	504
3.97	$\int \frac{x \cosh(c+dx)}{a+bx^3} dx$	509
3.98	$\int \frac{\cosh(c+dx)}{a+bx^3} dx$	514
3.99	$\int \frac{\cosh(c+dx)}{x(a+bx^3)} dx$	518
3.100	$\int \frac{\cosh(c+dx)}{x^2(a+bx^3)} dx$	523
3.101	$\int \frac{\cosh(c+dx)}{x^3(a+bx^3)} dx$	528
3.102	$\int \frac{x^3 \cosh(c+dx)}{(a+bx^3)^2} dx$	533
3.103	$\int \frac{x^2 \cosh(c+dx)}{(a+bx^3)^2} dx$	540
3.104	$\int \frac{x \cosh(c+dx)}{(a+bx^3)^2} dx$	546

3.105	$\int \frac{\cosh(c+dx)}{(a+bx^3)^2} dx$	553
3.106	$\int \frac{\cosh(c+dx)}{x(a+bx^3)^2} dx$	560
3.107	$\int \frac{x^5 \cosh(c+dx)}{(a+bx^3)^3} dx$	567
3.108	$\int \frac{x^4 \cosh(c+dx)}{(a+bx^3)^3} dx$	576
3.109	$\int \frac{x^3 \cosh(c+dx)}{(a+bx^3)^3} dx$	586
3.110	$\int \frac{x^2 \cosh(c+dx)}{(a+bx^3)^3} dx$	594
3.111	$\int \frac{x \cosh(c+dx)}{(a+bx^3)^3} dx$	602
4	Listing of Grading functions	613
4.0.1	Mathematica and Rubi grading function	613
4.0.2	Maple grading function	615
4.0.3	Sympy grading function	620
4.0.4	SageMath grading function	623

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from <https://rulebasedintegration.org>

The number of integrals in this report is [111]. This is test number [166].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or Hypergeometric₂F₁ functions. RootSum and RootOf are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100.00 (111)	% 0.00 (0)
Mathematica	% 100.00 (111)	% 0.00 (0)
Maple	% 100.00 (111)	% 0.00 (0)
Maxima	% 57.66 (64)	% 42.34 (47)
Fricas	% 100.00 (111)	% 0.00 (0)
Sympy	% 23.42 (26)	% 76.58 (85)
Giac	% 63.96 (71)	% 36.04 (40)
Mupad	% 18.02 (20)	% 81.98 (91)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons <ol style="list-style-type: none"> 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

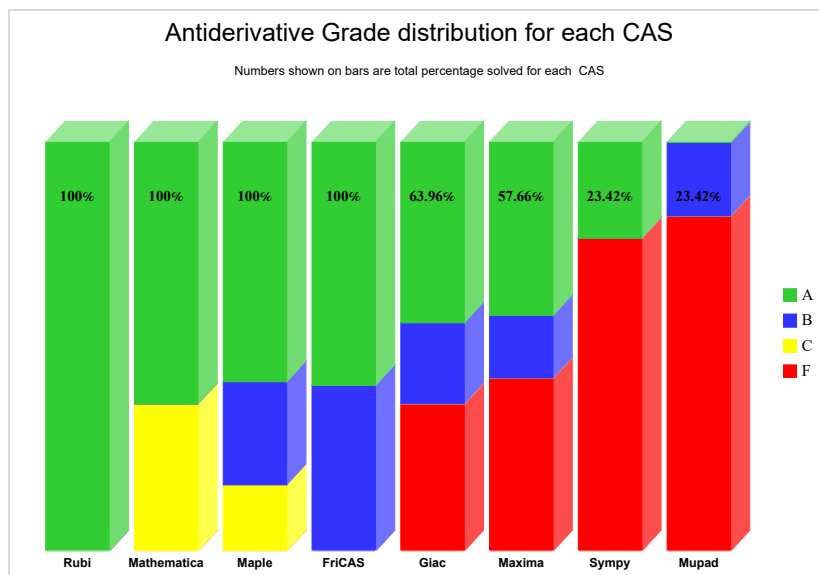
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

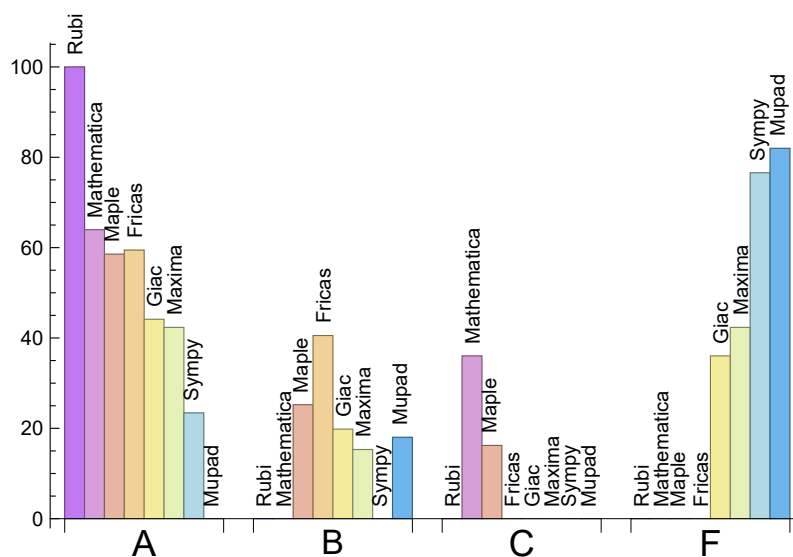
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	63.96	0.00	36.04	0.00
Maple	58.56	25.23	16.22	0.00
Maxima	42.34	15.32	0.00	42.34
Fricas	59.46	40.54	0.00	0.00
Sympy	23.42	0.00	0.00	76.58
Giac	44.14	19.82	0.00	36.04
Mupad	0.00	18.02	0.00	81.98

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This is the typical normal failure **F**.

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	0	0.00 %	0.00 %	0.00 %
Maxima	47	87.23 %	12.77 %	0.00 %
Fricas	0	0.00 %	0.00 %	0.00 %
Sympy	85	64.71 %	35.29 %	0.00 %
Giac	40	100.00 %	0.00 %	0.00 %
Mupad	91	100.00 %	0.00 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

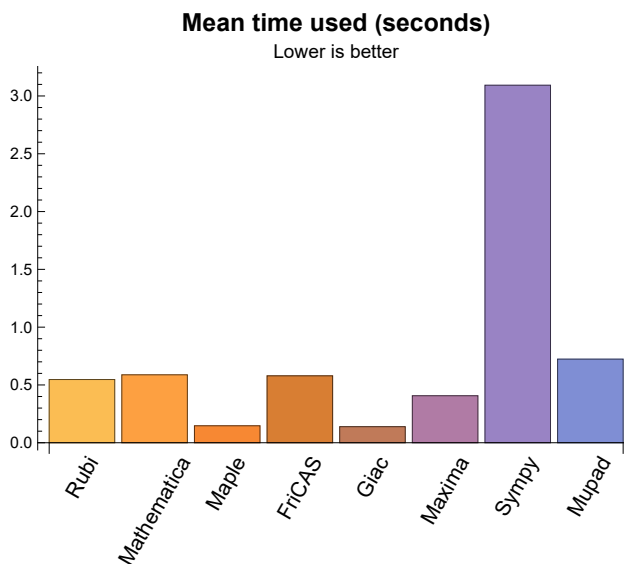
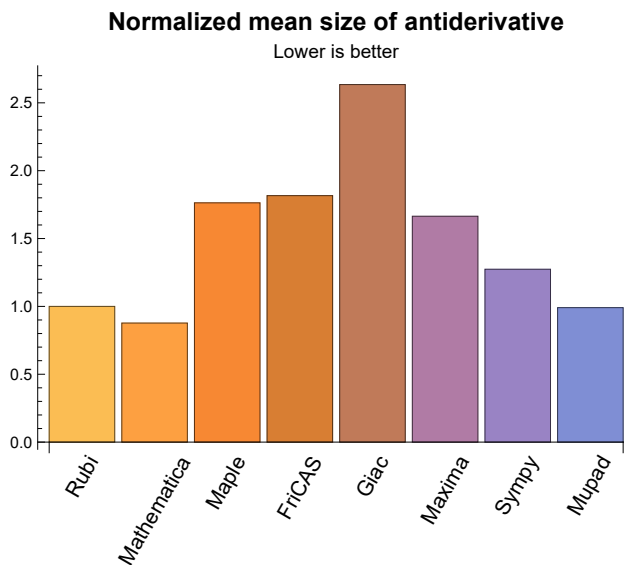
1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.55	268.55	1.00	178.00	1.00
Mathematica	0.59	233.19	0.88	140.00	0.86
Maple	0.15	412.83	1.76	288.00	1.69
Maxima	0.41	183.64	1.66	170.00	1.71
Fricas	0.58	652.41	1.82	200.00	1.68
Sympy	3.09	138.96	1.27	127.50	1.23
Giac	0.14	398.31	2.63	199.00	1.66
Mupad	0.72	118.50	0.99	115.00	0.97

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.



1.4 list of integrals that has no closed form antiderivative

{

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 104}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime')
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using `sagemath` (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See <https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/> for reference.

1.9.2 Important note about FriCAS and Giac/X-CAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the builtin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at <https://>

ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
    # 1.7 is a fudge factor since it is low side from actual leaf count
    leafCount = round(1.7*count_ops(anti))

except Exception as ee:
    leafCount = 1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

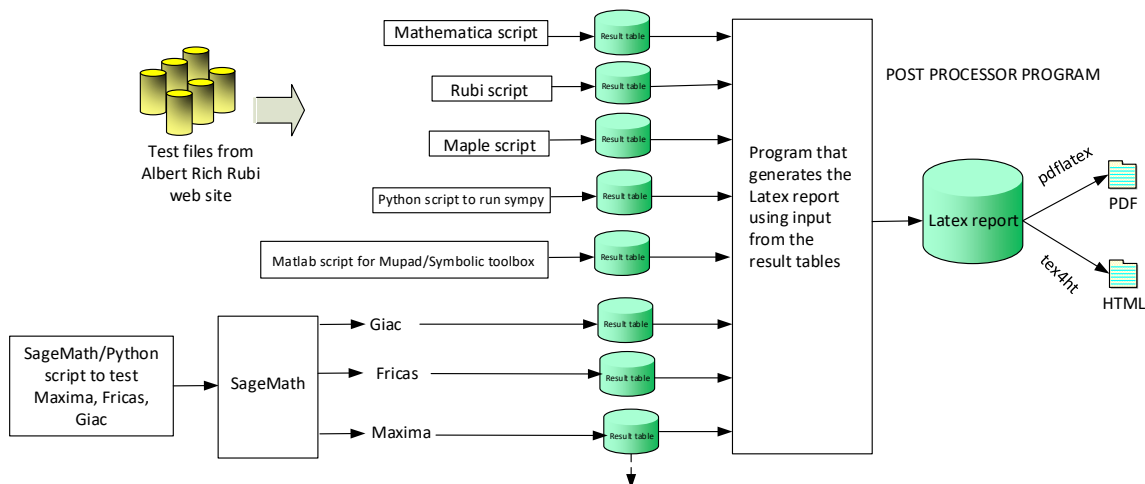
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine, 'cos(x)*sin(x)')
the_variable = evalin(symengine, 'x')
anti = int(integrand, the_variable)
```

Which gives $\sin(x)^2/2$

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
 2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
 3. integer. Leaf size of result.
 4. integer. Leaf size of the optimal antiderivative.
 5. number. CPU time used to solve this integral. 0 if failed.
 6. string. The integral in Latex format
 7. string. The input used in CAS own syntax.
 8. string. The result (antiderivative) produced by CAS in Latex format
 9. string. The optimal antiderivative in Latex format.
 10. integer. 0 or 1. Indicates if problem has known antiderivative or not
 11. String. The result (antiderivative) in CAS own syntax.
 12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
- The following field present only in Rubi and Mathematica Tables*
13. integer. 1 if result was verified or 0 if not verified.
- The following fields present only in Rubi Tables*
14. integer. Number of rules used.
 15. integer. Integrand leaf size.
 16. real number. Ratio of field 14 over field 15
 17. integer. 1 if result was verified or 0 if not verified.
 18. String of form "{n,n,...}" which is list of the rules used by Rubi

High level overview of the CAS independent integration test build system

Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93 }

B grade: { }

C grade: { 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111 }

F grade: { }

2.1.3 Maple

A grade: { 3, 4, 5, 6, 7, 8, 9, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 37, 43, 44, 45, 46, 47, 48, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 73, 74, 75, 76, 77, 83, 84, 85, 86, 91, 92, 93 }

B grade: { 1, 2, 10, 11, 12, 33, 34, 35, 36, 38, 39, 40, 41, 42, 49, 50, 51, 52, 72, 78, 79, 80, 81, 82, 87, 88, 89, 90 }

C grade: { 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111 }

F grade: { }

2.1.4 Maxima

A grade: { 1, 6, 7, 8, 9, 10, 14, 15, 16, 17, 18, 22, 24, 25, 26, 27, 28, 29, 30, 31, 36, 40, 41, 43, 45, 46, 47, 48, 49, 50, 51, 53, 54, 55, 56, 79, 82, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93 }

B grade: { 2, 3, 4, 5, 11, 12, 13, 19, 20, 21, 23, 42, 44, 52, 80, 81, 83 }

C grade: { }

F grade: { 32, 33, 34, 35, 37, 38, 39, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 31, 32, 34, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 60, 62, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93 }

B grade: { 30, 33, 35, 36, 37, 38, 39, 57, 58, 59, 61, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111 }

C grade: { }

F grade: { }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 10, 11, 12, 13, 40, 41, 42, 43, 44, 49, 50, 51, 52, 79, 80, 81, 82, 83, 87, 88, 89 }

B grade: { }

C grade: { }

F grade: { 6, 7, 8, 9, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 45, 46, 47, 48, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 84, 85, 86, 91, 92, 93 }

73, 74, 75, 76, 77, 78, 84, 85, 86, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 54, 55, 56, 79, 80, 81, 82, 83, 85, 86, 87, 88, 92 }

B grade: { 12, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 52, 53, 84, 89, 90, 91, 93 }

C grade: { }

F grade: { 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111 }

2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 10, 11, 12, 40, 41, 42, 43, 49, 50, 51, 79, 80, 81, 82, 87, 88 }

C grade: { }

F grade: { 5, 6, 7, 8, 9, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 44, 45, 46, 47, 48, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 83, 84, 85, 86, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table, the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	82	356	232	85	151	152	122
normalized size	1	1.00	0.66	2.87	1.87	0.69	1.22	1.23	0.98
time (sec)	N/A	0.321	0.161	0.043	0.369	0.431	1.881	0.136	0.958
Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	65	224	196	67	117	116	92
normalized size	1	1.00	0.69	2.38	2.09	0.71	1.24	1.23	0.98
time (sec)	N/A	0.217	0.137	0.041	0.409	0.999	0.938	0.113	0.962
Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	45	122	160	48	82	79	62
normalized size	1	1.00	0.70	1.91	2.50	0.75	1.28	1.23	0.97
time (sec)	N/A	0.114	0.097	0.050	0.395	0.654	0.487	0.118	0.922

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	27	53	68	30	46	46	35
normalized size	1	1.00	0.96	1.89	2.43	1.07	1.64	1.64	1.25
time (sec)	N/A	0.021	0.055	0.045	0.316	0.440	0.210	0.115	0.062

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	39	52	97	54	34	47	-1
normalized size	1	1.00	1.39	1.86	3.46	1.93	1.21	1.68	-0.04
time (sec)	N/A	0.149	0.029	0.088	0.532	0.762	3.192	0.138	0.000

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	59	77	82	76	0	72	-1
normalized size	1	1.00	1.26	1.64	1.74	1.62	0.00	1.53	-0.02
time (sec)	N/A	0.228	0.136	0.097	0.410	0.544	0.000	0.118	0.000

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	78	139	66	116	0	134	-1
normalized size	1	1.00	0.89	1.58	0.75	1.32	0.00	1.52	-0.01
time (sec)	N/A	0.281	0.166	0.098	0.429	0.521	0.000	0.123	0.000

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	110	205	78	143	0	199	-1
normalized size	1	1.00	0.83	1.55	0.59	1.08	0.00	1.51	-0.01
time (sec)	N/A	0.340	0.292	0.116	0.414	0.493	0.000	0.146	0.000

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	140	271	82	161	0	266	-1
normalized size	1	1.00	0.84	1.63	0.49	0.97	0.00	1.60	-0.01
time (sec)	N/A	0.406	0.350	0.117	0.408	0.554	0.000	0.144	0.000

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	100	463	329	127	228	236	168
normalized size	1	1.00	0.54	2.52	1.79	0.69	1.24	1.28	0.91
time (sec)	N/A	0.346	0.277	0.043	0.339	0.493	2.238	0.136	0.161

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	87	283	275	95	172	171	125
normalized size	1	1.00	0.65	2.11	2.05	0.71	1.28	1.28	0.93
time (sec)	N/A	0.214	0.217	0.041	0.333	0.535	1.154	0.117	0.923

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	56	147	135	64	112	112	82
normalized size	1	1.00	1.14	3.00	2.76	1.31	2.29	2.29	1.67
time (sec)	N/A	0.048	0.154	0.041	0.339	0.396	0.552	0.133	0.903

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	51	121	175	94	73	113	-1
normalized size	1	1.00	0.82	1.95	2.82	1.52	1.18	1.82	-0.02
time (sec)	N/A	0.185	0.250	0.108	0.400	0.622	3.700	0.143	0.000

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	62	118	136	122	0	119	-1
normalized size	1	1.00	0.89	1.69	1.94	1.74	0.00	1.70	-0.01
time (sec)	N/A	0.247	0.245	0.117	0.417	0.537	0.000	0.135	0.000

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	93	181	126	153	0	181	-1
normalized size	1	1.00	0.77	1.50	1.04	1.26	0.00	1.50	-0.01
time (sec)	N/A	0.339	0.399	0.113	0.442	0.627	0.000	0.119	0.000

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	154	287	117	194	0	285	-1
normalized size	1	1.00	0.90	1.67	0.68	1.13	0.00	1.66	-0.01
time (sec)	N/A	0.426	0.487	0.129	0.438	0.475	0.000	0.138	0.000

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	206	396	128	231	0	395	-1
normalized size	1	1.00	0.83	1.60	0.52	0.93	0.00	1.59	-0.00
time (sec)	N/A	0.521	0.586	0.135	0.438	0.471	0.000	0.143	0.000

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	159	442	437	236	0	407	-1
normalized size	1	1.00	0.73	2.02	2.00	1.08	0.00	1.86	-0.00
time (sec)	N/A	0.484	0.652	0.194	0.405	0.446	0.000	0.124	0.000

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	118	292	328	190	0	256	-1
normalized size	1	1.00	0.79	1.95	2.19	1.27	0.00	1.71	-0.01
time (sec)	N/A	0.329	0.451	0.139	0.413	0.911	0.000	0.146	0.000

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	89	184	233	156	0	148	-1
normalized size	1	1.00	0.89	1.84	2.33	1.56	0.00	1.48	-0.01
time (sec)	N/A	0.262	0.308	0.110	0.392	0.564	0.000	0.142	0.000

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	64	114	156	118	0	83	-1
normalized size	1	1.00	0.94	1.68	2.29	1.74	0.00	1.22	-0.01
time (sec)	N/A	0.176	0.147	0.089	0.380	0.451	0.000	0.143	0.000

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	49	81	57	95	0	56	-1
normalized size	1	1.00	0.96	1.59	1.12	1.86	0.00	1.10	-0.02
time (sec)	N/A	0.080	0.065	0.066	0.362	0.508	0.000	0.139	0.000

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	63	108	155	123	0	75	-1
normalized size	1	1.00	0.86	1.48	2.12	1.68	0.00	1.03	-0.01
time (sec)	N/A	0.259	0.150	0.082	0.413	0.638	0.000	0.141	0.000

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	101	172	192	179	0	129	-1
normalized size	1	1.00	0.89	1.52	1.70	1.58	0.00	1.14	-0.01
time (sec)	N/A	0.369	0.401	0.098	0.455	0.484	0.000	0.153	0.000

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	178	281	242	275	0	248	-1
normalized size	1	1.00	0.94	1.48	1.27	1.45	0.00	1.31	-0.01
time (sec)	N/A	0.485	0.499	0.104	0.485	0.516	0.000	0.141	0.000

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	173	431	406	371	0	2979	-1
normalized size	1	1.00	0.75	1.87	1.76	1.61	0.00	12.90	-0.00
time (sec)	N/A	0.536	1.263	0.210	0.421	0.468	0.000	0.241	0.000

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	156	325	312	333	0	1991	-1
normalized size	1	1.00	0.86	1.79	1.71	1.83	0.00	10.94	-0.01
time (sec)	N/A	0.423	0.956	0.149	0.421	0.536	0.000	0.212	0.000

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	115	254	236	274	0	1308	-1
normalized size	1	1.00	0.78	1.73	1.61	1.86	0.00	8.90	-0.01
time (sec)	N/A	0.375	0.713	0.120	0.412	0.473	0.000	0.179	0.000

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	97	215	178	200	0	994	-1
normalized size	1	1.00	0.78	1.72	1.42	1.60	0.00	7.95	-0.01
time (sec)	N/A	0.297	0.443	0.102	0.406	0.385	0.000	0.189	0.000

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	65	132	81	149	0	615	-1
normalized size	1	1.00	0.92	1.86	1.14	2.10	0.00	8.66	-0.01
time (sec)	N/A	0.110	0.211	0.079	0.361	0.566	0.000	0.171	0.000

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	139	254	227	270	0	1329	-1
normalized size	1	1.00	0.93	1.69	1.51	1.80	0.00	8.86	-0.01
time (sec)	N/A	0.400	1.212	0.098	0.465	0.612	0.000	0.196	0.000

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	183	312	0	377	0	3353	-1
normalized size	1	1.00	0.98	1.68	0.00	2.03	0.00	18.03	-0.01
time (sec)	N/A	0.501	1.420	0.106	0.000	0.503	0.000	0.223	0.000

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	236	571	0	566	0	879	-1
normalized size	1	1.00	0.89	2.16	0.00	2.14	0.00	3.33	-0.00
time (sec)	N/A	0.618	1.047	0.195	0.000	0.870	0.000	0.145	0.000

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	153	527	0	475	0	741	-1
normalized size	1	1.00	0.63	2.19	0.00	1.97	0.00	3.07	-0.00
time (sec)	N/A	0.530	0.973	0.132	0.000	0.615	0.000	0.131	0.000

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	158	435	0	373	0	529	-1
normalized size	1	1.00	0.89	2.44	0.00	2.10	0.00	2.97	-0.01
time (sec)	N/A	0.370	0.597	0.107	0.000	0.533	0.000	0.138	0.000

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	88	276	95	253	0	298	-1
normalized size	1	1.00	0.85	2.65	0.91	2.43	0.00	2.87	-0.01
time (sec)	N/A	0.146	0.511	0.087	0.363	0.484	0.000	0.125	0.000

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	451	488	0	601	0	837	-1
normalized size	1	1.00	1.72	1.86	0.00	2.29	0.00	3.19	-0.00
time (sec)	N/A	0.561	1.376	0.110	0.000	0.479	0.000	0.143	0.000

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	541	643	0	762	0	1006	-1
normalized size	1	1.00	1.82	2.16	0.00	2.56	0.00	3.38	-0.00
time (sec)	N/A	0.690	1.572	0.131	0.000	0.537	0.000	0.146	0.000

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	377	377	627	760	0	892	0	1169	-1
normalized size	1	1.00	1.66	2.02	0.00	2.37	0.00	3.10	-0.00
time (sec)	N/A	0.821	1.869	0.143	0.000	0.561	0.000	0.139	0.000

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	92	447	250	95	168	174	116
normalized size	1	1.00	0.66	3.22	1.80	0.68	1.21	1.25	0.83
time (sec)	N/A	0.255	0.170	0.042	0.324	0.528	3.424	0.123	0.991

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	74	298	214	78	134	138	93
normalized size	1	1.00	0.68	2.73	1.96	0.72	1.23	1.27	0.85
time (sec)	N/A	0.187	0.138	0.044	0.329	0.466	1.963	0.120	0.934

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	57	183	213	60	99	101	70
normalized size	1	1.00	0.72	2.32	2.70	0.76	1.25	1.28	0.89
time (sec)	N/A	0.121	0.122	0.041	0.325	0.423	0.993	0.114	0.975

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	40	97	86	42	65	70	47
normalized size	1	1.00	0.78	1.90	1.69	0.82	1.27	1.37	0.92
time (sec)	N/A	0.067	0.074	0.044	0.326	0.441	0.485	0.146	0.925

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	55	81	122	73	49	76	-1
normalized size	1	1.00	1.34	1.98	2.98	1.78	1.20	1.85	-0.02
time (sec)	N/A	0.098	0.119	0.101	0.391	0.540	3.414	0.121	0.000

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	81	80	82	0	80	-1
normalized size	1	1.00	1.00	1.93	1.90	1.95	0.00	1.90	-0.02
time (sec)	N/A	0.107	0.091	0.110	0.383	0.575	0.000	0.121	0.000

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	80	110	90	107	0	109	-1
normalized size	1	1.00	1.08	1.49	1.22	1.45	0.00	1.47	-0.01
time (sec)	N/A	0.178	0.160	0.112	0.410	0.499	0.000	0.119	0.000

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	95	172	73	127	0	170	-1
normalized size	1	1.00	0.90	1.64	0.70	1.21	0.00	1.62	-0.01
time (sec)	N/A	0.234	0.237	0.120	0.410	0.507	0.000	0.120	0.000

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	127	238	76	152	0	237	-1
normalized size	1	1.00	0.85	1.60	0.51	1.02	0.00	1.59	-0.01
time (sec)	N/A	0.290	0.286	0.122	0.414	0.483	0.000	0.144	0.000

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	138	738	383	155	286	304	182
normalized size	1	1.00	0.59	3.15	1.64	0.66	1.22	1.30	0.78
time (sec)	N/A	0.388	0.349	0.046	0.339	0.623	6.018	0.143	1.031

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	113	513	353	126	226	239	148
normalized size	1	1.00	0.61	2.79	1.92	0.68	1.23	1.30	0.80
time (sec)	N/A	0.277	0.248	0.045	0.332	0.615	3.577	0.144	0.982

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	85	332	189	98	172	180	114
normalized size	1	1.00	0.62	2.44	1.39	0.72	1.26	1.32	0.84
time (sec)	N/A	0.181	0.197	0.046	0.336	0.437	2.105	0.137	0.118

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	82	226	235	130	121	222	-1
normalized size	1	1.00	0.75	2.05	2.14	1.18	1.10	2.02	-0.01
time (sec)	N/A	0.189	0.390	0.147	0.400	0.588	4.773	0.127	0.000

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	95	190	179	127	0	197	-1
normalized size	1	1.00	1.00	2.00	1.88	1.34	0.00	2.07	-0.01
time (sec)	N/A	0.177	0.281	0.164	0.502	0.763	0.000	0.125	0.000

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	97	188	165	164	0	206	-1
normalized size	1	1.00	0.85	1.65	1.45	1.44	0.00	1.81	-0.01
time (sec)	N/A	0.225	0.396	0.171	0.427	0.895	0.000	0.127	0.000

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	114	222	135	171	0	236	-1
normalized size	1	1.00	0.86	1.67	1.02	1.29	0.00	1.77	-0.01
time (sec)	N/A	0.259	0.436	0.183	0.421	0.482	0.000	0.124	0.000

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	124	291	139	194	0	294	-1
normalized size	1	1.00	0.71	1.66	0.79	1.11	0.00	1.68	-0.01
time (sec)	N/A	0.360	0.488	0.196	0.452	0.546	0.000	0.142	0.000

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	273	273	274	369	0	605	0	0	-1
normalized size	1	1.00	1.00	1.35	0.00	2.22	0.00	0.00	-0.00
time (sec)	N/A	0.729	0.454	0.293	0.000	0.800	0.000	0.000	0.000

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	210	268	0	502	0	0	-1
normalized size	1	1.00	1.00	1.28	0.00	2.40	0.00	0.00	-0.00
time (sec)	N/A	0.358	0.390	0.214	0.000	1.114	0.000	0.000	0.000

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	226	226	213	259	0	496	0	0	-1
normalized size	1	1.00	0.94	1.15	0.00	2.19	0.00	0.00	-0.00
time (sec)	N/A	0.376	0.270	0.148	0.000	0.743	0.000	0.000	0.000

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	171	200	0	219	0	0	-1
normalized size	1	1.00	0.97	1.13	0.00	1.24	0.00	0.00	-0.01
time (sec)	N/A	0.254	0.186	0.110	0.000	0.659	0.000	0.000	0.000

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	180	212	0	316	0	0	-1
normalized size	1	1.00	0.85	1.00	0.00	1.48	0.00	0.00	-0.00
time (sec)	N/A	0.259	0.205	0.102	0.000	0.728	0.000	0.000	0.000

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	187	227	0	249	0	0	-1
normalized size	1	1.00	0.95	1.15	0.00	1.26	0.00	0.00	-0.01
time (sec)	N/A	0.372	0.334	0.103	0.000	0.689	0.000	0.000	0.000

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	249	249	243	288	0	599	0	0	-1
normalized size	1	1.00	0.98	1.16	0.00	2.41	0.00	0.00	-0.00
time (sec)	N/A	0.496	0.380	0.122	0.000	0.511	0.000	0.000	0.000

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	257	330	0	583	0	0	-1
normalized size	1	1.00	0.95	1.22	0.00	2.16	0.00	0.00	-0.00
time (sec)	N/A	0.504	0.569	0.115	0.000	0.491	0.000	0.000	0.000

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	449	449	621	532	0	1179	0	0	-1
normalized size	1	1.00	1.38	1.18	0.00	2.63	0.00	0.00	-0.00
time (sec)	N/A	0.859	1.626	0.406	0.000	0.639	0.000	0.000	0.000

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	431	431	582	495	0	931	0	0	-1
normalized size	1	1.00	1.35	1.15	0.00	2.16	0.00	0.00	-0.00
time (sec)	N/A	0.701	0.794	0.278	0.000	0.521	0.000	0.000	0.000

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	416	416	364	491	0	1162	0	0	-1
normalized size	1	1.00	0.88	1.18	0.00	2.79	0.00	0.00	-0.00
time (sec)	N/A	0.611	1.064	0.226	0.000	0.580	0.000	0.000	0.000

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	239	291	0	641	0	0	-1
normalized size	1	1.00	1.00	1.22	0.00	2.68	0.00	0.00	-0.00
time (sec)	N/A	0.325	0.534	0.182	0.000	0.573	0.000	0.000	0.000

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	476	476	590	503	0	1162	0	0	-1
normalized size	1	1.00	1.24	1.06	0.00	2.44	0.00	0.00	-0.00
time (sec)	N/A	0.818	0.740	0.111	0.000	0.508	0.000	0.000	0.000

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	435	435	501	546	0	992	0	0	-1
normalized size	1	1.00	1.15	1.26	0.00	2.28	0.00	0.00	-0.00
time (sec)	N/A	0.839	1.333	0.157	0.000	0.673	0.000	0.000	0.000

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	500	500	675	595	0	1310	0	0	-1
normalized size	1	1.00	1.35	1.19	0.00	2.62	0.00	0.00	-0.00
time (sec)	N/A	1.253	1.091	0.174	0.000	0.640	0.000	0.000	0.000

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	476	476	648	820	0	1620	0	0	-1
normalized size	1	1.00	1.36	1.72	0.00	3.40	0.00	0.00	-0.00
time (sec)	N/A	1.062	1.967	0.477	0.000	0.509	0.000	0.000	0.000

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	746	746	932	1064	0	2047	0	0	-1
normalized size	1	1.00	1.25	1.43	0.00	2.74	0.00	0.00	-0.00
time (sec)	N/A	1.100	2.671	0.327	0.000	0.476	0.000	0.000	0.000

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	512	512	637	743	0	1607	0	0	-1
normalized size	1	1.00	1.24	1.45	0.00	3.14	0.00	0.00	-0.00
time (sec)	N/A	0.831	1.847	0.227	0.000	0.587	0.000	0.000	0.000

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	856	856	933	1064	0	2116	0	0	-1
normalized size	1	1.00	1.09	1.24	0.00	2.47	0.00	0.00	-0.00
time (sec)	N/A	1.255	2.601	0.146	0.000	0.665	0.000	0.000	0.000

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	730	730	1046	1090	0	2076	0	0	-1
normalized size	1	1.00	1.43	1.49	0.00	2.84	0.00	0.00	-0.00
time (sec)	N/A	1.682	2.614	0.178	0.000	0.553	0.000	0.000	0.000

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	874	874	1009	1178	0	2346	0	0	-1
normalized size	1	1.00	1.15	1.35	0.00	2.68	0.00	0.00	-0.00
time (sec)	N/A	2.689	2.945	0.201	0.000	0.609	0.000	0.000	0.000

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	791	791	998	1294	0	2363	0	0	-1
normalized size	1	1.00	1.26	1.64	0.00	2.99	0.00	0.00	-0.00
time (sec)	N/A	1.868	4.020	0.233	0.000	0.529	0.000	0.000	0.000

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	100	551	268	105	185	192	152
normalized size	1	1.00	0.65	3.58	1.74	0.68	1.20	1.25	0.99
time (sec)	N/A	0.297	0.185	0.042	1.090	0.466	5.708	0.119	1.046

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	84	389	267	87	151	156	122
normalized size	1	1.00	0.68	3.14	2.15	0.70	1.22	1.26	0.98
time (sec)	N/A	0.228	0.147	0.046	0.340	0.514	3.252	0.125	0.960

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	66	257	196	68	116	119	92
normalized size	1	1.00	0.70	2.73	2.09	0.72	1.23	1.27	0.98
time (sec)	N/A	0.159	0.124	0.041	0.346	0.595	1.920	0.132	0.924

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	49	158	104	53	82	88	65
normalized size	1	1.00	0.74	2.39	1.58	0.80	1.24	1.33	0.98
time (sec)	N/A	0.103	0.097	0.043	0.341	0.572	0.940	0.124	0.098

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	49	113	140	86	66	109	-1
normalized size	1	1.00	0.88	2.02	2.50	1.54	1.18	1.95	-0.02
time (sec)	N/A	0.128	0.182	0.116	0.397	0.549	3.906	0.146	0.000

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	110	102	90	0	111	-1
normalized size	1	1.00	1.00	2.00	1.85	1.64	0.00	2.02	-0.02
time (sec)	N/A	0.129	0.142	0.139	0.398	0.591	0.000	0.140	0.000

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	86	114	87	101	0	118	-1
normalized size	1	1.00	1.25	1.65	1.26	1.46	0.00	1.71	-0.01
time (sec)	N/A	0.139	0.148	0.129	0.396	0.445	0.000	0.121	0.000

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	73	143	95	118	0	141	-1
normalized size	1	1.00	0.80	1.57	1.04	1.30	0.00	1.55	-0.01
time (sec)	N/A	0.215	0.263	0.132	0.419	0.483	0.000	0.117	0.000

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	139	818	383	161	284	303	301
normalized size	1	1.00	0.59	3.50	1.64	0.69	1.21	1.29	1.29
time (sec)	N/A	0.395	0.320	0.045	0.346	0.470	9.632	0.118	0.354

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	111	592	243	130	226	244	182
normalized size	1	1.00	0.60	3.18	1.31	0.70	1.22	1.31	0.98
time (sec)	N/A	0.287	0.244	0.042	0.345	0.654	5.855	0.136	0.241

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	108	335	289	161	168	331	-1
normalized size	1	1.00	0.68	2.09	1.81	1.01	1.05	2.07	-0.01
time (sec)	N/A	0.282	0.515	0.188	0.410	0.576	8.103	0.152	0.000

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	143	296	235	160	0	308	-1
normalized size	1	1.00	1.00	2.07	1.64	1.12	0.00	2.15	-0.01
time (sec)	N/A	0.252	0.377	0.207	0.403	0.553	0.000	0.124	0.000

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	136	265	203	161	0	280	-1
normalized size	1	1.00	0.96	1.88	1.44	1.14	0.00	1.99	-0.01
time (sec)	N/A	0.233	0.376	0.219	0.394	0.440	0.000	0.127	0.000

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	135	261	188	187	0	279	-1
normalized size	1	1.00	0.90	1.74	1.25	1.25	0.00	1.86	-0.01
time (sec)	N/A	0.279	0.633	0.237	0.439	0.453	0.000	0.129	0.000

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	150	292	154	196	0	315	-1
normalized size	1	1.00	0.90	1.75	0.92	1.17	0.00	1.89	-0.01
time (sec)	N/A	0.308	0.576	0.257	0.459	0.403	0.000	0.138	0.000

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	373	373	213	925	0	989	0	0	-1
normalized size	1	1.00	0.57	2.48	0.00	2.65	0.00	0.00	-0.00
time (sec)	N/A	0.937	0.511	0.187	0.000	0.534	0.000	0.000	0.000

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	358	358	198	671	0	977	0	0	-1
normalized size	1	1.00	0.55	1.87	0.00	2.73	0.00	0.00	-0.00
time (sec)	N/A	0.648	0.284	0.143	0.000	0.585	0.000	0.000	0.000

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	283	170	423	0	500	0	0	-1
normalized size	1	1.00	0.60	1.49	0.00	1.77	0.00	0.00	-0.00
time (sec)	N/A	0.455	0.240	0.120	0.000	0.450	0.000	0.000	0.000

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	345	345	180	280	0	671	0	0	-1
normalized size	1	1.00	0.52	0.81	0.00	1.94	0.00	0.00	-0.00
time (sec)	N/A	0.412	0.223	0.119	0.000	0.573	0.000	0.000	0.000

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	345	345	180	143	0	673	0	0	-1
normalized size	1	1.00	0.52	0.41	0.00	1.95	0.00	0.00	-0.00
time (sec)	N/A	0.401	0.136	0.094	0.000	0.652	0.000	0.000	0.000

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	303	303	186	138	0	530	0	0	-1
normalized size	1	1.00	0.61	0.46	0.00	1.75	0.00	0.00	-0.00
time (sec)	N/A	0.523	0.283	0.111	0.000	0.524	0.000	0.000	0.000

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	381	381	215	187	0	1154	0	0	-1
normalized size	1	1.00	0.56	0.49	0.00	3.03	0.00	0.00	-0.00
time (sec)	N/A	0.600	0.435	0.132	0.000	0.690	0.000	0.000	0.000

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	410	410	237	240	0	1251	0	0	-1
normalized size	1	1.00	0.58	0.59	0.00	3.05	0.00	0.00	-0.00
time (sec)	N/A	0.639	0.407	0.135	0.000	0.566	0.000	0.000	0.000

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	718	718	363	877	0	2050	0	0	-1
normalized size	1	1.00	0.51	1.22	0.00	2.86	0.00	0.00	-0.00
time (sec)	N/A	1.081	0.367	0.250	0.000	0.605	0.000	0.000	0.000

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	373	373	203	594	0	1276	0	0	-1
normalized size	1	1.00	0.54	1.59	0.00	3.42	0.00	0.00	-0.00
time (sec)	N/A	0.601	0.178	0.194	0.000	0.469	0.000	0.000	0.000

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	695	695	387	395	0	2135	0	0	-1
normalized size	1	1.00	0.56	0.57	0.00	3.07	0.00	0.00	-0.00
time (sec)	N/A	1.324	0.204	0.158	0.000	0.944	0.000	0.000	0.000

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	739	739	387	226	0	2048	0	0	-1
normalized size	1	1.00	0.52	0.31	0.00	2.77	0.00	0.00	-0.00
time (sec)	N/A	1.323	0.210	0.124	0.000	0.721	0.000	0.000	0.000

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	697	697	411	338	0	1773	0	0	-1
normalized size	1	1.00	0.59	0.48	0.00	2.54	0.00	0.00	-0.00
time (sec)	N/A	1.472	0.826	0.147	0.000	0.683	0.000	0.000	0.000

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	784	784	397	2448	0	2980	0	0	-1
normalized size	1	1.00	0.51	3.12	0.00	3.80	0.00	0.00	-0.00
time (sec)	N/A	1.626	0.677	0.683	0.000	1.004	0.000	0.000	0.000

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1105	1105	675	1927	0	4691	0	0	-1
normalized size	1	1.00	0.61	1.74	0.00	4.25	0.00	0.00	-0.00
time (sec)	N/A	1.853	0.577	0.479	0.000	0.589	0.000	0.000	0.000

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	776	776	429	1456	0	2962	0	0	-1
normalized size	1	1.00	0.55	1.88	0.00	3.82	0.00	0.00	-0.00
time (sec)	N/A	2.659	0.705	0.405	0.000	0.648	0.000	0.000	0.000
Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	781	781	423	994	0	2972	0	0	-1
normalized size	1	1.00	0.54	1.27	0.00	3.81	0.00	0.00	-0.00
time (sec)	N/A	1.449	0.458	0.346	0.000	0.692	0.000	0.000	0.000
Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1147	1147	669	810	0	4691	0	0	-1
normalized size	1	1.00	0.58	0.71	0.00	4.09	0.00	0.00	-0.00
time (sec)	N/A	3.198	0.502	0.258	0.000	0.683	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [111] had the largest ratio of [.5294]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	11	4	1.00	15	0.267
2	A	9	4	1.00	15	0.267
3	A	7	4	1.00	13	0.308
4	A	2	2	1.00	12	0.167
5	A	6	5	1.00	15	0.333
6	A	9	5	1.00	15	0.333
7	A	11	5	1.00	15	0.333
8	A	13	5	1.00	15	0.333
9	A	15	5	1.00	15	0.333
10	A	14	4	1.00	17	0.235
11	A	11	4	1.00	15	0.267
12	A	3	2	1.00	14	0.143
13	A	8	7	1.00	17	0.412
14	A	10	6	1.00	17	0.353
15	A	14	5	1.00	17	0.294
16	A	17	5	1.00	17	0.294
17	A	20	5	1.00	17	0.294
18	A	15	7	1.00	17	0.412
19	A	11	7	1.00	17	0.412
20	A	8	7	1.00	17	0.412
21	A	6	5	1.00	15	0.333
22	A	3	3	1.00	14	0.214
23	A	8	4	1.00	17	0.235
24	A	12	5	1.00	17	0.294
25	A	17	5	1.00	17	0.294

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	15	8	1.00	17	0.471
27	A	12	8	1.00	17	0.471
28	A	10	6	1.00	17	0.353
29	A	9	5	1.00	15	0.333
30	A	4	4	1.00	14	0.286
31	A	12	5	1.00	17	0.294
32	A	16	5	1.00	17	0.294
33	A	15	6	1.00	17	0.353
34	A	14	5	1.00	17	0.294
35	A	11	5	1.00	15	0.333
36	A	5	4	1.00	14	0.286
37	A	17	5	1.00	17	0.294
38	A	21	5	1.00	17	0.294
39	A	26	5	1.00	17	0.294
40	A	12	3	1.00	17	0.176
41	A	10	3	1.00	17	0.176
42	A	8	3	1.00	15	0.200
43	A	6	3	1.00	14	0.214
44	A	7	6	1.00	17	0.353
45	A	7	6	1.00	17	0.353
46	A	10	5	1.00	17	0.294
47	A	12	5	1.00	17	0.294
48	A	14	5	1.00	17	0.294
49	A	17	3	1.00	19	0.158
50	A	14	3	1.00	17	0.176
51	A	11	3	1.00	16	0.188

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
52	A	11	6	1.00	19	0.316
53	A	10	7	1.00	19	0.368
54	A	12	7	1.00	19	0.368
55	A	13	6	1.00	19	0.316
56	A	17	5	1.00	19	0.263
57	A	14	7	1.00	19	0.368
58	A	12	6	1.00	19	0.316
59	A	11	6	1.00	19	0.316
60	A	8	4	1.00	17	0.235
61	A	8	4	1.00	16	0.250
62	A	13	4	1.00	19	0.210
63	A	14	6	1.00	19	0.316
64	A	18	5	1.00	19	0.263
65	A	24	9	1.00	19	0.474
66	A	20	8	1.00	19	0.421
67	A	17	6	1.00	19	0.316
68	A	9	5	1.00	17	0.294
69	A	18	5	1.00	16	0.312
70	A	22	6	1.00	19	0.316
71	A	32	6	1.00	19	0.316
72	A	27	8	1.00	19	0.421
73	A	28	7	1.00	19	0.368
74	A	19	6	1.00	17	0.353
75	A	28	5	1.00	16	0.312
76	A	41	7	1.00	19	0.368
77	A	60	6	1.00	19	0.316

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
78	A	46	7	1.00	19	0.368
79	A	13	4	1.00	17	0.235
80	A	11	4	1.00	17	0.235
81	A	9	4	1.00	15	0.267
82	A	7	4	1.00	14	0.286
83	A	8	6	1.00	17	0.353
84	A	8	7	1.00	17	0.412
85	A	8	6	1.00	17	0.353
86	A	11	5	1.00	17	0.294
87	A	17	4	1.00	17	0.235
88	A	14	4	1.00	16	0.250
89	A	14	7	1.00	19	0.368
90	A	13	8	1.00	19	0.421
91	A	12	8	1.00	19	0.421
92	A	14	7	1.00	19	0.368
93	A	15	7	1.00	19	0.368
94	A	15	6	1.00	19	0.316
95	A	14	6	1.00	19	0.316
96	A	11	4	1.00	19	0.210
97	A	11	4	1.00	17	0.235
98	A	11	4	1.00	16	0.250
99	A	16	4	1.00	19	0.210
100	A	17	5	1.00	19	0.263
101	A	18	6	1.00	19	0.316
102	A	23	6	1.00	19	0.316
103	A	12	5	1.00	19	0.263

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
104	A	34	7	1.00	17	0.412
105	A	36	8	1.00	16	0.500
106	A	41	8	1.00	19	0.421
107	A	36	8	1.00	19	0.421
108	A	47	9	1.00	19	0.474
109	A	71	10	1.00	19	0.526
110	A	37	9	1.00	19	0.474
111	A	89	9	1.00	17	0.529

Chapter 3

Listing of integrals

3.1 $\int x^3(a + bx) \cosh(c + dx) dx$

Optimal. Leaf size=124

$$-\frac{6a \cosh(c + dx)}{d^4} + \frac{6ax \sinh(c + dx)}{d^3} - \frac{3ax^2 \cosh(c + dx)}{d^2} + \frac{ax^3 \sinh(c + dx)}{d} + \frac{24b \sinh(c + dx)}{d^5} - \frac{24bx \cosh(c + dx)}{d^4}$$

[Out] $-6*a*\cosh(d*x+c)/d^4-24*b*x*\cosh(d*x+c)/d^4-3*a*x^2*\cosh(d*x+c)/d^2-4*b*x^3*\cosh(d*x+c)/d^2+24*b*\sinh(d*x+c)/d^5+6*a*x*\sinh(d*x+c)/d^3+12*b*x^2*\sinh(d*x+c)/d^3+a*x^3*\sinh(d*x+c)/d+b*x^4*\sinh(d*x+c)/d$

Rubi [A] time = 0.32, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {6742, 3296, 2638, 2637}

$$-\frac{3ax^2 \cosh(c + dx)}{d^2} + \frac{6ax \sinh(c + dx)}{d^3} - \frac{6a \cosh(c + dx)}{d^4} + \frac{ax^3 \sinh(c + dx)}{d} + \frac{12bx^2 \sinh(c + dx)}{d^3} - \frac{4bx^3 \cosh(c + dx)}{d^2}$$

Antiderivative was successfully verified.

[In] `Int[x^3*(a + b*x)*Cosh[c + d*x], x]`

[Out] $(-6*a*\cosh[c + d*x])/d^4 - (24*b*x*\cosh[c + d*x])/d^4 - (3*a*x^2*\cosh[c + d*x])/d^2 - (4*b*x^3*\cosh[c + d*x])/d^2 + (24*b*\sinh[c + d*x])/d^5 + (6*a*x*\sinh[c + d*x])/d^3 + (12*b*x^2*\sinh[c + d*x])/d^3 + (a*x^3*\sinh[c + d*x])/d + (b*x^4*\sinh[c + d*x])/d$

Rule 2637

`Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;`
`FreeQ[{c, d}, x]`

Rule 2638

`Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]`

Rule 3296

`Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]`

Rule 6742

`Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]`

Rubi steps

$$\begin{aligned}
 \int x^3(a + bx) \cosh(c + dx) dx &= \int (ax^3 \cosh(c + dx) + bx^4 \cosh(c + dx)) dx \\
 &= a \int x^3 \cosh(c + dx) dx + b \int x^4 \cosh(c + dx) dx \\
 &= \frac{ax^3 \sinh(c + dx)}{d} + \frac{bx^4 \sinh(c + dx)}{d} - \frac{(3a) \int x^2 \sinh(c + dx) dx}{d} - \frac{(4b) \int x^3 \sinh(c + dx) dx}{d} \\
 &= -\frac{3ax^2 \cosh(c + dx)}{d^2} - \frac{4bx^3 \cosh(c + dx)}{d^2} + \frac{ax^3 \sinh(c + dx)}{d} + \frac{bx^4 \sinh(c + dx)}{d} \\
 &= -\frac{3ax^2 \cosh(c + dx)}{d^2} - \frac{4bx^3 \cosh(c + dx)}{d^2} + \frac{6ax \sinh(c + dx)}{d^3} + \frac{12bx^2 \sinh(c + dx)}{d^3} \\
 &= -\frac{6a \cosh(c + dx)}{d^4} - \frac{24bx \cosh(c + dx)}{d^4} - \frac{3ax^2 \cosh(c + dx)}{d^2} - \frac{4bx^3 \cosh(c + dx)}{d^2} \\
 &= -\frac{6a \cosh(c + dx)}{d^4} - \frac{24bx \cosh(c + dx)}{d^4} - \frac{3ax^2 \cosh(c + dx)}{d^2} - \frac{4bx^3 \cosh(c + dx)}{d^2}
 \end{aligned}$$

Mathematica [A] time = 0.16, size = 82, normalized size = 0.66

$$\frac{(ad^2x(d^2x^2 + 6) + b(d^4x^4 + 12d^2x^2 + 24)) \sinh(c + dx) - d(3a(d^2x^2 + 2) + 4bx(d^2x^2 + 6)) \cosh(c + dx)}{d^5}$$

Antiderivative was successfully verified.

[In] Integrate[x^3*(a + b*x)*Cosh[c + d*x], x]

[Out] $(-(d*(3*a*(2 + d^2*x^2) + 4*b*x*(6 + d^2*x^2))*\text{Cosh}[c + d*x]) + (a*d^2*x*(6 + d^2*x^2) + b*(24 + 12*d^2*x^2 + d^4*x^4))*\text{Sinh}[c + d*x])/d^5$

fricas [A] time = 0.43, size = 85, normalized size = 0.69

$$\frac{(4bd^3x^3 + 3ad^3x^2 + 24bdx + 6ad)\cosh(dx + c) - (bd^4x^4 + ad^4x^3 + 12bd^2x^2 + 6ad^2x + 24b)\sinh(dx + c)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*(b*x+a)*cosh(d*x+c),x, algorithm="fricas")`

[Out] $-((4*b*d^3*x^3 + 3*a*d^3*x^2 + 24*b*d*x + 6*a*d)*\cosh(d*x + c) - (b*d^4*x^4 + a*d^4*x^3 + 12*b*d^2*x^2 + 6*a*d^2*x + 24*b)*\sinh(d*x + c))/d^5$

giac [A] time = 0.14, size = 152, normalized size = 1.23

$$\frac{(bd^4x^4 + ad^4x^3 - 4bd^3x^3 - 3ad^3x^2 + 12bd^2x^2 + 6ad^2x - 24bdx - 6ad + 24b)e^{(dx+c)} - (bd^4x^4 + ad^4x^3 + 4bd^3x^3 + 3ad^3x^2 + 12bd^2x^2 + 6ad^2x + 24b)e^{-d*x - c}}{2d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*(b*x+a)*cosh(d*x+c),x, algorithm="giac")`

[Out] $1/2*(b*d^4*x^4 + a*d^4*x^3 - 4*b*d^3*x^3 - 3*a*d^3*x^2 + 12*b*d^2*x^2 + 6*a*d^2*x - 24*b*d*x - 6*a*d + 24*b)*e^{(d*x + c)}/d^5 - 1/2*(b*d^4*x^4 + a*d^4*x^3 + 4*b*d^3*x^3 + 3*a*d^3*x^2 + 12*b*d^2*x^2 + 6*a*d^2*x + 24*b*d*x + 6*a*d + 24*b)*e^{(-d*x - c)}/d^5$

maple [B] time = 0.04, size = 356, normalized size = 2.87

$$\frac{b((dx+c)^4 \sinh(dx+c) - 4(dx+c)^3 \cosh(dx+c) + 12(dx+c)^2 \sinh(dx+c) - 24(dx+c) \cosh(dx+c) + 24 \sinh(dx+c)) - 4bc((dx+c)^3 \sinh(dx+c) - 3(dx+c)^2 \cosh(dx+c) + 6(dx+c) \sinh(dx+c) - 6 \cosh(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*(b*x+a)*cosh(d*x+c),x)`

[Out] $1/d^4*(b/d*((d*x+c)^4*\sinh(d*x+c)-4*(d*x+c)^3*\cosh(d*x+c)+12*(d*x+c)^2*\sinh(d*x+c)-24*(d*x+c)*\cosh(d*x+c)+24*\sinh(d*x+c))-4*b*c/d*((d*x+c)^3*\sinh(d*x+c)-3*(d*x+c)^2*\cosh(d*x+c)+6*(d*x+c)*\sinh(d*x+c)-6*\cosh(d*x+c))+6*b/d*c^2*((d*x+c)^2*\sinh(d*x+c)-2*(d*x+c)*\cosh(d*x+c)+2*\sinh(d*x+c))-4*b/d*c^3*((d*x+c)*\sinh(d*x+c)-\cosh(d*x+c))+b*c^4/d*\sinh(d*x+c)+a*((d*x+c)^3*\sinh(d*x+c)-3*(d*x+c)^2*\cosh(d*x+c)+6*(d*x+c)*\sinh(d*x+c)-6*\cosh(d*x+c))-3*a*c*((d*x+c)^2*\sinh(d*x+c)-2*(d*x+c)*\cosh(d*x+c)+2*\sinh(d*x+c))+3*a*c^2*((d*x+c)*\sinh(d*x+c)-\cosh(d*x+c))-a*c^3*\sinh(d*x+c))$

maxima [A] time = 0.37, size = 232, normalized size = 1.87

$$-\frac{1}{40}d \left(\frac{5(d^4x^4e^c - 4d^3x^3e^c + 12d^2x^2e^c - 24dxe^c + 24e^c)ae^{(dx)}}{d^5} + \frac{5(d^4x^4 + 4d^3x^3 + 12d^2x^2 + 24dx + 24)ae^{(-dx)}}{d^5} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(b*x+a)*cosh(d*x+c),x, algorithm="maxima")

[Out] $-\frac{1}{40}d(5(d^4x^4e^c - 4d^3x^3e^c + 12d^2x^2e^c - 24dxe^c + 24e^c)ae^{(dx)}/d^5 + 5(d^4x^4 + 4d^3x^3 + 12d^2x^2 + 24dx + 24)ae^{(-dx)}/d^5 + 4(d^5x^5e^c - 5d^4x^4e^c + 20d^3x^3e^c - 60d^2x^2e^c + 120dxe^c - 120e^c)be^{(dx)}/d^6 + 4(d^5x^5 + 5d^4x^4 + 20d^3x^3 + 60d^2x^2 + 120dx + 120)be^{(-dx)}/d^6 + 1/20(4bx^5 + 5ax^4)cosh(dx + c))$

mupad [B] time = 0.96, size = 122, normalized size = 0.98

$$\frac{12bx^2 \sinh(c+dx) + 6ax \sinh(c+dx)}{d^3} - \frac{6a \cosh(c+dx) + 24bx \cosh(c+dx)}{d^4} - \frac{3ax^2 \cosh(c+dx) + 4bx^3 \cosh(c+dx)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*cosh(c + d*x)*(a + b*x),x)

[Out] $(12bx^2 \sinh(c+dx) + 6ax \sinh(c+dx))/d^3 - (6a \cosh(c+dx) + 24bx \cosh(c+dx))/d^4 - (3ax^2 \cosh(c+dx) + 4bx^3 \cosh(c+dx))/d^5 + (ax^3 \sinh(c+dx) + bx^4 \sinh(c+dx))/d + (24b \sinh(c+dx))/d^5$

sympy [A] time = 1.88, size = 151, normalized size = 1.22

$$\left\{ \begin{array}{l} \frac{ax^3 \sinh(c+dx)}{d} - \frac{3ax^2 \cosh(c+dx)}{d^2} + \frac{6ax \sinh(c+dx)}{d^3} - \frac{6a \cosh(c+dx)}{d^4} + \frac{bx^4 \sinh(c+dx)}{d} - \frac{4bx^3 \cosh(c+dx)}{d^2} + \frac{12bx^2 \sinh(c+dx)}{d^3} - \frac{24bx \cosh(c+dx)}{d^4} \\ \left(\frac{ax^4}{4} + \frac{bx^5}{5} \right) \cosh(c) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*(b*x+a)*cosh(d*x+c),x)

[Out] Piecewise((a*x**3*sinh(c + d*x)/d - 3*a*x**2*cosh(c + d*x)/d**2 + 6*a*x*sinh(c + d*x)/d**3 - 6*a*cosh(c + d*x)/d**4 + b*x**4*sinh(c + d*x)/d - 4*b*x**3*cosh(c + d*x)/d**2 + 12*b*x**2*sinh(c + d*x)/d**3 - 24*b*x*cosh(c + d*x)/d**4 + 24*b*sinh(c + d*x)/d**5, Ne(d, 0)), ((a*x**4/4 + b*x**5/5)*cosh(c), True))

3.2 $\int x^2(a + bx) \cosh(c + dx) dx$

Optimal. Leaf size=94

$$\frac{2a \sinh(c + dx)}{d^3} - \frac{2ax \cosh(c + dx)}{d^2} + \frac{ax^2 \sinh(c + dx)}{d} - \frac{6b \cosh(c + dx)}{d^4} + \frac{6bx \sinh(c + dx)}{d^3} - \frac{3bx^2 \cosh(c + dx)}{d^2} + \dots$$

[Out] $-6*b*\cosh(d*x+c)/d^4-2*a*x*\cosh(d*x+c)/d^2-3*b*x^2*\cosh(d*x+c)/d^2+2*a*\sinh(d*x+c)/d^3+6*b*x*\sinh(d*x+c)/d^3+a*x^2*\sinh(d*x+c)/d+b*x^3*\sinh(d*x+c)/d$

Rubi [A] time = 0.22, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {6742, 3296, 2637, 2638}

$$\frac{2a \sinh(c + dx)}{d^3} - \frac{2ax \cosh(c + dx)}{d^2} + \frac{ax^2 \sinh(c + dx)}{d} - \frac{3bx^2 \cosh(c + dx)}{d^2} + \frac{6bx \sinh(c + dx)}{d^3} - \frac{6b \cosh(c + dx)}{d^4} + \dots$$

Antiderivative was successfully verified.

[In] `Int[x^2*(a + b*x)*Cosh[c + d*x], x]`

[Out] $(-6*b*\cosh[c + d*x])/d^4 - (2*a*x*\cosh[c + d*x])/d^2 - (3*b*x^2*\cosh[c + d*x])/d^2 + (2*a*\sinh[c + d*x])/d^3 + (6*b*x*\sinh[c + d*x])/d^3 + (a*x^2*\sinh[c + d*x])/d + (b*x^3*\sinh[c + d*x])/d$

Rule 2637

`Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;`
`FreeQ[{c, d}, x]`

Rule 2638

`Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /;`
`FreeQ[{c, d}, x]`

Rule 3296

`Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[`
`((c + d*x)^m*cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[`
`e + f*x], x], x] /;`
`FreeQ[{c, d, e, f}, x] && GtQ[m, 0]`

Rule 6742

`Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /;`
`SumQ[v]`
`]`

Rubi steps

$$\begin{aligned}
\int x^2(a + bx) \cosh(c + dx) dx &= \int (ax^2 \cosh(c + dx) + bx^3 \cosh(c + dx)) dx \\
&= a \int x^2 \cosh(c + dx) dx + b \int x^3 \cosh(c + dx) dx \\
&= \frac{ax^2 \sinh(c + dx)}{d} + \frac{bx^3 \sinh(c + dx)}{d} - \frac{(2a) \int x \sinh(c + dx) dx}{d} - \frac{(3b) \int x^2 \sinh(c + dx) dx}{d} \\
&= -\frac{2ax \cosh(c + dx)}{d^2} - \frac{3bx^2 \cosh(c + dx)}{d^2} + \frac{ax^2 \sinh(c + dx)}{d} + \frac{bx^3 \sinh(c + dx)}{d} \\
&= -\frac{2ax \cosh(c + dx)}{d^2} - \frac{3bx^2 \cosh(c + dx)}{d^2} + \frac{2a \sinh(c + dx)}{d^3} + \frac{6bx \sinh(c + dx)}{d^3} + \frac{bx^2 \cosh(c + dx)}{d^3} \\
&= -\frac{6b \cosh(c + dx)}{d^4} - \frac{2ax \cosh(c + dx)}{d^2} - \frac{3bx^2 \cosh(c + dx)}{d^2} + \frac{2a \sinh(c + dx)}{d^3} + \frac{6bx \sinh(c + dx)}{d^3} + \frac{bx^2 \cosh(c + dx)}{d^3}
\end{aligned}$$

Mathematica [A] time = 0.14, size = 65, normalized size = 0.69

$$\frac{d \left(a \left(d^2 x^2 + 2 \right) + bx \left(d^2 x^2 + 6 \right) \right) \sinh(c + dx) - \left(2ad^2 x + 3b \left(d^2 x^2 + 2 \right) \right) \cosh(c + dx)}{d^4}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*(a + b*x)*Cosh[c + d*x], x]

[Out] (-((2*a*d^2*x + 3*b*(2 + d^2*x^2))*Cosh[c + d*x]) + d*(a*(2 + d^2*x^2) + b*x*(6 + d^2*x^2))*Sinh[c + d*x])/d^4

fricas [A] time = 1.00, size = 67, normalized size = 0.71

$$\frac{(3bd^2x^2 + 2ad^2x + 6b) \cosh(dx + c) - (bd^3x^3 + ad^3x^2 + 6bdx + 2ad) \sinh(dx + c)}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(b*x+a)*cosh(d*x+c), x, algorithm="fricas")

[Out] -((3*b*d^2*x^2 + 2*a*d^2*x + 6*b)*cosh(d*x + c) - (b*d^3*x^3 + a*d^3*x^2 + 6*b*d*x + 2*a*d)*sinh(d*x + c))/d^4

giac [A] time = 0.11, size = 116, normalized size = 1.23

$$\frac{(bd^3x^3 + ad^3x^2 - 3bd^2x^2 - 2ad^2x + 6bdx + 2ad - 6b)e^{(dx+c)}}{2d^4} - \frac{(bd^3x^3 + ad^3x^2 + 3bd^2x^2 + 2ad^2x + 6bdx + 2ad)}{2d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(b*x+a)*cosh(d*x+c),x, algorithm="giac")

[Out] $\frac{1}{2}*(b*d^3*x^3 + a*d^3*x^2 - 3*b*d^2*x^2 - 2*a*d^2*x + 6*b*d*x + 2*a*d - 6*b)*e^{(d*x + c)}/d^4 - \frac{1}{2}*(b*d^3*x^3 + a*d^3*x^2 + 3*b*d^2*x^2 + 2*a*d^2*x + 6*b*d*x + 2*a*d + 6*b)*e^{(-d*x - c)}/d^4$

maple [B] time = 0.04, size = 224, normalized size = 2.38

$$\frac{b((dx+c)^3 \sinh(dx+c) - 3(dx+c)^2 \cosh(dx+c) + 6(dx+c) \sinh(dx+c) - 6 \cosh(dx+c))}{d} - \frac{3bc((dx+c)^2 \sinh(dx+c) - 2(dx+c) \cosh(dx+c) + 2 \sinh(dx+c))}{d} +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(b*x+a)*cosh(d*x+c),x)

[Out] $\frac{1}{d^3}*(\frac{b}{d}*((d*x+c)^3*\sinh(d*x+c) - 3*(d*x+c)^2*\cosh(d*x+c) + 6*(d*x+c)*\sinh(d*x+c) - 6*\cosh(d*x+c)) - 3*b*c/d*((d*x+c)^2*\sinh(d*x+c) - 2*(d*x+c)*\cosh(d*x+c) + 2*\sinh(d*x+c)) + 3*b/d*c^2*((d*x+c)*\sinh(d*x+c) - \cosh(d*x+c)) - b*c^3/d*\sinh(d*x+c) + a*((d*x+c)^2*\sinh(d*x+c) - 2*(d*x+c)*\cosh(d*x+c) + 2*\sinh(d*x+c)) - 2*a*c*((d*x+c)*\sinh(d*x+c) - \cosh(d*x+c)) + a*c^2*\sinh(d*x+c))$

maxima [B] time = 0.41, size = 196, normalized size = 2.09

$$-\frac{1}{24}d \left(\frac{4(d^3x^3e^c - 3d^2x^2e^c + 6dxe^c - 6e^c)ae^{(dx)}}{d^4} + \frac{4(d^3x^3 + 3d^2x^2 + 6dx + 6)ae^{(-dx-c)}}{d^4} + \frac{3(d^4x^4e^c - 4d^3x^3e^c}{d^4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(b*x+a)*cosh(d*x+c),x, algorithm="maxima")

[Out] $-1/24*d*(4*(d^3*x^3*e^c - 3*d^2*x^2*e^c + 6*d*x*e^c - 6*e^c)*a*e^{(d*x)}/d^4 + 4*(d^3*x^3 + 3*d^2*x^2 + 6*d*x + 6)*a*e^{(-d*x - c)}/d^4 + 3*(d^4*x^4*e^c - 4*d^3*x^3*e^c + 12*d^2*x^2*e^c - 24*d*x*e^c + 24*e^c)*b*e^{(d*x)}/d^5 + 3*(d^4*x^4 + 4*d^3*x^3 + 12*d^2*x^2 + 24*d*x + 24)*b*e^{(-d*x - c)}/d^5) + 1/12*(3*b*x^4 + 4*a*x^3)*\cosh(d*x + c)$

mupad [B] time = 0.96, size = 92, normalized size = 0.98

$$\frac{2ax \sinh(c+dx) + 6bx \sinh(c+dx)}{d^3} - \frac{2ax \cosh(c+dx) + 3bx^2 \cosh(c+dx)}{d^2} + \frac{ax^2 \sinh(c+dx) + bx^3 \sinh(c+dx)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*cosh(c+d*x)*(a+b*x),x)

```
[Out] (2*a*sinh(c + d*x) + 6*b*x*sinh(c + d*x))/d^3 - (2*a*x*cosh(c + d*x) + 3*b*x^2*cosh(c + d*x))/d^2 + (a*x^2*sinh(c + d*x) + b*x^3*sinh(c + d*x))/d - (6*b*cosh(c + d*x))/d^4
```

```
sympy [A] time = 0.94, size = 117, normalized size = 1.24
```

$$\begin{cases} \frac{ax^2 \sinh(c+dx)}{d} - \frac{2ax \cosh(c+dx)}{d^2} + \frac{2a \sinh(c+dx)}{d^3} + \frac{bx^3 \sinh(c+dx)}{d} - \frac{3bx^2 \cosh(c+dx)}{d^2} + \frac{6bx \sinh(c+dx)}{d^3} - \frac{6b \cosh(c+dx)}{d^4} & \text{for } d \neq 0 \\ \left(\frac{ax^3}{3} + \frac{bx^4}{4}\right) \cosh(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)*cosh(d*x+c), x)
```

```
[Out] Piecewise((a*x**2*sinh(c + d*x)/d - 2*a*x*cosh(c + d*x)/d**2 + 2*a*sinh(c + d*x)/d**3 + b*x**3*sinh(c + d*x)/d - 3*b*x**2*cosh(c + d*x)/d**2 + 6*b*x*sinh(c + d*x)/d**3 - 6*b*cosh(c + d*x)/d**4, Ne(d, 0)), ((a*x**3/3 + b*x**4/4)*cosh(c), True))
```

3.3 $\int x(a + bx) \cosh(c + dx) dx$

Optimal. Leaf size=64

$$-\frac{a \cosh(c + dx)}{d^2} + \frac{ax \sinh(c + dx)}{d} + \frac{2b \sinh(c + dx)}{d^3} - \frac{2bx \cosh(c + dx)}{d^2} + \frac{bx^2 \sinh(c + dx)}{d}$$

[Out] $-a*\cosh(d*x+c)/d^2-2*b*x*\cosh(d*x+c)/d^2+2*b*\sinh(d*x+c)/d^3+a*x*\sinh(d*x+c)/d+b*x^2*\sinh(d*x+c)/d$

Rubi [A] time = 0.11, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {6742, 3296, 2638, 2637}

$$-\frac{a \cosh(c + dx)}{d^2} + \frac{ax \sinh(c + dx)}{d} + \frac{2b \sinh(c + dx)}{d^3} - \frac{2bx \cosh(c + dx)}{d^2} + \frac{bx^2 \sinh(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[x*(a + b*x)*Cosh[c + d*x], x]

[Out] $-((a*\cosh[c + d*x])/d^2) - (2*b*x*\cosh[c + d*x])/d^2 + (2*b*\sinh[c + d*x])/d^3 + (a*x*\sinh[c + d*x])/d + (b*x^2*\sinh[c + d*x])/d$

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 2638

Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3296

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[((c + d*x)^m*cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 6742

Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

Rubi steps

$$\begin{aligned}
\int x(a + bx) \cosh(c + dx) dx &= \int (ax \cosh(c + dx) + bx^2 \cosh(c + dx)) dx \\
&= a \int x \cosh(c + dx) dx + b \int x^2 \cosh(c + dx) dx \\
&= \frac{ax \sinh(c + dx)}{d} + \frac{bx^2 \sinh(c + dx)}{d} - \frac{a \int \sinh(c + dx) dx}{d} - \frac{(2b) \int x \sinh(c + dx) dx}{d} \\
&= -\frac{a \cosh(c + dx)}{d^2} - \frac{2bx \cosh(c + dx)}{d^2} + \frac{ax \sinh(c + dx)}{d} + \frac{bx^2 \sinh(c + dx)}{d} + \frac{(2b) \int x \sinh(c + dx) dx}{d} \\
&= -\frac{a \cosh(c + dx)}{d^2} - \frac{2bx \cosh(c + dx)}{d^2} + \frac{2b \sinh(c + dx)}{d^3} + \frac{ax \sinh(c + dx)}{d} + \frac{bx^2 \sinh(c + dx)}{d}
\end{aligned}$$

Mathematica [A] time = 0.10, size = 45, normalized size = 0.70

$$\frac{(ad^2x + b(d^2x^2 + 2)) \sinh(c + dx) - d(a + 2bx) \cosh(c + dx)}{d^3}$$

Antiderivative was successfully verified.

[In] Integrate[x*(a + b*x)*Cosh[c + d*x], x]

[Out] $-(d*(a + 2*b*x)*Cosh[c + d*x]) + (a*d^2*x + b*(2 + d^2*x^2))*Sinh[c + d*x] / d^3$

fricas [A] time = 0.65, size = 48, normalized size = 0.75

$$\frac{(2 b d x + a d) \cosh (d x + c) - (b d^2 x^2 + a d^2 x + 2 b) \sinh (d x + c)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x+a)*cosh(d*x+c), x, algorithm="fricas")

[Out] $-((2*b*d*x + a*d)*cosh(d*x + c) - (b*d^2*x^2 + a*d^2*x + 2*b)*sinh(d*x + c)) / d^3$

giac [A] time = 0.12, size = 79, normalized size = 1.23

$$\frac{(bd^2x^2 + ad^2x - 2bdx - ad + 2b)e^{(dx+c)}}{2d^3} - \frac{(bd^2x^2 + ad^2x + 2bdx + ad + 2b)e^{(-dx-c)}}{2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x+a)*cosh(d*x+c), x, algorithm="giac")

[Out] $1/2*(b*d^2*x^2 + a*d^2*x - 2*b*d*x - a*d + 2*b)*e^{(d*x + c)}/d^3 - 1/2*(b*d^2*x^2 + a*d^2*x + 2*b*d*x + a*d + 2*b)*e^{(-d*x - c)}/d^3$

maple [A] time = 0.05, size = 122, normalized size = 1.91

$$\frac{b((dx+c)^2 \sinh(dx+c) - 2(dx+c) \cosh(dx+c) + 2 \sinh(dx+c))}{d} - \frac{2bc((dx+c) \sinh(dx+c) - \cosh(dx+c))}{d} + \frac{a((dx+c) \sinh(dx+c) - \cosh(dx+c))}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(b*x+a)*cosh(d*x+c),x)`

[Out] $1/d^2*(b/d*((d*x+c)^2*\sinh(d*x+c)-2*(d*x+c)*\cosh(d*x+c)+2*\sinh(d*x+c))-2*b*c/d*((d*x+c)*\sinh(d*x+c)-\cosh(d*x+c))+a*((d*x+c)*\sinh(d*x+c)-\cosh(d*x+c))+b*c^2/d*\sinh(d*x+c)-a*c*\sinh(d*x+c))$

maxima [B] time = 0.39, size = 160, normalized size = 2.50

$$-\frac{1}{12}d \left(\frac{3(d^2x^2e^c - 2dxe^c + 2e^c)ae^{(dx)}}{d^3} + \frac{3(d^2x^2 + 2dx + 2)ae^{(-dx-c)}}{d^3} + \frac{2(d^3x^3e^c - 3d^2x^2e^c + 6dxe^c - 6e^c)be^{(dx)}}{d^4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(b*x+a)*cosh(d*x+c),x, algorithm="maxima")`

[Out] $-1/12*d*(3*(d^2*x^2*e^c - 2*d*x*e^c + 2*e^c)*a*e^{(d*x)}/d^3 + 3*(d^2*x^2 + 2*d*x + 2)*a*e^{(-d*x - c)}/d^3 + 2*(d^3*x^3*e^c - 3*d^2*x^2*e^c + 6*d*x*e^c - 6*e^c)*b*e^{(d*x)}/d^4 + 2*(d^3*x^3 + 3*d^2*x^2 + 6*d*x + 6)*b*e^{(-d*x - c)}/d^4 + 1/6*(2*b*x^3 + 3*a*x^2)*\cosh(d*x + c)$

mupad [B] time = 0.92, size = 62, normalized size = 0.97

$$\frac{bx^2 \sinh(c + dx) + ax \sinh(c + dx)}{d} - \frac{a \cosh(c + dx) + 2bx \cosh(c + dx)}{d^2} + \frac{2b \sinh(c + dx)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*cosh(c + d*x)*(a + b*x),x)`

[Out] $(b*x^2*\sinh(c + d*x) + a*x*\sinh(c + d*x))/d - (a*\cosh(c + d*x) + 2*b*x*\cosh(c + d*x))/d^2 + (2*b*\sinh(c + d*x))/d^3$

sympy [A] time = 0.49, size = 82, normalized size = 1.28

$$\begin{cases} \frac{ax \sinh(c+dx)}{d} - \frac{a \cosh(c+dx)}{d^2} + \frac{bx^2 \sinh(c+dx)}{d} - \frac{2bx \cosh(c+dx)}{d^2} + \frac{2b \sinh(c+dx)}{d^3} & \text{for } d \neq 0 \\ \left(\frac{ax^2}{2} + \frac{bx^3}{3} \right) \cosh(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)*cosh(d*x+c),x)
```

```
[Out] Piecewise((a*x*sinh(c + d*x)/d - a*cosh(c + d*x)/d**2 + b*x**2*sinh(c + d*x)/d - 2*b*x*cosh(c + d*x)/d**2 + 2*b*sinh(c + d*x)/d**3, Ne(d, 0)), ((a*x**2/2 + b*x**3/3)*cosh(c), True))
```

3.4 $\int (a + bx) \cosh(c + dx) dx$

Optimal. Leaf size=28

$$\frac{(a + bx) \sinh(c + dx)}{d} - \frac{b \cosh(c + dx)}{d^2}$$

[Out] $-b \cosh(d*x+c)/d^2 + (b*x+a)*\sinh(d*x+c)/d$

Rubi [A] time = 0.02, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3296, 2638}

$$\frac{(a + bx) \sinh(c + dx)}{d} - \frac{b \cosh(c + dx)}{d^2}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + b*x)*\text{Cosh}[c + d*x], x]$

[Out] $-((b*\text{Cosh}[c + d*x])/d^2) + ((a + b*x)*\text{Sinh}[c + d*x])/d$

Rule 2638

$\text{Int}[\sin[(c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow -\text{Simp}[\text{Cos}[c + d*x]/d, x] /; \text{FreeQ}[\{c, d\}, x]$

Rule 3296

$\text{Int}[((c_.) + (d_.)*(x_.))^{(m_.)} \sin[(e_.) + (f_.)*(x_.)], x_Symbol] \rightarrow -\text{Simp}[(c + d*x)^m \text{Cos}[e + f*x]/f, x] + \text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{(m-1)} \text{Cos}[e + f*x], x], x] /; \text{FreeQ}[\{c, d, e, f\}, x] \ \&\& \ \text{GtQ}[m, 0]$

Rubi steps

$$\begin{aligned} \int (a + bx) \cosh(c + dx) dx &= \frac{(a + bx) \sinh(c + dx)}{d} - \frac{b \int \sinh(c + dx) dx}{d} \\ &= -\frac{b \cosh(c + dx)}{d^2} + \frac{(a + bx) \sinh(c + dx)}{d} \end{aligned}$$

Mathematica [A] time = 0.06, size = 27, normalized size = 0.96

$$\frac{d(a + bx) \sinh(c + dx) - b \cosh(c + dx)}{d^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)*Cosh[c + d*x],x]

[Out] $(-(b*\text{Cosh}[c + d*x]) + d*(a + b*x)*\text{Sinh}[c + d*x])/d^2$

fricas [A] time = 0.44, size = 30, normalized size = 1.07

$$\frac{b \cosh(dx + c) - (bdx + ad) \sinh(dx + c)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)*cosh(d*x+c),x, algorithm="fricas")

[Out] $-(b*\cosh(d*x + c) - (b*d*x + a*d)*\sinh(d*x + c))/d^2$

giac [A] time = 0.11, size = 46, normalized size = 1.64

$$\frac{(bdx + ad - b)e^{(dx+c)}}{2d^2} - \frac{(bdx + ad + b)e^{(-dx-c)}}{2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)*cosh(d*x+c),x, algorithm="giac")

[Out] $1/2*(b*d*x + a*d - b)*e^{(d*x + c)}/d^2 - 1/2*(b*d*x + a*d + b)*e^{(-d*x - c)}/d^2$

maple [A] time = 0.04, size = 53, normalized size = 1.89

$$\frac{\frac{b((dx+c) \sinh(dx+c) - \cosh(dx+c))}{d} - \frac{bc \sinh(dx+c)}{d} + a \sinh(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x+a)*cosh(d*x+c),x)

[Out] $1/d*(b/d*((d*x+c)*\sinh(d*x+c) - \cosh(d*x+c)) - b*c/d*\sinh(d*x+c) + a*\sinh(d*x+c))$

maxima [B] time = 0.32, size = 68, normalized size = 2.43

$$\frac{ae^{(dx+c)}}{2d} + \frac{(dxe^c - e^c)be^{(dx)}}{2d^2} - \frac{(dx + 1)be^{(-dx-c)}}{2d^2} - \frac{ae^{(-dx-c)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)*cosh(d*x+c),x, algorithm="maxima")

[Out] $1/2*a*e^{(d*x + c)}/d + 1/2*(d*x*e^c - e^c)*b*e^{(d*x)}/d^2 - 1/2*(d*x + 1)*b*e^{(-d*x - c)}/d^2 - 1/2*a*e^{(-d*x - c)}/d$

mupad [B] time = 0.06, size = 35, normalized size = 1.25

$$\frac{a \sinh(c + dx) + b x \sinh(c + dx)}{d} - \frac{b \cosh(c + dx)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cosh(c + d*x)*(a + b*x),x)`

[Out] $(a*\sinh(c + d*x) + b*x*\sinh(c + d*x))/d - (b*\cosh(c + d*x))/d^2$

sympy [A] time = 0.21, size = 46, normalized size = 1.64

$$\begin{cases} \frac{a \sinh(c+dx)}{d} + \frac{bx \sinh(c+dx)}{d} - \frac{b \cosh(c+dx)}{d^2} & \text{for } d \neq 0 \\ \left(ax + \frac{bx^2}{2}\right) \cosh(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x+a)*cosh(d*x+c),x)`

[Out] `Piecewise((a*sinh(c + d*x)/d + b*x*sinh(c + d*x)/d - b*cosh(c + d*x)/d**2, Ne(d, 0)), ((a*x + b*x**2/2)*cosh(c), True))`

$$3.5 \quad \int \frac{(a+bx) \cosh(c+dx)}{x} dx$$

Optimal. Leaf size=28

$$a \cosh(c) \text{Chi}(dx) + a \sinh(c) \text{Shi}(dx) + \frac{b \sinh(c+dx)}{d}$$

[Out] a*Chi(d*x)*cosh(c)+a*Shi(d*x)*sinh(c)+b*sinh(d*x+c)/d

Rubi [A] time = 0.15, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6742, 2637, 3303, 3298, 3301}

$$a \cosh(c) \text{Chi}(dx) + a \sinh(c) \text{Shi}(dx) + \frac{b \sinh(c+dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[((a + b*x)*Cosh[c + d*x])/x,x]

[Out] a*Cosh[c]*CoshIntegral[d*x] + (b*Sinh[c + d*x])/d + a*Sinh[c]*SinhIntegral[d*x]

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3298

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]

Rule 3301

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]

Rule 3303

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&

NeQ[d*e - c*f, 0]

Rule 6742

Int[u_, x_Symbol] :=> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

Rubi steps

$$\begin{aligned} \int \frac{(a + bx) \cosh(c + dx)}{x} dx &= \int \left(b \cosh(c + dx) + \frac{a \cosh(c + dx)}{x} \right) dx \\ &= a \int \frac{\cosh(c + dx)}{x} dx + b \int \cosh(c + dx) dx \\ &= \frac{b \sinh(c + dx)}{d} + (a \cosh(c)) \int \frac{\cosh(dx)}{x} dx + (a \sinh(c)) \int \frac{\sinh(dx)}{x} dx \\ &= a \cosh(c) \text{Chi}(dx) + \frac{b \sinh(c + dx)}{d} + a \sinh(c) \text{Shi}(dx) \end{aligned}$$

Mathematica [A] time = 0.03, size = 39, normalized size = 1.39

$$a \cosh(c) \text{Chi}(dx) + a \sinh(c) \text{Shi}(dx) + \frac{b \sinh(c) \cosh(dx)}{d} + \frac{b \cosh(c) \sinh(dx)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x)*Cosh[c + d*x])/x,x]

[Out] a*Cosh[c]*CoshIntegral[d*x] + (b*Cosh[d*x]*Sinh[c])/d + (b*Cosh[c]*Sinh[d*x])/d + a*Sinh[c]*SinhIntegral[d*x]

fricas [A] time = 0.76, size = 54, normalized size = 1.93

$$\frac{(ad\text{Ei}(dx) + ad\text{Ei}(-dx)) \cosh(c) + 2b \sinh(dx + c) + (ad\text{Ei}(dx) - ad\text{Ei}(-dx)) \sinh(c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)*cosh(d*x+c)/x,x, algorithm="fricas")

[Out] 1/2*((a*d*Ei(d*x) + a*d*Ei(-d*x))*cosh(c) + 2*b*sinh(d*x + c) + (a*d*Ei(d*x) - a*d*Ei(-d*x))*sinh(c))/d

giac [A] time = 0.14, size = 47, normalized size = 1.68

$$\frac{ad\text{Ei}(-dx) e^{(-c)} + ad\text{Ei}(dx) e^c + be^{(dx+c)} - be^{(-dx-c)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)*cosh(d*x+c)/x,x, algorithm="giac")

[Out] $1/2*(a*d*Ei(-d*x)*e^{-c} + a*d*Ei(d*x)*e^c + b*e^{(d*x + c)} - b*e^{(-d*x - c)})/d$

maple [A] time = 0.09, size = 52, normalized size = 1.86

$$-\frac{a e^{-c} Ei(1, dx)}{2} - \frac{b e^{-dx-c}}{2d} - \frac{a e^c Ei(1, -dx)}{2} + \frac{b e^{dx+c}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x+a)*cosh(d*x+c)/x,x)

[Out] $-1/2*a*\exp(-c)*Ei(1,d*x)-1/2*b/d*\exp(-d*x-c)-1/2*a*\exp(c)*Ei(1,-d*x)+1/2*b/d*\exp(d*x+c)$

maxima [B] time = 0.53, size = 97, normalized size = 3.46

$$-\frac{1}{2} \left(b \left(\frac{(dx e^c - e^c) e^{dx}}{d^2} + \frac{(dx + 1) e^{(-dx-c)}}{d^2} \right) + \frac{2 a \cosh(dx + c) \log(x)}{d} - \frac{(Ei(-dx) e^{(-c)} + Ei(dx) e^c) a}{d} \right) d + (bx + a \log(x)) \cosh(dx + c)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)*cosh(d*x+c)/x,x, algorithm="maxima")

[Out] $-1/2*(b*((d*x*e^c - e^c)*e^{(d*x)}/d^2 + (d*x + 1)*e^{(-d*x - c)}/d^2) + 2*a*\cosh(d*x + c)*\log(x)/d - (Ei(-d*x)*e^{-c} + Ei(d*x)*e^c)*a/d)*d + (b*x + a*\log(x))*\cosh(d*x + c)$

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$a \operatorname{coshint}(dx) \cosh(c) + a \operatorname{sinhint}(dx) \sinh(c) + \frac{b \sinh(c + dx)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cosh(c + d*x)*(a + b*x))/x,x)

[Out] $a*\operatorname{coshint}(d*x)*\cosh(c) + a*\operatorname{sinhint}(d*x)*\sinh(c) + (b*\sinh(c + d*x))/d$

sympy [A] time = 3.19, size = 34, normalized size = 1.21

$$a \sinh(c) \operatorname{Shi}(dx) + a \cosh(c) \operatorname{Chi}(dx) + b \begin{cases} \frac{\sinh(c+dx)}{d} & \text{for } d \neq 0 \\ x \cosh(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*cosh(d*x+c)/x,x)
```

```
[Out] a*sinh(c)*Shi(d*x) + a*cosh(c)*Chi(d*x) + b*Piecewise((sinh(c + d*x)/d, Ne(d, 0)), (x*cosh(c), True))
```

$$3.6 \quad \int \frac{(a+bx) \cosh(c+dx)}{x^2} dx$$

Optimal. Leaf size=47

$$ad \sinh(c)\text{Chi}(dx) + ad \cosh(c)\text{Shi}(dx) - \frac{a \cosh(c+dx)}{x} + b \cosh(c)\text{Chi}(dx) + b \sinh(c)\text{Shi}(dx)$$

[Out] b*Chi(d*x)*cosh(c)-a*cosh(d*x+c)/x+a*d*cosh(c)*Shi(d*x)+a*d*Chi(d*x)*sinh(c)+b*Shi(d*x)*sinh(c)

Rubi [A] time = 0.23, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6742, 3297, 3303, 3298, 3301}

$$ad \sinh(c)\text{Chi}(dx) + ad \cosh(c)\text{Shi}(dx) - \frac{a \cosh(c+dx)}{x} + b \cosh(c)\text{Chi}(dx) + b \sinh(c)\text{Shi}(dx)$$

Antiderivative was successfully verified.

[In] Int[((a + b*x)*Cosh[c + d*x])/x^2,x]

[Out] -((a*Cosh[c + d*x])/x) + b*Cosh[c]*CoshIntegral[d*x] + a*d*CoshIntegral[d*x]*Sinh[c] + a*d*Cosh[c]*SinhIntegral[d*x] + b*Sinh[c]*SinhIntegral[d*x]

Rule 3297

Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]

Rule 3298

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)])/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]

Rule 3301

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)])/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]

Rule 3303

Int[sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f

)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]

Rule 6742

Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]

Rubi steps

$$\begin{aligned} \int \frac{(a + bx) \cosh(c + dx)}{x^2} dx &= \int \left(\frac{a \cosh(c + dx)}{x^2} + \frac{b \cosh(c + dx)}{x} \right) dx \\ &= a \int \frac{\cosh(c + dx)}{x^2} dx + b \int \frac{\cosh(c + dx)}{x} dx \\ &= -\frac{a \cosh(c + dx)}{x} + (ad) \int \frac{\sinh(c + dx)}{x} dx + (b \cosh(c)) \int \frac{\cosh(dx)}{x} dx + (b \sinh(c)) \int \frac{\sinh(dx)}{x} dx \\ &= -\frac{a \cosh(c + dx)}{x} + b \cosh(c) \text{Chi}(dx) + b \sinh(c) \text{Shi}(dx) + (ad \cosh(c)) \int \frac{\sinh(dx)}{x} dx \\ &= -\frac{a \cosh(c + dx)}{x} + b \cosh(c) \text{Chi}(dx) + ad \text{Chi}(dx) \sinh(c) + ad \cosh(c) \text{Shi}(dx) + \end{aligned}$$

Mathematica [A] time = 0.14, size = 59, normalized size = 1.26

$$ad(\sinh(c)\text{Chi}(dx)+\cosh(c)\text{Shi}(dx))-\frac{a \sinh(c) \sinh(dx)}{x}-\frac{a \cosh(c) \cosh(dx)}{x}+b \cosh(c)\text{Chi}(dx)+b \sinh(c)\text{Shi}(dx)$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x)*Cosh[c + d*x])/x^2,x]

[Out] -((a*Cosh[c]*Cosh[d*x])/x) + b*Cosh[c]*CoshIntegral[d*x] - (a*Sinh[c]*Sinh[d*x])/x + b*Sinh[c]*SinhIntegral[d*x] + a*d*(CoshIntegral[d*x]*Sinh[c] + Cosh[c]*SinhIntegral[d*x])

fricas [A] time = 0.54, size = 76, normalized size = 1.62

$$\frac{2 a \cosh (d x + c) - ((a d + b) x \text{Ei}(d x) - (a d - b) x \text{Ei}(-d x)) \cosh (c) - ((a d + b) x \text{Ei}(d x) + (a d - b) x \text{Ei}(-d x)) \sinh (c)}{2 x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)*cosh(d*x+c)/x^2,x, algorithm="fricas")

[Out] $-1/2*(2*a*cosh(d*x + c) - ((a*d + b)*x*Ei(d*x) - (a*d - b)*x*Ei(-d*x))*cosh(c) - ((a*d + b)*x*Ei(d*x) + (a*d - b)*x*Ei(-d*x))*sinh(c))/x$

giac [A] time = 0.12, size = 72, normalized size = 1.53

$$\frac{-adx\text{Ei}(-dx)e^{(-c)} - adx\text{Ei}(dx)e^c - bx\text{Ei}(-dx)e^{(-c)} - bx\text{Ei}(dx)e^c + ae^{(dx+c)} + ae^{(-dx-c)}}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x+a)*cosh(d*x+c)/x^2,x, algorithm="giac")`

[Out] $-1/2*(a*d*x*Ei(-d*x)*e^{(-c)} - a*d*x*Ei(d*x)*e^c - b*x*Ei(-d*x)*e^{(-c)} - b*x*Ei(d*x)*e^c + a*e^{(d*x + c)} + a*e^{(-d*x - c)})/x$

maple [A] time = 0.10, size = 77, normalized size = 1.64

$$-\frac{ae^{-dx-c}}{2x} + \frac{dae^{-c}\text{Ei}(1,dx)}{2} - \frac{be^{-c}\text{Ei}(1,dx)}{2} - \frac{ae^{dx+c}}{2x} - \frac{dae^c\text{Ei}(1,-dx)}{2} - \frac{be^c\text{Ei}(1,-dx)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x+a)*cosh(d*x+c)/x^2,x)`

[Out] $-1/2*a*\exp(-d*x-c)/x+1/2*d*a*\exp(-c)*\text{Ei}(1,d*x)-1/2*b*\exp(-c)*\text{Ei}(1,d*x)-1/2*a/x*\exp(d*x+c)-1/2*d*a*\exp(c)*\text{Ei}(1,-d*x)-1/2*b*\exp(c)*\text{Ei}(1,-d*x)$

maxima [A] time = 0.41, size = 82, normalized size = 1.74

$$-\frac{1}{2} \left((\text{Ei}(-dx)e^{(-c)} - \text{Ei}(dx)e^c)a + \frac{2b \cosh(dx+c) \log(x)}{d} - \frac{(\text{Ei}(-dx)e^{(-c)} + \text{Ei}(dx)e^c)b}{d} \right) d + \left(b \log(x) - \frac{a}{x} \right) \cosh(dx+c)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x+a)*cosh(d*x+c)/x^2,x, algorithm="maxima")`

[Out] $-1/2*((\text{Ei}(-d*x)*e^{(-c)} - \text{Ei}(d*x)*e^c)*a + 2*b*cosh(d*x + c)*log(x)/d - (\text{Ei}(-d*x)*e^{(-c)} + \text{Ei}(d*x)*e^c)*b/d)*d + (b*log(x) - a/x)*cosh(d*x + c)$

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cosh(c + dx)(a + bx)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((cosh(c + d*x)*(a + b*x))/x^2,x)`


```
[Out] int((cosh(c + d*x)*(a + b*x))/x^2, x)
```

```
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int \frac{(a + bx) \cosh(c + dx)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*cosh(d*x+c)/x**2,x)
```

```
[Out] Integral((a + b*x)*cosh(c + d*x)/x**2, x)
```

3.7 $\int \frac{(a+bx) \cosh(c+dx)}{x^3} dx$

Optimal. Leaf size=88

$$\frac{1}{2}ad^2 \cosh(c)\text{Chi}(dx) + \frac{1}{2}ad^2 \sinh(c)\text{Shi}(dx) - \frac{a \cosh(c+dx)}{2x^2} - \frac{ad \sinh(c+dx)}{2x} + bd \sinh(c)\text{Chi}(dx) + bd \cosh(c)\text{Shi}(dx)$$

[Out] $\frac{1}{2}a*d^2*\text{Chi}(d*x)*\cosh(c) - \frac{1}{2}a*\cosh(d*x+c)/x^2 - b*\cosh(d*x+c)/x + b*d*\cosh(c)*\text{Shi}(d*x) + b*d*\text{Chi}(d*x)*\sinh(c) + \frac{1}{2}a*d^2*\text{Shi}(d*x)*\sinh(c) - \frac{1}{2}a*d*\sinh(d*x+c)/x$

Rubi [A] time = 0.28, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6742, 3297, 3303, 3298, 3301}

$$\frac{1}{2}ad^2 \cosh(c)\text{Chi}(dx) + \frac{1}{2}ad^2 \sinh(c)\text{Shi}(dx) - \frac{a \cosh(c+dx)}{2x^2} - \frac{ad \sinh(c+dx)}{2x} + bd \sinh(c)\text{Chi}(dx) + bd \cosh(c)\text{Shi}(dx)$$

Antiderivative was successfully verified.

[In] Int[((a + b*x)*Cosh[c + d*x])/x^3,x]

[Out] $-(a*\text{Cosh}[c + d*x])/(2*x^2) - (b*\text{Cosh}[c + d*x])/x + (a*d^2*\text{Cosh}[c]*\text{CoshIntegral}[d*x])/2 + b*d*\text{CoshIntegral}[d*x]*\text{Sinh}[c] - (a*d*\text{Sinh}[c + d*x])/(2*x) + b*d*\text{Cosh}[c]*\text{SinhIntegral}[d*x] + (a*d^2*\text{Sinh}[c]*\text{SinhIntegral}[d*x])/2$

Rule 3297

Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]

Rule 3298

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)])/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]

Rule 3301

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)])/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + bx) \cosh(c + dx)}{x^3} dx &= \int \left(\frac{a \cosh(c + dx)}{x^3} + \frac{b \cosh(c + dx)}{x^2} \right) dx \\
&= a \int \frac{\cosh(c + dx)}{x^3} dx + b \int \frac{\cosh(c + dx)}{x^2} dx \\
&= -\frac{a \cosh(c + dx)}{2x^2} - \frac{b \cosh(c + dx)}{x} + \frac{1}{2}(ad) \int \frac{\sinh(c + dx)}{x^2} dx + (bd) \int \frac{\sinh(c + dx)}{x} dx \\
&= -\frac{a \cosh(c + dx)}{2x^2} - \frac{b \cosh(c + dx)}{x} - \frac{ad \sinh(c + dx)}{2x} + \frac{1}{2}(ad^2) \int \frac{\cosh(c + dx)}{x} dx \\
&= -\frac{a \cosh(c + dx)}{2x^2} - \frac{b \cosh(c + dx)}{x} + bd \operatorname{Chi}(dx) \sinh(c) - \frac{ad \sinh(c + dx)}{2x} + bd \operatorname{Chi}(dx) \cosh(c) \\
&= -\frac{a \cosh(c + dx)}{2x^2} - \frac{b \cosh(c + dx)}{x} + \frac{1}{2}ad^2 \cosh(c) \operatorname{Chi}(dx) + bd \operatorname{Chi}(dx) \sinh(c) - \frac{ad \sinh(c + dx)}{2x}
\end{aligned}$$

Mathematica [A] time = 0.17, size = 78, normalized size = 0.89

$$\frac{dx^2 \operatorname{Chi}(dx)(ad \cosh(c) + 2b \sinh(c)) + dx^2 \operatorname{Shi}(dx)(ad \sinh(c) + 2b \cosh(c)) - adx \sinh(c + dx) - a \cosh(c + dx)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)*Cosh[c + d*x])/x^3, x]
```

```
[Out] (- (a*Cosh[c + d*x]) - 2*b*x*Cosh[c + d*x] + d*x^2*CoshIntegral[d*x]*(a*d*Co
sh[c] + 2*b*Sinh[c]) - a*d*x*Sinh[c + d*x] + d*x^2*(2*b*Cosh[c] + a*d*Sinh[
c])*SinhIntegral[d*x])/(2*x^2)
```

fricas [A] time = 0.52, size = 116, normalized size = 1.32

$$\frac{2 adx \sinh(dx + c) + 2(2bx + a) \cosh(dx + c) - ((ad^2 + 2bd)x^2 \operatorname{Ei}(dx) + (ad^2 - 2bd)x^2 \operatorname{Ei}(-dx)) \cosh(c) - (ad \sinh(c + dx) + a \cosh(c + dx))}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)*cosh(d*x+c)/x^3,x, algorithm="fricas")

[Out] $-1/4*(2*a*d*x*\sinh(d*x + c) + 2*(2*b*x + a)*\cosh(d*x + c) - ((a*d^2 + 2*b*d)*x^2*Ei(d*x) + (a*d^2 - 2*b*d)*x^2*Ei(-d*x))*\cosh(c) - ((a*d^2 + 2*b*d)*x^2*Ei(d*x) - (a*d^2 - 2*b*d)*x^2*Ei(-d*x))*\sinh(c))/x^2$

giac [A] time = 0.12, size = 134, normalized size = 1.52

$$\frac{ad^2x^2Ei(-dx)e^{(-c)} + ad^2x^2Ei(dx)e^c - 2bdx^2Ei(-dx)e^{(-c)} + 2bdx^2Ei(dx)e^c - adxe^{(dx+c)} + adxe^{(-dx-c)} - 2bxe^{(dx+c)}}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)*cosh(d*x+c)/x^3,x, algorithm="giac")

[Out] $1/4*(a*d^2*x^2*Ei(-d*x)*e^{(-c)} + a*d^2*x^2*Ei(d*x)*e^c - 2*b*d*x^2*Ei(-d*x)*e^{(-c)} + 2*b*d*x^2*Ei(d*x)*e^c - a*d*x*e^{(d*x + c)} + a*d*x*e^{(-d*x - c)} - 2*b*x*e^{(d*x + c)} - 2*b*x*e^{(-d*x - c)} - a*e^{(d*x + c)} - a*e^{(-d*x - c)})/x^2$

maple [A] time = 0.10, size = 139, normalized size = 1.58

$$\frac{da e^{-dx-c}}{4x} - \frac{a e^{-dx-c}}{4x^2} - \frac{d^2 a e^{-c} Ei(1, dx)}{4} - \frac{b e^{-dx-c}}{2x} + \frac{db e^{-c} Ei(1, dx)}{2} - \frac{a e^{dx+c}}{4x^2} - \frac{da e^{dx+c}}{4x} - \frac{d^2 a e^c Ei(1, -dx)}{4} - \frac{b e^{dx+c}}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x+a)*cosh(d*x+c)/x^3,x)

[Out] $1/4*d*a*\exp(-d*x-c)/x - 1/4*a*\exp(-d*x-c)/x^2 - 1/4*d^2*a*\exp(-c)*Ei(1,d*x) - 1/2*b*\exp(-d*x-c)/x + 1/2*d*b*\exp(-c)*Ei(1,d*x) - 1/4*a/x^2*\exp(d*x+c) - 1/4*d*a/x*\exp(d*x+c) - 1/4*d^2*a*\exp(c)*Ei(1,-d*x) - 1/2*b/x*\exp(d*x+c) - 1/2*d*b*\exp(c)*Ei(1,-d*x)$

maxima [A] time = 0.43, size = 66, normalized size = 0.75

$$\frac{1}{4} \left(ade^{(-c)}\Gamma(-1, dx) + ade^c\Gamma(-1, -dx) - 2bEi(-dx)e^{(-c)} + 2bEi(dx)e^c \right) d - \frac{(2bx+a)\cosh(dx+c)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)*cosh(d*x+c)/x^3,x, algorithm="maxima")

[Out] $1/4*(a*d*e^{(-c)}*\gamma(-1, d*x) + a*d*e^c*\gamma(-1, -d*x) - 2*b*Ei(-d*x)*e^{(-c)} + 2*b*Ei(d*x)*e^c)*d - 1/2*(2*b*x + a)*\cosh(d*x + c)/x^2$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c + dx)(a + bx)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cosh(c + d*x)*(a + b*x))/x^3,x)

[Out] int((cosh(c + d*x)*(a + b*x))/x^3, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)*cosh(d*x+c)/x**3,x)

[Out] Timed out

3.8 $\int \frac{(a+bx) \cosh(c+dx)}{x^4} dx$

Optimal. Leaf size=132

$$\frac{1}{6}ad^3 \sinh(c)\text{Chi}(dx) + \frac{1}{6}ad^3 \cosh(c)\text{Shi}(dx) - \frac{ad^2 \cosh(c+dx)}{6x} - \frac{a \cosh(c+dx)}{3x^3} - \frac{ad \sinh(c+dx)}{6x^2} + \frac{1}{2}bd^2 \cosh(c)C$$

[Out] $1/2*b*d^2*Chi(d*x)*cosh(c) - 1/3*a*cosh(d*x+c)/x^3 - 1/2*b*cosh(d*x+c)/x^2 - 1/6*a*d^2*cosh(d*x+c)/x + 1/6*a*d^3*cosh(c)*Shi(d*x) + 1/6*a*d^3*Chi(d*x)*sinh(c) + 1/2*b*d^2*Shi(d*x)*sinh(c) - 1/6*a*d*sinh(d*x+c)/x^2 - 1/2*b*d*sinh(d*x+c)/x$

Rubi [A] time = 0.34, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6742, 3297, 3303, 3298, 3301}

$$\frac{1}{6}ad^3 \sinh(c)\text{Chi}(dx) + \frac{1}{6}ad^3 \cosh(c)\text{Shi}(dx) - \frac{ad^2 \cosh(c+dx)}{6x} - \frac{ad \sinh(c+dx)}{6x^2} - \frac{a \cosh(c+dx)}{3x^3} + \frac{1}{2}bd^2 \cosh(c)C$$

Antiderivative was successfully verified.

[In] Int[((a + b*x)*Cosh[c + d*x])/x^4, x]

[Out] $-(a*\text{Cosh}[c + d*x])/(3*x^3) - (b*\text{Cosh}[c + d*x])/(2*x^2) - (a*d^2*\text{Cosh}[c + d*x])/(6*x) + (b*d^2*\text{Cosh}[c]*\text{CoshIntegral}[d*x])/2 + (a*d^3*\text{CoshIntegral}[d*x]*\text{Sinh}[c])/6 - (a*d*\text{Sinh}[c + d*x])/(6*x^2) - (b*d*\text{Sinh}[c + d*x])/(2*x) + (a*d^3*\text{Cosh}[c]*\text{SinhIntegral}[d*x])/6 + (b*d^2*\text{Sinh}[c]*\text{SinhIntegral}[d*x])/2$

Rule 3297

Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]

Rule 3298

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)])/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]

Rule 3301

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)])/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + bx) \cosh(c + dx)}{x^4} dx &= \int \left(\frac{a \cosh(c + dx)}{x^4} + \frac{b \cosh(c + dx)}{x^3} \right) dx \\
&= a \int \frac{\cosh(c + dx)}{x^4} dx + b \int \frac{\cosh(c + dx)}{x^3} dx \\
&= -\frac{a \cosh(c + dx)}{3x^3} - \frac{b \cosh(c + dx)}{2x^2} + \frac{1}{3}(ad) \int \frac{\sinh(c + dx)}{x^3} dx + \frac{1}{2}(bd) \int \frac{\sinh(c + dx)}{x^2} dx \\
&= -\frac{a \cosh(c + dx)}{3x^3} - \frac{b \cosh(c + dx)}{2x^2} - \frac{ad \sinh(c + dx)}{6x^2} - \frac{bd \sinh(c + dx)}{2x} + \frac{1}{6}(ad^2) \int \frac{\cosh(c + dx)}{x^2} dx \\
&= -\frac{a \cosh(c + dx)}{3x^3} - \frac{b \cosh(c + dx)}{2x^2} - \frac{ad^2 \cosh(c + dx)}{6x} - \frac{ad \sinh(c + dx)}{6x^2} - \frac{bd \sinh(c + dx)}{2x} \\
&= -\frac{a \cosh(c + dx)}{3x^3} - \frac{b \cosh(c + dx)}{2x^2} - \frac{ad^2 \cosh(c + dx)}{6x} + \frac{1}{2}bd^2 \cosh(c) \text{Chi}(dx) - \frac{1}{2}bd \sinh(c) \text{Chi}(dx) \\
&= -\frac{a \cosh(c + dx)}{3x^3} - \frac{b \cosh(c + dx)}{2x^2} - \frac{ad^2 \cosh(c + dx)}{6x} + \frac{1}{2}bd^2 \cosh(c) \text{Chi}(dx) + \frac{1}{2}bd \sinh(c) \text{Chi}(dx)
\end{aligned}$$

Mathematica [A] time = 0.29, size = 110, normalized size = 0.83

$$\frac{-d^2 x^3 \text{Chi}(dx)(ad \sinh(c) + 3b \cosh(c)) - d^2 x^3 \text{Shi}(dx)(ad \cosh(c) + 3b \sinh(c)) + ad^2 x^2 \cosh(c + dx) + adx \sinh(c + dx)}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)*Cosh[c + d*x])/x^4, x]
```

```
[Out] -1/6*(2*a*Cosh[c + d*x] + 3*b*x*Cosh[c + d*x] + a*d^2*x^2*Cosh[c + d*x] - d
^2*x^3*CoshIntegral[d*x]*(3*b*Cosh[c] + a*d*Sinh[c]) + a*d*x*Sinh[c + d*x]
+ 3*b*d*x^2*Sinh[c + d*x] - d^2*x^3*(a*d*Cosh[c] + 3*b*Sinh[c])*SinhIntegra
l[d*x])/x^3
```

fricas [A] time = 0.49, size = 143, normalized size = 1.08

$$\frac{2(ad^2x^2 + 3bx + 2a) \cosh(dx + c) - ((ad^3 + 3bd^2)x^3 \text{Ei}(dx) - (ad^3 - 3bd^2)x^3 \text{Ei}(-dx)) \cosh(c) + 2(3bdx^2 + 2ax + a) \sinh(dx + c) - ((ad^3 + 3bd^2)x^3 \text{Ei}(dx) + (ad^3 - 3bd^2)x^3 \text{Ei}(-dx)) \sinh(c)}{12x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)*cosh(d*x+c)/x^4,x, algorithm="fricas")

[Out] -1/12*(2*(a*d^2*x^2 + 3*b*x + 2*a)*cosh(d*x + c) - ((a*d^3 + 3*b*d^2)*x^3*Ei(d*x) - (a*d^3 - 3*b*d^2)*x^3*Ei(-d*x))*cosh(c) + 2*(3*b*d*x^2 + a*d*x)*sinh(d*x + c) - ((a*d^3 + 3*b*d^2)*x^3*Ei(d*x) + (a*d^3 - 3*b*d^2)*x^3*Ei(-d*x))*sinh(c))/x^3

giac [A] time = 0.15, size = 199, normalized size = 1.51

$$\frac{ad^3x^3\text{Ei}(-dx)e^{(-c)} - ad^3x^3\text{Ei}(dx)e^c - 3bd^2x^3\text{Ei}(-dx)e^{(-c)} - 3bd^2x^3\text{Ei}(dx)e^c + ad^2x^2e^{(dx+c)} + ad^2x^2e^{(-dx-c)} + 2(3bdx^2 + 2ax + a)\sinh(dx+c) - ((ad^3 + 3bd^2)x^3\text{Ei}(dx) + (ad^3 - 3bd^2)x^3\text{Ei}(-dx))\sinh(c)}{12x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)*cosh(d*x+c)/x^4,x, algorithm="giac")

[Out] -1/12*(a*d^3*x^3*Ei(-d*x)*e^(-c) - a*d^3*x^3*Ei(d*x)*e^c - 3*b*d^2*x^3*Ei(-d*x)*e^(-c) - 3*b*d^2*x^3*Ei(d*x)*e^c + a*d^2*x^2*e^(d*x + c) + a*d^2*x^2*e^(-d*x - c) + 3*b*d*x^2*e^(d*x + c) - 3*b*d*x^2*e^(-d*x - c) + a*d*x*e^(d*x + c) - a*d*x*e^(-d*x - c) + 3*b*x*e^(d*x + c) + 3*b*x*e^(-d*x - c) + 2*a*e^(d*x + c) + 2*a*e^(-d*x - c))/x^3

maple [A] time = 0.12, size = 205, normalized size = 1.55

$$\frac{d^2ae^{-dx-c}}{12x} + \frac{dae^{-dx-c}}{12x^2} - \frac{ae^{-dx-c}}{6x^3} + \frac{d^3ae^{-c}\text{Ei}(1,dx)}{12} + \frac{dbe^{-dx-c}}{4x} - \frac{be^{-dx-c}}{4x^2} - \frac{d^2be^{-c}\text{Ei}(1,dx)}{4} - \frac{ae^{dx+c}}{6x^3} - \frac{dae^{dx+c}}{12x^2} - \frac{d^2e^{dx+c}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x+a)*cosh(d*x+c)/x^4,x)

[Out] -1/12*d^2*a*exp(-d*x-c)/x+1/12*d*a*exp(-d*x-c)/x^2-1/6*a*exp(-d*x-c)/x^3+1/12*d^3*a*exp(-c)*Ei(1,d*x)+1/4*d*b*exp(-d*x-c)/x-1/4*b*exp(-d*x-c)/x^2-1/4*d^2*b*exp(-c)*Ei(1,d*x)-1/6*a/x^3*exp(d*x+c)-1/12*d*a/x^2*exp(d*x+c)-1/12*d^2*a/x*exp(d*x+c)-1/12*d^3*a*exp(c)*Ei(1,-d*x)-1/4*b/x^2*exp(d*x+c)-1/4*d*b/x*exp(d*x+c)-1/4*d^2*b*exp(c)*Ei(1,-d*x)

maxima [A] time = 0.41, size = 78, normalized size = 0.59

$$\frac{1}{12} \left(2ad^2e^{(-c)}\Gamma(-2,dx) - 2ad^2e^c\Gamma(-2,-dx) + 3bde^{(-c)}\Gamma(-1,dx) + 3bde^c\Gamma(-1,-dx) \right) d - \frac{(3bx + 2a) \cosh(dx + c)}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)*cosh(d*x+c)/x^4,x, algorithm="maxima")

[Out] $1/12*(2*a*d^2*e^{-c}*\gamma(-2, d*x) - 2*a*d^2*e^c*\gamma(-2, -d*x) + 3*b*d*e^{-c}*\gamma(-1, d*x) + 3*b*d*e^c*\gamma(-1, -d*x))*d - 1/6*(3*b*x + 2*a)*\cosh(d*x + c)/x^3$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c + dx)(a + bx)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cosh(c + d*x)*(a + b*x))/x^4,x)

[Out] int((cosh(c + d*x)*(a + b*x))/x^4, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)*cosh(d*x+c)/x**4,x)

[Out] Timed out

3.9 $\int \frac{(a+bx) \cosh(c+dx)}{x^5} dx$

Optimal. Leaf size=166

$$\frac{1}{24}ad^4 \cosh(c)\text{Chi}(dx) + \frac{1}{24}ad^4 \sinh(c)\text{Shi}(dx) - \frac{ad^3 \sinh(c+dx)}{24x} - \frac{ad^2 \cosh(c+dx)}{24x^2} - \frac{a \cosh(c+dx)}{4x^4} - \frac{ad \sinh(c+dx)}{12x^3}$$

[Out] $1/24*a*d^4*\text{Chi}(d*x)*\cosh(c) - 1/4*a*\cosh(d*x+c)/x^4 - 1/3*b*\cosh(d*x+c)/x^3 - 1/24*a*d^2*\cosh(d*x+c)/x^2 - 1/6*b*d^2*\cosh(d*x+c)/x + 1/6*b*d^3*\cosh(c)*\text{Shi}(d*x) + 1/6*b*d^3*\text{Chi}(d*x)*\sinh(c) + 1/24*a*d^4*\text{Shi}(d*x)*\sinh(c) - 1/12*a*d*\sinh(d*x+c)/x^3 - 1/6*b*d*\sinh(d*x+c)/x^2 - 1/24*a*d^3*\sinh(d*x+c)/x$

Rubi [A] time = 0.41, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6742, 3297, 3303, 3298, 3301}

$$\frac{1}{24}ad^4 \cosh(c)\text{Chi}(dx) + \frac{1}{24}ad^4 \sinh(c)\text{Shi}(dx) - \frac{ad^2 \cosh(c+dx)}{24x^2} - \frac{ad^3 \sinh(c+dx)}{24x} - \frac{ad \sinh(c+dx)}{12x^3} - \frac{a \cosh(c+dx)}{4x^4}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + b*x)*\text{Cosh}[c + d*x])/x^5, x]$

[Out] $-(a*\text{Cosh}[c + d*x])/(4*x^4) - (b*\text{Cosh}[c + d*x])/(3*x^3) - (a*d^2*\text{Cosh}[c + d*x])/(24*x^2) - (b*d^2*\text{Cosh}[c + d*x])/(6*x) + (a*d^4*\text{Cosh}[c]*\text{CoshIntegral}[d*x])/24 + (b*d^3*\text{CoshIntegral}[d*x]*\text{Sinh}[c])/6 - (a*d*\text{Sinh}[c + d*x])/(12*x^3) - (b*d*\text{Sinh}[c + d*x])/(6*x^2) - (a*d^3*\text{Sinh}[c + d*x])/(24*x) + (b*d^3*\text{Cosh}[c]*\text{SinhIntegral}[d*x])/6 + (a*d^4*\text{Sinh}[c]*\text{SinhIntegral}[d*x])/24$

Rule 3297

$\text{Int}[(c_. + (d_.)*(x_.))^(m_)*\sin[(e_.) + (f_.)*(x_.)], x_Symbol] \rightarrow \text{Simp}[(c + d*x)^(m + 1)*\text{Sin}[e + f*x]/(d*(m + 1)), x] - \text{Dist}[f/(d*(m + 1)), \text{Int}[(c + d*x)^(m + 1)*\text{Cos}[e + f*x], x], x] /;$ $\text{FreeQ}\{c, d, e, f\}, x \ \&\& \ \text{LtQ}[m, -1]$

Rule 3298

$\text{Int}[\sin[(e_.) + (\text{Complex}[0, fz_])*(f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x_Symbol] \rightarrow \text{Simp}[(I*\text{SinhIntegral}[(c*f*fz)/d + f*fz*x])/d, x] /;$ $\text{FreeQ}\{c, d, e, f, fz\}, x \ \&\& \ \text{EqQ}[d*e - c*f*fz*I, 0]$

Rule 3301

$\text{Int}[\sin[(e_.) + (\text{Complex}[0, fz_])*(f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x_Symbol] \rightarrow \text{Simp}[\text{CoshIntegral}[(c*f*fz)/d + f*fz*x]/d, x] /;$ $\text{FreeQ}\{c, d, e, f, fz\}, x$

}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]

Rule 3303

Int[sin[(e_.) + (f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x_Symbol] :> Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]

Rule 6742

Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

Rubi steps

$$\begin{aligned}
 \int \frac{(a + bx) \cosh(c + dx)}{x^5} dx &= \int \left(\frac{a \cosh(c + dx)}{x^5} + \frac{b \cosh(c + dx)}{x^4} \right) dx \\
 &= a \int \frac{\cosh(c + dx)}{x^5} dx + b \int \frac{\cosh(c + dx)}{x^4} dx \\
 &= -\frac{a \cosh(c + dx)}{4x^4} - \frac{b \cosh(c + dx)}{3x^3} + \frac{1}{4}(ad) \int \frac{\sinh(c + dx)}{x^4} dx + \frac{1}{3}(bd) \int \frac{\sinh(c + dx)}{x^3} dx \\
 &= -\frac{a \cosh(c + dx)}{4x^4} - \frac{b \cosh(c + dx)}{3x^3} - \frac{ad \sinh(c + dx)}{12x^3} - \frac{bd \sinh(c + dx)}{6x^2} + \frac{1}{12} (ad) \int \frac{\sinh(c + dx)}{x^2} dx \\
 &= -\frac{a \cosh(c + dx)}{4x^4} - \frac{b \cosh(c + dx)}{3x^3} - \frac{ad^2 \cosh(c + dx)}{24x^2} - \frac{bd^2 \cosh(c + dx)}{6x} - \frac{ad \sinh(c + dx)}{6x} \\
 &= -\frac{a \cosh(c + dx)}{4x^4} - \frac{b \cosh(c + dx)}{3x^3} - \frac{ad^2 \cosh(c + dx)}{24x^2} - \frac{bd^2 \cosh(c + dx)}{6x} - \frac{ad \sinh(c + dx)}{6x} + \frac{1}{6} bd \int \frac{\sinh(c + dx)}{x} dx \\
 &= -\frac{a \cosh(c + dx)}{4x^4} - \frac{b \cosh(c + dx)}{3x^3} - \frac{ad^2 \cosh(c + dx)}{24x^2} - \frac{bd^2 \cosh(c + dx)}{6x} + \frac{1}{24} ad^3 \int \frac{\sinh(c + dx)}{x} dx
 \end{aligned}$$

Mathematica [A] time = 0.35, size = 140, normalized size = 0.84

$$\frac{-d^3 x^4 \text{Chi}(dx)(ad \cosh(c) + 4b \sinh(c)) - d^3 x^4 \text{Shi}(dx)(ad \sinh(c) + 4b \cosh(c)) + ad^3 x^3 \sinh(c + dx) + ad^2 x^2 \cosh(c + dx)}{24}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x)*Cosh[c + d*x])/x^5, x]

[Out] $-1/24*(6*a*\text{Cosh}[c + d*x] + 8*b*x*\text{Cosh}[c + d*x] + a*d^2*x^2*\text{Cosh}[c + d*x] + 4*b*d^2*x^3*\text{Cosh}[c + d*x] - d^3*x^4*\text{CoshIntegral}[d*x]*(a*d*\text{Cosh}[c] + 4*b*\text{Sinh}[c])) + 2*a*d*x*\text{Sinh}[c + d*x] + 4*b*d*x^2*\text{Sinh}[c + d*x] + a*d^3*x^3*\text{Sinh}[c + d*x] - d^3*x^4*(4*b*\text{Cosh}[c] + a*d*\text{Sinh}[c])*\text{SinhIntegral}[d*x])/x^4$

fricas [A] time = 0.55, size = 161, normalized size = 0.97

$$\frac{2(4bd^2x^3 + ad^2x^2 + 8bx + 6a)\cosh(dx + c) - ((ad^4 + 4bd^3)x^4\text{Ei}(dx) + (ad^4 - 4bd^3)x^4\text{Ei}(-dx))\cosh(c) + 2ad^3x^3e^{(-dx-c)} + 2ad^3x^3e^{(dx+c)}}{48x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x+a)*cosh(d*x+c)/x^5,x, algorithm="fricas")`

[Out] $-1/48*(2*(4*b*d^2*x^3 + a*d^2*x^2 + 8*b*x + 6*a)*\cosh(d*x + c) - ((a*d^4 + 4*b*d^3)*x^4*\text{Ei}(d*x) + (a*d^4 - 4*b*d^3)*x^4*\text{Ei}(-d*x))*\cosh(c) + 2*(a*d^3*x^3 + 4*b*d*x^2 + 2*a*d*x)*\sinh(d*x + c) - ((a*d^4 + 4*b*d^3)*x^4*\text{Ei}(d*x) - (a*d^4 - 4*b*d^3)*x^4*\text{Ei}(-d*x))*\sinh(c))/x^4$

giac [A] time = 0.14, size = 266, normalized size = 1.60

$$\frac{ad^4x^4\text{Ei}(-dx)e^{(-c)} + ad^4x^4\text{Ei}(dx)e^c - 4bd^3x^4\text{Ei}(-dx)e^{(-c)} + 4bd^3x^4\text{Ei}(dx)e^c - ad^3x^3e^{(dx+c)} + ad^3x^3e^{(-dx-c)} - 4ad^3x^3e^{(-dx-c)} + 4ad^3x^3e^{(dx+c)}}{48x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x+a)*cosh(d*x+c)/x^5,x, algorithm="giac")`

[Out] $1/48*(a*d^4*x^4*\text{Ei}(-d*x)*e^{(-c)} + a*d^4*x^4*\text{Ei}(d*x)*e^c - 4*b*d^3*x^4*\text{Ei}(-d*x)*e^{(-c)} + 4*b*d^3*x^4*\text{Ei}(d*x)*e^c - a*d^3*x^3*e^{(d*x + c)} + a*d^3*x^3*e^{(-d*x - c)} - 4*b*d^2*x^3*e^{(d*x + c)} - 4*b*d^2*x^3*e^{(-d*x - c)} - a*d^2*x^2*e^{(d*x + c)} - a*d^2*x^2*e^{(-d*x - c)} - 4*b*d*x^2*e^{(d*x + c)} + 4*b*d*x^2*e^{(-d*x - c)} - 2*a*d*x*e^{(d*x + c)} + 2*a*d*x*e^{(-d*x - c)} - 8*b*x*e^{(d*x + c)} - 8*b*x*e^{(-d*x - c)} - 6*a*e^{(d*x + c)} - 6*a*e^{(-d*x - c)})/x^4$

maple [A] time = 0.12, size = 271, normalized size = 1.63

$$\frac{d^3ae^{-dx-c}}{48x} - \frac{d^2ae^{-dx-c}}{48x^2} + \frac{dae^{-dx-c}}{24x^3} - \frac{ae^{-dx-c}}{8x^4} - \frac{d^4ae^{-c}\text{Ei}(1,dx)}{48} - \frac{d^2be^{-dx-c}}{12x} + \frac{dbe^{-dx-c}}{12x^2} - \frac{be^{-dx-c}}{6x^3} + \frac{d^3be^{-c}\text{Ei}(1,dx)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x+a)*cosh(d*x+c)/x^5,x)`

[Out] $1/48*d^3*a*\exp(-d*x-c)/x - 1/48*d^2*a*\exp(-d*x-c)/x^2 + 1/24*d*a*\exp(-d*x-c)/x^3 - 1/8*a*\exp(-d*x-c)/x^4 - 1/48*d^4*a*\exp(-c)*\text{Ei}(1,d*x) - 1/12*d^2*b*\exp(-d*x-c)/x + 1/12*d*b*\exp(-d*x-c)/x^2 - 1/6*b*\exp(-d*x-c)/x^3 + 1/12*d^3*b*\exp(-c)*\text{Ei}(1,d$

$*x) - 1/8*a/x^4*\exp(d*x+c) - 1/24*d*a/x^3*\exp(d*x+c) - 1/48*d^2*a/x^2*\exp(d*x+c) - 1/48*d^3*a/x*\exp(d*x+c) - 1/48*d^4*a*\exp(c)*\text{Ei}(1, -d*x) - 1/6*b/x^3*\exp(d*x+c) - 1/12*d*b/x^2*\exp(d*x+c) - 1/12*d^2*b/x*\exp(d*x+c) - 1/12*d^3*b*\exp(c)*\text{Ei}(1, -d*x)$

maxima [A] time = 0.41, size = 82, normalized size = 0.49

$$\frac{1}{24} \left(3ad^3e^{(-c)}\Gamma(-3, dx) + 3ad^3e^c\Gamma(-3, -dx) + 4bd^2e^{(-c)}\Gamma(-2, dx) - 4bd^2e^c\Gamma(-2, -dx) \right) d - \frac{(4bx + 3a) \cosh(dx)}{12x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)*cosh(d*x+c)/x^5,x, algorithm="maxima")

[Out] $1/24*(3*a*d^3*e^{(-c)}*\text{gamma}(-3, d*x) + 3*a*d^3*e^c*\text{gamma}(-3, -d*x) + 4*b*d^2*e^{(-c)}*\text{gamma}(-2, d*x) - 4*b*d^2*e^c*\text{gamma}(-2, -d*x))*d - 1/12*(4*b*x + 3*a)*\cosh(d*x + c)/x^4$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c + dx)(a + bx)}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cosh(c + d*x)*(a + b*x))/x^5,x)

[Out] int((cosh(c + d*x)*(a + b*x))/x^5, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)*cosh(d*x+c)/x**5,x)

[Out] Timed out

3.10 $\int x^2(a + bx)^2 \cosh(c + dx) dx$

Optimal. Leaf size=184

$$\frac{2a^2 \sinh(c + dx)}{d^3} - \frac{2a^2 x \cosh(c + dx)}{d^2} + \frac{a^2 x^2 \sinh(c + dx)}{d} - \frac{12ab \cosh(c + dx)}{d^4} + \frac{12abx \sinh(c + dx)}{d^3} - \frac{6abx^2 \cosh(c + dx)}{d^2}$$

[Out] $-12*a*b*\cosh(d*x+c)/d^4-24*b^2*x*\cosh(d*x+c)/d^4-2*a^2*x*\cosh(d*x+c)/d^2-6*a*b*x^2*\cosh(d*x+c)/d^2-4*b^2*x^3*\cosh(d*x+c)/d^2+24*b^2*\sinh(d*x+c)/d^5+2*a^2*\sinh(d*x+c)/d^3+12*a*b*x*\sinh(d*x+c)/d^3+12*b^2*x^2*\sinh(d*x+c)/d^3+a^2*x^2*\sinh(d*x+c)/d+2*a*b*x^3*\sinh(d*x+c)/d+b^2*x^4*\sinh(d*x+c)/d$

Rubi [A] time = 0.35, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {6742, 3296, 2637, 2638}

$$\frac{2a^2 \sinh(c + dx)}{d^3} - \frac{2a^2 x \cosh(c + dx)}{d^2} + \frac{a^2 x^2 \sinh(c + dx)}{d} - \frac{6abx^2 \cosh(c + dx)}{d^2} + \frac{12abx \sinh(c + dx)}{d^3} - \frac{12ab \cosh(c + dx)}{d^4}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^2*(a + b*x)^2*\text{Cosh}[c + d*x], x]$

[Out] $(-12*a*b*\text{Cosh}[c + d*x])/d^4 - (24*b^2*x*\text{Cosh}[c + d*x])/d^4 - (2*a^2*x*\text{Cosh}[c + d*x])/d^2 - (6*a*b*x^2*\text{Cosh}[c + d*x])/d^2 - (4*b^2*x^3*\text{Cosh}[c + d*x])/d^2 + (24*b^2*\text{Sinh}[c + d*x])/d^5 + (2*a^2*\text{Sinh}[c + d*x])/d^3 + (12*a*b*x*\text{Sinh}[c + d*x])/d^3 + (12*b^2*x^2*\text{Sinh}[c + d*x])/d^3 + (a^2*x^2*\text{Sinh}[c + d*x])/d + (2*a*b*x^3*\text{Sinh}[c + d*x])/d + (b^2*x^4*\text{Sinh}[c + d*x])/d$

Rule 2637

$\text{Int}[\sin[\text{Pi}/2 + (c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow \text{Simp}[\sin[c + d*x]/d, x] /;$
FreeQ[{c, d}, x]

Rule 2638

$\text{Int}[\sin[(c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow -\text{Simp}[\cos[c + d*x]/d, x] /;$ FreeQ[{c, d}, x]

Rule 3296

$\text{Int}[((c_.) + (d_.)*(x_.))^{(m_.)}*\sin[(e_.) + (f_.)*(x_.)], x_Symbol] \rightarrow -\text{Simp}[(c + d*x)^m*\cos[e + f*x]/f, x] + \text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{(m-1)}*\cos[e + f*x], x], x] /;$ FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int x^2(a+bx)^2 \cosh(c+dx) dx &= \int (a^2x^2 \cosh(c+dx) + 2abx^3 \cosh(c+dx) + b^2x^4 \cosh(c+dx)) dx \\
&= a^2 \int x^2 \cosh(c+dx) dx + (2ab) \int x^3 \cosh(c+dx) dx + b^2 \int x^4 \cosh(c+dx) dx \\
&= \frac{a^2x^2 \sinh(c+dx)}{d} + \frac{2abx^3 \sinh(c+dx)}{d} + \frac{b^2x^4 \sinh(c+dx)}{d} - \frac{(2a^2) \int x \sinh(c+dx) dx}{d} \\
&= -\frac{2a^2x \cosh(c+dx)}{d^2} - \frac{6abx^2 \cosh(c+dx)}{d^2} - \frac{4b^2x^3 \cosh(c+dx)}{d^2} + \frac{a^2x^2 \sinh(c+dx)}{d} \\
&= -\frac{2a^2x \cosh(c+dx)}{d^2} - \frac{6abx^2 \cosh(c+dx)}{d^2} - \frac{4b^2x^3 \cosh(c+dx)}{d^2} + \frac{2a^2 \sinh(c+dx)}{d^3} \\
&= -\frac{12ab \cosh(c+dx)}{d^4} - \frac{24b^2x \cosh(c+dx)}{d^4} - \frac{2a^2x \cosh(c+dx)}{d^2} - \frac{6abx^2 \cosh(c+dx)}{d^2} \\
&= -\frac{12ab \cosh(c+dx)}{d^4} - \frac{24b^2x \cosh(c+dx)}{d^4} - \frac{2a^2x \cosh(c+dx)}{d^2} - \frac{6abx^2 \cosh(c+dx)}{d^2}
\end{aligned}$$

Mathematica [A] time = 0.28, size = 100, normalized size = 0.54

$$\frac{(a^2d^2(d^2x^2 + 2) + 2abd^2x(d^2x^2 + 6) + b^2(d^4x^4 + 12d^2x^2 + 24)) \sinh(c+dx) - 2d(a+2bx)(ad^2x + b(d^2x^2 + 2))}{d^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x)^2*Cosh[c + d*x], x]
```

```
[Out] (-2*d*(a + 2*b*x)*(a*d^2*x + b*(6 + d^2*x^2))*Cosh[c + d*x] + (a^2*d^2*(2 + d^2*x^2) + 2*a*b*d^2*x*(6 + d^2*x^2) + b^2*(24 + 12*d^2*x^2 + d^4*x^4))*Sinh[c + d*x])/d^5
```

fricas [A] time = 0.49, size = 127, normalized size = 0.69

$$\frac{2(b^2d^3x^3 + 3abd^3x^2 + 6abd + (a^2d^3 + 12b^2d)x) \cosh(dx+c) - (b^2d^4x^4 + 2abd^4x^3 + 12abd^2x + 2a^2d^2 + 2ad^2)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^2*cosh(d*x+c), x, algorithm="fricas")
```

[Out] $-(2*(2*b^2*d^3*x^3 + 3*a*b*d^3*x^2 + 6*a*b*d + (a^2*d^3 + 12*b^2*d)*x)*\cosh(d*x + c) - (b^2*d^4*x^4 + 2*a*b*d^4*x^3 + 12*a*b*d^2*x + 2*a^2*d^2 + (a^2*d^4 + 12*b^2*d^2)*x^2 + 24*b^2)*\sinh(d*x + c))/d^5$

giac [A] time = 0.14, size = 236, normalized size = 1.28

$$\frac{(b^2d^4x^4 + 2abd^4x^3 + a^2d^4x^2 - 4b^2d^3x^3 - 6abd^3x^2 - 2a^2d^3x + 12b^2d^2x^2 + 12abd^2x + 2a^2d^2 - 24b^2dx - 12abd)}{2d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(b*x+a)^2*cosh(d*x+c),x, algorithm="giac")`

[Out] $1/2*(b^2*d^4*x^4 + 2*a*b*d^4*x^3 + a^2*d^4*x^2 - 4*b^2*d^3*x^3 - 6*a*b*d^3*x^2 - 2*a^2*d^3*x + 12*b^2*d^2*x^2 + 12*a*b*d^2*x + 2*a^2*d^2 - 24*b^2*d*x - 12*a*b*d + 24*b^2)*e^{(d*x + c)}/d^5 - 1/2*(b^2*d^4*x^4 + 2*a*b*d^4*x^3 + a^2*d^4*x^2 + 4*b^2*d^3*x^3 + 6*a*b*d^3*x^2 + 2*a^2*d^3*x + 12*b^2*d^2*x^2 + 12*a*b*d^2*x + 2*a^2*d^2 + 24*b^2*d*x + 12*a*b*d + 24*b^2)*e^{(-d*x - c)}/d^5$

maple [B] time = 0.04, size = 463, normalized size = 2.52

$$\frac{b^2((dx+c)^4 \sinh(dx+c) - 4(dx+c)^3 \cosh(dx+c) + 12(dx+c)^2 \sinh(dx+c) - 24(dx+c) \cosh(dx+c) + 24 \sinh(dx+c))}{d^2} - \frac{4b^2c((dx+c)^3 \sinh(dx+c) - 3(dx+c)^2 \cosh(dx+c) + 6(dx+c) \sinh(dx+c) - 6 \cosh(dx+c))}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(b*x+a)^2*cosh(d*x+c),x)`

[Out] $1/d^3*(b^2/d^2*((d*x+c)^4*\sinh(d*x+c)-4*(d*x+c)^3*\cosh(d*x+c)+12*(d*x+c)^2*\sinh(d*x+c)-24*(d*x+c)*\cosh(d*x+c)+24*\sinh(d*x+c))-4*b^2/d^2*c*((d*x+c)^3*\sinh(d*x+c)-3*(d*x+c)^2*\cosh(d*x+c)+6*(d*x+c)*\sinh(d*x+c)-6*\cosh(d*x+c))+6*b^2*c^2/d^2*((d*x+c)^2*\sinh(d*x+c)-2*(d*x+c)*\cosh(d*x+c)+2*\sinh(d*x+c))-4*b^2/d^2*c^3*((d*x+c)*\sinh(d*x+c)-\cosh(d*x+c))+2*b/d*a*((d*x+c)^3*\sinh(d*x+c)-3*(d*x+c)^2*\cosh(d*x+c)+6*(d*x+c)*\sinh(d*x+c)-6*\cosh(d*x+c))-6*b*c/d*a*((d*x+c)^2*\sinh(d*x+c)-2*(d*x+c)*\cosh(d*x+c)+2*\sinh(d*x+c))+6*b/d*a*c^2*((d*x+c)*\sinh(d*x+c)-\cosh(d*x+c))+b^2*c^4/d^2*\sinh(d*x+c)-2*b*c^3/d*a*\sinh(d*x+c)+a^2*((d*x+c)^2*\sinh(d*x+c)-2*(d*x+c)*\cosh(d*x+c)+2*\sinh(d*x+c))-2*a^2*c*((d*x+c)*\sinh(d*x+c)-\cosh(d*x+c))+a^2*c^2*\sinh(d*x+c))$

maxima [A] time = 0.34, size = 329, normalized size = 1.79

$$-\frac{1}{60}d\left(\frac{10(d^3x^3e^c - 3d^2x^2e^c + 6dxe^c - 6e^c)a^2e^{(dx)}}{d^4} + \frac{10(d^3x^3 + 3d^2x^2 + 6dx + 6)a^2e^{(-dx-c)}}{d^4} + \frac{15(d^4x^4e^c - 4d^3x^3e^c + 6d^2x^2e^c - 6dxe^c + 6e^c)a^2e^{(dx)}}{d^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(b*x+a)^2*cosh(d*x+c),x, algorithm="maxima")

[Out]
$$-1/60*d*(10*(d^3*x^3*e^c - 3*d^2*x^2*e^c + 6*d*x*e^c - 6*e^c)*a^2*e^{(d*x)}/d^4 + 10*(d^3*x^3 + 3*d^2*x^2 + 6*d*x + 6)*a^2*e^{-(d*x - c)}/d^4 + 15*(d^4*x^4*4*e^c - 4*d^3*x^3*e^c + 12*d^2*x^2*e^c - 24*d*x*e^c + 24*e^c)*a*b*e^{(d*x)}/d^5 + 15*(d^4*x^4 + 4*d^3*x^3 + 12*d^2*x^2 + 24*d*x + 24)*a*b*e^{-(d*x - c)}/d^5 + 6*(d^5*x^5*e^c - 5*d^4*x^4*e^c + 20*d^3*x^3*e^c - 60*d^2*x^2*e^c + 120*d*x*e^c - 120*e^c)*b^2*e^{(d*x)}/d^6 + 6*(d^5*x^5 + 5*d^4*x^4 + 20*d^3*x^3 + 60*d^2*x^2 + 120*d*x + 120)*b^2*e^{-(d*x - c)}/d^6) + 1/30*(6*b^2*x^5 + 15*a*b*x^4 + 10*a^2*x^3)*cosh(d*x + c)$$

mupad [B] time = 0.16, size = 168, normalized size = 0.91

$$\frac{2 \sinh(c + dx) (a^2 d^2 + 12 b^2)}{d^5} - \frac{4 b^2 x^3 \cosh(c + dx)}{d^2} + \frac{b^2 x^4 \sinh(c + dx)}{d} - \frac{12 a b \cosh(c + dx)}{d^4} - \frac{2 x \cosh(c + dx)}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*cosh(c + d*x)*(a + b*x)^2,x)

[Out]
$$(2*\sinh(c + d*x)*(12*b^2 + a^2*d^2))/d^5 - (4*b^2*x^3*cosh(c + d*x))/d^2 + (b^2*x^4*sinh(c + d*x))/d - (12*a*b*cosh(c + d*x))/d^4 - (2*x*cosh(c + d*x)*(12*b^2 + a^2*d^2))/d^4 + (x^2*sinh(c + d*x)*(12*b^2 + a^2*d^2))/d^3 - (6*a*b*x^2*cosh(c + d*x))/d^2 + (2*a*b*x^3*sinh(c + d*x))/d + (12*a*b*x*sinh(c + d*x))/d^3$$

sympy [A] time = 2.24, size = 228, normalized size = 1.24

$$\left\{ \begin{array}{l} \frac{a^2 x^2 \sinh(c+dx)}{d} - \frac{2a^2 x \cosh(c+dx)}{d^2} + \frac{2a^2 \sinh(c+dx)}{d^3} + \frac{2abx^3 \sinh(c+dx)}{d} - \frac{6abx^2 \cosh(c+dx)}{d^2} + \frac{12abx \sinh(c+dx)}{d^3} - \frac{12ab \cosh(c+dx)}{d^4} \\ \left(\frac{a^2 x^3}{3} + \frac{abx^4}{2} + \frac{b^2 x^5}{5} \right) \cosh(c) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(b*x+a)**2*cosh(d*x+c),x)

[Out] Piecewise((a**2*x**2*sinh(c + d*x)/d - 2*a**2*x*cosh(c + d*x)/d**2 + 2*a**2*x*sinh(c + d*x)/d**3 + 2*a*b*x**3*sinh(c + d*x)/d - 6*a*b*x**2*cosh(c + d*x)/d**2 + 12*a*b*x*sinh(c + d*x)/d**3 - 12*a*b*cosh(c + d*x)/d**4 + b**2*x**4*sinh(c + d*x)/d - 4*b**2*x**3*cosh(c + d*x)/d**2 + 12*b**2*x**2*sinh(c + d*x)/d**3 - 24*b**2*x*cosh(c + d*x)/d**4 + 24*b**2*sinh(c + d*x)/d**5, Ne(d, 0)), ((a**2*x**3/3 + a*b*x**4/2 + b**2*x**5/5)*cosh(c), True))

3.11 $\int x(a + bx)^2 \cosh(c + dx) dx$

Optimal. Leaf size=134

$$-\frac{a^2 \cosh(c + dx)}{d^2} + \frac{a^2 x \sinh(c + dx)}{d} + \frac{4ab \sinh(c + dx)}{d^3} - \frac{4abx \cosh(c + dx)}{d^2} + \frac{2abx^2 \sinh(c + dx)}{d} - \frac{6b^2 \cosh(c + dx)}{d^4}$$

[Out] $-6*b^2*\cosh(d*x+c)/d^4-a^2*\cosh(d*x+c)/d^2-4*a*b*x*\cosh(d*x+c)/d^2-3*b^2*x^2*\cosh(d*x+c)/d^2+4*a*b*\sinh(d*x+c)/d^3+6*b^2*x*\sinh(d*x+c)/d^3+a^2*x*\sinh(d*x+c)/d+2*a*b*x^2*\sinh(d*x+c)/d+b^2*x^3*\sinh(d*x+c)/d$

Rubi [A] time = 0.21, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {6742, 3296, 2638, 2637}

$$-\frac{a^2 \cosh(c + dx)}{d^2} + \frac{a^2 x \sinh(c + dx)}{d} + \frac{4ab \sinh(c + dx)}{d^3} - \frac{4abx \cosh(c + dx)}{d^2} + \frac{2abx^2 \sinh(c + dx)}{d} - \frac{3b^2 x^2 \cosh(c + dx)}{d^2}$$

Antiderivative was successfully verified.

[In] Int[x*(a + b*x)^2*Cosh[c + d*x], x]

[Out] $(-6*b^2*Cosh[c + d*x])/d^4 - (a^2*Cosh[c + d*x])/d^2 - (4*a*b*x*Cosh[c + d*x])/d^2 - (3*b^2*x^2*Cosh[c + d*x])/d^2 + (4*a*b*Sinh[c + d*x])/d^3 + (6*b^2*x*Sinh[c + d*x])/d^3 + (a^2*x*Sinh[c + d*x])/d + (2*a*b*x^2*Sinh[c + d*x])/d + (b^2*x^3*Sinh[c + d*x])/d$

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 2638

Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3296

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
 \int x(a + bx)^2 \cosh(c + dx) dx &= \int (a^2 x \cosh(c + dx) + 2abx^2 \cosh(c + dx) + b^2 x^3 \cosh(c + dx)) dx \\
 &= a^2 \int x \cosh(c + dx) dx + (2ab) \int x^2 \cosh(c + dx) dx + b^2 \int x^3 \cosh(c + dx) dx \\
 &= \frac{a^2 x \sinh(c + dx)}{d} + \frac{2abx^2 \sinh(c + dx)}{d} + \frac{b^2 x^3 \sinh(c + dx)}{d} - \frac{a^2 \int \sinh(c + dx) dx}{d} \\
 &= -\frac{a^2 \cosh(c + dx)}{d^2} - \frac{4abx \cosh(c + dx)}{d^2} - \frac{3b^2 x^2 \cosh(c + dx)}{d^2} + \frac{a^2 x \sinh(c + dx)}{d} \\
 &= -\frac{a^2 \cosh(c + dx)}{d^2} - \frac{4abx \cosh(c + dx)}{d^2} - \frac{3b^2 x^2 \cosh(c + dx)}{d^2} + \frac{4ab \sinh(c + dx)}{d^3} \\
 &= -\frac{6b^2 \cosh(c + dx)}{d^4} - \frac{a^2 \cosh(c + dx)}{d^2} - \frac{4abx \cosh(c + dx)}{d^2} - \frac{3b^2 x^2 \cosh(c + dx)}{d^2}
 \end{aligned}$$

Mathematica [A] time = 0.22, size = 87, normalized size = 0.65

$$\frac{d(a^2 d^2 x + 2ab(d^2 x^2 + 2) + b^2 x(d^2 x^2 + 6)) \sinh(c + dx) - (a^2 d^2 + 4abd^2 x + 3b^2(d^2 x^2 + 2)) \cosh(c + dx)}{d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)^2*Cosh[c + d*x], x]
```

```
[Out] (-(a^2*d^2 + 4*a*b*d^2*x + 3*b^2*(2 + d^2*x^2))*Cosh[c + d*x]) + d*(a^2*d^2*x + 2*a*b*(2 + d^2*x^2) + b^2*x*(6 + d^2*x^2))*Sinh[c + d*x])/d^4
```

fricas [A] time = 0.53, size = 95, normalized size = 0.71

$$\frac{(3b^2 d^2 x^2 + 4abd^2 x + a^2 d^2 + 6b^2) \cosh(dx + c) - (b^2 d^3 x^3 + 2abd^3 x^2 + 4abd + (a^2 d^3 + 6b^2 d)x) \sinh(dx + c)}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^2*cosh(d*x+c), x, algorithm="fricas")
```

```
[Out] -((3*b^2*d^2*x^2 + 4*a*b*d^2*x + a^2*d^2 + 6*b^2)*cosh(d*x + c) - (b^2*d^3*x^3 + 2*a*b*d^3*x^2 + 4*a*b*d + (a^2*d^3 + 6*b^2*d)*x)*sinh(d*x + c))/d^4
```

giac [A] time = 0.12, size = 171, normalized size = 1.28

$$\frac{(b^2 d^3 x^3 + 2 a b d^3 x^2 + a^2 d^3 x - 3 b^2 d^2 x^2 - 4 a b d^2 x - a^2 d^2 + 6 b^2 d x + 4 a b d - 6 b^2) e^{(d x + c)}}{2 d^4} \frac{(b^2 d^3 x^3 + 2 a b d^3 x^2 + a^2 d^3 x - 3 b^2 d^2 x^2 - 4 a b d^2 x - a^2 d^2 + 6 b^2 d x + 4 a b d - 6 b^2) e^{(d x + c)}}{2 d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x+a)^2*cosh(d*x+c),x, algorithm="giac")

[Out] 1/2*(b^2*d^3*x^3 + 2*a*b*d^3*x^2 + a^2*d^3*x - 3*b^2*d^2*x^2 - 4*a*b*d^2*x - a^2*d^2 + 6*b^2*d*x + 4*a*b*d - 6*b^2)*e^(d*x + c)/d^4 - 1/2*(b^2*d^3*x^3 + 2*a*b*d^3*x^2 + a^2*d^3*x + 3*b^2*d^2*x^2 + 4*a*b*d^2*x + a^2*d^2 + 6*b^2*d*x + 4*a*b*d + 6*b^2)*e^(-d*x - c)/d^4

maple [B] time = 0.04, size = 283, normalized size = 2.11

$$\frac{b^2((d x + c)^3 \sinh(d x + c) - 3(d x + c)^2 \cosh(d x + c) + 6(d x + c) \sinh(d x + c) - 6 \cosh(d x + c))}{d^2} - \frac{3 b^2 c((d x + c)^2 \sinh(d x + c) - 2(d x + c) \cosh(d x + c) + 2 \sinh(d x + c))}{d^2} +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(b*x+a)^2*cosh(d*x+c),x)

[Out] 1/d^2*(b^2/d^2*((d*x+c)^3*sinh(d*x+c)-3*(d*x+c)^2*cosh(d*x+c)+6*(d*x+c)*sinh(d*x+c)-6*cosh(d*x+c))-3*b^2/d^2*c*((d*x+c)^2*sinh(d*x+c)-2*(d*x+c)*cosh(d*x+c)+2*sinh(d*x+c))+2*b/d*a*((d*x+c)^2*sinh(d*x+c)-2*(d*x+c)*cosh(d*x+c)+2*sinh(d*x+c))+3*b^2*c^2/d^2*((d*x+c)*sinh(d*x+c)-cosh(d*x+c))-4*b*c/d*a*((d*x+c)*sinh(d*x+c)-cosh(d*x+c))+a^2*((d*x+c)*sinh(d*x+c)-cosh(d*x+c))-b^2*c^3/d^2*sinh(d*x+c)+2*b*c^2/d*a*sinh(d*x+c)-c*a^2*sinh(d*x+c))

maxima [B] time = 0.33, size = 275, normalized size = 2.05

$$-\frac{1}{24} d \left(\frac{6(d^2 x^2 e^c - 2 d x e^c + 2 e^c) a^2 e^{(d x)}}{d^3} + \frac{6(d^2 x^2 + 2 d x + 2) a^2 e^{(-d x - c)}}{d^3} + \frac{8(d^3 x^3 e^c - 3 d^2 x^2 e^c + 6 d x e^c - 6 e^c) a b e^{(d x + c)}}{d^4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x+a)^2*cosh(d*x+c),x, algorithm="maxima")

[Out] -1/24*d*(6*(d^2*x^2*e^c - 2*d*x*e^c + 2*e^c)*a^2*e^(d*x)/d^3 + 6*(d^2*x^2 + 2*d*x + 2)*a^2*e^(-d*x - c)/d^3 + 8*(d^3*x^3*e^c - 3*d^2*x^2*e^c + 6*d*x*e^c - 6*e^c)*a*b*e^(d*x)/d^4 + 8*(d^3*x^3 + 3*d^2*x^2 + 6*d*x + 6)*a*b*e^(-d*x - c)/d^4 + 3*(d^4*x^4*e^c - 4*d^3*x^3*e^c + 12*d^2*x^2*e^c - 24*d*x*e^c + 24*e^c)*b^2*e^(d*x)/d^5 + 3*(d^4*x^4 + 4*d^3*x^3 + 12*d^2*x^2 + 24*d*x + 24)*b^2*e^(-d*x - c)/d^5 + 1/12*(3*b^2*x^4 + 8*a*b*x^3 + 6*a^2*x^2)*cosh(d*x + c)

mupad [B] time = 0.92, size = 125, normalized size = 0.93

$$\frac{b^2 x^3 \sinh(c + dx)}{d} - \frac{3b^2 x^2 \cosh(c + dx)}{d^2} - \frac{\cosh(c + dx) (a^2 d^2 + 6b^2)}{d^4} + \frac{4ab \sinh(c + dx)}{d^3} + \frac{x \sinh(c + dx)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*cosh(c + d*x)*(a + b*x)^2,x)`

[Out] $(b^2 x^3 \sinh(c + dx))/d - (3b^2 x^2 \cosh(c + dx))/d^2 - (\cosh(c + dx) * (6b^2 + a^2 d^2))/d^4 + (4a * b * \sinh(c + dx))/d^3 + (x * \sinh(c + dx) * (6b^2 + a^2 d^2))/d^3 + (2a * b * x^2 * \sinh(c + dx))/d - (4a * b * x * \cosh(c + dx))/d^2$

sympy [A] time = 1.15, size = 172, normalized size = 1.28

$$\left\{ \begin{array}{l} \frac{a^2 x \sinh(c+dx)}{d} - \frac{a^2 \cosh(c+dx)}{d^2} + \frac{2abx^2 \sinh(c+dx)}{d} - \frac{4abx \cosh(c+dx)}{d^2} + \frac{4ab \sinh(c+dx)}{d^3} + \frac{b^2 x^3 \sinh(c+dx)}{d} - \frac{3b^2 x^2 \cosh(c+dx)}{d^2} + \frac{6b^2 x \sinh(c+dx)}{d} - \frac{6b^2 \cosh(c+dx)}{d^2} \\ \left(\frac{a^2 x^2}{2} + \frac{2abx^3}{3} + \frac{b^2 x^4}{4} \right) \cosh(c) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(b*x+a)**2*cosh(d*x+c),x)`

[Out] `Piecewise((a**2*x*sinh(c + d*x)/d - a**2*cosh(c + d*x)/d**2 + 2*a*b*x**2*sinh(c + d*x)/d - 4*a*b*x*cosh(c + d*x)/d**2 + 4*a*b*sinh(c + d*x)/d**3 + b**2*x**3*sinh(c + d*x)/d - 3*b**2*x**2*cosh(c + d*x)/d**2 + 6*b**2*x*sinh(c + d*x)/d**3 - 6*b**2*cosh(c + d*x)/d**4, Ne(d, 0)), ((a**2*x**2/2 + 2*a*b*x**3/3 + b**2*x**4/4)*cosh(c), True))`

3.12 $\int (a + bx)^2 \cosh(c + dx) dx$

Optimal. Leaf size=49

$$-\frac{2b(a + bx) \cosh(c + dx)}{d^2} + \frac{(a + bx)^2 \sinh(c + dx)}{d} + \frac{2b^2 \sinh(c + dx)}{d^3}$$

[Out] $-2*b*(b*x+a)*\cosh(d*x+c)/d^2+2*b^2*\sinh(d*x+c)/d^3+(b*x+a)^2*\sinh(d*x+c)/d$

Rubi [A] time = 0.05, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3296, 2637}

$$-\frac{2b(a + bx) \cosh(c + dx)}{d^2} + \frac{(a + bx)^2 \sinh(c + dx)}{d} + \frac{2b^2 \sinh(c + dx)}{d^3}$$

Antiderivative was successfully verified.

[In] `Int[(a + b*x)^2*Cosh[c + d*x], x]`

[Out] $(-2*b*(a + b*x)*\text{Cosh}[c + d*x])/d^2 + (2*b^2*\text{Sinh}[c + d*x])/d^3 + ((a + b*x)^2*\text{Sinh}[c + d*x])/d$

Rule 2637

`Int[sin[Pi/2 + (c_.) + (d_.)*(x_.)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;`
`FreeQ[{c, d}, x]`

Rule 3296

`Int[((c_.) + (d_.)*(x_.))^(m_.)*sin[(e_.) + (f_.)*(x_.)], x_Symbol] := -Simp[`
`((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[`
`e + f*x], x], x] /;` `FreeQ[{c, d, e, f}, x] && GtQ[m, 0]`

Rubi steps

$$\begin{aligned} \int (a + bx)^2 \cosh(c + dx) dx &= \frac{(a + bx)^2 \sinh(c + dx)}{d} - \frac{(2b) \int (a + bx) \sinh(c + dx) dx}{d} \\ &= -\frac{2b(a + bx) \cosh(c + dx)}{d^2} + \frac{(a + bx)^2 \sinh(c + dx)}{d} + \frac{(2b^2) \int \cosh(c + dx) dx}{d^2} \\ &= -\frac{2b(a + bx) \cosh(c + dx)}{d^2} + \frac{2b^2 \sinh(c + dx)}{d^3} + \frac{(a + bx)^2 \sinh(c + dx)}{d} \end{aligned}$$

Mathematica [A] time = 0.15, size = 56, normalized size = 1.14

$$\frac{(a^2 d^2 + 2abd^2 x + b^2 (d^2 x^2 + 2)) \sinh(c + dx) - 2bd(a + bx) \cosh(c + dx)}{d^3}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)^2*Cosh[c + d*x],x]

[Out] (-2*b*d*(a + b*x)*Cosh[c + d*x] + (a^2*d^2 + 2*a*b*d^2*x + b^2*(2 + d^2*x^2))*Sinh[c + d*x])/d^3

fricas [A] time = 0.40, size = 64, normalized size = 1.31

$$\frac{2(b^2 dx + abd) \cosh(dx + c) - (b^2 d^2 x^2 + 2abd^2 x + a^2 d^2 + 2b^2) \sinh(dx + c)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^2*cosh(d*x+c),x, algorithm="fricas")

[Out] -(2*(b^2*d*x + a*b*d)*cosh(d*x + c) - (b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2 + 2*b^2)*sinh(d*x + c))/d^3

giac [B] time = 0.13, size = 112, normalized size = 2.29

$$\frac{(b^2 d^2 x^2 + 2abd^2 x + a^2 d^2 - 2b^2 dx - 2abd + 2b^2) e^{(dx+c)}}{2d^3} - \frac{(b^2 d^2 x^2 + 2abd^2 x + a^2 d^2 + 2b^2 dx + 2abd + 2b^2) e^{(-d*x - c)}}{2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^2*cosh(d*x+c),x, algorithm="giac")

[Out] 1/2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2 - 2*b^2*d*x - 2*a*b*d + 2*b^2)*e^(d*x + c)/d^3 - 1/2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2 + 2*b^2*d*x + 2*a*b*d + 2*b^2)*e^(-d*x - c)/d^3

maple [B] time = 0.04, size = 147, normalized size = 3.00

$$\frac{b^2((dx+c)^2 \sinh(dx+c) - 2(dx+c) \cosh(dx+c) + 2 \sinh(dx+c))}{d^2} - \frac{2b^2c((dx+c) \sinh(dx+c) - \cosh(dx+c))}{d^2} + \frac{2ba((dx+c) \sinh(dx+c) - \cosh(dx+c))}{d} + \frac{b^2}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x+a)^2*cosh(d*x+c),x)

[Out] $1/d*(b^2/d^2*((d*x+c)^2*\sinh(d*x+c)-2*(d*x+c)*\cosh(d*x+c)+2*\sinh(d*x+c))-2*b^2/d^2*c*((d*x+c)*\sinh(d*x+c)-\cosh(d*x+c))+2*b/d*a*((d*x+c)*\sinh(d*x+c)-\cosh(d*x+c))+b^2*c^2/d^2*\sinh(d*x+c)-2*b*c/d*a*\sinh(d*x+c)+a^2*\sinh(d*x+c))$

maxima [B] time = 0.34, size = 135, normalized size = 2.76

$$\frac{a^2 e^{(dx+c)}}{2d} + \frac{(dx e^c - e^c) a b e^{(dx)}}{d^2} - \frac{(dx+1) a b e^{(-dx-c)}}{d^2} - \frac{a^2 e^{(-dx-c)}}{2d} + \frac{(d^2 x^2 e^c - 2 dx e^c + 2 e^c) b^2 e^{(dx)}}{2d^3} - \frac{(d^2 x^2 + 2 dx + 2) b^2 e^{(-dx-c)}}{2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^2*cosh(d*x+c),x, algorithm="maxima")

[Out] $1/2*a^2*e^{(d*x+c)}/d + (d*x*e^c - e^c)*a*b*e^{(d*x)}/d^2 - (d*x+1)*a*b*e^{(-d*x-c)}/d^2 - 1/2*a^2*e^{(-d*x-c)}/d + 1/2*(d^2*x^2*e^c - 2*d*x*e^c + 2*e^c)*b^2*e^{(d*x)}/d^3 - 1/2*(d^2*x^2 + 2*d*x + 2)*b^2*e^{(-d*x-c)}/d^3$

mupad [B] time = 0.90, size = 82, normalized size = 1.67

$$\frac{\sinh(c+dx) (a^2 d^2 + 2 b^2)}{d^3} + \frac{b^2 x^2 \sinh(c+dx)}{d} - \frac{2 a b \cosh(c+dx)}{d^2} - \frac{2 b^2 x \cosh(c+dx)}{d^2} + \frac{2 a b x \sinh(c+dx)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(c+d*x)*(a+b*x)^2,x)

[Out] $(\sinh(c+d*x)*(2*b^2+a^2*d^2))/d^3 + (b^2*x^2*\sinh(c+d*x))/d - (2*a*b*\cosh(c+d*x))/d^2 - (2*b^2*x*\cosh(c+d*x))/d^2 + (2*a*b*x*\sinh(c+d*x))/d$

sympy [A] time = 0.55, size = 112, normalized size = 2.29

$$\begin{cases} \frac{a^2 \sinh(c+dx)}{d} + \frac{2abx \sinh(c+dx)}{d} - \frac{2ab \cosh(c+dx)}{d^2} + \frac{b^2 x^2 \sinh(c+dx)}{d} - \frac{2b^2 x \cosh(c+dx)}{d^2} + \frac{2b^2 \sinh(c+dx)}{d^3} & \text{for } d \neq 0 \\ \left(a^2 x + abx^2 + \frac{b^2 x^3}{3} \right) \cosh(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)**2*cosh(d*x+c),x)

[Out] Piecewise((a**2*sinh(c+d*x)/d + 2*a*b*x*sinh(c+d*x)/d - 2*a*b*cosh(c+d*x)/d**2 + b**2*x**2*sinh(c+d*x)/d - 2*b**2*x*cosh(c+d*x)/d**2 + 2*b**2*sinh(c+d*x)/d**3, Ne(d, 0)), ((a**2*x + a*b*x**2 + b**2*x**3/3)*cosh(c), True))

$$3.13 \quad \int \frac{(a+bx)^2 \cosh(c+dx)}{x} dx$$

Optimal. Leaf size=62

$$a^2 \cosh(c) \text{Chi}(dx) + a^2 \sinh(c) \text{Shi}(dx) + \frac{2ab \sinh(c+dx)}{d} - \frac{b^2 \cosh(c+dx)}{d^2} + \frac{b^2 x \sinh(c+dx)}{d}$$

[Out] $a^2 \text{Chi}(d*x) * \cosh(c) - b^2 * \cosh(d*x+c) / d^2 + a^2 * \text{Shi}(d*x) * \sinh(c) + 2*a*b*\sinh(d*x+c)/d + b^2*x*\sinh(d*x+c)/d$

Rubi [A] time = 0.18, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {6742, 2637, 3303, 3298, 3301, 3296, 2638}

$$a^2 \cosh(c) \text{Chi}(dx) + a^2 \sinh(c) \text{Shi}(dx) + \frac{2ab \sinh(c+dx)}{d} - \frac{b^2 \cosh(c+dx)}{d^2} + \frac{b^2 x \sinh(c+dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[((a + b*x)^2*Cosh[c + d*x])/x,x]

[Out] $-((b^2 * \text{Cosh}[c + d*x]) / d^2) + a^2 * \text{Cosh}[c] * \text{CoshIntegral}[d*x] + (2*a*b*\text{Sinh}[c + d*x]) / d + (b^2*x*\text{Sinh}[c + d*x]) / d + a^2 * \text{Sinh}[c] * \text{SinhIntegral}[d*x]$

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 2638

Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3296

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[((c + d*x)^m * Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1) * Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 3298

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_) / ((c_.) + (d_.)*(x_))], x_Symbol] := Simp[(I * SinhIntegral[(c*f*fz)/d + f*fz*x]) / d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x]
&& EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[
Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

Rule 6742

```
Int[u_, x_Symbol]
:> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]
```

Rubi steps

$$\begin{aligned} \int \frac{(a + bx)^2 \cosh(c + dx)}{x} dx &= \int \left(2ab \cosh(c + dx) + \frac{a^2 \cosh(c + dx)}{x} + b^2 x \cosh(c + dx) \right) dx \\ &= a^2 \int \frac{\cosh(c + dx)}{x} dx + (2ab) \int \cosh(c + dx) dx + b^2 \int x \cosh(c + dx) dx \\ &= \frac{2ab \sinh(c + dx)}{d} + \frac{b^2 x \sinh(c + dx)}{d} - \frac{b^2 \int \sinh(c + dx) dx}{d} + \left(a^2 \cosh(c) \right) \int \frac{dx}{x} \\ &= -\frac{b^2 \cosh(c + dx)}{d^2} + a^2 \cosh(c) \text{Chi}(dx) + \frac{2ab \sinh(c + dx)}{d} + \frac{b^2 x \sinh(c + dx)}{d} + \end{aligned}$$

Mathematica [A] time = 0.25, size = 51, normalized size = 0.82

$$a^2 \cosh(c) \text{Chi}(dx) + a^2 \sinh(c) \text{Shi}(dx) + \frac{b(d(2a + bx) \sinh(c + dx) - b \cosh(c + dx))}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)^2*Cosh[c + d*x])/x,x]
```

```
[Out] a^2*Cosh[c]*CoshIntegral[d*x] + (b*(-(b*Cosh[c + d*x]) + d*(2*a + b*x)*Sinh[c + d*x]))/d^2 + a^2*Sinh[c]*SinhIntegral[d*x]
```

fricas [A] time = 0.62, size = 94, normalized size = 1.52

$$\frac{2b^2 \cosh(dx+c) - (a^2 d^2 \operatorname{Ei}(dx) + a^2 d^2 \operatorname{Ei}(-dx)) \cosh(c) - 2(b^2 dx + 2abd) \sinh(dx+c) - (a^2 d^2 \operatorname{Ei}(dx) - a^2 d^2 \operatorname{Ei}(-dx)) \sinh(c)}{2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^2*cosh(d*x+c)/x,x, algorithm="fricas")

[Out] $-1/2*(2*b^2*\cosh(d*x+c) - (a^2*d^2*\operatorname{Ei}(d*x) + a^2*d^2*\operatorname{Ei}(-d*x))*\cosh(c) - 2*(b^2*d*x + 2*a*b*d)*\sinh(d*x+c) - (a^2*d^2*\operatorname{Ei}(d*x) - a^2*d^2*\operatorname{Ei}(-d*x))*\sinh(c))/d^2$

giac [A] time = 0.14, size = 113, normalized size = 1.82

$$\frac{a^2 d^2 \operatorname{Ei}(-dx) e^{(-c)} + a^2 d^2 \operatorname{Ei}(dx) e^c + b^2 dx e^{(dx+c)} - b^2 dx e^{(-dx-c)} + 2 ab d e^{(dx+c)} - 2 ab d e^{(-dx-c)} - b^2 e^{(dx+c)} - b^2 e^{(-dx-c)}}{2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^2*cosh(d*x+c)/x,x, algorithm="giac")

[Out] $1/2*(a^2*d^2*\operatorname{Ei}(-d*x)*e^{(-c)} + a^2*d^2*\operatorname{Ei}(d*x)*e^c + b^2*d*x*e^{(d*x+c)} - b^2*d*x*e^{(-d*x-c)} + 2*a*b*d*e^{(d*x+c)} - 2*a*b*d*e^{(-d*x-c)} - b^2*e^{(d*x+c)} - b^2*e^{(-d*x-c)})/d^2$

maple [A] time = 0.11, size = 121, normalized size = 1.95

$$\frac{a^2 e^{-c} \operatorname{Ei}(1, dx)}{2} - \frac{b^2 e^{-dx-c} x}{2d} - \frac{b^2 e^{-dx-c}}{2d^2} - \frac{ab e^{-dx-c}}{d} - \frac{a^2 e^c \operatorname{Ei}(1, -dx)}{2} + \frac{b^2 e^{dx+c} x}{2d} - \frac{b^2 e^{dx+c}}{2d^2} + \frac{ab e^{dx+c}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x+a)^2*cosh(d*x+c)/x,x)

[Out] $-1/2*a^2*\exp(-c)*\operatorname{Ei}(1,d*x) - 1/2*b^2/d*\exp(-d*x-c)*x - 1/2*b^2/d^2*\exp(-d*x-c) - a*b/d*\exp(-d*x-c) - 1/2*a^2*\exp(c)*\operatorname{Ei}(1,-d*x) + 1/2*b^2/d*\exp(d*x+c)*x - 1/2*b^2/d^2*\exp(d*x+c) + a*b/d*\exp(d*x+c)$

maxima [B] time = 0.40, size = 175, normalized size = 2.82

$$-\frac{1}{4} \left(4ab \left(\frac{(dxe^c - e^c)e^{(dx)}}{d^2} + \frac{(dx+1)e^{(-dx-c)}}{d^2} \right) + b^2 \left(\frac{(d^2x^2e^c - 2dxe^c + 2e^c)e^{(dx)}}{d^3} + \frac{(d^2x^2 + 2dx + 2)e^{(-dx-c)}}{d^3} \right) + 4 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^2*cosh(d*x+c)/x,x, algorithm="maxima")

[Out] $-1/4*(4*a*b*((d*x*e^c - e^c)*e^{(d*x)}/d^2 + (d*x + 1)*e^{(-d*x - c)}/d^2) + b^2*((d^2*x^2*e^c - 2*d*x*e^c + 2*e^c)*e^{(d*x)}/d^3 + (d^2*x^2 + 2*d*x + 2)*e^{(-d*x - c)}/d^3) + 4*a^2*\cosh(d*x + c)*\log(x)/d - 2*(\text{Ei}(-d*x)*e^{-c} + \text{Ei}(d*x)*e^c)*a^2/d)*d + 1/2*(b^2*x^2 + 4*a*b*x + 2*a^2*\log(x))*\cosh(d*x + c)$

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cosh(c + dx) (a + bx)^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((cosh(c + d*x)*(a + b*x)^2)/x,x)`

[Out] `int((cosh(c + d*x)*(a + b*x)^2)/x, x)`

sympy [A] time = 3.70, size = 73, normalized size = 1.18

$$a^2 \sinh(c) \text{Shi}(dx) + a^2 \cosh(c) \text{Chi}(dx) + 2ab \begin{cases} \frac{\sinh(c+dx)}{d} & \text{for } d \neq 0 \\ x \cosh(c) & \text{otherwise} \end{cases} + b^2 \begin{cases} \frac{x \sinh(c+dx)}{d} - \frac{\cosh(c+dx)}{d^2} & \text{for } d \neq 0 \\ \frac{x^2 \cosh(c)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x+a)**2*cosh(d*x+c)/x,x)`

[Out] `a**2*sinh(c)*Shi(d*x) + a**2*cosh(c)*Chi(d*x) + 2*a*b*Piecewise((sinh(c + d*x)/d, Ne(d, 0)), (x*cosh(c), True)) + b**2*Piecewise((x*sinh(c + d*x)/d - cosh(c + d*x)/d**2, Ne(d, 0)), (x**2*cosh(c)/2, True))`

$$3.14 \quad \int \frac{(a+bx)^2 \cosh(c+dx)}{x^2} dx$$

Optimal. Leaf size=70

$$a^2 d \sinh(c) \operatorname{Chi}(dx) + a^2 d \cosh(c) \operatorname{Shi}(dx) - \frac{a^2 \cosh(c+dx)}{x} + 2ab \cosh(c) \operatorname{Chi}(dx) + 2ab \sinh(c) \operatorname{Shi}(dx) + \frac{b^2 \sinh(c)}{d}$$

[Out] 2*a*b*Chi(d*x)*cosh(c)-a^2*cosh(d*x+c)/x+a^2*d*cosh(c)*Shi(d*x)+a^2*d*Chi(d*x)*sinh(c)+2*a*b*Shi(d*x)*sinh(c)+b^2*sinh(d*x+c)/d

Rubi [A] time = 0.25, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {6742, 2637, 3297, 3303, 3298, 3301}

$$a^2 d \sinh(c) \operatorname{Chi}(dx) + a^2 d \cosh(c) \operatorname{Shi}(dx) - \frac{a^2 \cosh(c+dx)}{x} + 2ab \cosh(c) \operatorname{Chi}(dx) + 2ab \sinh(c) \operatorname{Shi}(dx) + \frac{b^2 \sinh(c)}{d}$$

Antiderivative was successfully verified.

[In] Int[((a + b*x)^2*Cosh[c + d*x])/x^2,x]

[Out] -((a^2*Cosh[c + d*x])/x) + 2*a*b*Cosh[c]*CoshIntegral[d*x] + a^2*d*CoshIntegral[d*x]*Sinh[c] + (b^2*Sinh[c + d*x])/d + a^2*d*Cosh[c]*SinhIntegral[d*x] + 2*a*b*Sinh[c]*SinhIntegral[d*x]

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3297

Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]

Rule 3298

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x]
&& EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x]
+ Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /;
FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]
```

Rule 6742

```
Int[u_, x_Symbol]
:> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]
```

Rubi steps

$$\begin{aligned} \int \frac{(a + bx)^2 \cosh(c + dx)}{x^2} dx &= \int \left(b^2 \cosh(c + dx) + \frac{a^2 \cosh(c + dx)}{x^2} + \frac{2ab \cosh(c + dx)}{x} \right) dx \\ &= a^2 \int \frac{\cosh(c + dx)}{x^2} dx + (2ab) \int \frac{\cosh(c + dx)}{x} dx + b^2 \int \cosh(c + dx) dx \\ &= -\frac{a^2 \cosh(c + dx)}{x} + \frac{b^2 \sinh(c + dx)}{d} + (a^2 d) \int \frac{\sinh(c + dx)}{x} dx + (2ab \cosh(c)) \\ &= -\frac{a^2 \cosh(c + dx)}{x} + 2ab \cosh(c) \text{Chi}(dx) + \frac{b^2 \sinh(c + dx)}{d} + 2ab \sinh(c) \text{Shi}(dx) \\ &= -\frac{a^2 \cosh(c + dx)}{x} + 2ab \cosh(c) \text{Chi}(dx) + a^2 d \text{Chi}(dx) \sinh(c) + \frac{b^2 \sinh(c + dx)}{d} \end{aligned}$$

Mathematica [A] time = 0.25, size = 62, normalized size = 0.89

$$-\frac{a^2 \cosh(c + dx)}{x} + a \text{Chi}(dx) (ad \sinh(c) + 2b \cosh(c)) + a \text{Shi}(dx) (ad \cosh(c) + 2b \sinh(c)) + \frac{b^2 \sinh(c + dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)^2*Cosh[c + d*x])/x^2,x]
```

```
[Out] -((a^2*Cosh[c + d*x])/x) + a*CoshIntegral[d*x]*(2*b*Cosh[c] + a*d*Sinh[c])
+ (b^2*Sinh[c + d*x])/d + a*(a*d*Cosh[c] + 2*b*Sinh[c])*SinhIntegral[d*x]
```

fricas [A] time = 0.54, size = 122, normalized size = 1.74

$$\frac{2 a^2 d \cosh(dx + c) - 2 b^2 x \sinh(dx + c) - \left((a^2 d^2 + 2 abd) x \operatorname{Ei}(dx) - (a^2 d^2 - 2 abd) x \operatorname{Ei}(-dx) \right) \cosh(c) - \left((a^2 d^2 + 2 abd) x \operatorname{Ei}(dx) + (a^2 d^2 - 2 abd) x \operatorname{Ei}(-dx) \right) \sinh(c)}{2 dx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^2*cosh(d*x+c)/x^2,x, algorithm="fricas")

[Out] $-1/2*(2*a^2*d*\cosh(d*x + c) - 2*b^2*x*\sinh(d*x + c) - ((a^2*d^2 + 2*a*b*d)*x*\operatorname{Ei}(d*x) - (a^2*d^2 - 2*a*b*d)*x*\operatorname{Ei}(-d*x))*\cosh(c) - ((a^2*d^2 + 2*a*b*d)*x*\operatorname{Ei}(d*x) + (a^2*d^2 - 2*a*b*d)*x*\operatorname{Ei}(-d*x))*\sinh(c))/(d*x)$

giac [A] time = 0.13, size = 119, normalized size = 1.70

$$\frac{a^2 d^2 x \operatorname{Ei}(-dx) e^{-c} - a^2 d^2 x \operatorname{Ei}(dx) e^c - 2 ab dx \operatorname{Ei}(-dx) e^{-c} - 2 ab dx \operatorname{Ei}(dx) e^c + a^2 d e^{(dx+c)} - b^2 x e^{(dx+c)} + a^2 d e^{-(dx+c)}}{2 dx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^2*cosh(d*x+c)/x^2,x, algorithm="giac")

[Out] $-1/2*(a^2*d^2*x*\operatorname{Ei}(-d*x)*e^{-c} - a^2*d^2*x*\operatorname{Ei}(d*x)*e^c - 2*a*b*d*x*\operatorname{Ei}(-d*x)*e^{-c} - 2*a*b*d*x*\operatorname{Ei}(d*x)*e^c + a^2*d*e^{(d*x + c)} - b^2*x*e^{(d*x + c)} + a^2*d*e^{-(d*x - c)} + b^2*x*e^{-(d*x - c)})/(d*x)$

maple [A] time = 0.12, size = 118, normalized size = 1.69

$$-\frac{a^2 e^{-dx-c}}{2x} + \frac{d a^2 e^{-c} \operatorname{Ei}(1, dx)}{2} - \frac{b^2 e^{-dx-c}}{2d} - ab e^{-c} \operatorname{Ei}(1, dx) - \frac{a^2 e^{dx+c}}{2x} - \frac{d a^2 e^c \operatorname{Ei}(1, -dx)}{2} + \frac{b^2 e^{dx+c}}{2d} - ab e^c \operatorname{Ei}(1, -dx)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x+a)^2*cosh(d*x+c)/x^2,x)

[Out] $-1/2*a^2*\exp(-d*x-c)/x+1/2*d*a^2*\exp(-c)*\operatorname{Ei}(1,d*x)-1/2*b^2/d*\exp(-d*x-c)-a*b*\exp(-c)*\operatorname{Ei}(1,d*x)-1/2*a^2/x*\exp(d*x+c)-1/2*d*a^2*\exp(c)*\operatorname{Ei}(1,-d*x)+1/2*b^2/d*\exp(d*x+c)-a*b*\exp(c)*\operatorname{Ei}(1,-d*x)$

maxima [A] time = 0.42, size = 136, normalized size = 1.94

$$-\frac{1}{2} \left((\operatorname{Ei}(-dx) e^{-c} - \operatorname{Ei}(dx) e^c) a^2 + b^2 \left(\frac{(dx e^c - e^c) e^{(dx)}}{d^2} + \frac{(dx + 1) e^{(-dx-c)}}{d^2} \right) \right) + \frac{4 ab \cosh(dx + c) \log(x)}{d} - \frac{2 (\operatorname{Ei}(-dx) e^{-c} + \operatorname{Ei}(dx) e^c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^2*cosh(d*x+c)/x^2,x, algorithm="maxima")

[Out] $-1/2*((\text{Ei}(-d*x)*e^{-c}) - \text{Ei}(d*x)*e^c)*a^2 + b^2*((d*x*e^c - e^c)*e^{(d*x)/d^2} + (d*x + 1)*e^{-(d*x - c)}/d^2) + 4*a*b*\cosh(d*x + c)*\log(x)/d - 2*(\text{Ei}(-d*x)*e^{-c}) + \text{Ei}(d*x)*e^c)*a*b/d)*d + (b^2*x + 2*a*b*\log(x) - a^2/x)*\cosh(d*x + c)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c + dx) (a + bx)^2}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((cosh(c + d*x)*(a + b*x)^2)/x^2, x)`

[Out] `int((cosh(c + d*x)*(a + b*x)^2)/x^2, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx)^2 \cosh(c + dx)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x+a)**2*cosh(d*x+c)/x**2, x)`

[Out] `Integral((a + b*x)**2*cosh(c + d*x)/x**2, x)`

$$3.15 \quad \int \frac{(a+bx)^2 \cosh(c+dx)}{x^3} dx$$

Optimal. Leaf size=121

$$\frac{1}{2}a^2d^2 \cosh(c)\text{Chi}(dx) + \frac{1}{2}a^2d^2 \sinh(c)\text{Shi}(dx) - \frac{a^2 \cosh(c+dx)}{2x^2} - \frac{a^2d \sinh(c+dx)}{2x} + 2abd \sinh(c)\text{Chi}(dx) + 2abd$$

[Out] $b^2 \text{Chi}(d*x) * \cosh(c) + 1/2 * a^2 * d^2 * \text{Chi}(d*x) * \cosh(c) - 1/2 * a^2 * \cosh(d*x+c) / x^2 - 2 * a * b * \cosh(d*x+c) / x + 2 * a * b * d * \cosh(c) * \text{Shi}(d*x) + 2 * a * b * d * \text{Chi}(d*x) * \sinh(c) + b^2 * \text{Shi}(d*x) * \sinh(c) + 1/2 * a^2 * d^2 * \text{Shi}(d*x) * \sinh(c) - 1/2 * a^2 * d * \sinh(d*x+c) / x$

Rubi [A] time = 0.34, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {6742, 3297, 3303, 3298, 3301}

$$\frac{1}{2}a^2d^2 \cosh(c)\text{Chi}(dx) + \frac{1}{2}a^2d^2 \sinh(c)\text{Shi}(dx) - \frac{a^2 \cosh(c+dx)}{2x^2} - \frac{a^2d \sinh(c+dx)}{2x} + 2abd \sinh(c)\text{Chi}(dx) + 2abd$$

Antiderivative was successfully verified.

[In] Int[((a + b*x)^2*Cosh[c + d*x])/x^3,x]

[Out] $-(a^2 * \text{Cosh}[c + d*x]) / (2 * x^2) - (2 * a * b * \text{Cosh}[c + d*x]) / x + b^2 * \text{Cosh}[c] * \text{CoshIntegral}[d*x] + (a^2 * d^2 * \text{Cosh}[c] * \text{CoshIntegral}[d*x]) / 2 + 2 * a * b * d * \text{CoshIntegral}[d*x] * \text{Sinh}[c] - (a^2 * d * \text{Sinh}[c + d*x]) / (2 * x) + 2 * a * b * d * \text{Cosh}[c] * \text{SinhIntegral}[d*x] + b^2 * \text{Sinh}[c] * \text{SinhIntegral}[d*x] + (a^2 * d^2 * \text{Sinh}[c] * \text{SinhIntegral}[d*x]) / 2$

Rule 3297

Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]

Rule 3298

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)])/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]

Rule 3301

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)])/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + bx)^2 \cosh(c + dx)}{x^3} dx &= \int \left(\frac{a^2 \cosh(c + dx)}{x^3} + \frac{2ab \cosh(c + dx)}{x^2} + \frac{b^2 \cosh(c + dx)}{x} \right) dx \\
&= a^2 \int \frac{\cosh(c + dx)}{x^3} dx + (2ab) \int \frac{\cosh(c + dx)}{x^2} dx + b^2 \int \frac{\cosh(c + dx)}{x} dx \\
&= -\frac{a^2 \cosh(c + dx)}{2x^2} - \frac{2ab \cosh(c + dx)}{x} + \frac{1}{2} (a^2 d) \int \frac{\sinh(c + dx)}{x^2} dx + (2abd) \int \frac{\sinh(c + dx)}{x} dx \\
&= -\frac{a^2 \cosh(c + dx)}{2x^2} - \frac{2ab \cosh(c + dx)}{x} + b^2 \cosh(c) \text{Chi}(dx) - \frac{a^2 d \sinh(c + dx)}{2x} + \frac{2abd \sinh(c + dx)}{2} \\
&= -\frac{a^2 \cosh(c + dx)}{2x^2} - \frac{2ab \cosh(c + dx)}{x} + b^2 \cosh(c) \text{Chi}(dx) + 2abd \text{Chi}(dx) \sinh(c) \\
&= -\frac{a^2 \cosh(c + dx)}{2x^2} - \frac{2ab \cosh(c + dx)}{x} + b^2 \cosh(c) \text{Chi}(dx) + \frac{1}{2} a^2 d^2 \cosh(c) \text{Chi}(dx)
\end{aligned}$$

Mathematica [A] time = 0.40, size = 93, normalized size = 0.77

$$\frac{1}{2} \left(\text{Chi}(dx) \left(\cosh(c) (a^2 d^2 + 2b^2) + 4abd \sinh(c) \right) + \text{Shi}(dx) \left(\sinh(c) (a^2 d^2 + 2b^2) + 4abd \cosh(c) \right) - \frac{a((a + 4bx)^2 \cosh(c + dx) + a^2 d^2 \cosh(c) \text{Chi}(dx) + 2abd \sinh(c) \text{Chi}(dx))}{4x^2} \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)^2*Cosh[c + d*x])/x^3,x]
```

```
[Out] (CoshIntegral[d*x]*((2*b^2 + a^2*d^2)*Cosh[c] + 4*a*b*d*Sinh[c]) - (a*((a +
4*b*x)*Cosh[c + d*x] + a*d*x*Sinh[c + d*x]))/x^2 + (4*a*b*d*Cosh[c] + (2*b
^2 + a^2*d^2)*Sinh[c])*SinhIntegral[d*x])/2
```

fricas [A] time = 0.63, size = 153, normalized size = 1.26

$$\frac{2 a^2 dx \sinh(dx + c) + 2 (4 abx + a^2) \cosh(dx + c) - \left((a^2 d^2 + 4 abd + 2 b^2) x^2 \text{Ei}(dx) + (a^2 d^2 - 4 abd + 2 b^2) x^2 \text{Ei}(dx) \right)}{4 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^2*cosh(d*x+c)/x^3,x, algorithm="fricas")

[Out] $-1/4*(2*a^2*d*x*\sinh(d*x + c) + 2*(4*a*b*x + a^2)*\cosh(d*x + c) - ((a^2*d^2 + 4*a*b*d + 2*b^2)*x^2*Ei(d*x) + (a^2*d^2 - 4*a*b*d + 2*b^2)*x^2*Ei(-d*x)) * \cosh(c) - ((a^2*d^2 + 4*a*b*d + 2*b^2)*x^2*Ei(d*x) - (a^2*d^2 - 4*a*b*d + 2*b^2)*x^2*Ei(-d*x))*\sinh(c))/x^2$

giac [A] time = 0.12, size = 181, normalized size = 1.50

$$\frac{a^2 d^2 x^2 Ei(-dx) e^{(-c)} + a^2 d^2 x^2 Ei(dx) e^c - 4 ab dx^2 Ei(-dx) e^{(-c)} + 4 ab dx^2 Ei(dx) e^c + 2 b^2 x^2 Ei(-dx) e^{(-c)} + 2 b^2 x^2 Ei(dx) e^c}{4 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^2*cosh(d*x+c)/x^3,x, algorithm="giac")

[Out] $1/4*(a^2*d^2*x^2*Ei(-d*x)*e^{(-c)} + a^2*d^2*x^2*Ei(d*x)*e^c - 4*a*b*d*x^2*Ei(-d*x)*e^{(-c)} + 4*a*b*d*x^2*Ei(d*x)*e^c + 2*b^2*x^2*Ei(-d*x)*e^{(-c)} + 2*b^2*x^2*Ei(d*x)*e^c - a^2*d*x*e^{(d*x + c)} + a^2*d*x*e^{(-d*x - c)} - 4*a*b*x*e^{(d*x + c)} - 4*a*b*x*e^{(-d*x - c)} - a^2*e^{(d*x + c)} - a^2*e^{(-d*x - c)})/x^2$

maple [A] time = 0.11, size = 181, normalized size = 1.50

$$\frac{b^2 e^{-c} Ei(1, dx)}{2} - \frac{a^2 e^{-dx-c}}{4x^2} + \frac{d a^2 e^{-dx-c}}{4x} - \frac{ab e^{-dx-c}}{x} + dab e^{-c} Ei(1, dx) - \frac{d^2 a^2 e^{-c} Ei(1, dx)}{4} - \frac{b^2 e^c Ei(1, -dx)}{2} - \frac{a^2 e^{dx-c}}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x+a)^2*cosh(d*x+c)/x^3,x)

[Out] $-1/2*b^2*\exp(-c)*Ei(1,d*x)-1/4*a^2*\exp(-d*x-c)/x^2+1/4*d*a^2*\exp(-d*x-c)/x-a*b*\exp(-d*x-c)/x+d*a*b*\exp(-c)*Ei(1,d*x)-1/4*d^2*a^2*\exp(-c)*Ei(1,d*x)-1/2*b^2*\exp(c)*Ei(1,-d*x)-1/4*a^2/x^2*\exp(d*x+c)-1/4*d*a^2/x*\exp(d*x+c)-a*b/x*\exp(d*x+c)-d*a*b*\exp(c)*Ei(1,-d*x)-1/4*d^2*a^2*\exp(c)*Ei(1,-d*x)$

maxima [A] time = 0.44, size = 126, normalized size = 1.04

$$\frac{1}{4} \left((de^{(-c)}\Gamma(-1, dx) + de^c\Gamma(-1, -dx))a^2 - 4(Ei(-dx)e^{(-c)} - Ei(dx)e^c)ab - \frac{4b^2 \cosh(dx + c) \log(x)}{d} + \frac{2(Ei(-c) - Ei(dx)e^c)}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^2*cosh(d*x+c)/x^3,x, algorithm="maxima")

[Out] $1/4*((d*e^{(-c)}*\gamma(-1, d*x) + d*e^c*\gamma(-1, -d*x))*a^2 - 4*(Ei(-d*x)*e^{(-c)} - Ei(d*x)*e^c)*a*b - 4*b^2*\cosh(d*x + c)*\log(x)/d + 2*(Ei(-d*x)*e^{(-c)} - Ei(d*x)*e^c))$

+ Ei(d*x)*e^c)*b^2/d)*d + 1/2*(2*b^2*log(x) - (4*a*b*x + a^2)/x^2)*cosh(d*x + c)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c + dx) (a + bx)^2}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cosh(c + d*x)*(a + b*x)^2)/x^3, x)

[Out] int((cosh(c + d*x)*(a + b*x)^2)/x^3, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx)^2 \cosh(c + dx)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)**2*cosh(d*x+c)/x**3, x)

[Out] Integral((a + b*x)**2*cosh(c + d*x)/x**3, x)

$$3.16 \quad \int \frac{(a+bx)^2 \cosh(c+dx)}{x^4} dx$$

Optimal. Leaf size=172

$$\frac{1}{6}a^2d^3 \sinh(c)\text{Chi}(dx) + \frac{1}{6}a^2d^3 \cosh(c)\text{Shi}(dx) - \frac{a^2d^2 \cosh(c+dx)}{6x} - \frac{a^2 \cosh(c+dx)}{3x^3} - \frac{a^2d \sinh(c+dx)}{6x^2} + abd^2 \cos$$

[Out] a*b*d^2*Chi(d*x)*cosh(c)-1/3*a^2*cosh(d*x+c)/x^3-a*b*cosh(d*x+c)/x^2-b^2*co
sh(d*x+c)/x-1/6*a^2*d^2*cosh(d*x+c)/x+b^2*d*cosh(c)*Shi(d*x)+1/6*a^2*d^3*co
sh(c)*Shi(d*x)+b^2*d*Chi(d*x)*sinh(c)+1/6*a^2*d^3*Chi(d*x)*sinh(c)+a*b*d^2*
Shi(d*x)*sinh(c)-1/6*a^2*d*sinh(d*x+c)/x^2-a*b*d*sinh(d*x+c)/x

Rubi [A] time = 0.43, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {6742, 3297, 3303, 3298, 3301}

$$\frac{1}{6}a^2d^3 \sinh(c)\text{Chi}(dx) + \frac{1}{6}a^2d^3 \cosh(c)\text{Shi}(dx) - \frac{a^2d^2 \cosh(c+dx)}{6x} - \frac{a^2d \sinh(c+dx)}{6x^2} - \frac{a^2 \cosh(c+dx)}{3x^3} + abd^2 \cos$$

Antiderivative was successfully verified.

[In] Int[((a + b*x)^2*Cosh[c + d*x])/x^4,x]

[Out] -(a^2*Cosh[c + d*x])/(3*x^3) - (a*b*Cosh[c + d*x])/x^2 - (b^2*Cosh[c + d*x])/x - (a^2*d^2*Cosh[c + d*x])/(6*x) + a*b*d^2*Cosh[c]*CoshIntegral[d*x] + b^2*d*CoshIntegral[d*x]*Sinh[c] + (a^2*d^3*CoshIntegral[d*x]*Sinh[c])/6 - (a^2*d*Sinh[c + d*x])/(6*x^2) - (a*b*d*Sinh[c + d*x])/x + b^2*d*Cosh[c]*SinhIntegral[d*x] + (a^2*d^3*Cosh[c]*SinhIntegral[d*x])/6 + a*b*d^2*Sinh[c]*SinhIntegral[d*x]

Rule 3297

Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]

Rule 3298

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)])/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x]
&& EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x]
+ Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /;
FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]
```

Rule 6742

```
Int[u_, x_Symbol]
:> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + bx)^2 \cosh(c + dx)}{x^4} dx &= \int \left(\frac{a^2 \cosh(c + dx)}{x^4} + \frac{2ab \cosh(c + dx)}{x^3} + \frac{b^2 \cosh(c + dx)}{x^2} \right) dx \\
&= a^2 \int \frac{\cosh(c + dx)}{x^4} dx + (2ab) \int \frac{\cosh(c + dx)}{x^3} dx + b^2 \int \frac{\cosh(c + dx)}{x^2} dx \\
&= -\frac{a^2 \cosh(c + dx)}{3x^3} - \frac{ab \cosh(c + dx)}{x^2} - \frac{b^2 \cosh(c + dx)}{x} + \frac{1}{3} (a^2 d) \int \frac{\sinh(c + dx)}{x^3} dx \\
&= -\frac{a^2 \cosh(c + dx)}{3x^3} - \frac{ab \cosh(c + dx)}{x^2} - \frac{b^2 \cosh(c + dx)}{x} - \frac{a^2 d \sinh(c + dx)}{6x^2} - \frac{abd \cosh(c + dx)}{6x} \\
&= -\frac{a^2 \cosh(c + dx)}{3x^3} - \frac{ab \cosh(c + dx)}{x^2} - \frac{b^2 \cosh(c + dx)}{x} - \frac{a^2 d^2 \cosh(c + dx)}{6x} + b^2 d \int \frac{\cosh(c + dx)}{x} dx \\
&= -\frac{a^2 \cosh(c + dx)}{3x^3} - \frac{ab \cosh(c + dx)}{x^2} - \frac{b^2 \cosh(c + dx)}{x} - \frac{a^2 d^2 \cosh(c + dx)}{6x} + ab d \int \frac{\cosh(c + dx)}{x} dx \\
&= -\frac{a^2 \cosh(c + dx)}{3x^3} - \frac{ab \cosh(c + dx)}{x^2} - \frac{b^2 \cosh(c + dx)}{x} - \frac{a^2 d^2 \cosh(c + dx)}{6x} + ab d \int \frac{\cosh(c + dx)}{x} dx
\end{aligned}$$

Mathematica [A] time = 0.49, size = 154, normalized size = 0.90

$$-dx^3 \text{Chi}(dx) (\sinh(c) (a^2 d^2 + 6b^2) + 6abd \cosh(c)) - dx^3 \text{Shi}(dx) (a^2 d^2 \cosh(c) + 6abd \sinh(c) + 6b^2 \cosh(c)) -$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)^2*Cosh[c + d*x])/x^4,x]
```

[Out] $-1/6*(2*a^2*Cosh[c + d*x] + 6*a*b*x*Cosh[c + d*x] + 6*b^2*x^2*Cosh[c + d*x] + a^2*d^2*x^2*Cosh[c + d*x] - d*x^3*CoshIntegral[d*x]*(6*a*b*d*Cosh[c] + (6*b^2 + a^2*d^2)*Sinh[c]) + a^2*d*x*Sinh[c + d*x] + 6*a*b*d*x^2*Sinh[c + d*x] - d*x^3*(6*b^2*Cosh[c] + a^2*d^2*Cosh[c] + 6*a*b*d*Sinh[c])*SinhIntegral[d*x])/x^3$

fricas [A] time = 0.47, size = 194, normalized size = 1.13

$$\frac{2(6abx + (a^2d^2 + 6b^2)x^2 + 2a^2)\cosh(dx + c) - ((a^2d^3 + 6abd^2 + 6b^2d)x^3\text{Ei}(dx) - (a^2d^3 - 6abd^2 + 6b^2d))}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x+a)^2*cosh(d*x+c)/x^4,x, algorithm="fricas")`

[Out] $-1/12*(2*(6*a*b*x + (a^2*d^2 + 6*b^2)*x^2 + 2*a^2)*\cosh(d*x + c) - ((a^2*d^3 + 6*a*b*d^2 + 6*b^2*d)*x^3*\text{Ei}(d*x) - (a^2*d^3 - 6*a*b*d^2 + 6*b^2*d)*x^3*\text{Ei}(-d*x))*\cosh(c) + 2*(6*a*b*d*x^2 + a^2*d*x)*\sinh(d*x + c) - ((a^2*d^3 + 6*a*b*d^2 + 6*b^2*d)*x^3*\text{Ei}(d*x) + (a^2*d^3 - 6*a*b*d^2 + 6*b^2*d)*x^3*\text{Ei}(-d*x))*\sinh(c))/x^3$

giac [A] time = 0.14, size = 285, normalized size = 1.66

$$\frac{a^2d^3x^3\text{Ei}(-dx)e^{(-c)} - a^2d^3x^3\text{Ei}(dx)e^c - 6abd^2x^3\text{Ei}(-dx)e^{(-c)} - 6abd^2x^3\text{Ei}(dx)e^c + 6b^2dx^3\text{Ei}(-dx)e^{(-c)} - 6b^2dx^3\text{Ei}(dx)e^c}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x+a)^2*cosh(d*x+c)/x^4,x, algorithm="giac")`

[Out] $-1/12*(a^2*d^3*x^3*\text{Ei}(-d*x)*e^{(-c)} - a^2*d^3*x^3*\text{Ei}(d*x)*e^c - 6*a*b*d^2*x^3*\text{Ei}(-d*x)*e^{(-c)} - 6*a*b*d^2*x^3*\text{Ei}(d*x)*e^c + 6*b^2*d*x^3*\text{Ei}(-d*x)*e^{(-c)} - 6*b^2*d*x^3*\text{Ei}(d*x)*e^c + a^2*d^2*x^2*e^{(d*x + c)} + a^2*d^2*x^2*e^{(-d*x - c)} + 6*a*b*d*x^2*e^{(d*x + c)} - 6*a*b*d*x^2*e^{(-d*x - c)} + a^2*d*x*e^{(d*x + c)} + 6*b^2*x^2*e^{(d*x + c)} - a^2*d*x*e^{(-d*x - c)} + 6*b^2*x^2*e^{(-d*x - c)} + 6*a*b*x*e^{(d*x + c)} + 6*a*b*x*e^{(-d*x - c)} + 2*a^2*e^{(d*x + c)} + 2*a^2*e^{(-d*x - c)})/x^3$

maple [A] time = 0.13, size = 287, normalized size = 1.67

$$\frac{d^2a^2e^{-dx-c}}{12x} + \frac{da^2e^{-dx-c}}{12x^2} - \frac{a^2e^{-dx-c}}{6x^3} - \frac{b^2e^{-dx-c}}{2x} + \frac{db^2e^{-c}\text{Ei}(1,dx)}{2} + \frac{dabe^{-dx-c}}{2x} - \frac{abe^{-dx-c}}{2x^2} - \frac{d^2abe^{-c}\text{Ei}(1,dx)}{2} + \frac{d^3a^2e^{-dx-c}}{12x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x+a)^2*cosh(d*x+c)/x^4,x)`

```
[Out] -1/12*d^2*a^2*exp(-d*x-c)/x+1/12*d*a^2*exp(-d*x-c)/x^2-1/6*a^2*exp(-d*x-c)/
x^3-1/2*b^2*exp(-d*x-c)/x+1/2*d*b^2*exp(-c)*Ei(1,d*x)+1/2*d*a*b*exp(-d*x-c)
/x-1/2*a*b*exp(-d*x-c)/x^2-1/2*d^2*a*b*exp(-c)*Ei(1,d*x)+1/12*d^3*a^2*exp(-
c)*Ei(1,d*x)-1/6*a^2/x^3*exp(d*x+c)-1/2*a*b/x^2*exp(d*x+c)-1/2*d*a*b/x*exp(
d*x+c)-1/2*d^2*a*b*exp(c)*Ei(1,-d*x)-1/12*d^2*a^2/x*exp(d*x+c)-1/12*d^3*a^2
*exp(c)*Ei(1,-d*x)-1/12*d*a^2/x^2*exp(d*x+c)-1/2*b^2/x*exp(d*x+c)-1/2*d*b^2
*exp(c)*Ei(1,-d*x)
```

maxima [A] time = 0.44, size = 117, normalized size = 0.68

$$\frac{1}{6} \left(a^2 d^2 e^{(-c)} \Gamma(-2, dx) - a^2 d^2 e^c \Gamma(-2, -dx) + 3 abde^{(-c)} \Gamma(-1, dx) + 3 abde^c \Gamma(-1, -dx) - 3 b^2 \text{Ei}(-dx) e^{(-c)} + 3 b^2 \text{Ei}(dx) e^c \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*cosh(d*x+c)/x^4,x, algorithm="maxima")
```

```
[Out] 1/6*(a^2*d^2*e^(-c)*gamma(-2, d*x) - a^2*d^2*e^c*gamma(-2, -d*x) + 3*a*b*d*
e^(-c)*gamma(-1, d*x) + 3*a*b*d*e^c*gamma(-1, -d*x) - 3*b^2*Ei(-d*x)*e^(-c)
+ 3*b^2*Ei(d*x)*e^c)*d - 1/3*(3*b^2*x^2 + 3*a*b*x + a^2)*cosh(d*x + c)/x^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c + dx) (a + bx)^2}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)*(a + b*x)^2)/x^4, x)
```

```
[Out] int((cosh(c + d*x)*(a + b*x)^2)/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx)^2 \cosh(c + dx)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*cosh(d*x+c)/x**4,x)
```

```
[Out] Integral((a + b*x)**2*cosh(c + d*x)/x**4, x)
```


$$3.17 \quad \int \frac{(a+bx)^2 \cosh(c+dx)}{x^5} dx$$

Optimal. Leaf size=248

$$\frac{1}{24} a^2 d^4 \cosh(c) \text{Chi}(dx) + \frac{1}{24} a^2 d^4 \sinh(c) \text{Shi}(dx) - \frac{a^2 d^3 \sinh(c+dx)}{24x} - \frac{a^2 d^2 \cosh(c+dx)}{24x^2} - \frac{a^2 \cosh(c+dx)}{4x^4} - \frac{a^2 d \sinh(c+dx)}{12x^3}$$

[Out] $\frac{1}{2} b^2 d^2 \text{Chi}(d*x) \cosh(c) + \frac{1}{24} a^2 d^4 \text{Chi}(d*x) \cosh(c) - \frac{1}{4} a^2 \cosh(d*x+c) / x^4 - \frac{2}{3} a*b \cosh(d*x+c) / x^3 - \frac{1}{2} b^2 \cosh(d*x+c) / x^2 - \frac{1}{24} a^2 d^2 \cosh(d*x+c) / x^2 - \frac{1}{3} a*b d^2 \cosh(d*x+c) / x + \frac{1}{3} a*b d^3 \cosh(c) \text{Shi}(d*x) + \frac{1}{3} a*b d^3 \text{Chi}(d*x) \sinh(c) + \frac{1}{2} b^2 d^2 \text{Shi}(d*x) \sinh(c) + \frac{1}{24} a^2 d^4 \text{Shi}(d*x) \sinh(c) - \frac{1}{12} a^2 d \sinh(d*x+c) / x^3 - \frac{1}{3} a*b d \sinh(d*x+c) / x^2 - \frac{1}{2} b^2 d \sinh(d*x+c) / x - \frac{1}{24} a^2 d^3 \sinh(d*x+c) / x$

Rubi [A] time = 0.52, antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {6742, 3297, 3303, 3298, 3301}

$$\frac{1}{24} a^2 d^4 \cosh(c) \text{Chi}(dx) + \frac{1}{24} a^2 d^4 \sinh(c) \text{Shi}(dx) - \frac{a^2 d^2 \cosh(c+dx)}{24x^2} - \frac{a^2 d^3 \sinh(c+dx)}{24x} - \frac{a^2 d \sinh(c+dx)}{12x^3} - \frac{a^2 d \cosh(c+dx)}{24x^4}$$

Antiderivative was successfully verified.

[In] Int[((a + b*x)^2*Cosh[c + d*x])/x^5,x]

[Out] $-(a^2 \text{Cosh}[c + d*x]) / (4*x^4) - (2*a*b \text{Cosh}[c + d*x]) / (3*x^3) - (b^2 \text{Cosh}[c + d*x]) / (2*x^2) - (a^2 d^2 \text{Cosh}[c + d*x]) / (24*x^2) - (a*b d^2 \text{Cosh}[c + d*x]) / (3*x) + (b^2 d^2 \text{Cosh}[c] \text{CoshIntegral}[d*x]) / 2 + (a^2 d^4 \text{Cosh}[c] \text{CoshIntegral}[d*x]) / 24 + (a*b d^3 \text{CoshIntegral}[d*x] \text{Sinh}[c]) / 3 - (a^2 d \text{Sinh}[c + d*x]) / (12*x^3) - (a*b d \text{Sinh}[c + d*x]) / (3*x^2) - (b^2 d \text{Sinh}[c + d*x]) / (2*x) - (a^2 d^3 \text{Sinh}[c + d*x]) / (24*x) + (a*b d^3 \text{Cosh}[c] \text{SinhIntegral}[d*x]) / 3 + (b^2 d^2 \text{Sinh}[c] \text{SinhIntegral}[d*x]) / 2 + (a^2 d^4 \text{Sinh}[c] \text{SinhIntegral}[d*x]) / 24$

Rule 3297

Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((c + d*x)^(m + 1)*Sin[e + f*x]) / (d*(m + 1)), x] - Dist[f / (d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]

Rule 3298

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_) / ((c_.) + (d_.)*(x_))], x_Symbol] := Simp[(I*SinhIntegral[(c*f*fz) / d + f*fz*x]) / d, x] /; FreeQ[{c, d, e, f}

, fz}, x] && EqQ[d*e - c*f*fz*I, 0]

Rule 3301

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]

Rule 3303

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]

Rule 6742

Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

Rubi steps

$$\begin{aligned}
 \int \frac{(a + bx)^2 \cosh(c + dx)}{x^5} dx &= \int \left(\frac{a^2 \cosh(c + dx)}{x^5} + \frac{2ab \cosh(c + dx)}{x^4} + \frac{b^2 \cosh(c + dx)}{x^3} \right) dx \\
 &= a^2 \int \frac{\cosh(c + dx)}{x^5} dx + (2ab) \int \frac{\cosh(c + dx)}{x^4} dx + b^2 \int \frac{\cosh(c + dx)}{x^3} dx \\
 &= -\frac{a^2 \cosh(c + dx)}{4x^4} - \frac{2ab \cosh(c + dx)}{3x^3} - \frac{b^2 \cosh(c + dx)}{2x^2} + \frac{1}{4} (a^2 d) \int \frac{\sinh(c + dx)}{x^4} dx \\
 &= -\frac{a^2 \cosh(c + dx)}{4x^4} - \frac{2ab \cosh(c + dx)}{3x^3} - \frac{b^2 \cosh(c + dx)}{2x^2} - \frac{a^2 d \sinh(c + dx)}{12x^3} - \frac{ab d^2 \cosh(c + dx)}{12x^2} \\
 &= -\frac{a^2 \cosh(c + dx)}{4x^4} - \frac{2ab \cosh(c + dx)}{3x^3} - \frac{b^2 \cosh(c + dx)}{2x^2} - \frac{a^2 d^2 \cosh(c + dx)}{24x^2} - \frac{ab d^2 \sinh(c + dx)}{12x} \\
 &= -\frac{a^2 \cosh(c + dx)}{4x^4} - \frac{2ab \cosh(c + dx)}{3x^3} - \frac{b^2 \cosh(c + dx)}{2x^2} - \frac{a^2 d^2 \cosh(c + dx)}{24x^2} - \frac{ab d^2 \sinh(c + dx)}{12x} \\
 &= -\frac{a^2 \cosh(c + dx)}{4x^4} - \frac{2ab \cosh(c + dx)}{3x^3} - \frac{b^2 \cosh(c + dx)}{2x^2} - \frac{a^2 d^2 \cosh(c + dx)}{24x^2} - \frac{ab d^2 \sinh(c + dx)}{12x}
 \end{aligned}$$

Mathematica [A] time = 0.59, size = 206, normalized size = 0.83

$$\frac{-d^2 x^4 \operatorname{Chi}(dx) \left(\cosh(c) (a^2 d^2 + 12 b^2) + 8 a b d \sinh(c) \right) - d^2 x^4 \operatorname{Shi}(dx) \left(a^2 d^2 \sinh(c) + 8 a b d \cosh(c) + 12 b^2 \sinh(c) \right)}{1}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x)^2*Cosh[c + d*x])/x^5,x]

[Out] $-1/24*(6*a^2*Cosh[c + d*x] + 16*a*b*x*Cosh[c + d*x] + 12*b^2*x^2*Cosh[c + d*x] + a^2*d^2*x^2*Cosh[c + d*x] + 8*a*b*d^2*x^3*Cosh[c + d*x] - d^2*x^4*\operatorname{CoshIntegral}[d*x]*((12*b^2 + a^2*d^2)*Cosh[c] + 8*a*b*d*\sinh[c]) + 2*a^2*d*x*\sinh[c + d*x] + 8*a*b*d*x^2*\sinh[c + d*x] + 12*b^2*d*x^3*\sinh[c + d*x] + a^2*d^3*x^3*\sinh[c + d*x] - d^2*x^4*(8*a*b*d*Cosh[c] + 12*b^2*\sinh[c] + a^2*d^2*\sinh[c]))*\operatorname{SinhIntegral}[d*x])/x^4$

fricas [A] time = 0.47, size = 231, normalized size = 0.93

$$\frac{2 \left(8 a b d^2 x^3 + 16 a b x + (a^2 d^2 + 12 b^2) x^2 + 6 a^2 \right) \cosh(dx + c) - \left((a^2 d^4 + 8 a b d^3 + 12 b^2 d^2) x^4 \operatorname{Ei}(dx) + (a^2 d^4 - 8 a b d^3 + 12 b^2 d^2) x^4 \operatorname{Ei}(-dx) \right) \cosh(c) + 2 \left((a^2 d^3 + 12 b^2 d^2) x^3 \sinh(dx + c) - (a^2 d^3 + 12 b^2 d^2) x^3 \sinh(-dx) \right) \cosh(c) - \left((a^2 d^4 + 8 a b d^3 + 12 b^2 d^2) x^4 \operatorname{Ei}(dx) - (a^2 d^4 - 8 a b d^3 + 12 b^2 d^2) x^4 \operatorname{Ei}(-dx) \right) \sinh(c)}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^2*cosh(d*x+c)/x^5,x, algorithm="fricas")

[Out] $-1/48*(2*(8*a*b*d^2*x^3 + 16*a*b*x + (a^2*d^2 + 12*b^2)*x^2 + 6*a^2)*\cosh(d*x + c) - ((a^2*d^4 + 8*a*b*d^3 + 12*b^2*d^2)*x^4*\operatorname{Ei}(d*x) + (a^2*d^4 - 8*a*b*d^3 + 12*b^2*d^2)*x^4*\operatorname{Ei}(-d*x))*\cosh(c) + 2*((8*a*b*d*x^2 + 2*a^2*d*x + (a^2*d^3 + 12*b^2*d)*x^3)*\sinh(d*x + c) - ((a^2*d^4 + 8*a*b*d^3 + 12*b^2*d^2)*x^4*\operatorname{Ei}(d*x) - (a^2*d^4 - 8*a*b*d^3 + 12*b^2*d^2)*x^4*\operatorname{Ei}(-d*x))*\sinh(c))/x^4$

giac [A] time = 0.14, size = 395, normalized size = 1.59

$$\frac{a^2 d^4 x^4 \operatorname{Ei}(-dx) e^{-c} + a^2 d^4 x^4 \operatorname{Ei}(dx) e^c - 8 a b d^3 x^4 \operatorname{Ei}(-dx) e^{-c} + 8 a b d^3 x^4 \operatorname{Ei}(dx) e^c + 12 b^2 d^2 x^4 \operatorname{Ei}(-dx) e^{-c} + 12 b^2 d^2 x^4 \operatorname{Ei}(dx) e^c - 8 a b d^2 x^3 e^{-c} + 8 a b d^2 x^3 e^c - 12 b^2 d x^2 e^{-c} + 12 b^2 d x^2 e^c - 8 a^2 d x e^{-c} + 8 a^2 d x e^c - 6 a^2 e^{-c} + 6 a^2 e^c}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^2*cosh(d*x+c)/x^5,x, algorithm="giac")

[Out] $1/48*(a^2*d^4*x^4*\operatorname{Ei}(-d*x)*e^{-c} + a^2*d^4*x^4*\operatorname{Ei}(d*x)*e^c - 8*a*b*d^3*x^4*\operatorname{Ei}(-d*x)*e^{-c} + 8*a*b*d^3*x^4*\operatorname{Ei}(d*x)*e^c + 12*b^2*d^2*x^4*\operatorname{Ei}(-d*x)*e^{-c} + 12*b^2*d^2*x^4*\operatorname{Ei}(d*x)*e^c - a^2*d^3*x^3*e^{(d*x + c)} + a^2*d^3*x^3*e^{(-d*x - c)} - 8*a*b*d^2*x^3*e^{(d*x + c)} - 8*a*b*d^2*x^3*e^{(-d*x - c)} - a^2*d^2*x^2*e^{(d*x + c)} - 12*b^2*d*x^3*e^{(d*x + c)} - a^2*d^2*x^2*e^{(-d*x - c)} + 12*b^2*d*x^3*e^{(-d*x - c)} - 8*a*b*d*x^2*e^{(d*x + c)} + 8*a*b*d*x^2*e^{(-d*x - c)} - 6*a^2*d*x*e^{(d*x + c)} - 6*a^2*d*x*e^{(-d*x - c)} + 6*a^2*e^{(d*x + c)} + 6*a^2*e^{(-d*x - c)})$

$$c) - 2*a^2*d*x*e^{(d*x + c)} - 12*b^2*x^2*e^{(d*x + c)} + 2*a^2*d*x*e^{(-d*x - c)} - 12*b^2*x^2*e^{(-d*x - c)} - 16*a*b*x*e^{(d*x + c)} - 16*a*b*x*e^{(-d*x - c)} - 6*a^2*e^{(d*x + c)} - 6*a^2*e^{(-d*x - c)})/x^4$$

maple [A] time = 0.14, size = 396, normalized size = 1.60

$$-\frac{a^2e^{-dx-c}}{8x^4} + \frac{d^3a^2e^{-dx-c}}{48x} - \frac{d^2a^2e^{-dx-c}}{48x^2} + \frac{da^2e^{-dx-c}}{24x^3} + \frac{db^2e^{-dx-c}}{4x} - \frac{b^2e^{-dx-c}}{4x^2} - \frac{d^2b^2e^{-c} \operatorname{Ei}(1, dx)}{4} - \frac{d^2abe^{-dx-c}}{6x} + \frac{dabe^{-dx-c}}{6x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x+a)^2*cosh(d*x+c)/x^5,x)

[Out] $-1/8*a^2*\exp(-d*x-c)/x^4 + 1/48*d^3*a^2*\exp(-d*x-c)/x - 1/48*d^2*a^2*\exp(-d*x-c)/x^2 + 1/24*d*a^2*\exp(-d*x-c)/x^3 + 1/4*d*b^2*\exp(-d*x-c)/x - 1/4*b^2*\exp(-d*x-c)/x^2 - 1/4*d^2*b^2*\exp(-c)*\operatorname{Ei}(1, d*x) - 1/6*d^2*a*b*\exp(-d*x-c)/x + 1/6*d*a*b*\exp(-d*x-c)/x^2 - 1/3*a*b*\exp(-d*x-c)/x^3 + 1/6*d^3*a*b*\exp(-c)*\operatorname{Ei}(1, d*x) - 1/48*d^4*a^2*\exp(-c)*\operatorname{Ei}(1, d*x) - 1/8*a^2/x^4*\exp(d*x+c) - 1/24*d*a^2/x^3*\exp(d*x+c) - 1/4*8*d^2*a^2/x^2*\exp(d*x+c) - 1/48*d^3*a^2/x*\exp(d*x+c) - 1/4*b^2/x^2*\exp(d*x+c) - 1/4*d*b^2/x*\exp(d*x+c) - 1/4*d^2*b^2*\exp(c)*\operatorname{Ei}(1, -d*x) - 1/48*d^4*a^2*\exp(c)*\operatorname{Ei}(1, -d*x) - 1/3*a*b/x^3*\exp(d*x+c) - 1/6*d*a*b/x^2*\exp(d*x+c) - 1/6*d^2*a*b/x*\exp(d*x+c) - 1/6*d^3*a*b*\exp(c)*\operatorname{Ei}(1, -d*x)$

maxima [A] time = 0.44, size = 128, normalized size = 0.52

$$\frac{1}{24} (3a^2d^3e^{(-c)}\Gamma(-3, dx) + 3a^2d^3e^c\Gamma(-3, -dx) + 8abd^2e^{(-c)}\Gamma(-2, dx) - 8abd^2e^c\Gamma(-2, -dx) + 6b^2de^{(-c)}\Gamma(-1, dx))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^2*cosh(d*x+c)/x^5,x, algorithm="maxima")

[Out] $1/24*(3*a^2*d^3*e^{(-c)}*\operatorname{gamma}(-3, d*x) + 3*a^2*d^3*e^c*\operatorname{gamma}(-3, -d*x) + 8*a*b*d^2*e^{(-c)}*\operatorname{gamma}(-2, d*x) - 8*a*b*d^2*e^c*\operatorname{gamma}(-2, -d*x) + 6*b^2*d*e^{(-c)}*\operatorname{gamma}(-1, d*x) + 6*b^2*d*e^c*\operatorname{gamma}(-1, -d*x))*d - 1/12*(6*b^2*x^2 + 8*a*b*x + 3*a^2)*\cosh(d*x + c)/x^4$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(c + dx) (a + bx)^2}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cosh(c + d*x)*(a + b*x)^2)/x^5,x)

[Out] int((cosh(c + d*x)*(a + b*x)^2)/x^5, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx)^2 \cosh(c + dx)}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)**2*cosh(d*x+c)/x**5,x)

[Out] Integral((a + b*x)**2*cosh(c + d*x)/x**5, x)

$$3.18 \quad \int \frac{x^4 \cosh(c+dx)}{a+bx} dx$$

Optimal. Leaf size=219

$$\frac{a^4 \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{b^5} + \frac{a^4 \sinh\left(c - \frac{ad}{b}\right) \text{Shi}\left(xd + \frac{ad}{b}\right)}{b^5} - \frac{a^3 \sinh(c+dx)}{b^4 d} - \frac{a^2 \cosh(c+dx)}{b^3 d^2} + \frac{a^2 x \sinh(c+dx)}{b^3 d}$$

[Out] $a^4 \text{Chi}(a*d/b+d*x) * \cosh(-c+a*d/b) / b^5 - 6 * \cosh(d*x+c) / b / d^4 - a^2 * \cosh(d*x+c) / b^3 / d^2 + 2 * a * x * \cosh(d*x+c) / b^2 / d^2 - 3 * x^2 * \cosh(d*x+c) / b / d^2 - a^4 * \text{Shi}(a*d/b+d*x) * \sinh(-c+a*d/b) / b^5 - 2 * a * \sinh(d*x+c) / b^2 / d^3 - a^3 * \sinh(d*x+c) / b^4 / d + 6 * x * \sinh(d*x+c) / b / d^3 + a^2 * x * \sinh(d*x+c) / b^3 / d - a * x^2 * \sinh(d*x+c) / b^2 / d + x^3 * \sinh(d*x+c) / b / d$

Rubi [A] time = 0.48, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {6742, 2637, 3296, 2638, 3303, 3298, 3301}

$$\frac{a^4 \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{b^5} - \frac{a^2 \cosh(c+dx)}{b^3 d^2} + \frac{a^4 \sinh\left(c - \frac{ad}{b}\right) \text{Shi}\left(xd + \frac{ad}{b}\right)}{b^5} - \frac{a^3 \sinh(c+dx)}{b^4 d} + \frac{a^2 x \sinh(c+dx)}{b^3 d}$$

Antiderivative was successfully verified.

[In] Int[(x^4*Cosh[c + d*x])/(a + b*x),x]

[Out] $(-6 * \text{Cosh}[c + d*x]) / (b*d^4) - (a^2 * \text{Cosh}[c + d*x]) / (b^3*d^2) + (2*a*x * \text{Cosh}[c + d*x]) / (b^2*d^2) - (3*x^2 * \text{Cosh}[c + d*x]) / (b*d^2) + (a^4 * \text{Cosh}[c - (a*d)/b] * \text{CoshIntegral}[(a*d)/b + d*x]) / b^5 - (2*a * \text{Sinh}[c + d*x]) / (b^2*d^3) - (a^3 * \text{Sinh}[c + d*x]) / (b^4*d) + (6*x * \text{Sinh}[c + d*x]) / (b*d^3) + (a^2 * x * \text{Sinh}[c + d*x]) / (b^3*d) - (a * x^2 * \text{Sinh}[c + d*x]) / (b^2*d) + (x^3 * \text{Sinh}[c + d*x]) / (b*d) + (a^4 * \text{Sinh}[c - (a*d)/b] * \text{SinhIntegral}[(a*d)/b + d*x]) / b^5$

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 2638

Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3296

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[((c + d*x)^m * Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m-1) * Cos[e + f*x], x], x]

$e + f*x]$, $x]$, $x]$ /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 3298

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]

Rule 3301

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]

Rule 3303

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]

Rule 6742

Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

Rubi steps

$$\begin{aligned}
\int \frac{x^4 \cosh(c + dx)}{a + bx} dx &= \int \left(-\frac{a^3 \cosh(c + dx)}{b^4} + \frac{a^2 x \cosh(c + dx)}{b^3} - \frac{ax^2 \cosh(c + dx)}{b^2} + \frac{x^3 \cosh(c + dx)}{b} + \frac{a^4 \cosh(c + dx)}{b^4} \right) dx \\
&= -\frac{a^3 \int \cosh(c + dx) dx}{b^4} + \frac{a^4 \int \frac{\cosh(c+dx)}{a+bx} dx}{b^4} + \frac{a^2 \int x \cosh(c + dx) dx}{b^3} - \frac{a \int x^2 \cosh(c + dx) dx}{b^2} + \frac{a^4 \int \cosh(c + dx) dx}{b^4} \\
&= -\frac{a^3 \sinh(c + dx)}{b^4 d} + \frac{a^2 x \sinh(c + dx)}{b^3 d} - \frac{ax^2 \sinh(c + dx)}{b^2 d} + \frac{x^3 \sinh(c + dx)}{bd} - \frac{a^2 \int \sinh(c + dx) dx}{b^4} \\
&= -\frac{a^2 \cosh(c + dx)}{b^3 d^2} + \frac{2ax \cosh(c + dx)}{b^2 d^2} - \frac{3x^2 \cosh(c + dx)}{bd^2} + \frac{a^4 \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(\frac{ad}{b} + x\right)}{b^5} \\
&= -\frac{a^2 \cosh(c + dx)}{b^3 d^2} + \frac{2ax \cosh(c + dx)}{b^2 d^2} - \frac{3x^2 \cosh(c + dx)}{bd^2} + \frac{a^4 \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(\frac{ad}{b} + x\right)}{b^5} \\
&= -\frac{6 \cosh(c + dx)}{bd^4} - \frac{a^2 \cosh(c + dx)}{b^3 d^2} + \frac{2ax \cosh(c + dx)}{b^2 d^2} - \frac{3x^2 \cosh(c + dx)}{bd^2} + \frac{a^4 \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(\frac{ad}{b} + x\right)}{b^5}
\end{aligned}$$

Mathematica [A] time = 0.65, size = 159, normalized size = 0.73

$$\frac{a^4 d^4 \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(d\left(\frac{a}{b} + x\right)\right) + a^4 d^4 \sinh\left(c - \frac{ad}{b}\right) \text{Shi}\left(d\left(\frac{a}{b} + x\right)\right) - b\left(b\left(a^2 d^2 - 2abd^2 x + 3b^2\left(d^2 x^2 + 2\right)\right) \cosh(c + dx) + a^4 d^4 \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(\frac{ad}{b} + x\right) - b^5 d^4}{b^5 d^4}$$

Antiderivative was successfully verified.

[In] Integrate[(x^4*Cosh[c + d*x])/(a + b*x),x]

[Out] (a^4*d^4*Cosh[c - (a*d)/b]*CoshIntegral[d*(a/b + x)] - b*(b*(a^2*d^2 - 2*a*b*d^2*x + 3*b^2*(2 + d^2*x^2))*Cosh[c + d*x] + d*(a^3*d^2 - a^2*b*d^2*x + a*b^2*(2 + d^2*x^2) - b^3*x*(6 + d^2*x^2))*Sinh[c + d*x]) + a^4*d^4*Sinh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)]/(b^5*d^4)

fricas [A] time = 0.45, size = 236, normalized size = 1.08

$$\frac{2\left(3b^4d^2x^2 - 2ab^3d^2x + a^2b^2d^2 + 6b^4\right) \cosh(dx + c) - \left(a^4d^4\text{Ei}\left(\frac{bdx+ad}{b}\right) + a^4d^4\text{Ei}\left(-\frac{bdx+ad}{b}\right)\right) \cosh\left(-\frac{bc-ad}{b}\right) - 2a^4d^4\text{Ei}\left(\frac{bdx+ad}{b}\right) \cosh\left(-\frac{bc-ad}{b}\right)}{b^5d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*cosh(d*x+c)/(b*x+a),x, algorithm="fricas")

[Out] -1/2*(2*(3*b^4*d^2*x^2 - 2*a*b^3*d^2*x + a^2*b^2*d^2 + 6*b^4)*cosh(d*x + c) - (a^4*d^4*Ei((b*d*x + a*d)/b) + a^4*d^4*Ei(-(b*d*x + a*d)/b))*cosh(-(b*c

$- a*d)/b) - 2*(b^4*d^3*x^3 - a*b^3*d^3*x^2 - a^3*b*d^3 - 2*a*b^3*d + (a^2*b^2*d^3 + 6*b^4*d)*x)*\sinh(d*x + c) + (a^4*d^4*Ei((b*d*x + a*d)/b) - a^4*d^4*Ei(-(b*d*x + a*d)/b))*\sinh(-(b*c - a*d)/b))/(b^5*d^4)$

giac [A] time = 0.12, size = 407, normalized size = 1.86

$$b^4 d^3 x^3 e^{(dx+c)} - b^4 d^3 x^3 e^{(-dx-c)} - ab^3 d^3 x^2 e^{(dx+c)} + ab^3 d^3 x^2 e^{(-dx-c)} + a^4 d^4 Ei\left(\frac{bdx+ad}{b}\right) e^{\left(c-\frac{ad}{b}\right)} + a^4 d^4 Ei\left(-\frac{bdx+ad}{b}\right) e^{\left(-\frac{ad-c}{b}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*cosh(d*x+c)/(b*x+a),x, algorithm="giac")

[Out] $1/2*(b^4*d^3*x^3*e^{(d*x + c)} - b^4*d^3*x^3*e^{(-d*x - c)} - a*b^3*d^3*x^2*e^{(d*x + c)} + a*b^3*d^3*x^2*e^{(-d*x - c)} + a^4*d^4*Ei((b*d*x + a*d)/b)*e^{(c - a*d/b)} + a^4*d^4*Ei(-(b*d*x + a*d)/b)*e^{(-c + a*d/b)} + a^2*b^2*d^3*x*e^{(d*x + c)} - 3*b^4*d^2*x^2*e^{(d*x + c)} - a^2*b^2*d^3*x*e^{(-d*x - c)} - 3*b^4*d^2*x^2*e^{(-d*x - c)} - a^3*b*d^3*e^{(d*x + c)} + 2*a*b^3*d^2*x*e^{(d*x + c)} + a^3*b*d^3*e^{(-d*x - c)} + 2*a*b^3*d^2*x*e^{(-d*x - c)} - a^2*b^2*d^2*e^{(d*x + c)} + 6*b^4*d*x*e^{(d*x + c)} - a^2*b^2*d^2*e^{(-d*x - c)} - 6*b^4*d*x*e^{(-d*x - c)} - 2*a*b^3*d*e^{(d*x + c)} + 2*a*b^3*d*e^{(-d*x - c)} - 6*b^4*e^{(d*x + c)} - 6*b^4*e^{(-d*x - c)})/(b^5*d^4)$

maple [A] time = 0.19, size = 442, normalized size = 2.02

$$\frac{3e^{-dx-c}}{d^4b} - \frac{e^{\frac{da-cb}{b}} Ei\left(1, dx + c + \frac{da-cb}{b}\right) a^4}{2b^5} - \frac{3e^{-dx-c}x^2}{2d^2b} - \frac{e^{-dx-c}a^2}{2d^2b^3} - \frac{e^{-dx-c}x^3}{2db} + \frac{e^{-dx-c}a^3}{2db^4} - \frac{3e^{-dx-c}x}{d^3b} + \frac{e^{-dx-c}a}{d^3b^2} + \frac{e^{-dx-c}}{d^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4*cosh(d*x+c)/(b*x+a),x)

[Out] $-3/d^4*\exp(-d*x-c)/b-1/2/b^5*\exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)*a^4-3/2/d^2*\exp(-d*x-c)/b*x^2-1/2/d^2*\exp(-d*x-c)/b^3*a^2-1/2/d*\exp(-d*x-c)/b*x^3+1/2/d*\exp(-d*x-c)/b^4*a^3-3/d^3*\exp(-d*x-c)/b*x+1/d^3*\exp(-d*x-c)/b^2*a+1/d^2*\exp(-d*x-c)/b^2*a*x+1/2/d*\exp(-d*x-c)/b^2*a*x^2-1/2/d*\exp(-d*x-c)/b^3*a^2*x+1/2/d/b*\exp(d*x+c)*x^3-3/2/d^2/b*\exp(d*x+c)*x^2+3/d^3/b*\exp(d*x+c)*x-1/2/d/b^4*a^3*\exp(d*x+c)-1/d^3/b^2*a*\exp(d*x+c)-1/2/d/b^2*a*\exp(d*x+c)*x^2+1/d^2/b^2*a*\exp(d*x+c)*x+1/2/d/b^3*a^2*\exp(d*x+c)*x-1/2/b^5*\exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)*a^4-1/2/d^2/b^3*a^2*\exp(d*x+c)-3/d^4/b*\exp(d*x+c)$

maxima [A] time = 0.41, size = 437, normalized size = 2.00

$$-\frac{1}{24}d \left(\frac{12a^4 \left(\frac{e^{(-c+\frac{ad}{b})} E_1\left(\frac{(bx+a)d}{b}\right)}{b} + \frac{e^{(c-\frac{ad}{b})} E_1\left(-\frac{(bx+a)d}{b}\right)}{b} \right)}{b^4 d} - \frac{12a^3 \left(\frac{(dx e^c - e^c) e^{(dx)}}{d^2} + \frac{(dx+1) e^{(-dx-c)}}{d^2} \right)}{b^4} + \frac{6a^2 \left(\frac{(d^2 x^2 e^c - 2 dx e^c + 2 e^c) e^{(dx)}}{d^3} \right)}{b^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*cosh(d*x+c)/(b*x+a),x, algorithm="maxima")

[Out] $-1/24*d*(12*a^4*(e^{(-c+a*d/b)}*exp_integral_e(1,(b*x+a)*d/b)/b + e^{(c-a*d/b)}*exp_integral_e(1,-(b*x+a)*d/b)/b)/(b^4*d) - 12*a^3*((d*x*e^c - e^c)*e^{(d*x)}/d^2 + (d*x+1)*e^{(-d*x-c)}/d^2)/b^4 + 6*a^2*((d^2*x^2*e^c - 2*d*x*e^c + 2*e^c)*e^{(d*x)}/d^3 + (d^2*x^2 + 2*d*x + 2)*e^{(-d*x-c)}/d^3)/b^3 - 4*a*((d^3*x^3*e^c - 3*d^2*x^2*e^c + 6*d*x*e^c - 6*e^c)*e^{(d*x)}/d^4 + (d^3*x^3 + 3*d^2*x^2 + 6*d*x + 6)*e^{(-d*x-c)}/d^4)/b^2 + 3*((d^4*x^4*e^c - 4*d^3*x^3*e^c + 12*d^2*x^2*e^c - 24*d*x*e^c + 24*e^c)*e^{(d*x)}/d^5 + (d^4*x^4 + 4*d^3*x^3 + 12*d^2*x^2 + 24*d*x + 24)*e^{(-d*x-c)}/d^5)/b + 24*a^4*cosh(d*x+c)*log(b*x+a)/(b^5*d) + 1/12*(12*a^4*log(b*x+a)/b^5 + (3*b^3*x^4 - 4*a*b^2*x^3 + 6*a^2*b*x^2 - 12*a^3*x)/b^4)*cosh(d*x+c)$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4 \cosh(c + dx)}{a + bx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4*cosh(c+d*x))/(a+b*x),x)

[Out] int((x^4*cosh(c+d*x))/(a+b*x), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 \cosh(c + dx)}{a + bx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4*cosh(d*x+c)/(b*x+a),x)

[Out] Integral(x**4*cosh(c+d*x)/(a+b*x), x)

$$3.19 \quad \int \frac{x^3 \cosh(c+dx)}{a+bx} dx$$

Optimal. Leaf size=150

$$\frac{a^3 \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{b^4} - \frac{a^3 \sinh\left(c - \frac{ad}{b}\right) \text{Shi}\left(xd + \frac{ad}{b}\right)}{b^4} + \frac{a^2 \sinh(c+dx)}{b^3 d} + \frac{a \cosh(c+dx)}{b^2 d^2} - \frac{ax \sinh(c+dx)}{b^2 d}$$

[Out] $-a^3 \text{Chi}(a*d/b+d*x) * \cosh(-c+a*d/b) / b^4 + a * \cosh(d*x+c) / b^2 / d^2 - 2*x * \cosh(d*x+c) / b / d^2 + a^3 * \text{Shi}(a*d/b+d*x) * \sinh(-c+a*d/b) / b^4 + 2 * \sinh(d*x+c) / b / d^3 + a^2 * \sinh(d*x+c) / b^3 / d - a*x * \sinh(d*x+c) / b^2 / d + x^2 * \sinh(d*x+c) / b / d$

Rubi [A] time = 0.33, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {6742, 2637, 3296, 2638, 3303, 3298, 3301}

$$\frac{a^3 \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{b^4} - \frac{a^3 \sinh\left(c - \frac{ad}{b}\right) \text{Shi}\left(xd + \frac{ad}{b}\right)}{b^4} + \frac{a^2 \sinh(c+dx)}{b^3 d} + \frac{a \cosh(c+dx)}{b^2 d^2} - \frac{ax \sinh(c+dx)}{b^2 d}$$

Antiderivative was successfully verified.

[In] Int[(x^3*Cosh[c + d*x])/(a + b*x), x]

[Out] $(a * \text{Cosh}[c + d*x]) / (b^2 * d^2) - (2 * x * \text{Cosh}[c + d*x]) / (b * d^2) - (a^3 * \text{Cosh}[c - (a*d)/b] * \text{CoshIntegral}[(a*d)/b + d*x]) / b^4 + (2 * \text{Sinh}[c + d*x]) / (b * d^3) + (a^2 * \text{Sinh}[c + d*x]) / (b^3 * d) - (a * x * \text{Sinh}[c + d*x]) / (b^2 * d) + (x^2 * \text{Sinh}[c + d*x]) / (b * d) - (a^3 * \text{Sinh}[c - (a*d)/b] * \text{SinhIntegral}[(a*d)/b + d*x]) / b^4$

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 2638

Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3296

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[((c + d*x)^m * Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m-1) * Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x]
&& EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x]
&& EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d],
Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

Rule 6742

```
Int[u_, x_Symbol]
:> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]
```

Rubi steps

$$\begin{aligned} \int \frac{x^3 \cosh(c + dx)}{a + bx} dx &= \int \left(\frac{a^2 \cosh(c + dx)}{b^3} - \frac{ax \cosh(c + dx)}{b^2} + \frac{x^2 \cosh(c + dx)}{b} - \frac{a^3 \cosh(c + dx)}{b^3(a + bx)} \right) dx \\ &= \frac{a^2 \int \cosh(c + dx) dx}{b^3} - \frac{a^3 \int \frac{\cosh(c+dx)}{a+bx} dx}{b^3} - \frac{a \int x \cosh(c + dx) dx}{b^2} + \frac{\int x^2 \cosh(c + dx) dx}{b} \\ &= \frac{a^2 \sinh(c + dx)}{b^3 d} - \frac{ax \sinh(c + dx)}{b^2 d} + \frac{x^2 \sinh(c + dx)}{bd} + \frac{a \int \sinh(c + dx) dx}{b^2 d} - \frac{2 \int x \sinh(c + dx) dx}{b^2 d} \\ &= \frac{a \cosh(c + dx)}{b^2 d^2} - \frac{2x \cosh(c + dx)}{bd^2} - \frac{a^3 \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(\frac{ad}{b} + dx\right)}{b^4} + \frac{a^2 \sinh(c + dx)}{b^3 d} \\ &= \frac{a \cosh(c + dx)}{b^2 d^2} - \frac{2x \cosh(c + dx)}{bd^2} - \frac{a^3 \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(\frac{ad}{b} + dx\right)}{b^4} + \frac{2 \sinh(c + dx)}{bd^3} + \end{aligned}$$

Mathematica [A] time = 0.45, size = 118, normalized size = 0.79

$$\frac{-a^3 d^3 \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(d\left(\frac{a}{b} + x\right)\right) - a^3 d^3 \sinh\left(c - \frac{ad}{b}\right) \text{Shi}\left(d\left(\frac{a}{b} + x\right)\right) + b\left(\left(a^2 d^2 - abd^2 x + b^2\left(d^2 x^2 + 2\right)\right) \sin\right)}{b^4 d^3}$$

Antiderivative was successfully verified.

[In] Integrate[(x^3*Cosh[c + d*x])/(a + b*x),x]

[Out] $(-a^3 d^3 \text{Cosh}[c - (a*d)/b] * \text{CoshIntegral}[d*(a/b + x)]) + b*(b*d*(a - 2*b*x) * \text{Cosh}[c + d*x] + (a^2*d^2 - a*b*d^2*x + b^2*(2 + d^2*x^2)) * \text{Sinh}[c + d*x]) - a^3*d^3*\text{Sinh}[c - (a*d)/b] * \text{SinhIntegral}[d*(a/b + x)]/(b^4*d^3)$

fricas [A] time = 0.91, size = 190, normalized size = 1.27

$$\frac{2(2b^3 dx - ab^2 d) \cosh(dx + c) + \left(a^3 d^3 \text{Ei}\left(\frac{bdx+ad}{b}\right) + a^3 d^3 \text{Ei}\left(-\frac{bdx+ad}{b}\right)\right) \cosh\left(-\frac{bc-ad}{b}\right) - 2(b^3 d^2 x^2 - ab^2 d^2 x + a^2 d^2)}{2b^4 d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*cosh(d*x+c)/(b*x+a),x, algorithm="fricas")

[Out] $-1/2*(2*(2*b^3*d*x - a*b^2*d)*\cosh(d*x + c) + (a^3*d^3*\text{Ei}((b*d*x + a*d)/b) + a^3*d^3*\text{Ei}(-(b*d*x + a*d)/b))*\cosh(-(b*c - a*d)/b) - 2*(b^3*d^2*x^2 - a*b^2*d^2*x + a^2*b*d^2 + 2*b^3)*\sinh(d*x + c) - (a^3*d^3*\text{Ei}((b*d*x + a*d)/b) - a^3*d^3*\text{Ei}(-(b*d*x + a*d)/b))*\sinh(-(b*c - a*d)/b))/(b^4*d^3)$

giac [A] time = 0.15, size = 256, normalized size = 1.71

$$\frac{b^3 d^2 x^2 e^{(dx+c)} - b^3 d^2 x^2 e^{(-dx-c)} - a^3 d^3 \text{Ei}\left(\frac{bdx+ad}{b}\right) e^{\left(\frac{c-ad}{b}\right)} - a^3 d^3 \text{Ei}\left(-\frac{bdx+ad}{b}\right) e^{\left(-c+\frac{ad}{b}\right)} - ab^2 d^2 x e^{(dx+c)} + ab^2 d^2 x e^{(-dx-c)}}{2b^4 d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*cosh(d*x+c)/(b*x+a),x, algorithm="giac")

[Out] $1/2*(b^3*d^2*x^2*e^{(d*x + c)} - b^3*d^2*x^2*e^{(-d*x - c)} - a^3*d^3*\text{Ei}((b*d*x + a*d)/b)*e^{(c - a*d/b)} - a^3*d^3*\text{Ei}(-(b*d*x + a*d)/b)*e^{(-c + a*d/b)} - a*b^2*d^2*x*e^{(d*x + c)} + a*b^2*d^2*x*e^{(-d*x - c)} + a^2*b*d^2*e^{(d*x + c)} - 2*b^3*d*x*e^{(d*x + c)} - a^2*b*d^2*e^{(-d*x - c)} - 2*b^3*d*x*e^{(-d*x - c)} + a*b^2*d*e^{(d*x + c)} + a*b^2*d*e^{(-d*x - c)} + 2*b^3*e^{(d*x + c)} - 2*b^3*e^{(-d*x - c)})/(b^4*d^3)$

maple [A] time = 0.14, size = 292, normalized size = 1.95

$$\frac{-\frac{e^{-dx-c}x^2}{2db} + \frac{e^{-dx-c}ax}{2db^2} - \frac{e^{-dx-c}a^2}{2db^3} - \frac{e^{-dx-c}x}{d^2b} + \frac{e^{-dx-c}a}{2d^2b^2} - \frac{e^{-dx-c}}{d^3b} + \frac{e^{\frac{da-cb}{b}} \text{Ei}\left(1, dx + c + \frac{da-cb}{b}\right) a^3}{2b^4} + \frac{ae^{dx+c}}{2d^2b^2} + \frac{e^{dx+c}x^2}{2db} - \frac{e^{dx+c}x}{d^2b} + \frac{e^{dx+c}a}{2db} - \frac{e^{dx+c}}{d^2b}}{2b^4 d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*cosh(d*x+c)/(b*x+a),x)`

[Out]
$$-1/2/d*\exp(-d*x-c)/b*x^2+1/2/d*\exp(-d*x-c)/b^2*a*x-1/2/d*\exp(-d*x-c)/b^3*a^2-1/d^2*\exp(-d*x-c)/b*x+1/2/d^2*\exp(-d*x-c)/b^2*a-1/d^3*\exp(-d*x-c)/b+1/2/b^4*\exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)*a^3+1/2/d^2/b^2*a*\exp(d*x+c)+1/2/d/b*\exp(d*x+c)*x^2-1/d^2/b*\exp(d*x+c)*x+1/2/d/b^3*a^2*\exp(d*x+c)+1/d^3/b*\exp(d*x+c)-1/2/d/b^2*a*\exp(d*x+c)*x+1/2/b^4*\exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)*a^3$$

maxima [B] time = 0.41, size = 328, normalized size = 2.19

$$\frac{1}{12}d \left(\frac{6a^3 \left(\frac{e^{(-c+\frac{ad}{b})} E_1\left(\frac{(bx+a)d}{b}\right)}{b} + \frac{e^{(c-\frac{ad}{b})} E_1\left(-\frac{(bx+a)d}{b}\right)}{b} \right)}{b^3d} - \frac{6a^2 \left(\frac{(dxe^c - e^c)e^{(dx)}}{d^2} + \frac{(dx+1)e^{(-dx-c)}}{d^2} \right)}{b^3} + \frac{3a \left(\frac{(d^2x^2e^c - 2dxe^c + 2e^c)e^{(dx)}}{d^3} + \frac{(d^2x^2e^{-c} - 2dxe^{-c} + 2e^{-c})e^{(-dx-c)}}{d^3} \right)}{b^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*cosh(d*x+c)/(b*x+a),x, algorithm="maxima")`

[Out]
$$\frac{1}{12}d*(6*a^3*(e^{(-c+a*d/b)}*\exp_integral_e(1,(b*x+a)*d/b)/b + e^{(c-a*d/b)}*\exp_integral_e(1,-(b*x+a)*d/b)/b)/(b^3*d) - 6*a^2*((d*x*e^c - e^c)*e^{(d*x)}/d^2 + (d*x+1)*e^{(-d*x-c)}/d^2)/b^3 + 3*a*((d^2*x^2*e^c - 2*d*x*e^c + 2*e^c)*e^{(d*x)}/d^3 + (d^2*x^2 + 2*d*x + 2)*e^{(-d*x-c)}/d^3)/b^2 - 2*((d^3*x^3*e^c - 3*d^2*x^2*e^c + 6*d*x*e^c - 6*e^c)*e^{(d*x)}/d^4 + (d^3*x^3 + 3*d^2*x^2 + 6*d*x + 6)*e^{(-d*x-c)}/d^4)/b + 12*a^3*cosh(d*x+c)*log(b*x+a)/(b^4*d) - 1/6*(6*a^3*log(b*x+a)/b^4 - (2*b^2*x^3 - 3*a*b*x^2 + 6*a^2*x)/b^3)*cosh(d*x+c)$$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3 \cosh(c + dx)}{a + bx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^3*cosh(c+d*x))/(a+b*x),x)`

[Out] `int((x^3*cosh(c+d*x))/(a+b*x),x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \cosh(c + dx)}{a + bx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cosh(d*x+c)/(b*x+a),x)
```

```
[Out] Integral(x**3*cosh(c + d*x)/(a + b*x), x)
```

$$3.20 \quad \int \frac{x^2 \cosh(c+dx)}{a+bx} dx$$

Optimal. Leaf size=100

$$\frac{a^2 \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{b^3} + \frac{a^2 \sinh\left(c - \frac{ad}{b}\right) \text{Shi}\left(xd + \frac{ad}{b}\right)}{b^3} - \frac{a \sinh(c+dx)}{b^2 d} - \frac{\cosh(c+dx)}{bd^2} + \frac{x \sinh(c+dx)}{bd}$$

[Out] $a^2 \text{Chi}(a*d/b+d*x) * \cosh(-c+a*d/b) / b^3 - \cosh(d*x+c) / b / d^2 - a^2 \text{Shi}(a*d/b+d*x) * \sinh(-c+a*d/b) / b^3 - a * \sinh(d*x+c) / b^2 / d + x * \sinh(d*x+c) / b / d$

Rubi [A] time = 0.26, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {6742, 2637, 3296, 2638, 3303, 3298, 3301}

$$\frac{a^2 \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{b^3} + \frac{a^2 \sinh\left(c - \frac{ad}{b}\right) \text{Shi}\left(xd + \frac{ad}{b}\right)}{b^3} - \frac{a \sinh(c+dx)}{b^2 d} - \frac{\cosh(c+dx)}{bd^2} + \frac{x \sinh(c+dx)}{bd}$$

Antiderivative was successfully verified.

[In] Int[(x^2*Cosh[c + d*x])/(a + b*x),x]

[Out] $-(\text{Cosh}[c + d*x] / (b*d^2)) + (a^2 * \text{Cosh}[c - (a*d)/b] * \text{CoshIntegral}[(a*d)/b + d*x]) / b^3 - (a * \text{Sinh}[c + d*x]) / (b^2 * d) + (x * \text{Sinh}[c + d*x]) / (b*d) + (a^2 * \text{Sinh}[c - (a*d)/b] * \text{SinhIntegral}[(a*d)/b + d*x]) / b^3$

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 2638

Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3296

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[((c + d*x)^m * Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m-1) * Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 3298

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_) / ((c_.) + (d_.)*(x_))], x_Symbol] := Simp[(I * SinhIntegral[(c*f*fz)/d + f*fz*x]) / d, x] /; FreeQ[{c, d, e, f}

, fz}, x] && EqQ[d*e - c*f*fz*I, 0]

Rule 3301

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]

Rule 3303

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]

Rule 6742

Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

Rubi steps

$$\begin{aligned} \int \frac{x^2 \cosh(c + dx)}{a + bx} dx &= \int \left(-\frac{a \cosh(c + dx)}{b^2} + \frac{x \cosh(c + dx)}{b} + \frac{a^2 \cosh(c + dx)}{b^2(a + bx)} \right) dx \\ &= -\frac{a \int \cosh(c + dx) dx}{b^2} + \frac{a^2 \int \frac{\cosh(c+dx)}{a+bx} dx}{b^2} + \frac{\int x \cosh(c + dx) dx}{b} \\ &= -\frac{a \sinh(c + dx)}{b^2 d} + \frac{x \sinh(c + dx)}{bd} - \frac{\int \sinh(c + dx) dx}{bd} + \frac{\left(a^2 \cosh\left(c - \frac{ad}{b}\right) \right) \int \frac{\cosh\left(\frac{ad}{b} + \dots}{a+bx}}{b^2} \\ &= -\frac{\cosh(c + dx)}{bd^2} + \frac{a^2 \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(\frac{ad}{b} + dx\right)}{b^3} - \frac{a \sinh(c + dx)}{b^2 d} + \frac{x \sinh(c + dx)}{bd} + \end{aligned}$$

Mathematica [A] time = 0.31, size = 89, normalized size = 0.89

$$\frac{a^2 d^2 \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(d\left(\frac{a}{b} + x\right)\right) + a^2 d^2 \sinh\left(c - \frac{ad}{b}\right) \text{Shi}\left(d\left(\frac{a}{b} + x\right)\right) + b(d(bx - a) \sinh(c + dx) - b \cosh(c + dx))}{b^3 d^2}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2*Cosh[c + d*x])/(a + b*x),x]

[Out] (a^2*d^2*Cosh[c - (a*d)/b]*CoshIntegral[d*(a/b + x)] + b*(-(b*Cosh[c + d*x]) + d*(-a + b*x)*Sinh[c + d*x]) + a^2*d^2*Sinh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)]/(b^3*d^2)

fricas [A] time = 0.56, size = 156, normalized size = 1.56

$$\frac{2b^2 \cosh(dx + c) - \left(a^2 d^2 \operatorname{Ei}\left(\frac{bdx+ad}{b}\right) + a^2 d^2 \operatorname{Ei}\left(-\frac{bdx+ad}{b}\right) \right) \cosh\left(-\frac{bc-ad}{b}\right) - 2(b^2 dx - abd) \sinh(dx + c) + \left(a^2 d^2 \right)}{2b^3 d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*cosh(d*x+c)/(b*x+a),x, algorithm="fricas")

[Out] -1/2*(2*b^2*cosh(d*x + c) - (a^2*d^2*Ei((b*d*x + a*d)/b) + a^2*d^2*Ei(-(b*d*x + a*d)/b))*cosh(-(b*c - a*d)/b) - 2*(b^2*d*x - a*b*d)*sinh(d*x + c) + (a^2*d^2*Ei((b*d*x + a*d)/b) - a^2*d^2*Ei(-(b*d*x + a*d)/b))*sinh(-(b*c - a*d)/b))/(b^3*d^2)

giac [A] time = 0.14, size = 148, normalized size = 1.48

$$\frac{a^2 d^2 \operatorname{Ei}\left(\frac{bdx+ad}{b}\right) e^{\left(c-\frac{ad}{b}\right)} + a^2 d^2 \operatorname{Ei}\left(-\frac{bdx+ad}{b}\right) e^{\left(-c+\frac{ad}{b}\right)} + b^2 dx e^{(dx+c)} - b^2 dx e^{(-dx-c)} - abde^{(dx+c)} + abde^{(-dx-c)} - b^2 e^{(dx+c)}}{2b^3 d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*cosh(d*x+c)/(b*x+a),x, algorithm="giac")

[Out] 1/2*(a^2*d^2*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + a^2*d^2*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) + b^2*d*x*e^(d*x + c) - b^2*d*x*e^(-d*x - c) - a*b*d*e^(d*x + c) + a*b*d*e^(-d*x - c) - b^2*e^(d*x + c) - b^2*e^(-d*x - c))/(b^3*d^2)

maple [A] time = 0.11, size = 184, normalized size = 1.84

$$-\frac{e^{-dx-c}x}{2db} + \frac{e^{-dx-c}a}{2db^2} - \frac{e^{-dx-c}}{2d^2b} - \frac{e^{\frac{da-cb}{b}} \operatorname{Ei}\left(1, dx + c + \frac{da-cb}{b}\right) a^2}{2b^3} + \frac{e^{dx+c}x}{2db} - \frac{e^{dx+c}}{2d^2b} - \frac{ae^{dx+c}}{2db^2} - \frac{e^{-\frac{da-cb}{b}} \operatorname{Ei}\left(1, -dx - c - \frac{da-cb}{b}\right)}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*cosh(d*x+c)/(b*x+a),x)

[Out] -1/2/d*exp(-d*x-c)/b*x+1/2/d*exp(-d*x-c)/b^2*a-1/2/d^2*exp(-d*x-c)/b-1/2/b^3*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)*a^2+1/2/d/b*exp(d*x+c)*x-1/2/d^2

$/b \exp(dx+c) - 1/2/d/b^2 * a \exp(dx+c) - 1/2/b^3 \exp(-(a*d-b*c)/b) * Ei(1, -d*x-c - (a*d-b*c)/b) * a^2$

maxima [B] time = 0.39, size = 233, normalized size = 2.33

$$-\frac{1}{4}d \left(\frac{2a^2 \left(\frac{e^{(-c+\frac{ad}{b})} E_1\left(\frac{(bx+a)d}{b}\right)}{b} + \frac{e^{(c-\frac{ad}{b})} E_1\left(-\frac{(bx+a)d}{b}\right)}{b} \right)}{b^2 d} - \frac{2a \left(\frac{(dx e^c - e^c) e^{(dx)}}{d^2} + \frac{(dx+1) e^{(-dx-c)}}{d^2} \right)}{b^2} + \frac{\left(d^2 x^2 e^c - 2 dx e^c + 2 e^c \right) e^{(dx)}}{d^3} + \frac{\left(d^2 x^2 + 2 dx + 2 \right) e^{(-dx-c)}}{d^3} \right) / b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*cosh(d*x+c)/(b*x+a),x, algorithm="maxima")

[Out] $-1/4*d*(2*a^2*(e^{(-c+a*d/b)}*\exp_integral_e(1,(b*x+a)*d/b)/b + e^{(c-a*d/b)}*\exp_integral_e(1,-(b*x+a)*d/b)/b)/(b^2*d) - 2*a*((d*x*e^c - e^c)*e^{(d*x)}/d^2 + (d*x+1)*e^{(-d*x-c)}/d^2)/b^2 + ((d^2*x^2*e^c - 2*d*x*e^c + 2*e^c)*e^{(d*x)}/d^3 + (d^2*x^2 + 2*d*x + 2)*e^{(-d*x-c)}/d^3)/b + 4*a^2*cosh(d*x+c)*log(b*x+a)/(b^3*d) + 1/2*(2*a^2*log(b*x+a)/b^3 + (b*x^2 - 2*a*x)/b^2)*cosh(d*x+c)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2 \cosh(c + dx)}{a + bx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2*cosh(c + d*x))/(a + b*x),x)

[Out] int((x^2*cosh(c + d*x))/(a + b*x), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \cosh(c + dx)}{a + bx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*cosh(d*x+c)/(b*x+a),x)

[Out] Integral(x**2*cosh(c + d*x)/(a + b*x), x)

3.21 $\int \frac{x \cosh(c+dx)}{a+bx} dx$

Optimal. Leaf size=68

$$-\frac{a \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{b^2} - \frac{a \sinh\left(c - \frac{ad}{b}\right) \text{Shi}\left(xd + \frac{ad}{b}\right)}{b^2} + \frac{\sinh(c + dx)}{bd}$$

[Out] $-a*\text{Chi}(a*d/b+d*x)*\cosh(-c+a*d/b)/b^2+a*\text{Shi}(a*d/b+d*x)*\sinh(-c+a*d/b)/b^2+\sinh(d*x+c)/b/d$

Rubi [A] time = 0.18, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6742, 2637, 3303, 3298, 3301}

$$-\frac{a \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{b^2} - \frac{a \sinh\left(c - \frac{ad}{b}\right) \text{Shi}\left(xd + \frac{ad}{b}\right)}{b^2} + \frac{\sinh(c + dx)}{bd}$$

Antiderivative was successfully verified.

[In] `Int[(x*Cosh[c + d*x])/(a + b*x),x]`

[Out] $-\left(\frac{a*\text{Cosh}\left[c - \frac{a*d}{b}\right]*\text{CoshIntegral}\left[\frac{a*d}{b} + d*x\right]}{b^2}\right) + \frac{\text{Sinh}\left[c + d*x\right]}{b*d} - \frac{a*\text{Sinh}\left[c - \frac{a*d}{b}\right]*\text{SinhIntegral}\left[\frac{a*d}{b} + d*x\right]}{b^2}$

Rule 2637

`Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;`
`FreeQ[{c, d}, x]`

Rule 3298

`Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /;`
`FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]`

Rule 3301

`Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /;`
`FreeQ[{c, d, e, f, fz}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]`

Rule 3303

`Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)`

)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]

Rule 6742

Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]

Rubi steps

$$\begin{aligned} \int \frac{x \cosh(c + dx)}{a + bx} dx &= \int \left(\frac{\cosh(c + dx)}{b} - \frac{a \cosh(c + dx)}{b(a + bx)} \right) dx \\ &= \frac{\int \cosh(c + dx) dx}{b} - \frac{a \int \frac{\cosh(c+dx)}{a+bx} dx}{b} \\ &= \frac{\sinh(c + dx)}{bd} - \frac{\left(a \cosh\left(c - \frac{ad}{b}\right) \int \frac{\cosh\left(\frac{ad}{b} + dx\right)}{a+bx} dx \right)}{b} - \frac{\left(a \sinh\left(c - \frac{ad}{b}\right) \int \frac{\sinh\left(\frac{ad}{b} + dx\right)}{a+bx} dx \right)}{b} \\ &= -\frac{a \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(\frac{ad}{b} + dx\right)}{b^2} + \frac{\sinh(c + dx)}{bd} - \frac{a \sinh\left(c - \frac{ad}{b}\right) \text{Shi}\left(\frac{ad}{b} + dx\right)}{b^2} \end{aligned}$$

Mathematica [A] time = 0.15, size = 64, normalized size = 0.94

$$\frac{-ad \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(d\left(\frac{a}{b} + x\right)\right) - ad \sinh\left(c - \frac{ad}{b}\right) \text{Shi}\left(d\left(\frac{a}{b} + x\right)\right) + b \sinh(c + dx)}{b^2 d}$$

Antiderivative was successfully verified.

[In] Integrate[(x*Cosh[c + d*x])/(a + b*x), x]

[Out] (-(a*d*Cosh[c - (a*d)/b]*CoshIntegral[d*(a/b + x)]) + b*Sinh[c + d*x] - a*d
*Sinh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)]/(b^2*d)

fricas [A] time = 0.45, size = 118, normalized size = 1.74

$$\frac{\left(ad\text{Ei}\left(\frac{bdx+ad}{b}\right) + ad\text{Ei}\left(-\frac{bdx+ad}{b}\right)\right) \cosh\left(-\frac{bc-ad}{b}\right) - 2b \sinh(dx + c) - \left(ad\text{Ei}\left(\frac{bdx+ad}{b}\right) - ad\text{Ei}\left(-\frac{bdx+ad}{b}\right)\right) \sinh\left(-\frac{bc-ad}{b}\right)}{2b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cosh(d*x+c)/(b*x+a), x, algorithm="fricas")

[Out] $-1/2*((a*d*Ei((b*d*x + a*d)/b) + a*d*Ei(-(b*d*x + a*d)/b))*\cosh(-(b*c - a*d)/b) - 2*b*\sinh(d*x + c) - (a*d*Ei((b*d*x + a*d)/b) - a*d*Ei(-(b*d*x + a*d)/b))*\sinh(-(b*c - a*d)/b))/(b^2*d)$

giac [A] time = 0.14, size = 83, normalized size = 1.22

$$\frac{adEi\left(\frac{bdx+ad}{b}\right)e^{\left(c-\frac{ad}{b}\right)} + adEi\left(-\frac{bdx+ad}{b}\right)e^{\left(-c+\frac{ad}{b}\right)} - be^{(dx+c)} + be^{(-dx-c)}}{2b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*cosh(d*x+c)/(b*x+a),x, algorithm="giac")`

[Out] $-1/2*(a*d*Ei((b*d*x + a*d)/b)*e^{(c - a*d/b)} + a*d*Ei(-(b*d*x + a*d)/b)*e^{(-c + a*d/b)} - b*e^{(d*x + c)} + b*e^{(-d*x - c)})/(b^2*d)$

maple [A] time = 0.09, size = 114, normalized size = 1.68

$$-\frac{e^{-dx-c}}{2db} + \frac{e^{\frac{da-cb}{b}} Ei\left(1, dx + c + \frac{da-cb}{b}\right) a}{2b^2} + \frac{e^{dx+c}}{2db} + \frac{e^{-\frac{da-cb}{b}} Ei\left(1, -dx - c - \frac{da-cb}{b}\right) a}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*cosh(d*x+c)/(b*x+a),x)`

[Out] $-1/2/d*\exp(-d*x-c)/b+1/2/b^2*\exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)*a+1/2/d/b*\exp(d*x+c)+1/2/b^2*\exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)*a$

maxima [B] time = 0.38, size = 156, normalized size = 2.29

$$\frac{1}{2}d \left(\frac{a \left(\frac{e^{(-c+\frac{ad}{b})} E_1\left(\frac{(bx+a)d}{b}\right)}{b} + \frac{e^{(c-\frac{ad}{b})} E_1\left(-\frac{(bx+a)d}{b}\right)}{b} \right)}{bd} - \frac{\frac{(dxe^c - e^c)e^{(dx)}}{d^2} + \frac{(dx+1)e^{(-dx-c)}}{d^2}}{b} + \frac{2a \cosh(dx+c) \log(bx+a)}{b^2d} \right) + \left(\frac{x}{b} - \frac{a}{b^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*cosh(d*x+c)/(b*x+a),x, algorithm="maxima")`

[Out] $1/2*d*(a*(e^{(-c + a*d/b)}*\exp_integral_e(1, (b*x + a)*d/b)/b + e^{(c - a*d/b)}*\exp_integral_e(1, -(b*x + a)*d/b)/b)/(b*d) - ((d*x*e^c - e^c)*e^{(d*x)}/d^2 + (d*x + 1)*e^{(-d*x - c)}/d^2)/b + 2*a*cosh(d*x + c)*log(b*x + a)/(b^2*d) + (x/b - a*log(b*x + a)/b^2)*cosh(d*x + c)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x \cosh(c + dx)}{a + bx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x*cosh(c + d*x))/(a + b*x), x)

[Out] int((x*cosh(c + d*x))/(a + b*x), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \cosh(c + dx)}{a + bx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cosh(d*x+c)/(b*x+a), x)

[Out] Integral(x*cosh(c + d*x)/(a + b*x), x)

$$3.22 \quad \int \frac{\cosh(c+dx)}{a+bx} dx$$

Optimal. Leaf size=51

$$\frac{\cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{b} + \frac{\sinh\left(c - \frac{ad}{b}\right) \text{Shi}\left(xd + \frac{ad}{b}\right)}{b}$$

[Out] Chi(a*d/b+d*x)*cosh(-c+a*d/b)/b-Shi(a*d/b+d*x)*sinh(-c+a*d/b)/b

Rubi [A] time = 0.08, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3303, 3298, 3301}

$$\frac{\cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{b} + \frac{\sinh\left(c - \frac{ad}{b}\right) \text{Shi}\left(xd + \frac{ad}{b}\right)}{b}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]/(a + b*x), x]

[Out] (Cosh[c - (a*d)/b]*CoshIntegral[(a*d)/b + d*x])/b + (Sinh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/b

Rule 3298

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]

Rule 3301

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]

Rule 3303

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]

Rubi steps

$$\int \frac{\cosh(c + dx)}{a + bx} dx = \cosh\left(c - \frac{ad}{b}\right) \int \frac{\cosh\left(\frac{ad}{b} + dx\right)}{a + bx} dx + \sinh\left(c - \frac{ad}{b}\right) \int \frac{\sinh\left(\frac{ad}{b} + dx\right)}{a + bx} dx$$

$$= \frac{\cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(\frac{ad}{b} + dx\right)}{b} + \frac{\sinh\left(c - \frac{ad}{b}\right) \text{Shi}\left(\frac{ad}{b} + dx\right)}{b}$$

Mathematica [A] time = 0.06, size = 49, normalized size = 0.96

$$\frac{\cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right) + \sinh\left(c - \frac{ad}{b}\right) \text{Shi}\left(xd + \frac{ad}{b}\right)}{b}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d*x]/(a + b*x), x]

[Out] (Cosh[c - (a*d)/b]*CoshIntegral[(a*d)/b + d*x] + Sinh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/b

fricas [A] time = 0.51, size = 95, normalized size = 1.86

$$\frac{\left(\text{Ei}\left(\frac{bdx+ad}{b}\right) + \text{Ei}\left(-\frac{bdx+ad}{b}\right)\right) \cosh\left(-\frac{bc-ad}{b}\right) - \left(\text{Ei}\left(\frac{bdx+ad}{b}\right) - \text{Ei}\left(-\frac{bdx+ad}{b}\right)\right) \sinh\left(-\frac{bc-ad}{b}\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/(b*x+a), x, algorithm="fricas")

[Out] 1/2*((Ei((b*d*x + a*d)/b) + Ei(-(b*d*x + a*d)/b))*cosh(-(b*c - a*d)/b) - (Ei((b*d*x + a*d)/b) - Ei(-(b*d*x + a*d)/b))*sinh(-(b*c - a*d)/b))/b

giac [A] time = 0.14, size = 56, normalized size = 1.10

$$\frac{\text{Ei}\left(\frac{bdx+ad}{b}\right) e^{\left(c - \frac{ad}{b}\right)} + \text{Ei}\left(-\frac{bdx+ad}{b}\right) e^{\left(-c + \frac{ad}{b}\right)}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/(b*x+a), x, algorithm="giac")

[Out] 1/2*(Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b))/b

maple [A] time = 0.07, size = 81, normalized size = 1.59

$$-\frac{e^{\frac{da-cb}{b}} \operatorname{Ei}\left(1, dx + c + \frac{da-cb}{b}\right)}{2b} - \frac{e^{-\frac{da-cb}{b}} \operatorname{Ei}\left(1, -dx - c - \frac{da-cb}{b}\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cosh(d*x+c)/(b*x+a), x)`

[Out] `-1/2/b*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)-1/2/b*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)`

maxima [A] time = 0.36, size = 57, normalized size = 1.12

$$-\frac{e^{\left(-c+\frac{ad}{b}\right)} E_1\left(\frac{(bx+a)d}{b}\right)}{2b} - \frac{e^{\left(c-\frac{ad}{b}\right)} E_1\left(-\frac{(bx+a)d}{b}\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(d*x+c)/(b*x+a), x, algorithm="maxima")`

[Out] `-1/2*e^(-c + a*d/b)*exp_integral_e(1, (b*x + a)*d/b)/b - 1/2*e^(c - a*d/b)*exp_integral_e(1, -(b*x + a)*d/b)/b`

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cosh(c + dx)}{a + bx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cosh(c + d*x)/(a + b*x), x)`

[Out] `int(cosh(c + d*x)/(a + b*x), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(c + dx)}{a + bx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(d*x+c)/(b*x+a), x)`

[Out] `Integral(cosh(c + d*x)/(a + b*x), x)`

3.23 $\int \frac{\cosh(c+dx)}{x(a+bx)} dx$

Optimal. Leaf size=73

$$\frac{\cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{a} - \frac{\sinh\left(c - \frac{ad}{b}\right) \text{Shi}\left(xd + \frac{ad}{b}\right)}{a} + \frac{\cosh(c) \text{Chi}(dx)}{a} + \frac{\sinh(c) \text{Shi}(dx)}{a}$$

[Out] Chi(d*x)*cosh(c)/a-Chi(a*d/b+d*x)*cosh(-c+a*d/b)/a+Shi(d*x)*sinh(c)/a+Shi(a*d/b+d*x)*sinh(-c+a*d/b)/a

Rubi [A] time = 0.26, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {6742, 3303, 3298, 3301}

$$\frac{\cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{a} - \frac{\sinh\left(c - \frac{ad}{b}\right) \text{Shi}\left(xd + \frac{ad}{b}\right)}{a} + \frac{\cosh(c) \text{Chi}(dx)}{a} + \frac{\sinh(c) \text{Shi}(dx)}{a}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]/(x*(a + b*x)), x]

[Out] (Cosh[c]*CoshIntegral[d*x])/a - (Cosh[c - (a*d)/b]*CoshIntegral[(a*d)/b + d*x])/a + (Sinh[c]*SinhIntegral[d*x])/a - (Sinh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/a

Rule 3298

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]

Rule 3301

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]

Rule 3303

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]

Rule 6742

`Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]`
`]`

Rubi steps

$$\begin{aligned}
 \int \frac{\cosh(c + dx)}{x(a + bx)} dx &= \int \left(\frac{\cosh(c + dx)}{ax} - \frac{b \cosh(c + dx)}{a(a + bx)} \right) dx \\
 &= \frac{\int \frac{\cosh(c+dx)}{x} dx}{a} - \frac{b \int \frac{\cosh(c+dx)}{a+bx} dx}{a} \\
 &= \frac{\cosh(c) \int \frac{\cosh(dx)}{x} dx}{a} - \frac{\left(b \cosh\left(c - \frac{ad}{b}\right) \right) \int \frac{\cosh\left(\frac{ad}{b} + dx\right)}{a+bx} dx}{a} + \frac{\sinh(c) \int \frac{\sinh(dx)}{x} dx}{a} - \frac{\left(b \sinh\left(c - \frac{ad}{b}\right) \right) \int \frac{\sinh\left(\frac{ad}{b} + dx\right)}{a+bx} dx}{a} \\
 &= \frac{\cosh(c)\text{Chi}(dx)}{a} - \frac{\cosh\left(c - \frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b} + dx\right)}{a} + \frac{\sinh(c)\text{Shi}(dx)}{a} - \frac{\sinh\left(c - \frac{ad}{b}\right)\text{Shi}\left(\frac{ad}{b} + dx\right)}{a}
 \end{aligned}$$

Mathematica [A] time = 0.15, size = 63, normalized size = 0.86

$$\frac{-\cosh\left(c - \frac{ad}{b}\right)\text{Chi}\left(d\left(\frac{a}{b} + x\right)\right) - \sinh\left(c - \frac{ad}{b}\right)\text{Shi}\left(d\left(\frac{a}{b} + x\right)\right) + \cosh(c)\text{Chi}(dx) + \sinh(c)\text{Shi}(dx)}{a}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d*x]/(x*(a + b*x)),x]

[Out] (Cosh[c]*CoshIntegral[d*x] - Cosh[c - (a*d)/b]*CoshIntegral[d*(a/b + x)] + Sinh[c]*SinhIntegral[d*x] - Sinh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)])/a

fricas [A] time = 0.64, size = 123, normalized size = 1.68

$$\frac{(\text{Ei}(dx) + \text{Ei}(-dx))\cosh(c) - \left(\text{Ei}\left(\frac{bdx+ad}{b}\right) + \text{Ei}\left(-\frac{bdx+ad}{b}\right)\right)\cosh\left(-\frac{bc-ad}{b}\right) + (\text{Ei}(dx) - \text{Ei}(-dx))\sinh(c) + \left(\text{Ei}\left(\frac{bdx+ad}{b}\right) - \text{Ei}\left(-\frac{bdx+ad}{b}\right)\right)\sinh\left(-\frac{bc-ad}{b}\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/x/(b*x+a),x, algorithm="fricas")

[Out] 1/2*((Ei(d*x) + Ei(-d*x))*cosh(c) - (Ei((b*d*x + a*d)/b) + Ei(-(b*d*x + a*d)/b))*cosh(-(b*c - a*d)/b) + (Ei(d*x) - Ei(-d*x))*sinh(c) + (Ei((b*d*x + a*d)/b) - Ei(-(b*d*x + a*d)/b))*sinh(-(b*c - a*d)/b))/a

giac [A] time = 0.14, size = 75, normalized size = 1.03

$$\frac{\operatorname{Ei}(-dx)e^{-c} - \operatorname{Ei}\left(\frac{bdx+ad}{b}\right)e^{\left(c-\frac{ad}{b}\right)} + \operatorname{Ei}(dx)e^c - \operatorname{Ei}\left(-\frac{bdx+ad}{b}\right)e^{\left(-c+\frac{ad}{b}\right)}}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/x/(b*x+a),x, algorithm="giac")

[Out] 1/2*(Ei(-d*x)*e^(-c) - Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + Ei(d*x)*e^c - Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b))/a

maple [A] time = 0.08, size = 108, normalized size = 1.48

$$\frac{e^{\frac{da-cb}{b}} \operatorname{Ei}\left(1, dx + c + \frac{da-cb}{b}\right)}{2a} - \frac{e^{-c} \operatorname{Ei}(1, dx)}{2a} + \frac{e^{-\frac{da-cb}{b}} \operatorname{Ei}\left(1, -dx - c - \frac{da-cb}{b}\right)}{2a} - \frac{e^c \operatorname{Ei}(1, -dx)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d*x+c)/x/(b*x+a),x)

[Out] 1/2/a*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)-1/2/a*exp(-c)*Ei(1,d*x)+1/2/a*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)-1/2/a*exp(c)*Ei(1,-d*x)

maxima [B] time = 0.41, size = 155, normalized size = 2.12

$$\frac{1}{2}d \left(\frac{b \left(\frac{e^{\left(-c+\frac{ad}{b}\right)} E_1\left(\frac{(bx+a)d}{b}\right)} + \frac{e^{\left(c-\frac{ad}{b}\right)} E_1\left(-\frac{(bx+a)d}{b}\right)} \right)}{ad} + \frac{2 \cosh(dx+c) \log(bx+a)}{ad} - \frac{2 \cosh(dx+c) \log(x)}{ad} + \frac{\operatorname{Ei}(-dx)}{a} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/x/(b*x+a),x, algorithm="maxima")

[Out] 1/2*d*(b*(e^(-c + a*d/b)*exp_integral_e(1, (b*x + a)*d/b)/b + e^(c - a*d/b)*exp_integral_e(1, -(b*x + a)*d/b)/b)/(a*d) + 2*cosh(d*x + c)*log(b*x + a)/(a*d) - 2*cosh(d*x + c)*log(x)/(a*d) + (Ei(-d*x)*e^(-c) + Ei(d*x)*e^c)/(a*d)) - (log(b*x + a)/a - log(x)/a)*cosh(d*x + c)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c + dx)}{x(a + bx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cosh(c + d*x)/(x*(a + b*x)),x)`

[Out] `int(cosh(c + d*x)/(x*(a + b*x)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(c + dx)}{x(a + bx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(d*x+c)/x/(b*x+a),x)`

[Out] `Integral(cosh(c + d*x)/(x*(a + b*x)), x)`

3.24 $\int \frac{\cosh(c+dx)}{x^2(a+bx)} dx$

Optimal. Leaf size=113

$$-\frac{b \cosh(c) \operatorname{Chi}(dx)}{a^2} + \frac{b \cosh\left(c - \frac{ad}{b}\right) \operatorname{Chi}\left(xd + \frac{ad}{b}\right)}{a^2} - \frac{b \sinh(c) \operatorname{Shi}(dx)}{a^2} + \frac{b \sinh\left(c - \frac{ad}{b}\right) \operatorname{Shi}\left(xd + \frac{ad}{b}\right)}{a^2} + \frac{d \sinh(c)}{a}$$

[Out] $-b \operatorname{Chi}(d*x) \cosh(c) / a^2 + b \operatorname{Chi}(a*d/b + d*x) \cosh(-c + a*d/b) / a^2 - \cosh(d*x + c) / a / x + d \cosh(c) \operatorname{Shi}(d*x) / a + d \operatorname{Chi}(d*x) \sinh(c) / a - b \operatorname{Shi}(d*x) \sinh(c) / a^2 - b \operatorname{Shi}(a*d/b + d*x) \sinh(-c + a*d/b) / a^2$

Rubi [A] time = 0.37, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {6742, 3297, 3303, 3298, 3301}

$$-\frac{b \cosh(c) \operatorname{Chi}(dx)}{a^2} + \frac{b \cosh\left(c - \frac{ad}{b}\right) \operatorname{Chi}\left(xd + \frac{ad}{b}\right)}{a^2} - \frac{b \sinh(c) \operatorname{Shi}(dx)}{a^2} + \frac{b \sinh\left(c - \frac{ad}{b}\right) \operatorname{Shi}\left(xd + \frac{ad}{b}\right)}{a^2} + \frac{d \sinh(c)}{a}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Cosh}[c + d*x] / (x^2 * (a + b*x)), x]$

[Out] $-(\operatorname{Cosh}[c + d*x] / (a*x)) - (b \operatorname{Cosh}[c] \operatorname{CoshIntegral}[d*x]) / a^2 + (b \operatorname{Cosh}[c - (a*d)/b] \operatorname{CoshIntegral}[(a*d)/b + d*x]) / a^2 + (d \operatorname{CoshIntegral}[d*x] \operatorname{Sinh}[c]) / a + (d \operatorname{Cosh}[c] \operatorname{SinhIntegral}[d*x]) / a - (b \operatorname{Sinh}[c] \operatorname{SinhIntegral}[d*x]) / a^2 + (b \operatorname{Sinh}[c - (a*d)/b] \operatorname{SinhIntegral}[(a*d)/b + d*x]) / a^2$

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((c + d*x)^(m + 1)*Sin[e + f*x]) / (d*(m + 1)), x] - Dist[f / (d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_) / ((c_.) + (d_.)*(x_))], x_Symbol] := Simp[(I*SinhIntegral[(c*f*fz) / d + f*fz*x]) / d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_) / ((c_.) + (d_.)*(x_))], x_Symbol] := Simp[CoshIntegral[(c*f*fz) / d + f*fz*x] / d, x] /; FreeQ[{c, d, e, f, fz}
```

}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]

Rule 3303

Int[sin[(e_.) + (f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x_Symbol] := Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]

Rule 6742

Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

Rubi steps

$$\begin{aligned}
 \int \frac{\cosh(c + dx)}{x^2(a + bx)} dx &= \int \left(\frac{\cosh(c + dx)}{ax^2} - \frac{b \cosh(c + dx)}{a^2x} + \frac{b^2 \cosh(c + dx)}{a^2(a + bx)} \right) dx \\
 &= \frac{\int \frac{\cosh(c+dx)}{x^2} dx}{a} - \frac{b \int \frac{\cosh(c+dx)}{x} dx}{a^2} + \frac{b^2 \int \frac{\cosh(c+dx)}{a+bx} dx}{a^2} \\
 &= -\frac{\cosh(c + dx)}{ax} + \frac{d \int \frac{\sinh(c+dx)}{x} dx}{a} - \frac{(b \cosh(c)) \int \frac{\cosh(dx)}{x} dx}{a^2} + \frac{\left(b^2 \cosh\left(c - \frac{ad}{b}\right)\right) \int \frac{\cosh\left(\frac{ad}{b}\right)}{a+bx} dx}{a^2} \\
 &= -\frac{\cosh(c + dx)}{ax} - \frac{b \cosh(c) \text{Chi}(dx)}{a^2} + \frac{b \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(\frac{ad}{b} + dx\right)}{a^2} - \frac{b \sinh(c) \text{Shi}(dx)}{a^2} + \frac{ad \cosh\left(c - \frac{ad}{b}\right) \text{Shi}\left(\frac{ad}{b} + dx\right)}{a^2} \\
 &= -\frac{\cosh(c + dx)}{ax} - \frac{b \cosh(c) \text{Chi}(dx)}{a^2} + \frac{b \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(\frac{ad}{b} + dx\right)}{a^2} + \frac{d \text{Chi}(dx) \sinh(c)}{a} + \frac{ad \cosh\left(c - \frac{ad}{b}\right) \text{Shi}\left(\frac{ad}{b} + dx\right)}{a^2}
 \end{aligned}$$

Mathematica [A] time = 0.40, size = 101, normalized size = 0.89

$$\frac{bx \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(d\left(\frac{a}{b} + x\right)\right) + \text{Chi}(dx)(adx \sinh(c) - bx \cosh(c)) + bx \sinh\left(c - \frac{ad}{b}\right) \text{Shi}\left(d\left(\frac{a}{b} + x\right)\right) + adx \cosh\left(c - \frac{ad}{b}\right) \text{Shi}\left(\frac{ad}{b} + dx\right)}{a^2x}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d*x]/(x^2*(a + b*x)),x]

[Out] (-a*Cosh[c + d*x]) + b*x*Cosh[c - (a*d)/b]*CoshIntegral[d*(a/b + x)] + CoshIntegral[d*x]*(-b*x*Cosh[c]) + a*d*x*Sinh[c] + a*d*x*Cosh[c]*SinhIntegral[d*(a/b + x)]

$1[d*x] - b*x*\text{Sinh}[c]*\text{SinhIntegral}[d*x] + b*x*\text{Sinh}[c - (a*d)/b]*\text{SinhIntegral}[d*(a/b + x)]/(a^2*x)$

fricas [A] time = 0.48, size = 179, normalized size = 1.58

$$\frac{2 a \cosh (d x+c)-((a d-b) x \operatorname{Ei}(d x)-(a d+b) x \operatorname{Ei}(-d x)) \cosh (c)-\left(b x \operatorname{Ei}\left(\frac{b d x+a d}{b}\right)+b x \operatorname{Ei}\left(-\frac{b d x+a d}{b}\right)\right) \cosh \left(-\frac{b c-a d}{b}\right)-((a d-b) x \operatorname{Ei}(d x)+(a d+b) x \operatorname{Ei}(-d x)) \sinh (c)+\left(b x \operatorname{Ei}\left(\frac{b d x+a d}{b}\right)-b x \operatorname{Ei}\left(-\frac{b d x+a d}{b}\right)\right) \sinh \left(-\frac{b c-a d}{b}\right)}{2 a^2 x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/x^2/(b*x+a),x, algorithm="fricas")

[Out] $-1/2*(2*a*\cosh(d*x + c) - ((a*d - b)*x*\operatorname{Ei}(d*x) - (a*d + b)*x*\operatorname{Ei}(-d*x))*\cosh(c) - (b*x*\operatorname{Ei}((b*d*x + a*d)/b) + b*x*\operatorname{Ei}(-(b*d*x + a*d)/b))*\cosh(-(b*c - a*d)/b) - ((a*d - b)*x*\operatorname{Ei}(d*x) + (a*d + b)*x*\operatorname{Ei}(-d*x))*\sinh(c) + (b*x*\operatorname{Ei}((b*d*x + a*d)/b) - b*x*\operatorname{Ei}(-(b*d*x + a*d)/b))*\sinh(-(b*c - a*d)/b))/(a^2*x)$

giac [A] time = 0.15, size = 129, normalized size = 1.14

$$\frac{a d x \operatorname{Ei}(-d x) e^{(-c)}-a d x \operatorname{Ei}(d x) e^c+b x \operatorname{Ei}(-d x) e^{(-c)}-b x \operatorname{Ei}\left(\frac{b d x+a d}{b}\right) e^{\left(c-\frac{a d}{b}\right)}+b x \operatorname{Ei}(d x) e^c-b x \operatorname{Ei}\left(-\frac{b d x+a d}{b}\right) e^{(-c)}}{2 a^2 x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/x^2/(b*x+a),x, algorithm="giac")

[Out] $-1/2*(a*d*x*\operatorname{Ei}(-d*x)*e^{(-c)} - a*d*x*\operatorname{Ei}(d*x)*e^c + b*x*\operatorname{Ei}(-d*x)*e^{(-c)} - b*x*\operatorname{Ei}((b*d*x + a*d)/b)*e^{(c - a*d/b)} + b*x*\operatorname{Ei}(d*x)*e^c - b*x*\operatorname{Ei}(-(b*d*x + a*d)/b)*e^{(-c + a*d/b)} + a*e^{(d*x + c)} + a*e^{(-d*x - c)})/(a^2*x)$

maple [A] time = 0.10, size = 172, normalized size = 1.52

$$\frac{e^{-d x-c}}{2 a x}-\frac{b e^{\frac{d a-c b}{b}} \operatorname{Ei}\left(1, d x+c+\frac{d a-c b}{b}\right)}{2 a^2}+\frac{d e^{-c} \operatorname{Ei}(1, d x)}{2 a}+\frac{e^{-c} \operatorname{Ei}(1, d x) b}{2 a^2}-\frac{b e^{-\frac{d a-c b}{b}} \operatorname{Ei}\left(1,-d x-c-\frac{d a-c b}{b}\right)}{2 a^2}-\frac{e^{d x+c}}{2 a x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d*x+c)/x^2/(b*x+a),x)

[Out] $-1/2*\exp(-d*x-c)/a/x-1/2*b/a^2*\exp((a*d-b*c)/b)*\operatorname{Ei}(1,d*x+c+(a*d-b*c)/b)+1/2*d/a*\exp(-c)*\operatorname{Ei}(1,d*x)+1/2/a^2*\exp(-c)*\operatorname{Ei}(1,d*x)*b-1/2/a^2*b*\exp(-(a*d-b*c)/b)*\operatorname{Ei}(1,-d*x-c-(a*d-b*c)/b)-1/2/a/x*\exp(d*x+c)-1/2*d/a*\exp(c)*\operatorname{Ei}(1,-d*x)+1/2*b/a^2*\exp(c)*\operatorname{Ei}(1,-d*x)$

maxima [A] time = 0.46, size = 192, normalized size = 1.70

$$-\frac{1}{2}d \left(\frac{\operatorname{Ei}(-dx)e^{-c} - \operatorname{Ei}(dx)e^c}{a} + \frac{b^2 \left(\frac{e^{\left(-c+\frac{ad}{b}\right)} \operatorname{E}_1\left(\frac{(bx+a)d}{b}\right)} + \frac{e^{\left(c-\frac{ad}{b}\right)} \operatorname{E}_1\left(-\frac{(bx+a)d}{b}\right)}{b} \right)}{a^2d} \right) + \frac{2b \cosh(dx+c) \log(bx+a)}{a^2d} - \frac{2bc}{a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/x^2/(b*x+a),x, algorithm="maxima")

[Out] $-1/2*d*((\operatorname{Ei}(-d*x)*e^{-c} - \operatorname{Ei}(d*x)*e^c)/a + b^2*(e^{-c + a*d/b}*\exp_integral_e(1, (b*x + a)*d/b)/b + e^{(c - a*d/b)}*\exp_integral_e(1, -(b*x + a)*d/b)/b)/(a^2*d) + 2*b*\cosh(d*x + c)*\log(b*x + a)/(a^2*d) - 2*b*\cosh(d*x + c)*\log(x)/(a^2*d) + (\operatorname{Ei}(-d*x)*e^{-c} + \operatorname{Ei}(d*x)*e^c)*b/(a^2*d)) + (b*\log(b*x + a)/a^2 - b*\log(x)/a^2 - 1/(a*x))*\cosh(d*x + c)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c + dx)}{x^2(a + bx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(c + d*x)/(x^2*(a + b*x)),x)

[Out] int(cosh(c + d*x)/(x^2*(a + b*x)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(c + dx)}{x^2(a + bx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/x**2/(b*x+a),x)

[Out] Integral(cosh(c + d*x)/(x**2*(a + b*x)), x)

3.25 $\int \frac{\cosh(c+dx)}{x^3(a+bx)} dx$

Optimal. Leaf size=190

$$\frac{b^2 \cosh(c) \operatorname{Chi}(dx)}{a^3} - \frac{b^2 \cosh\left(c - \frac{ad}{b}\right) \operatorname{Chi}\left(xd + \frac{ad}{b}\right)}{a^3} + \frac{b^2 \sinh(c) \operatorname{Shi}(dx)}{a^3} - \frac{b^2 \sinh\left(c - \frac{ad}{b}\right) \operatorname{Shi}\left(xd + \frac{ad}{b}\right)}{a^3} - \frac{bd \sinh(c)}{a^3}$$

[Out] $b^2 \operatorname{Chi}(d*x) \cosh(c) / a^3 + 1/2 d^2 \operatorname{Chi}(d*x) \cosh(c) / a - b^2 \operatorname{Chi}(a*d/b + d*x) \cosh(-c + a*d/b) / a^3 - 1/2 \cosh(d*x + c) / a / x^2 + b \cosh(d*x + c) / a^2 / x - b*d \cosh(c) \operatorname{Shi}(d*x) / a^2 - b*d \operatorname{Chi}(d*x) \sinh(c) / a^2 + b^2 \operatorname{Shi}(d*x) \sinh(c) / a^3 + 1/2 d^2 \operatorname{Shi}(d*x) \sinh(c) / a + b^2 \operatorname{Shi}(a*d/b + d*x) \sinh(-c + a*d/b) / a^3 - 1/2 d \sinh(d*x + c) / a / x$

Rubi [A] time = 0.49, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {6742, 3297, 3303, 3298, 3301}

$$\frac{b^2 \cosh(c) \operatorname{Chi}(dx)}{a^3} - \frac{b^2 \cosh\left(c - \frac{ad}{b}\right) \operatorname{Chi}\left(xd + \frac{ad}{b}\right)}{a^3} + \frac{b^2 \sinh(c) \operatorname{Shi}(dx)}{a^3} - \frac{b^2 \sinh\left(c - \frac{ad}{b}\right) \operatorname{Shi}\left(xd + \frac{ad}{b}\right)}{a^3} - \frac{bd \sinh(c)}{a^3}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]/(x^3*(a + b*x)), x]

[Out] $-\operatorname{Cosh}[c + d*x] / (2*a*x^2) + (b*\operatorname{Cosh}[c + d*x]) / (a^2*x) + (b^2*\operatorname{Cosh}[c]*\operatorname{CoshIntegral}[d*x]) / a^3 + (d^2*\operatorname{Cosh}[c]*\operatorname{CoshIntegral}[d*x]) / (2*a) - (b^2*\operatorname{Cosh}[c - (a*d)/b]*\operatorname{CoshIntegral}[(a*d)/b + d*x]) / a^3 - (b*d*\operatorname{CoshIntegral}[d*x]*\operatorname{Sinh}[c]) / a^2 - (d*\operatorname{Sinh}[c + d*x]) / (2*a*x) - (b*d*\operatorname{Cosh}[c]*\operatorname{SinhIntegral}[d*x]) / a^2 + (b^2*\operatorname{Sinh}[c]*\operatorname{SinhIntegral}[d*x]) / a^3 + (d^2*\operatorname{Sinh}[c]*\operatorname{SinhIntegral}[d*x]) / (2*a) - (b^2*\operatorname{Sinh}[c - (a*d)/b]*\operatorname{SinhIntegral}[(a*d)/b + d*x]) / a^3$

Rule 3297

Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((c + d*x)^(m + 1)*Sin[e + f*x]) / (d*(m + 1)), x] - Dist[f / (d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]

Rule 3298

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_) / ((c_.) + (d_.)*(x_))], x_Symbol] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x]) / d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x]
&& EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x]
+ Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /;
FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]
```

Rule 6742

```
Int[u_, x_Symbol]
:> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh(c+dx)}{x^3(a+bx)} dx &= \int \left(\frac{\cosh(c+dx)}{ax^3} - \frac{b \cosh(c+dx)}{a^2x^2} + \frac{b^2 \cosh(c+dx)}{a^3x} - \frac{b^3 \cosh(c+dx)}{a^3(a+bx)} \right) dx \\
&= \frac{\int \frac{\cosh(c+dx)}{x^3} dx}{a} - \frac{b \int \frac{\cosh(c+dx)}{x^2} dx}{a^2} + \frac{b^2 \int \frac{\cosh(c+dx)}{x} dx}{a^3} - \frac{b^3 \int \frac{\cosh(c+dx)}{a+bx} dx}{a^3} \\
&= -\frac{\cosh(c+dx)}{2ax^2} + \frac{b \cosh(c+dx)}{a^2x} + \frac{d \int \frac{\sinh(c+dx)}{x^2} dx}{2a} - \frac{(bd) \int \frac{\sinh(c+dx)}{x} dx}{a^2} + \frac{(b^2 \cosh(c)) \int}{a^3} \\
&= -\frac{\cosh(c+dx)}{2ax^2} + \frac{b \cosh(c+dx)}{a^2x} + \frac{b^2 \cosh(c) \text{Chi}(dx)}{a^3} - \frac{b^2 \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(\frac{ad}{b} + dx\right)}{a^3} - \\
&= -\frac{\cosh(c+dx)}{2ax^2} + \frac{b \cosh(c+dx)}{a^2x} + \frac{b^2 \cosh(c) \text{Chi}(dx)}{a^3} - \frac{b^2 \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(\frac{ad}{b} + dx\right)}{a^3} - \\
&= -\frac{\cosh(c+dx)}{2ax^2} + \frac{b \cosh(c+dx)}{a^2x} + \frac{b^2 \cosh(c) \text{Chi}(dx)}{a^3} + \frac{d^2 \cosh(c) \text{Chi}(dx)}{2a} - \frac{b^2 \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(\frac{ad}{b} + dx\right)}{a^3}
\end{aligned}$$

Mathematica [A] time = 0.50, size = 178, normalized size = 0.94

$$x^2 \text{Chi}(dx) \left(\cosh(c) (a^2 d^2 + 2b^2) - 2abd \sinh(c) \right) + a^2 d^2 x^2 \sinh(c) \text{Shi}(dx) - a^2 dx \sinh(c+dx) + a^2 (-\cosh(c+dx) - \cosh(c - \frac{ad}{b})) \text{Chi}\left(\frac{ad}{b} + dx\right)$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d*x]/(x^3*(a + b*x)),x]

[Out] $(-(a^2 \operatorname{Cosh}[c + d*x]) + 2*a*b*x*\operatorname{Cosh}[c + d*x] - 2*b^2*x^2*\operatorname{Cosh}[c - (a*d)/b] * \operatorname{CoshIntegral}[d*(a/b + x)] + x^2*\operatorname{CoshIntegral}[d*x]*((2*b^2 + a^2*d^2)*\operatorname{Cosh}[c] - 2*a*b*d*\operatorname{Sinh}[c]) - a^2*d*x*\operatorname{Sinh}[c + d*x] - 2*a*b*d*x^2*\operatorname{Cosh}[c]*\operatorname{SinhIntegral}[d*x] + 2*b^2*x^2*\operatorname{Sinh}[c]*\operatorname{SinhIntegral}[d*x] + a^2*d^2*x^2*\operatorname{Sinh}[c]*\operatorname{SinhIntegral}[d*x] - 2*b^2*x^2*\operatorname{Sinh}[c - (a*d)/b]*\operatorname{SinhIntegral}[d*(a/b + x)])/(2*a^3*x^2)$

fricas [A] time = 0.52, size = 275, normalized size = 1.45

$$2 a^2 dx \sinh(dx + c) - 2 (2 abx - a^2) \cosh(dx + c) - ((a^2 d^2 - 2 abd + 2 b^2) x^2 \operatorname{Ei}(dx) + (a^2 d^2 + 2 abd + 2 b^2) x^2) x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/x^3/(b*x+a),x, algorithm="fricas")

[Out] $-1/4*(2*a^2*d*x*\sinh(d*x + c) - 2*(2*a*b*x - a^2)*\cosh(d*x + c) - ((a^2*d^2 - 2*a*b*d + 2*b^2)*x^2*\operatorname{Ei}(d*x) + (a^2*d^2 + 2*a*b*d + 2*b^2)*x^2*\operatorname{Ei}(-d*x)) * \cosh(c) + 2*(b^2*x^2*\operatorname{Ei}((b*d*x + a*d)/b) + b^2*x^2*\operatorname{Ei}(-(b*d*x + a*d)/b)) * \cosh(-(b*c - a*d)/b) - ((a^2*d^2 - 2*a*b*d + 2*b^2)*x^2*\operatorname{Ei}(d*x) - (a^2*d^2 + 2*a*b*d + 2*b^2)*x^2*\operatorname{Ei}(-d*x))*\sinh(c) - 2*(b^2*x^2*\operatorname{Ei}((b*d*x + a*d)/b) - b^2*x^2*\operatorname{Ei}(-(b*d*x + a*d)/b))*\sinh(-(b*c - a*d)/b))/(a^3*x^2)$

giac [A] time = 0.14, size = 248, normalized size = 1.31

$$a^2 d^2 x^2 \operatorname{Ei}(-dx) e^{(-c)} + a^2 d^2 x^2 \operatorname{Ei}(dx) e^c + 2 abdx^2 \operatorname{Ei}(-dx) e^{(-c)} - 2 abdx^2 \operatorname{Ei}(dx) e^c + 2 b^2 x^2 \operatorname{Ei}(-dx) e^{(-c)} - 2 b^2 x^2 \operatorname{Ei}(dx) e^c$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/x^3/(b*x+a),x, algorithm="giac")

[Out] $1/4*(a^2*d^2*x^2*\operatorname{Ei}(-d*x)*e^{(-c)} + a^2*d^2*x^2*\operatorname{Ei}(d*x)*e^c + 2*a*b*d*x^2*\operatorname{Ei}(-d*x)*e^{(-c)} - 2*a*b*d*x^2*\operatorname{Ei}(d*x)*e^c + 2*b^2*x^2*\operatorname{Ei}(-d*x)*e^{(-c)} - 2*b^2*x^2*\operatorname{Ei}((b*d*x + a*d)/b)*e^{(c - a*d/b)} + 2*b^2*x^2*\operatorname{Ei}(d*x)*e^c - 2*b^2*x^2*\operatorname{Ei}(-(b*d*x + a*d)/b)*e^{(-c + a*d/b)} - a^2*d*x*e^{(d*x + c)} + a^2*d*x*e^{(-d*x - c)} + 2*a*b*x*e^{(d*x + c)} + 2*a*b*x*e^{(-d*x - c)} - a^2*e^{(d*x + c)} - a^2*e^{(-d*x - c)})/(a^3*x^2)$

maple [A] time = 0.10, size = 281, normalized size = 1.48

$$\frac{d e^{-dx-c}}{4ax} + \frac{e^{-dx-c} b}{2a^2 x} - \frac{e^{-dx-c}}{4a x^2} + \frac{b^2 e^{\frac{da-cb}{b}} \operatorname{Ei}\left(1, dx + c + \frac{da-cb}{b}\right)}{2a^3} - \frac{d^2 e^{-c} \operatorname{Ei}(1, dx)}{4a} - \frac{d e^{-c} \operatorname{Ei}(1, dx) b}{2a^2} - \frac{e^{-c} \operatorname{Ei}(1, dx) b^2}{2a^3} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cosh(d*x+c)/x^3/(b*x+a),x)`

[Out] $\frac{1}{4}d \exp(-d*x-c)/a/x + \frac{1}{2} \exp(-d*x-c)/a^2/x*b - \frac{1}{4} \exp(-d*x-c)/a/x^2 + \frac{1}{2} b^2/a^3 \exp((a*d-b*c)/b) * \text{Ei}(1, d*x+c+(a*d-b*c)/b) - \frac{1}{4} d^2/a \exp(-c) * \text{Ei}(1, d*x) - \frac{1}{2} d/a^2 \exp(-c) * \text{Ei}(1, d*x) * b - \frac{1}{2} a^3 \exp(-c) * \text{Ei}(1, d*x) * b^2 + \frac{1}{2} a^3 b^2 \exp(-(a*d-b*c)/b) * \text{Ei}(1, -d*x-c-(a*d-b*c)/b) + \frac{1}{2} b/a^2/x \exp(d*x+c) + \frac{1}{2} d*b/a^2 \exp(c) * \text{Ei}(1, -d*x) - \frac{1}{4} a/x^2 \exp(d*x+c) - \frac{1}{4} d/a/x \exp(d*x+c) - \frac{1}{4} d^2/a \exp(c) * \text{Ei}(1, -d*x) - \frac{1}{2} b^2/a^3 \exp(c) * \text{Ei}(1, -d*x)$

maxima [A] time = 0.48, size = 242, normalized size = 1.27

$$\frac{1}{4} d \left(\frac{d e^{(-c)} \Gamma(-1, dx) + d e^c \Gamma(-1, -dx)}{a} + \frac{2 (\text{Ei}(-dx) e^{(-c)} - \text{Ei}(dx) e^c) b}{a^2} + \frac{2 b^3 \left(\frac{e^{(-c+\frac{ad}{b})} \text{E}_1\left(\frac{(bx+a)d}{b}\right)}{b} + \frac{e^{(c-\frac{ad}{b})} \text{E}_1\left(-\frac{(bx+a)d}{b}\right)}{b} \right)}{a^3 d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(d*x+c)/x^3/(b*x+a),x, algorithm="maxima")`

[Out] $\frac{1}{4} d * ((d * e^{(-c)} * \text{gamma}(-1, d*x) + d * e^c * \text{gamma}(-1, -d*x)) / a + 2 * (\text{Ei}(-d*x) * e^{(-c)} - \text{Ei}(d*x) * e^c) * b / a^2 + 2 * b^3 * (e^{(-c + a*d/b)} * \text{exp_integral_e}(1, (b*x + a) * d / b) / b + e^{(c - a*d/b)} * \text{exp_integral_e}(1, -(b*x + a) * d / b) / b) / (a^3 * d) + 4 * b^2 * \cosh(d*x + c) * \log(b*x + a) / (a^3 * d) - 4 * b^2 * \cosh(d*x + c) * \log(x) / (a^3 * d) + 2 * (\text{Ei}(-d*x) * e^{(-c)} + \text{Ei}(d*x) * e^c) * b^2 / (a^3 * d)) - \frac{1}{2} * (2 * b^2 * \log(b*x + a) / a^3 - 2 * b^2 * \log(x) / a^3 - (2 * b * x - a) / (a^2 * x^2)) * \cosh(d*x + c)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c + dx)}{x^3 (a + bx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cosh(c + d*x)/(x^3*(a + b*x)),x)`

[Out] `int(cosh(c + d*x)/(x^3*(a + b*x)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(c + dx)}{x^3 (a + bx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x**3/(b*x+a),x)
```

```
[Out] Integral(cosh(c + d*x)/(x**3*(a + b*x)), x)
```

$$3.26 \quad \int \frac{x^4 \cosh(c+dx)}{(a+bx)^2} dx$$

Optimal. Leaf size=231

$$\frac{a^4 d \sinh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{b^6} + \frac{a^4 d \cosh\left(c - \frac{ad}{b}\right) \text{Shi}\left(xd + \frac{ad}{b}\right)}{b^6} - \frac{a^4 \cosh(c+dx)}{b^5(a+bx)} - \frac{4a^3 \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{b^5}$$

[Out] $-4*a^3*\text{Chi}(a*d/b+d*x)*\cosh(-c+a*d/b)/b^5+2*a*\cosh(d*x+c)/b^3/d^2-2*x*\cosh(d*x+c)/b^2/d^2-a^4*\cosh(d*x+c)/b^5/(b*x+a)+a^4*d*\cosh(-c+a*d/b)*\text{Shi}(a*d/b+d*x)/b^6-a^4*d*\text{Chi}(a*d/b+d*x)*\sinh(-c+a*d/b)/b^6+4*a^3*\text{Shi}(a*d/b+d*x)*\sinh(-c+a*d/b)/b^5+2*\sinh(d*x+c)/b^2/d^3+3*a^2*\sinh(d*x+c)/b^4/d-2*a*x*\sinh(d*x+c)/b^3/d+x^2*\sinh(d*x+c)/b^2/d$

Rubi [A] time = 0.54, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {6742, 2637, 3296, 2638, 3297, 3303, 3298, 3301}

$$\frac{a^4 d \sinh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{b^6} - \frac{4a^3 \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{b^5} - \frac{4a^3 \sinh\left(c - \frac{ad}{b}\right) \text{Shi}\left(xd + \frac{ad}{b}\right)}{b^5} + \frac{a^4 d \cosh\left(c - \frac{ad}{b}\right) \text{Shi}\left(xd + \frac{ad}{b}\right)}{b^5}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(x^4*\text{Cosh}[c + d*x])/(a + b*x)^2, x]$

[Out] $(2*a*\text{Cosh}[c + d*x])/(b^3*d^2) - (2*x*\text{Cosh}[c + d*x])/(b^2*d^2) - (a^4*\text{Cosh}[c + d*x])/(b^5*(a + b*x)) - (4*a^3*\text{Cosh}[c - (a*d)/b]*\text{CoshIntegral}[(a*d)/b + d*x])/b^5 + (a^4*d*\text{CoshIntegral}[(a*d)/b + d*x]*\text{Sinh}[c - (a*d)/b])/b^6 + (2*\text{Sinh}[c + d*x])/(b^2*d^3) + (3*a^2*\text{Sinh}[c + d*x])/(b^4*d) - (2*a*x*\text{Sinh}[c + d*x])/(b^3*d) + (x^2*\text{Sinh}[c + d*x])/(b^2*d) + (a^4*d*\text{Cosh}[c - (a*d)/b]*\text{SinhIntegral}[(a*d)/b + d*x])/b^6 - (4*a^3*\text{Sinh}[c - (a*d)/b]*\text{SinhIntegral}[(a*d)/b + d*x])/b^5$

Rule 2637

$\text{Int}[\sin[\text{Pi}/2 + (c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow \text{Simp}[\sin[c + d*x]/d, x] /;$
FreeQ[{c, d}, x]

Rule 2638

$\text{Int}[\sin[(c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow -\text{Simp}[\cos[c + d*x]/d, x] /;$ FreeQ[{c, d}, x]

Rule 3296


```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
+ d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:= Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:= Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4 \cosh(c + dx)}{(a + bx)^2} dx &= \int \left(\frac{3a^2 \cosh(c + dx)}{b^4} - \frac{2ax \cosh(c + dx)}{b^3} + \frac{x^2 \cosh(c + dx)}{b^2} + \frac{a^4 \cosh(c + dx)}{b^4(a + bx)^2} - \frac{4a^3 \cosh(c + dx)}{b^4} \right) dx \\
&= \frac{(3a^2) \int \cosh(c + dx) dx}{b^4} - \frac{(4a^3) \int \frac{\cosh(c+dx)}{a+bx} dx}{b^4} + \frac{a^4 \int \frac{\cosh(c+dx)}{(a+bx)^2} dx}{b^4} - \frac{(2a) \int x \cosh(c + dx) dx}{b^3} \\
&= -\frac{a^4 \cosh(c + dx)}{b^5(a + bx)} + \frac{3a^2 \sinh(c + dx)}{b^4 d} - \frac{2ax \sinh(c + dx)}{b^3 d} + \frac{x^2 \sinh(c + dx)}{b^2 d} + \frac{(2a) \int \sinh(c + dx) dx}{b^3} \\
&= \frac{2a \cosh(c + dx)}{b^3 d^2} - \frac{2x \cosh(c + dx)}{b^2 d^2} - \frac{a^4 \cosh(c + dx)}{b^5(a + bx)} - \frac{4a^3 \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(\frac{ad}{b} + dx\right)}{b^5} \\
&= \frac{2a \cosh(c + dx)}{b^3 d^2} - \frac{2x \cosh(c + dx)}{b^2 d^2} - \frac{a^4 \cosh(c + dx)}{b^5(a + bx)} - \frac{4a^3 \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(\frac{ad}{b} + dx\right)}{b^5}
\end{aligned}$$

Mathematica [A] time = 1.26, size = 173, normalized size = 0.75

$$\frac{a^3 \text{Chi}\left(d\left(\frac{a}{b} + x\right)\right) \left(ad \sinh\left(c - \frac{ad}{b}\right) - 4b \cosh\left(c - \frac{ad}{b}\right)\right) + a^3 \text{Shi}\left(d\left(\frac{a}{b} + x\right)\right) \left(ad \cosh\left(c - \frac{ad}{b}\right) - 4b \sinh\left(c - \frac{ad}{b}\right)\right)}{b^6}$$

Antiderivative was successfully verified.

[In] Integrate[(x^4*Cosh[c + d*x])/(a + b*x)^2,x]

[Out] $\left(-\left(\left(b(-2a^2b^2 + a^4d^2 + 2b^4x^2)\right) \text{Cosh}[c + d*x]\right) / \left(d^2(a + b*x)\right)\right) + a^3 \text{CoshIntegral}[d(a/b + x)] * \left(-4b \text{Cosh}[c - (a*d)/b] + a*d \text{Sinh}[c - (a*d)/b]\right) + \left(b^2(3a^2d^2 - 2a*b*d^2*x + b^2(2 + d^2*x^2)) \text{Sinh}[c + d*x]\right) / d^3 + a^3(a*d \text{Cosh}[c - (a*d)/b] - 4b \text{Sinh}[c - (a*d)/b]) * \text{SinhIntegral}[d(a/b + x)] / b^6$

fricas [A] time = 0.47, size = 371, normalized size = 1.61

$$\frac{2(a^4bd^3 + 2b^5dx^2 - 2a^2b^3d) \cosh(dx + c) - \left((a^5d^4 - 4a^4bd^3 + (a^4bd^4 - 4a^3b^2d^3)x\right) \text{Ei}\left(\frac{bdx+ad}{b}\right) - (a^5d^4 + 4a^4bd^3)}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*cosh(d*x+c)/(b*x+a)^2,x, algorithm="fricas")

[Out] $-1/2 * (2(a^4*b*d^3 + 2*b^5*d*x^2 - 2*a^2*b^3*d) * \cosh(d*x + c) - ((a^5*d^4 - 4*a^4*b*d^3 + (a^4*b*d^4 - 4*a^3*b^2*d^3)*x) * \text{Ei}((b*d*x + a*d)/b) - (a^5*d^4 + 4*a^4*b*d^3)) / b^6$

$$4 + 4*a^4*b*d^3 + (a^4*b*d^4 + 4*a^3*b^2*d^3)*x)*Ei(-(b*d*x + a*d)/b))*\cosh(-(b*c - a*d)/b) - 2*(b^5*d^2*x^3 - a*b^4*d^2*x^2 + 3*a^3*b^2*d^2 + 2*a*b^4 + (a^2*b^3*d^2 + 2*b^5)*x)*\sinh(d*x + c) + ((a^5*d^4 - 4*a^4*b*d^3 + (a^4*b*d^4 - 4*a^3*b^2*d^3)*x)*Ei((b*d*x + a*d)/b) + (a^5*d^4 + 4*a^4*b*d^3 + (a^4*b*d^4 + 4*a^3*b^2*d^3)*x)*Ei(-(b*d*x + a*d)/b))*\sinh(-(b*c - a*d)/b))/(b^7*d^3*x + a*b^6*d^3)$$

giac [B] time = 0.24, size = 2979, normalized size = 12.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*cosh(d*x+c)/(b*x+a)^2,x, algorithm="giac")

[Out] $\frac{1}{2}*((b*x + a)*a^4*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^4*Ei(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^{((b*c - a*d)/b) - a^4*b*c*d^4*Ei(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^{((b*c - a*d)/b) + a^5*d^5*Ei(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^{((b*c - a*d)/b) - (b*x + a)*a^4*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^4*Ei(-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^{-((b*c - a*d)/b) + a^4*b*c*d^4*Ei(-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^{-((b*c - a*d)/b) - a^5*d^5*Ei(-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^{-((b*c - a*d)/b) - 4*(b*x + a)*a^3*b*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^3*Ei(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^{((b*c - a*d)/b) + 4*a^3*b^2*c*d^3*Ei(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^{((b*c - a*d)/b) - 4*a^4*b*d^4*Ei(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^{((b*c - a*d)/b) - 4*(b*x + a)*a^3*b*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^3*Ei(-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^{-((b*c - a*d)/b) + 4*a^3*b^2*c*d^3*Ei(-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^{-((b*c - a*d)/b) - 4*a^4*b*d^4*Ei(-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b) - a^4*b*d^4*e^{((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) - a^4*b*d^4*e^{-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) + (b*x + a)^3*b^2*(b*c/(b*x + a) - a*d/(b*x + a) + d)^3*e^{((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) - 3*(b*x + a)^2*b^3*(b*c/(b*x + a) - a*d/(b*x + a) + d)^2*c*e^{((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) + 3*(b*x + a)*b^4*(b*c/(b*x + a) - a*d/(b*x + a) + d)*c^2*e^{((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) - a*d/(b*x + a) + d)/b) - b^5*c^3*e^{((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) - (b*x + a)^2*a*b^2*(b*c/(b*x + a) - a*d/(b*x + a) + d)^2*d*e^{((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) + 2*(b*x + a)*a*b^3*(b*c/(b*x + a) - a*d/(b*x + a) + d)*c*d*e^{((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) - a*b^4*c^2*d*e^{((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) + (b*x + a)*a^2*b^2*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^2*e^{((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) - a^2*b^3*c*d^2*e^{((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b)}}$

$$\begin{aligned}
& b*x + a) - a*d/(b*x + a) + d)/b) + 3*a^3*b^2*d^3*e^{((b*x + a)*(b*c/(b*x + a) \\
&) - a*d/(b*x + a) + d)/b) - (b*x + a)^3*b^2*(b*c/(b*x + a) - a*d/(b*x + a) \\
& + d)^3*e^{-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) + 3*(b*x + a)^2 \\
& *b^3*(b*c/(b*x + a) - a*d/(b*x + a) + d)^2*c*e^{-(b*x + a)*(b*c/(b*x + a) - \\
& a*d/(b*x + a) + d)/b) - 3*(b*x + a)*b^4*(b*c/(b*x + a) - a*d/(b*x + a) + d) \\
&)*c^2*e^{-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) + b^5*c^3*e^{-(b \\
& *x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) + (b*x + a)^2*a*b^2*(b*c/(b* \\
& x + a) - a*d/(b*x + a) + d)^2*d*e^{-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) \\
&) + d)/b) - 2*(b*x + a)*a*b^3*(b*c/(b*x + a) - a*d/(b*x + a) + d)*c*d*e^{-(\\
& b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) + a*b^4*c^2*d*e^{-(b*x + a) \\
& *(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) - (b*x + a)*a^2*b^2*(b*c/(b*x + a) \\
& - a*d/(b*x + a) + d)*d^2*e^{-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/ \\
& b) + a^2*b^3*c*d^2*e^{-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) - 3 \\
& *a^3*b^2*d^3*e^{-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) - 2*(b*x \\
& + a)^2*b^3*(b*c/(b*x + a) - a*d/(b*x + a) + d)^2*e^{((b*x + a)*(b*c/(b*x + a) \\
&) - a*d/(b*x + a) + d)/b) + 4*(b*x + a)*b^4*(b*c/(b*x + a) - a*d/(b*x + a) \\
& + d)*c*e^{((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) - 2*b^5*c^2*e^{((\\
& b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) + 2*a^2*b^3*d^2*e^{((b*x + a) \\
&)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) - 2*(b*x + a)^2*b^3*(b*c/(b*x + a) \\
& - a*d/(b*x + a) + d)^2*e^{-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b} \\
&) + 4*(b*x + a)*b^4*(b*c/(b*x + a) - a*d/(b*x + a) + d)*c*e^{-(b*x + a)*(b* \\
& c/(b*x + a) - a*d/(b*x + a) + d)/b) - 2*b^5*c^2*e^{-(b*x + a)*(b*c/(b*x + a) \\
&) - a*d/(b*x + a) + d)/b) + 2*a^2*b^3*d^2*e^{-(b*x + a)*(b*c/(b*x + a) - a* \\
& d/(b*x + a) + d)/b) + 2*(b*x + a)*b^4*(b*c/(b*x + a) - a*d/(b*x + a) + d)*e \\
& ^{((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) - 2*b^5*c*e^{((b*x + a)*(\\
& b*c/(b*x + a) - a*d/(b*x + a) + d)/b) + 2*a*b^4*d*e^{((b*x + a)*(b*c/(b*x + \\
& a) - a*d/(b*x + a) + d)/b) - 2*(b*x + a)*b^4*(b*c/(b*x + a) - a*d/(b*x + a) \\
& + d)*e^{-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) + 2*b^5*c*e^{-(b \\
& *x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) - 2*a*b^4*d*e^{-(b*x + a)*(b \\
& *c/(b*x + a) - a*d/(b*x + a) + d)/b))*b^2/(((b*x + a)*b^8*(b*c/(b*x + a) - \\
& a*d/(b*x + a) + d)*d^2 - b^9*c*d^2 + a*b^8*d^3)*d)
\end{aligned}$$

maple [A] time = 0.21, size = 431, normalized size = 1.87

$$-\frac{e^{-dx-c}x}{d^2b^2} + \frac{e^{-dx-c}a}{d^2b^3} - \frac{e^{-dx-c}}{d^3b^2} - \frac{e^{-dx-c}x^2}{2db^2} - \frac{3e^{-dx-c}a^2}{2db^4} + \frac{de^{\frac{da-cb}{b}} \operatorname{Ei}\left(1, dx + c + \frac{da-cb}{b}\right) a^4}{2b^6} + \frac{2e^{\frac{da-cb}{b}} \operatorname{Ei}\left(1, dx + c + \frac{da-cb}{b}\right)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4*cosh(d*x+c)/(b*x+a)^2,x)

[Out] $-1/d^2*\exp(-d*x-c)/b^2*x+1/d^2*\exp(-d*x-c)/b^3*a-1/d^3*\exp(-d*x-c)/b^2-1/2/d*\exp(-d*x-c)/b^2*x^2-3/2/d*\exp(-d*x-c)/b^4*a^2+1/2*d/b^6*\exp((a*d-b*c)/b)*\operatorname{Ei}(1,d*x+c+(a*d-b*c)/b)*a^4+2/b^5*\exp((a*d-b*c)/b)*\operatorname{Ei}(1,d*x+c+(a*d-b*c)/b)*$

$a^3 - \frac{1}{2}d \exp(-dx-c)/b^5 / (b dx + a d) * a^4 + \frac{1}{d} \exp(-dx-c)/b^3 * a * \exp(dx+c) * x^2 / b^5 * \exp(-(a d - b c)/b) * Ei(1, -dx-c-(a d - b c)/b) * a^3 - \frac{1}{2}d / b^6 * \exp(dx+c) / (a d / b + d x) * a^4 - \frac{1}{2}d / b^6 * \exp(-(a d - b c)/b) * Ei(1, -dx-c-(a d - b c)/b) * a^4 + \frac{1}{d^2} / b^3 * a * \exp(dx+c) + \frac{1}{2} / d / b^2 * \exp(dx+c) * x^2 - \frac{1}{d^2} / b^2 * \exp(dx+c) * x + \frac{1}{d^3} / b^2 * \exp(dx+c) + \frac{3}{2} / d / b^4 * a^2 * \exp(dx+c)$

maxima [A] time = 0.42, size = 406, normalized size = 1.76

$$\frac{1}{6} \left(3 a^4 \left(\frac{e^{\left(-c + \frac{ad}{b}\right)} E_1\left(\frac{(bx+a)d}{b}\right)} - e^{\left(c - \frac{ad}{b}\right)} E_1\left(-\frac{(bx+a)d}{b}\right)} \right) + \frac{12 a^3 \left(\frac{e^{\left(-c + \frac{ad}{b}\right)} E_1\left(\frac{(bx+a)d}{b}\right)} + \frac{e^{\left(c - \frac{ad}{b}\right)} E_1\left(-\frac{(bx+a)d}{b}\right)} \right)}{b^4 d} - \frac{9 a^2 \left(\frac{d x e^{-c} - e^c}{d^2} \right)}{d^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*cosh(dx+c)/(b*x+a)^2,x, algorithm="maxima")

[Out] 1/6*(3*a^4*(e^(-c + a*d/b)*exp_integral_e(1, (b*x + a)*d/b)/b^6 - e^(c - a*d/b)*exp_integral_e(1, -(b*x + a)*d/b)/b^6) + 12*a^3*(e^(-c + a*d/b)*exp_integral_e(1, (b*x + a)*d/b)/b + e^(c - a*d/b)*exp_integral_e(1, -(b*x + a)*d/b)/b)/(b^4*d) - 9*a^2*((d*x*e^c - e^c)*e^(d*x)/d^2 + (d*x + 1)*e^(-d*x - c)/d^2)/b^4 + 3*a*((d^2*x^2*e^c - 2*d*x*e^c + 2*e^c)*e^(d*x)/d^3 + (d^2*x^2 + 2*d*x + 2)*e^(-d*x - c)/d^3)/b^3 - ((d^3*x^3*e^c - 3*d^2*x^2*e^c + 6*d*x*e^c - 6*e^c)*e^(d*x)/d^4 + (d^3*x^3 + 3*d^2*x^2 + 6*d*x + 6)*e^(-d*x - c)/d^4)/b^2 + 24*a^3*cosh(d*x + c)*log(b*x + a)/(b^5*d)*d - 1/3*(3*a^4/(b^6*x + a*b^5) + 12*a^3*log(b*x + a)/b^5 - (b^2*x^3 - 3*a*b*x^2 + 9*a^2*x)/b^4)*cosh(d*x + c)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4 \cosh(c + dx)}{(a + bx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4*cosh(c + d*x))/(a + b*x)^2,x)

[Out] int((x^4*cosh(c + d*x))/(a + b*x)^2, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 \cosh(c + dx)}{(a + bx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*cosh(d*x+c)/(b*x+a)**2,x)
```

```
[Out] Integral(x**4*cosh(c + d*x)/(a + b*x)**2, x)
```

$$3.27 \quad \int \frac{x^3 \cosh(c+dx)}{(a+bx)^2} dx$$

Optimal. Leaf size=182

$$\frac{a^3 d \sinh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{b^5} - \frac{a^3 d \cosh\left(c - \frac{ad}{b}\right) \text{Shi}\left(xd + \frac{ad}{b}\right)}{b^5} + \frac{a^3 \cosh(c+dx)}{b^4(a+bx)} + \frac{3a^2 \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{b^4}$$

[Out] $3a^2 \text{Chi}(a*d/b+d*x) * \cosh(-c+a*d/b) / b^4 - \cosh(d*x+c) / b^2 / d^2 + a^3 \cosh(d*x+c) / b^4 / (b*x+a) - a^3*d*\cosh(-c+a*d/b) * \text{Shi}(a*d/b+d*x) / b^5 + a^3*d*\text{Chi}(a*d/b+d*x) * \sinh(-c+a*d/b) / b^5 - 3*a^2*\text{Shi}(a*d/b+d*x) * \sinh(-c+a*d/b) / b^4 - 2*a*\sinh(d*x+c) / b^3 / d + x*\sinh(d*x+c) / b^2 / d$

Rubi [A] time = 0.42, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {6742, 2637, 3296, 2638, 3297, 3303, 3298, 3301}

$$\frac{a^3 d \sinh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{b^5} + \frac{3a^2 \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{b^4} + \frac{3a^2 \sinh\left(c - \frac{ad}{b}\right) \text{Shi}\left(xd + \frac{ad}{b}\right)}{b^4} - \frac{a^3 d \cosh(c+dx)}{b^4(a+bx)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(x^3 * \text{Cosh}[c + d*x]) / (a + b*x)^2, x]$

[Out] $-(\text{Cosh}[c + d*x] / (b^2*d^2)) + (a^3*\text{Cosh}[c + d*x] / (b^4*(a + b*x))) + (3*a^2*\text{Cosh}[c - (a*d)/b] * \text{CoshIntegral}[(a*d)/b + d*x]) / b^4 - (a^3*d*\text{CoshIntegral}[(a*d)/b + d*x] * \text{Sinh}[c - (a*d)/b]) / b^5 - (2*a*\text{Sinh}[c + d*x]) / (b^3*d) + (x*\text{Sinh}[c + d*x]) / (b^2*d) - (a^3*d*\text{Cosh}[c - (a*d)/b] * \text{SinhIntegral}[(a*d)/b + d*x]) / b^5 + (3*a^2*\text{Sinh}[c - (a*d)/b] * \text{SinhIntegral}[(a*d)/b + d*x]) / b^4$

Rule 2637

$\text{Int}[\sin[\text{Pi}/2 + (c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow \text{Simp}[\sin[c + d*x] / d, x] /;$
FreeQ[{c, d}, x]

Rule 2638

$\text{Int}[\sin[(c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow -\text{Simp}[\cos[c + d*x] / d, x] /;$ FreeQ[{c, d}, x]

Rule 3296

$\text{Int}[(c_. + (d_.)*(x_.))^(m_.) * \sin[(e_.) + (f_.)*(x_.)], x_Symbol] \rightarrow -\text{Simp}[(c + d*x)^m * \cos[e + f*x] / f, x] + \text{Dist}[(d*m) / f, \text{Int}[(c + d*x)^(m-1) * \cos[e + f*x] / f, x]]$

$e + f*x], x], x] /; \text{FreeQ}\{c, d, e, f\}, x\} \&\& \text{GtQ}[m, 0]$

Rule 3297

$\text{Int}[(c_.) + (d_.)*(x_.))^{(m_.)}*\sin[(e_.) + (f_.)*(x_.)], x_Symbol] \rightarrow \text{Simp}[(c + d*x)^{(m + 1)}*\sin[e + f*x]/(d*(m + 1)), x] - \text{Dist}[f/(d*(m + 1)), \text{Int}[(c + d*x)^{(m + 1)}*\cos[e + f*x], x], x] /; \text{FreeQ}\{c, d, e, f\}, x\} \&\& \text{LtQ}[m, -1]$

Rule 3298

$\text{Int}[\sin[(e_.) + (\text{Complex}[0, fz_])*(f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x_Symbol] \rightarrow \text{Simp}[(I*\text{SinhIntegral}[(c*f*fz)/d + f*fz*x])/d, x] /; \text{FreeQ}\{c, d, e, f, fz\}, x\} \&\& \text{EqQ}[d*e - c*f*fz*I, 0]$

Rule 3301

$\text{Int}[\sin[(e_.) + (\text{Complex}[0, fz_])*(f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x_Symbol] \rightarrow \text{Simp}[\text{CoshIntegral}[(c*f*fz)/d + f*fz*x]/d, x] /; \text{FreeQ}\{c, d, e, f, fz\}, x\} \&\& \text{EqQ}[d*(e - \text{Pi}/2) - c*f*fz*I, 0]$

Rule 3303

$\text{Int}[\sin[(e_.) + (f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x_Symbol] \rightarrow \text{Dist}[\cos[(d*e - c*f)/d], \text{Int}[\sin[(c*f)/d + f*x]/(c + d*x), x], x] + \text{Dist}[\sin[(d*e - c*f)/d], \text{Int}[\cos[(c*f)/d + f*x]/(c + d*x), x], x] /; \text{FreeQ}\{c, d, e, f\}, x\} \&\& \text{NeQ}[d*e - c*f, 0]$

Rule 6742

$\text{Int}[u_, x_Symbol] \rightarrow \text{With}\{v = \text{ExpandIntegrand}[u, x]\}, \text{Int}[v, x] /; \text{SumQ}[v]$

Rubi steps

$$\begin{aligned}
\int \frac{x^3 \cosh(c + dx)}{(a + bx)^2} dx &= \int \left(-\frac{2a \cosh(c + dx)}{b^3} + \frac{x \cosh(c + dx)}{b^2} - \frac{a^3 \cosh(c + dx)}{b^3(a + bx)^2} + \frac{3a^2 \cosh(c + dx)}{b^3(a + bx)} \right) dx \\
&= -\frac{(2a) \int \cosh(c + dx) dx}{b^3} + \frac{(3a^2) \int \frac{\cosh(c+dx)}{a+bx} dx}{b^3} - \frac{a^3 \int \frac{\cosh(c+dx)}{(a+bx)^2} dx}{b^3} + \frac{\int x \cosh(c + dx)}{b^2} \\
&= \frac{a^3 \cosh(c + dx)}{b^4(a + bx)} - \frac{2a \sinh(c + dx)}{b^3 d} + \frac{x \sinh(c + dx)}{b^2 d} - \frac{\int \sinh(c + dx) dx}{b^2 d} - \frac{(a^3 d) \int \frac{\sinh(c + dx)}{a + bx} dx}{b^5} \\
&= -\frac{\cosh(c + dx)}{b^2 d^2} + \frac{a^3 \cosh(c + dx)}{b^4(a + bx)} + \frac{3a^2 \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(\frac{ad}{b} + dx\right)}{b^4} - \frac{2a \sinh(c + dx)}{b^3 d} \\
&= -\frac{\cosh(c + dx)}{b^2 d^2} + \frac{a^3 \cosh(c + dx)}{b^4(a + bx)} + \frac{3a^2 \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(\frac{ad}{b} + dx\right)}{b^4} - \frac{a^3 d \text{Chi}\left(\frac{ad}{b} + dx\right)}{b^5}
\end{aligned}$$

Mathematica [A] time = 0.96, size = 156, normalized size = 0.86

$$\frac{a^2 \text{Chi}\left(d\left(\frac{a}{b} + x\right)\right) \left(3b \cosh\left(c - \frac{ad}{b}\right) - ad \sinh\left(c - \frac{ad}{b}\right)\right) - a^2 \text{Shi}\left(d\left(\frac{a}{b} + x\right)\right) \left(ad \cosh\left(c - \frac{ad}{b}\right) - 3b \sinh\left(c - \frac{ad}{b}\right)\right)}{b^5}$$

Antiderivative was successfully verified.

[In] Integrate[(x^3*Cosh[c + d*x])/(a + b*x)^2,x]

[Out] (a^2*CoshIntegral[d*(a/b + x)]*(3*b*Cosh[c - (a*d)/b] - a*d*Sinh[c - (a*d)/b]) + (b*((-(a*b^2) + a^3*d^2 - b^3*x)*Cosh[c + d*x] + b*d*(-2*a^2 - a*b*x + b^2*x^2)*Sinh[c + d*x]))/(d^2*(a + b*x)) - a^2*(a*d*Cosh[c - (a*d)/b] - 3*b*Sinh[c - (a*d)/b])*SinhIntegral[d*(a/b + x)]/b^5

fricas [A] time = 0.54, size = 333, normalized size = 1.83

$$\frac{2(a^3 b d^2 - b^4 x - a b^3) \cosh(dx + c) - \left((a^4 d^3 - 3 a^3 b d^2 + (a^3 b d^3 - 3 a^2 b^2 d^2)x\right) \text{Ei}\left(\frac{b d x + a d}{b}\right) - (a^4 d^3 + 3 a^3 b d^2 + (a^3 b d^3 - 3 a^2 b^2 d^2)x) \text{Ei}\left(\frac{b d x + a d}{b}\right)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*cosh(d*x+c)/(b*x+a)^2,x, algorithm="fricas")

[Out] 1/2*(2*(a^3*b*d^2 - b^4*x - a*b^3)*cosh(d*x + c) - ((a^4*d^3 - 3*a^3*b*d^2 + (a^3*b*d^3 - 3*a^2*b^2*d^2)*x)*Ei((b*d*x + a*d)/b) - (a^4*d^3 + 3*a^3*b*d^2 + (a^3*b*d^3 - 3*a^2*b^2*d^2)*x)*Ei((b*d*x + a*d)/b))/b^5

$$\begin{aligned} &^2 + (a^3*b*d^3 + 3*a^2*b^2*d^2)*x)*Ei(-(b*d*x + a*d)/b))*\cosh(-(b*c - a*d) \\ &/b) + 2*(b^4*d*x^2 - a*b^3*d*x - 2*a^2*b^2*d)*\sinh(d*x + c) + ((a^4*d^3 - 3 \\ &*a^3*b*d^2 + (a^3*b*d^3 - 3*a^2*b^2*d^2)*x)*Ei((b*d*x + a*d)/b) + (a^4*d^3 \\ &+ 3*a^3*b*d^2 + (a^3*b*d^3 + 3*a^2*b^2*d^2)*x)*Ei(-(b*d*x + a*d)/b))*\sinh(- \\ &(b*c - a*d)/b))/(b^6*d^2*x + a*b^5*d^2) \end{aligned}$$

giac [B] time = 0.21, size = 1991, normalized size = 10.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*cosh(d*x+c)/(b*x+a)^2,x, algorithm="giac")

[Out]
$$\begin{aligned} &-1/2*((b*x + a)*a^3*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^3*Ei(((b*x + a)*(\\ &b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^{((b*c - a*d)/b)} - a^3* \\ &b*c*d^3*Ei(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e \\ &^{((b*c - a*d)/b)} + a^4*d^4*Ei(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d \\ &) - b*c + a*d)/b)*e^{((b*c - a*d)/b)} - (b*x + a)*a^3*(b*c/(b*x + a) - a*d/(b \\ &*x + a) + d)*d^3*Ei(-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + \\ &a*d)/b)*e^{-((b*c - a*d)/b)} + a^3*b*c*d^3*Ei(-((b*x + a)*(b*c/(b*x + a) - a \\ &*d/(b*x + a) + d) - b*c + a*d)/b)*e^{-((b*c - a*d)/b)} - a^4*d^4*Ei(-((b*x + \\ &a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^{-((b*c - a*d)/b)} - \\ &3*(b*x + a)*a^2*b*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^2*Ei(((b*x + a)*(b \\ &c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^{((b*c - a*d)/b)} + 3*a^2 \\ &*b^2*c*d^2*Ei(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b \\ &)*e^{((b*c - a*d)/b)} - 3*a^3*b*d^3*Ei(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + \\ &a) + d) - b*c + a*d)/b)*e^{((b*c - a*d)/b)} - 3*(b*x + a)*a^2*b*(b*c/(b*x + \\ &a) - a*d/(b*x + a) + d)*d^2*Ei(-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + \\ &d) - b*c + a*d)/b)*e^{-((b*c - a*d)/b)} + 3*a^2*b^2*c*d^2*Ei(-((b*x + a)*(b* \\ &c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^{-((b*c - a*d)/b)} - 3*a^3 \\ &*b*d^3*Ei(-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e \\ &^{-((b*c - a*d)/b)} - a^3*b*d^3*e^{((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + \\ &d)/b)} - a^3*b*d^3*e^{-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b)} - (\\ &b*x + a)^2*b^2*(b*c/(b*x + a) - a*d/(b*x + a) + d)^2*e^{((b*x + a)*(b*c/(b*x \\ &+ a) - a*d/(b*x + a) + d)/b)} + 2*(b*x + a)*b^3*(b*c/(b*x + a) - a*d/(b*x + \\ &a) + d)*c*e^{((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b)} - b^4*c^2*e^{ \\ &((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b)} + (b*x + a)*a*b^2*(b*c/(b \\ &*x + a) - a*d/(b*x + a) + d)*d*e^{((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) \\ &+ d)/b)} - a*b^3*c*d*e^{((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b)} + 2 \\ &*a^2*b^2*d^2*e^{((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b)} + (b*x + a \\ &)^2*b^2*(b*c/(b*x + a) - a*d/(b*x + a) + d)^2*e^{-((b*x + a)*(b*c/(b*x + a) \\ &- a*d/(b*x + a) + d)/b)} - 2*(b*x + a)*b^3*(b*c/(b*x + a) - a*d/(b*x + a) + \\ &d)*c*e^{-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b)} + b^4*c^2*e^{-((b \\ &x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b)} - (b*x + a)*a*b^2*(b*c/(b*x + \\ &a) - a*d/(b*x + a) + d)*d*e^{-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d} \end{aligned}$$

)/b) + a*b^3*c*d*e^(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) - 2*a^2*b^2*d^2*e^(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) + (b*x + a)*b^3*(b*c/(b*x + a) - a*d/(b*x + a) + d)*e^((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) - b^4*c*e^((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) + a*b^3*d*e^((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) + (b*x + a)*b^3*(b*c/(b*x + a) - a*d/(b*x + a) + d)*e^(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) - b^4*c*e^(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) + a*b^3*d*e^(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b))*b^2/((b*x + a)*b^7*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d - b^8*c*d + a*b^7*d^2)*d)

maple [A] time = 0.15, size = 325, normalized size = 1.79

$$\frac{e^{-dx-c}x}{2db^2} + \frac{e^{-dx-c}a}{db^3} - \frac{e^{-dx-c}}{2d^2b^2} - \frac{de^{\frac{da-cb}{b}} \operatorname{Ei}\left(1, dx + c + \frac{da-cb}{b}\right) a^3}{2b^5} - \frac{3e^{\frac{da-cb}{b}} \operatorname{Ei}\left(1, dx + c + \frac{da-cb}{b}\right) a^2}{2b^4} + \frac{de^{-dx-c}a^3}{2b^4(bdx + da)} +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*cosh(d*x+c)/(b*x+a)^2,x)

[Out] -1/2/d*exp(-d*x-c)/b^2*x+1/d*exp(-d*x-c)/b^3*a-1/2/d^2*exp(-d*x-c)/b^2-1/2*d/b^5*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)*a^3-3/2/b^4*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)*a^2+1/2*d*exp(-d*x-c)/b^4/(b*d*x+a*d)*a^3+1/2/d/b^2*exp(d*x+c)*x-3/2/b^4*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)*a^2-1/d/b^3*a*exp(d*x+c)-1/2/d^2/b^2*exp(d*x+c)+1/2*d/b^5*exp(d*x+c)/(a*d/b+d*x)*a^3+1/2*d/b^5*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)*a^3

maxima [A] time = 0.42, size = 312, normalized size = 1.71

$$-\frac{1}{4} \left(2a^3 \left(\frac{e^{\left(-c+\frac{ad}{b}\right)} E_1\left(\frac{(bx+a)d}{b}\right)} - \frac{e^{\left(c-\frac{ad}{b}\right)} E_1\left(-\frac{(bx+a)d}{b}\right)} \right) + \frac{6a^2 \left(\frac{e^{\left(-c+\frac{ad}{b}\right)} E_1\left(\frac{(bx+a)d}{b}\right)} + \frac{e^{\left(c-\frac{ad}{b}\right)} E_1\left(-\frac{(bx+a)d}{b}\right)} \right)}{b^3 d} - \frac{4a \left(\frac{dxe^c - e^c}{d^2} \right)}{d^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*cosh(d*x+c)/(b*x+a)^2,x, algorithm="maxima")

[Out] -1/4*(2*a^3*(e^(-c + a*d/b)*exp_integral_e(1, (b*x + a)*d/b)/b^5 - e^(c - a*d/b)*exp_integral_e(1, -(b*x + a)*d/b)/b^5) + 6*a^2*(e^(-c + a*d/b)*exp_integral_e(1, (b*x + a)*d/b)/b + e^(c - a*d/b)*exp_integral_e(1, -(b*x + a)*d/b)/b)/(b^3*d) - 4*a*((d*x*e^c - e^c)*e^(d*x)/d^2 + (d*x + 1)*e^(-d*x - c)/

$d^2)/b^3 + ((d^2*x^2*e^c - 2*d*x*e^c + 2*e^c)*e^{(d*x)}/d^3 + (d^2*x^2 + 2*d*x + 2)*e^{(-d*x - c)}/d^3)/b^2 + 12*a^2*cosh(d*x + c)*log(b*x + a)/(b^4*d)*d + 1/2*(2*a^3/(b^5*x + a*b^4) + 6*a^2*log(b*x + a)/b^4 + (b*x^2 - 4*a*x)/b^3)*cosh(d*x + c)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3 \cosh(c + dx)}{(a + bx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3*cosh(c + d*x))/(a + b*x)^2,x)

[Out] int((x^3*cosh(c + d*x))/(a + b*x)^2, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \cosh(c + dx)}{(a + bx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*cosh(d*x+c)/(b*x+a)**2,x)

[Out] Integral(x**3*cosh(c + d*x)/(a + b*x)**2, x)

$$3.28 \quad \int \frac{x^2 \cosh(c+dx)}{(a+bx)^2} dx$$

Optimal. Leaf size=147

$$\frac{a^2 d \sinh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{b^4} + \frac{a^2 d \cosh\left(c - \frac{ad}{b}\right) \text{Shi}\left(xd + \frac{ad}{b}\right)}{b^4} - \frac{a^2 \cosh(c+dx)}{b^3(a+bx)} - \frac{2a \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{b^3}$$

[Out] $-2*a*\text{Chi}(a*d/b+d*x)*\cosh(-c+a*d/b)/b^3-a^2*\cosh(d*x+c)/b^3/(b*x+a)+a^2*d*\cosh(-c+a*d/b)*\text{Shi}(a*d/b+d*x)/b^4-a^2*d*\text{Chi}(a*d/b+d*x)*\sinh(-c+a*d/b)/b^4+2*a*\text{Shi}(a*d/b+d*x)*\sinh(-c+a*d/b)/b^3+\sinh(d*x+c)/b^2/d$

Rubi [A] time = 0.37, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {6742, 2637, 3297, 3303, 3298, 3301}

$$\frac{a^2 d \sinh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{b^4} + \frac{a^2 d \cosh\left(c - \frac{ad}{b}\right) \text{Shi}\left(xd + \frac{ad}{b}\right)}{b^4} - \frac{a^2 \cosh(c+dx)}{b^3(a+bx)} - \frac{2a \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{b^3}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(x^2*\text{Cosh}[c + d*x])/(a + b*x)^2, x]$

[Out] $-((a^2*\text{Cosh}[c + d*x])/(b^3*(a + b*x))) - (2*a*\text{Cosh}[c - (a*d)/b]*\text{CoshIntegral}[(a*d)/b + d*x])/b^3 + (a^2*d*\text{CoshIntegral}[(a*d)/b + d*x]*\text{Sinh}[c - (a*d)/b])/b^4 + \text{Sinh}[c + d*x]/(b^2*d) + (a^2*d*\text{Cosh}[c - (a*d)/b]*\text{SinhIntegral}[(a*d)/b + d*x])/b^4 - (2*a*\text{Sinh}[c - (a*d)/b]*\text{SinhIntegral}[(a*d)/b + d*x])/b^3$

Rule 2637

$\text{Int}[\sin[\text{Pi}/2 + (c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow \text{Simp}[\sin[c + d*x]/d, x] /;$
FreeQ[{c, d}, x]

Rule 3297

$\text{Int}[((c_.) + (d_.)*(x_.))^{(m_.)}*\sin[(e_.) + (f_.)*(x_.)], x_Symbol] \rightarrow \text{Simp}[((c + d*x)^{(m+1)}*\sin[e + f*x])/(d*(m+1)), x] - \text{Dist}[f/(d*(m+1)), \text{Int}[(c + d*x)^{(m+1)}*\cos[e + f*x], x], x] /;$ FreeQ[{c, d, e, f}, x] && LtQ[m, -1]

Rule 3298

$\text{Int}[\sin[(e_.) + (\text{Complex}[0, fz_])*(f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x_Symbol] \rightarrow \text{Simp}[(I*\text{SinhIntegral}[(c*f+fz)/d + f*fz*x])/d, x] /;$ FreeQ[{c, d, e, f}

, fz}, x] && EqQ[d*e - c*f*fz*I, 0]

Rule 3301

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]

Rule 3303

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]

Rule 6742

Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

Rubi steps

$$\begin{aligned}
 \int \frac{x^2 \cosh(c + dx)}{(a + bx)^2} dx &= \int \left(\frac{\cosh(c + dx)}{b^2} + \frac{a^2 \cosh(c + dx)}{b^2(a + bx)^2} - \frac{2a \cosh(c + dx)}{b^2(a + bx)} \right) dx \\
 &= \frac{\int \cosh(c + dx) dx}{b^2} - \frac{(2a) \int \frac{\cosh(c+dx)}{a+bx} dx}{b^2} + \frac{a^2 \int \frac{\cosh(c+dx)}{(a+bx)^2} dx}{b^2} \\
 &= -\frac{a^2 \cosh(c + dx)}{b^3(a + bx)} + \frac{\sinh(c + dx)}{b^2 d} + \frac{(a^2 d) \int \frac{\sinh(c+dx)}{a+bx} dx}{b^3} - \frac{\left(2a \cosh\left(c - \frac{ad}{b}\right)\right) \int \frac{\cosh\left(\frac{ad}{b} + dx\right)}{a+bx} dx}{b^2} \\
 &= -\frac{a^2 \cosh(c + dx)}{b^3(a + bx)} - \frac{2a \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(\frac{ad}{b} + dx\right)}{b^3} + \frac{\sinh(c + dx)}{b^2 d} - \frac{2a \sinh\left(c - \frac{ad}{b}\right) \text{Shi}\left(\frac{ad}{b} + dx\right)}{b^3} \\
 &= -\frac{a^2 \cosh(c + dx)}{b^3(a + bx)} - \frac{2a \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(\frac{ad}{b} + dx\right)}{b^3} + \frac{a^2 d \text{Chi}\left(\frac{ad}{b} + dx\right) \sinh\left(c - \frac{ad}{b}\right)}{b^4} + \dots
 \end{aligned}$$

Mathematica [A] time = 0.71, size = 115, normalized size = 0.78

$$\frac{b \left(\frac{b \sinh(c+dx)}{d} - \frac{a^2 \cosh(c+dx)}{a+bx} \right) + a \text{Chi} \left(d \left(\frac{a}{b} + x \right) \right) \left(ad \sinh \left(c - \frac{ad}{b} \right) - 2b \cosh \left(c - \frac{ad}{b} \right) \right) + a \text{Shi} \left(d \left(\frac{a}{b} + x \right) \right) \left(ad \cosh \left(c - \frac{ad}{b} \right) - 2b \sinh \left(c - \frac{ad}{b} \right) \right)}{b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Cosh[c + d*x])/(a + b*x)^2,x]
```

```
[Out] (a*CoshIntegral[d*(a/b + x)]*(-2*b*Cosh[c - (a*d)/b] + a*d*Sinh[c - (a*d)/b]) + b*(-((a^2*Cosh[c + d*x])/(a + b*x)) + (b*Sinh[c + d*x])/d) + a*(a*d*Cosh[c - (a*d)/b] - 2*b*Sinh[c - (a*d)/b])*SinhIntegral[d*(a/b + x)]/b^4
```

fricas [A] time = 0.47, size = 274, normalized size = 1.86

$$2a^2bd \cosh(dx + c) - \left((a^3d^2 - 2a^2bd + (a^2bd^2 - 2ab^2d)x \right) \operatorname{Ei}\left(\frac{bdx+ad}{b}\right) - (a^3d^2 + 2a^2bd + (a^2bd^2 + 2ab^2d)x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(d*x+c)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^2*b*d*cosh(d*x + c) - ((a^3*d^2 - 2*a^2*b*d + (a^2*b*d^2 - 2*a*b^2*d)*x)*Ei((b*d*x + a*d)/b) - (a^3*d^2 + 2*a^2*b*d + (a^2*b*d^2 + 2*a*b^2*d)*x)*Ei(-(b*d*x + a*d)/b))*cosh(-(b*c - a*d)/b) - 2*(b^3*x + a*b^2)*sinh(d*x + c) + ((a^3*d^2 - 2*a^2*b*d + (a^2*b*d^2 - 2*a*b^2*d)*x)*Ei((b*d*x + a*d)/b) + (a^3*d^2 + 2*a^2*b*d + (a^2*b*d^2 + 2*a*b^2*d)*x)*Ei(-(b*d*x + a*d)/b))*sinh(-(b*c - a*d)/b))/(b^5*d*x + a*b^4*d)
```

giac [B] time = 0.18, size = 1308, normalized size = 8.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(d*x+c)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*((b*x + a)*a^2*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^2*Ei(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^((b*c - a*d)/b) - a^2*b*c*d^2*Ei(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^((b*c - a*d)/b) + a^3*d^3*Ei(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^((b*c - a*d)/b) - (b*x + a)*a^2*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^2*Ei(-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^(-(b*c - a*d)/b) + a^2*b*c*d^2*Ei(-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^(-(b*c - a*d)/b) - a^3*d^3*Ei(-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^(-(b*c - a*d)/b) - 2*(b*x + a)*a*b*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d*Ei(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^((b*c - a*d)/b) + 2*a*b^2*c*d*Ei(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^((b*c - a*d)/b) - 2*a^2*b*d^2*Ei(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^((b*c - a*d)/b) - 2*(b*x + a)*a*b*(b*c/(b*x + a) - a*d/(b
```

$x + a) + d) * d * \text{Ei}(-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b) * e^{-(b*c - a*d)/b} + 2*a*b^2*c*d * \text{Ei}(-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b) * e^{-(b*c - a*d)/b} - 2*a^2*b*d^2 * \text{Ei}(-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b) * e^{-(b*c - a*d)/b} - a^2*b*d^2 * e^{((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b)} - a^2*b*d^2 * e^{-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b} + (b*x + a)*b^2*(b*c/(b*x + a) - a*d/(b*x + a) + d) * e^{((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b)} - b^3*c * e^{((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b)} + a*b^2*d * e^{((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b)} - (b*x + a)*b^2*(b*c/(b*x + a) - a*d/(b*x + a) + d) * e^{-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b} + b^3*c * e^{-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b} - a*b^2*d * e^{-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b} * b^2 / (((b*x + a)*b^6*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b^7*c + a*b^6*d) * d)$

maple [A] time = 0.12, size = 254, normalized size = 1.73

$$\frac{e^{-dx-c}}{2db^2} - \frac{de^{-dx-c}a^2}{2b^3(bdx+da)} + \frac{de^{\frac{da-cb}{b}} \text{Ei}\left(1, dx+c+\frac{da-cb}{b}\right) a^2}{2b^4} + \frac{e^{\frac{da-cb}{b}} \text{Ei}\left(1, dx+c+\frac{da-cb}{b}\right) a}{b^3} + \frac{e^{dx+c}}{2db^2} + \frac{e^{-\frac{da-cb}{b}} \text{Ei}\left(1, -dx-c-\frac{da-cb}{b}\right) a^2}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*cosh(d*x+c)/(b*x+a)^2,x)

[Out] $-1/2/d*\exp(-d*x-c)/b^2 - 1/2*d*\exp(-d*x-c)/b^3/(b*d*x+a*d)*a^2 + 1/2*d/b^4*\exp((a*d-b*c)/b)*\text{Ei}(1, d*x+c+(a*d-b*c)/b)*a^2 + 1/b^3*\exp((a*d-b*c)/b)*\text{Ei}(1, d*x+c+(a*d-b*c)/b)*a + 1/2/d/b^2*\exp(d*x+c) + 1/b^3*\exp(-(a*d-b*c)/b)*\text{Ei}(1, -d*x-c-(a*d-b*c)/b)*a - 1/2*d/b^4*\exp(d*x+c)/(a*d/b+d*x)*a^2 - 1/2*d/b^4*\exp(-(a*d-b*c)/b)*\text{Ei}(1, -d*x-c-(a*d-b*c)/b)*a^2$

maxima [A] time = 0.41, size = 236, normalized size = 1.61

$$\frac{1}{2} \left(a^2 \left(\frac{e^{(-c+\frac{ad}{b})} E_1\left(\frac{(bx+a)d}{b}\right)}{b^4} - \frac{e^{(c-\frac{ad}{b})} E_1\left(-\frac{(bx+a)d}{b}\right)}{b^4} \right) + \frac{2a \left(\frac{e^{(-c+\frac{ad}{b})} E_1\left(\frac{(bx+a)d}{b}\right)}{b} + \frac{e^{(c-\frac{ad}{b})} E_1\left(-\frac{(bx+a)d}{b}\right)}{b} \right)}{b^2 d} - \frac{(dxe^c - e^c)e^{(dx)}}{d^2} + \frac{(dx+c)e^{(dx+c)}}{b^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*cosh(d*x+c)/(b*x+a)^2,x, algorithm="maxima")

[Out] $1/2*(a^2*(e^{-c+a*d/b}*\text{exp_integral_e}(1, (b*x+a)*d/b)/b^4 - e^{(c-a*d/b)}*\text{exp_integral_e}(1, -(b*x+a)*d/b)/b^4) + 2*a*(e^{-c+a*d/b}*\text{exp_integral_e}(1, (b*x+a)*d/b)/b^3 - e^{(c-a*d/b)}*\text{exp_integral_e}(1, -(b*x+a)*d/b)/b^3) + \frac{(dxe^c - e^c)e^{(dx)}}{d^2} + \frac{(dx+c)e^{(dx+c)}}{b^2}$


```
l_e(1, (b*x + a)*d/b)/b + e^(c - a*d/b)*exp_integral_e(1, -(b*x + a)*d/b)/b
)/(b^2*d) - ((d*x*e^c - e^c)*e^(d*x)/d^2 + (d*x + 1)*e^(-d*x - c)/d^2)/b^2
+ 4*a*cosh(d*x + c)*log(b*x + a)/(b^3*d))*d - (a^2/(b^4*x + a*b^3) - x/b^2
+ 2*a*log(b*x + a)/b^3)*cosh(d*x + c)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2 \cosh(c + dx)}{(a + bx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*cosh(c + d*x))/(a + b*x)^2, x)
```

```
[Out] int((x^2*cosh(c + d*x))/(a + b*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \cosh(c + dx)}{(a + bx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cosh(d*x+c)/(b*x+a)**2, x)
```

```
[Out] Integral(x**2*cosh(c + d*x)/(a + b*x)**2, x)
```

$$3.29 \quad \int \frac{x \cosh(c+dx)}{(a+bx)^2} dx$$

Optimal. Leaf size=125

$$\frac{ad \sinh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{b^3} - \frac{ad \cosh\left(c - \frac{ad}{b}\right) \text{Shi}\left(xd + \frac{ad}{b}\right)}{b^3} + \frac{\cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{b^2} + \frac{\sinh\left(c - \frac{ad}{b}\right) \text{Shi}\left(xd + \frac{ad}{b}\right)}{b^2}$$

[Out] Chi(a*d/b+d*x)*cosh(-c+a*d/b)/b^2+a*cosh(d*x+c)/b^2/(b*x+a)-a*d*cosh(-c+a*d/b)*Shi(a*d/b+d*x)/b^3+a*d*Chi(a*d/b+d*x)*sinh(-c+a*d/b)/b^3-Shi(a*d/b+d*x)*sinh(-c+a*d/b)/b^2

Rubi [A] time = 0.30, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6742, 3297, 3303, 3298, 3301}

$$-\frac{ad \sinh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{b^3} + \frac{\cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{b^2} + \frac{\sinh\left(c - \frac{ad}{b}\right) \text{Shi}\left(xd + \frac{ad}{b}\right)}{b^2} - \frac{ad \cosh\left(c - \frac{ad}{b}\right) \text{Shi}\left(xd + \frac{ad}{b}\right)}{b^3}$$

Antiderivative was successfully verified.

[In] Int[(x*Cosh[c + d*x])/(a + b*x)^2,x]

[Out] (a*Cosh[c + d*x])/(b^2*(a + b*x)) + (Cosh[c - (a*d)/b]*CoshIntegral[(a*d)/b + d*x])/b^2 - (a*d*CoshIntegral[(a*d)/b + d*x]*Sinh[c - (a*d)/b])/b^3 - (a*d*Cosh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/b^3 + (Sinh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/b^2

Rule 3297

Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]

Rule 3298

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)])/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]

Rule 3301

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)])/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}

}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]

Rule 3303

Int[sin[(e_.) + (f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x_Symbol] :> Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]

Rule 6742

Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

Rubi steps

$$\begin{aligned}
 \int \frac{x \cosh(c + dx)}{(a + bx)^2} dx &= \int \left(-\frac{a \cosh(c + dx)}{b(a + bx)^2} + \frac{\cosh(c + dx)}{b(a + bx)} \right) dx \\
 &= \frac{\int \frac{\cosh(c+dx)}{a+bx} dx}{b} - \frac{a \int \frac{\cosh(c+dx)}{(a+bx)^2} dx}{b} \\
 &= \frac{a \cosh(c + dx)}{b^2(a + bx)} - \frac{(ad) \int \frac{\sinh(c+dx)}{a+bx} dx}{b^2} + \frac{\cosh\left(c - \frac{ad}{b}\right) \int \frac{\cosh\left(\frac{ad}{b} + dx\right)}{a+bx} dx}{b} + \frac{\sinh\left(c - \frac{ad}{b}\right) \int \frac{\sinh\left(\frac{ad}{b} + dx\right)}{a+bx} dx}{b} \\
 &= \frac{a \cosh(c + dx)}{b^2(a + bx)} + \frac{\cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(\frac{ad}{b} + dx\right)}{b^2} + \frac{\sinh\left(c - \frac{ad}{b}\right) \text{Shi}\left(\frac{ad}{b} + dx\right)}{b^2} - \frac{(ad \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(\frac{ad}{b} + dx\right) + ad \sinh\left(c - \frac{ad}{b}\right) \text{Shi}\left(\frac{ad}{b} + dx\right))}{b^3} \\
 &= \frac{a \cosh(c + dx)}{b^2(a + bx)} + \frac{\cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(\frac{ad}{b} + dx\right)}{b^2} - \frac{ad \text{Chi}\left(\frac{ad}{b} + dx\right) \sinh\left(c - \frac{ad}{b}\right)}{b^3} - \frac{ad \cosh\left(c - \frac{ad}{b}\right) \text{Shi}\left(\frac{ad}{b} + dx\right)}{b^3}
 \end{aligned}$$

Mathematica [A] time = 0.44, size = 97, normalized size = 0.78

$$\frac{\text{Chi}\left(d\left(\frac{a}{b} + x\right)\right) \left(b \cosh\left(c - \frac{ad}{b}\right) - ad \sinh\left(c - \frac{ad}{b}\right)\right) + \text{Shi}\left(d\left(\frac{a}{b} + x\right)\right) \left(b \sinh\left(c - \frac{ad}{b}\right) - ad \cosh\left(c - \frac{ad}{b}\right)\right) + \frac{ad \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(\frac{ad}{b} + dx\right) + ad \sinh\left(c - \frac{ad}{b}\right) \text{Shi}\left(\frac{ad}{b} + dx\right)}{b^3}$$

Antiderivative was successfully verified.

[In] Integrate[(x*Cosh[c + d*x])/(a + b*x)^2,x]

[Out] $((a*b*\text{Cosh}[c + d*x])/(a + b*x) + \text{CoshIntegral}[d*(a/b + x)]*(b*\text{Cosh}[c - (a*d)/b] - a*d*\text{Sinh}[c - (a*d)/b]) + (-a*d*\text{Cosh}[c - (a*d)/b] + b*\text{Sinh}[c - (a*d)/b])*\text{SinhIntegral}[d*(a/b + x)]/b^3$

fricas [A] time = 0.38, size = 200, normalized size = 1.60

$$\frac{2ab \cosh(dx + c) - \left((a^2d - ab + (abd - b^2)x) \text{Ei}\left(\frac{bdx+ad}{b}\right) - (a^2d + ab + (abd + b^2)x) \text{Ei}\left(-\frac{bdx+ad}{b}\right) \right) \cosh\left(-\frac{bc-ad}{b}\right)}{2(b^4x + ab^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*cosh(d*x+c)/(b*x+a)^2,x, algorithm="fricas")`

[Out] $1/2*(2*a*b*\cosh(d*x + c) - ((a^2*d - a*b + (a*b*d - b^2)*x)*\text{Ei}((b*d*x + a*d)/b) - (a^2*d + a*b + (a*b*d + b^2)*x)*\text{Ei}(-(b*d*x + a*d)/b))*\cosh(-(b*c - a*d)/b) + ((a^2*d - a*b + (a*b*d - b^2)*x)*\text{Ei}((b*d*x + a*d)/b) + (a^2*d + a*b + (a*b*d + b^2)*x)*\text{Ei}(-(b*d*x + a*d)/b))*\sinh(-(b*c - a*d)/b))/(b^4*x + a*b^3)$

giac [B] time = 0.19, size = 994, normalized size = 7.95

$$\left((bx + a) a \left(\frac{bc}{bx+a} - \frac{ad}{bx+a} + d \right) d^2 \text{Ei} \left(\frac{(bx+a) \left(\frac{bc}{bx+a} - \frac{ad}{bx+a} + d \right) - bc + ad}{b} \right) e^{\left(\frac{bc-ad}{b} \right)} - abcd^2 \text{Ei} \left(\frac{(bx+a) \left(\frac{bc}{bx+a} - \frac{ad}{bx+a} + d \right) - bc + ad}{b} \right) e^{\left(\frac{bc-ad}{b} \right)} + \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*cosh(d*x+c)/(b*x+a)^2,x, algorithm="giac")`

[Out] $-1/2*((b*x + a)*a*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^2*\text{Ei}(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^{((b*c - a*d)/b)} - a*b*c*d^2*\text{Ei}(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^{((b*c - a*d)/b)} + a^2*d^3*\text{Ei}(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^{((b*c - a*d)/b)} - (b*x + a)*a*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^2*\text{Ei}(-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^{-((b*c - a*d)/b)} + a*b*c*d^2*\text{Ei}(-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^{-((b*c - a*d)/b)} - a^2*d^3*\text{Ei}(-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^{-((b*c - a*d)/b)} - (b*x + a)*b*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d*\text{Ei}(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^{((b*c - a*d)/b)} + b^2*c*d*\text{Ei}(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^{((b*c - a*d)/b)} - a*b*d^2*\text{Ei}(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^{((b*c - a*d)/b)}$

$(b*c - a*d)/b) - (b*x + a)*b*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d*Ei(-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b) + b^2*c*d*Ei(-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^{-((b*c - a*d)/b) - a*b*d^2*Ei(-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^{-((b*c - a*d)/b) - a*b*d^2*e^{((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) - a*b*d^2*e^{-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b)}}*b/(((b*x + a)*b^4*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b^5*c + a*b^4*d)*d)$

maple [A] time = 0.10, size = 215, normalized size = 1.72

$$\frac{d e^{-dx-c} a}{2b^2 (bdx + da)} - \frac{d e^{\frac{da-cb}{b}} Ei\left(1, dx + c + \frac{da-cb}{b}\right) a}{2b^3} - \frac{e^{\frac{da-cb}{b}} Ei\left(1, dx + c + \frac{da-cb}{b}\right)}{2b^2} - \frac{e^{-\frac{da-cb}{b}} Ei\left(1, -dx - c - \frac{da-cb}{b}\right)}{2b^2} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*cosh(d*x+c)/(b*x+a)^2,x)

[Out] $1/2*d*exp(-d*x-c)/b^2/(b*d*x+a*d)*a-1/2*d/b^3*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)*a-1/2/b^2*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)-1/2/b^2*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)+1/2*d/b^3*exp(d*x+c)/(a*d/b+d*x)*a+1/2*d/b^3*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)*a$

maxima [A] time = 0.41, size = 178, normalized size = 1.42

$$-\frac{1}{2} \left(a \left(\frac{e^{(-c+\frac{ad}{b})} E_1\left(\frac{(bx+a)d}{b}\right)}{b^3} - \frac{e^{(c-\frac{ad}{b})} E_1\left(-\frac{(bx+a)d}{b}\right)}{b^3} \right) + \frac{e^{(-c+\frac{ad}{b})} E_1\left(\frac{(bx+a)d}{b}\right)}{bd} + \frac{e^{(c-\frac{ad}{b})} E_1\left(-\frac{(bx+a)d}{b}\right)}{bd} + \frac{2 \cosh(dx+c) \log}{b^2 d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cosh(d*x+c)/(b*x+a)^2,x, algorithm="maxima")

[Out] $-1/2*(a*(e^{-c+a*d/b}*exp_integral_e(1,(b*x+a)*d/b)/b^3 - e^{(c-a*d/b)}*exp_integral_e(1,-(b*x+a)*d/b)/b^3) + (e^{-c+a*d/b}*exp_integral_e(1,(b*x+a)*d/b)/b + e^{(c-a*d/b)}*exp_integral_e(1,-(b*x+a)*d/b)/b)/(b*d) + 2*cosh(d*x+c)*log(b*x+a)/(b^2*d))*d + (a/(b^3*x+a*b^2) + log(b*x+a)/b^2)*cosh(d*x+c)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x \cosh(c + dx)}{(a + bx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*cosh(c + d*x))/(a + b*x)^2,x)
```

```
[Out] int((x*cosh(c + d*x))/(a + b*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \cosh(c + dx)}{(a + bx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(d*x+c)/(b*x+a)**2,x)
```

```
[Out] Integral(x*cosh(c + d*x)/(a + b*x)**2, x)
```

3.30 $\int \frac{\cosh(c+dx)}{(a+bx)^2} dx$

Optimal. Leaf size=71

$$\frac{d \sinh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{b^2} + \frac{d \cosh\left(c - \frac{ad}{b}\right) \text{Shi}\left(xd + \frac{ad}{b}\right)}{b^2} - \frac{\cosh(c + dx)}{b(a + bx)}$$

[Out] $-\cosh(d*x+c)/b/(b*x+a)+d*\cosh(-c+a*d/b)*\text{Shi}(a*d/b+d*x)/b^2-d*\text{Chi}(a*d/b+d*x)*\sinh(-c+a*d/b)/b^2$

Rubi [A] time = 0.11, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3297, 3303, 3298, 3301}

$$\frac{d \sinh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{b^2} + \frac{d \cosh\left(c - \frac{ad}{b}\right) \text{Shi}\left(xd + \frac{ad}{b}\right)}{b^2} - \frac{\cosh(c + dx)}{b(a + bx)}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]/(a + b*x)^2, x]

[Out] $-(\text{Cosh}[c + d*x]/(b*(a + b*x))) + (d*\text{CoshIntegral}[(a*d)/b + d*x]*\text{Sinh}[c - (a*d)/b])/b^2 + (d*\text{Cosh}[c - (a*d)/b]*\text{SinhIntegral}[(a*d)/b + d*x])/b^2$

Rule 3297

Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]

Rule 3298

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)])/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]

Rule 3301

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)])/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{\cosh(c + dx)}{(a + bx)^2} dx &= -\frac{\cosh(c + dx)}{b(a + bx)} + \frac{d \int \frac{\sinh(c+dx)}{a+bx} dx}{b} \\ &= -\frac{\cosh(c + dx)}{b(a + bx)} + \frac{\left(d \cosh\left(c - \frac{ad}{b}\right)\right) \int \frac{\sinh\left(\frac{ad}{b} + dx\right)}{a+bx} dx}{b} + \frac{\left(d \sinh\left(c - \frac{ad}{b}\right)\right) \int \frac{\cosh\left(\frac{ad}{b} + dx\right)}{a+bx} dx}{b} \\ &= -\frac{\cosh(c + dx)}{b(a + bx)} + \frac{d \operatorname{Chi}\left(\frac{ad}{b} + dx\right) \sinh\left(c - \frac{ad}{b}\right)}{b^2} + \frac{d \cosh\left(c - \frac{ad}{b}\right) \operatorname{Shi}\left(\frac{ad}{b} + dx\right)}{b^2} \end{aligned}$$

Mathematica [A] time = 0.21, size = 65, normalized size = 0.92

$$\frac{d \sinh\left(c - \frac{ad}{b}\right) \operatorname{Chi}\left(d\left(\frac{a}{b} + x\right)\right) + d \cosh\left(c - \frac{ad}{b}\right) \operatorname{Shi}\left(d\left(\frac{a}{b} + x\right)\right) - \frac{b \cosh(c+dx)}{a+bx}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]/(a + b*x)^2, x]
```

```
[Out] (-((b*Cosh[c + d*x])/(a + b*x)) + d*CoshIntegral[d*(a/b + x)]*Sinh[c - (a*d
)/b] + d*Cosh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)])/b^2
```

fricas [B] time = 0.57, size = 149, normalized size = 2.10

$$\frac{2 b \cosh(dx + c) - \left((bdx + ad) \operatorname{Ei}\left(\frac{bdx+ad}{b}\right) - (bdx + ad) \operatorname{Ei}\left(-\frac{bdx+ad}{b}\right)\right) \cosh\left(-\frac{bc-ad}{b}\right) + \left((bdx + ad) \operatorname{Ei}\left(\frac{bdx+ad}{b}\right) + (bdx + ad) \operatorname{Ei}\left(-\frac{bdx+ad}{b}\right)\right) \cosh\left(\frac{bc-ad}{b}\right)}{2(b^3x + ab^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*cosh(d*x + c) - ((b*d*x + a*d)*Ei((b*d*x + a*d)/b) - (b*d*x + a*d
)*Ei(-(b*d*x + a*d)/b))*cosh(-(b*c - a*d)/b) + ((b*d*x + a*d)*Ei((b*d*x + a

```


$*d)/b) + (b*d*x + a*d)*Ei(-(b*d*x + a*d)/b))*sinh(-(b*c - a*d)/b))/(b^3*x + a*b^2)$

giac [B] time = 0.17, size = 615, normalized size = 8.66

$$\frac{\left((bx+a) \left(\frac{bc}{bx+a} - \frac{ad}{bx+a} + d \right) d^2 Ei \left(\frac{(bx+a) \left(\frac{bc}{bx+a} - \frac{ad}{bx+a} + d \right) - bc + ad}{b} \right) e^{\left(\frac{bc-ad}{b} \right)} - bcd^2 Ei \left(\frac{(bx+a) \left(\frac{bc}{bx+a} - \frac{ad}{bx+a} + d \right) - bc + ad}{b} \right) e^{\left(\frac{bc-ad}{b} \right)} + ac \right)}{2 \left((bx+a) b^4 \left(\frac{bc}{bx+a} - \frac{ad}{bx+a} + d \right) - b^5 c + ab^4 d \right) d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/(b*x+a)^2,x, algorithm="giac")

[Out] $\frac{1}{2} * ((b*x + a) * (b*c / (b*x + a) - a*d / (b*x + a) + d) * d^2 * Ei(((b*x + a) * (b*c / (b*x + a) - a*d / (b*x + a) + d) - b*c + a*d) / b) * e^{((b*c - a*d) / b)} - b*c * d^2 * Ei(((b*x + a) * (b*c / (b*x + a) - a*d / (b*x + a) + d) - b*c + a*d) / b) * e^{((b*c - a*d) / b)} + a*d^3 * Ei(((b*x + a) * (b*c / (b*x + a) - a*d / (b*x + a) + d) - b*c + a*d) / b) * e^{((b*c - a*d) / b)} - b*d^2 * e^{((b*x + a) * (b*c / (b*x + a) - a*d / (b*x + a) + d) / b)} * b^2 / (((b*x + a) * b^4 * (b*c / (b*x + a) - a*d / (b*x + a) + d) - b^5 * c + a * b^4 * d) * d) - 1/2 * ((b*x + a) * (b*c / (b*x + a) - a*d / (b*x + a) + d) * d^2 * Ei(-((b*x + a) * (b*c / (b*x + a) - a*d / (b*x + a) + d) - b*c + a*d) / b) * e^{-((b*c - a*d) / b)} - b*c * d^2 * Ei(-((b*x + a) * (b*c / (b*x + a) - a*d / (b*x + a) + d) - b*c + a*d) / b) * e^{-((b*c - a*d) / b)} + a*d^3 * Ei(-((b*x + a) * (b*c / (b*x + a) - a*d / (b*x + a) + d) - b*c + a*d) / b) * e^{-((b*c - a*d) / b)} + b*d^2 * e^{-((b*x + a) * (b*c / (b*x + a) - a*d / (b*x + a) + d) / b)} * b^2 / (((b*x + a) * b^4 * (b*c / (b*x + a) - a*d / (b*x + a) + d) - b^5 * c + a * b^4 * d) * d))$

maple [A] time = 0.08, size = 132, normalized size = 1.86

$$-\frac{d e^{-dx-c}}{2b(bdx+da)} + \frac{d e^{\frac{da-cb}{b}} Ei\left(1, dx+c+\frac{da-cb}{b}\right)}{2b^2} - \frac{d e^{dx+c}}{2b^2\left(\frac{ad}{b}+dx\right)} - \frac{d e^{-\frac{da-cb}{b}} Ei\left(1, -dx-c-\frac{da-cb}{b}\right)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d*x+c)/(b*x+a)^2,x)

[Out] $-1/2*d*exp(-d*x-c)/b/(b*d*x+a*d)+1/2*d/b^2*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)-1/2*d/b^2*exp(d*x+c)/(a*d/b+d*x)-1/2*d/b^2*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)$

maxima [A] time = 0.36, size = 81, normalized size = 1.14

$$\frac{d \left(\frac{e^{\left(-c + \frac{ad}{b}\right)} E_1\left(\frac{(bx+a)d}{b}\right)} - \frac{e^{\left(c - \frac{ad}{b}\right)} E_1\left(-\frac{(bx+a)d}{b}\right)}{b} \right)}{2b} - \frac{\cosh(dx + c)}{(bx + a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/(b*x+a)^2,x, algorithm="maxima")

[Out] 1/2*d*(e^(-c + a*d/b)*exp_integral_e(1, (b*x + a)*d/b)/b - e^(c - a*d/b)*exp_integral_e(1, -(b*x + a)*d/b)/b - cosh(d*x + c)/((b*x + a)*b)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c + dx)}{(a + bx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(c + d*x)/(a + b*x)^2,x)

[Out] int(cosh(c + d*x)/(a + b*x)^2, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/(b*x+a)**2,x)

[Out] Timed out

3.31 $\int \frac{\cosh(c+dx)}{x(a+bx)^2} dx$

Optimal. Leaf size=150

$$\frac{\cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right) - \sinh\left(c - \frac{ad}{b}\right) \text{Shi}\left(xd + \frac{ad}{b}\right)}{a^2} + \frac{\cosh(c) \text{Chi}(dx)}{a^2} + \frac{\sinh(c) \text{Shi}(dx)}{a^2} - \frac{d \sinh\left(c - \frac{ad}{b}\right)}{ab}$$

[Out] Chi(d*x)*cosh(c)/a^2-Chi(a*d/b+d*x)*cosh(-c+a*d/b)/a^2+cosh(d*x+c)/a/(b*x+a)-d*cosh(-c+a*d/b)*Shi(a*d/b+d*x)/a/b+Shi(d*x)*sinh(c)/a^2+d*Chi(a*d/b+d*x)*sinh(-c+a*d/b)/a/b+Shi(a*d/b+d*x)*sinh(-c+a*d/b)/a^2

Rubi [A] time = 0.40, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {6742, 3303, 3298, 3301, 3297}

$$\frac{\cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right) - \sinh\left(c - \frac{ad}{b}\right) \text{Shi}\left(xd + \frac{ad}{b}\right)}{a^2} + \frac{\cosh(c) \text{Chi}(dx)}{a^2} + \frac{\sinh(c) \text{Shi}(dx)}{a^2} - \frac{d \sinh\left(c - \frac{ad}{b}\right)}{ab}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]/(x*(a + b*x)^2), x]

[Out] Cosh[c + d*x]/(a*(a + b*x)) + (Cosh[c]*CoshIntegral[d*x])/a^2 - (Cosh[c - (a*d)/b]*CoshIntegral[(a*d)/b + d*x])/a^2 - (d*CoshIntegral[(a*d)/b + d*x]*Sinh[c - (a*d)/b])/(a*b) + (Sinh[c]*SinhIntegral[d*x])/a^2 - (d*Cosh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/(a*b) - (Sinh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/a^2

Rule 3297

Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]

Rule 3298

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)])/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x]
&& EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x]
+ Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /;
FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]
```

Rule 6742

```
Int[u_, x_Symbol]
:> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh(c+dx)}{x(a+bx)^2} dx &= \int \left(\frac{\cosh(c+dx)}{a^2 x} - \frac{b \cosh(c+dx)}{a(a+bx)^2} - \frac{b \cosh(c+dx)}{a^2(a+bx)} \right) dx \\
&= \frac{\int \frac{\cosh(c+dx)}{x} dx}{a^2} - \frac{b \int \frac{\cosh(c+dx)}{a+bx} dx}{a^2} - \frac{b \int \frac{\cosh(c+dx)}{(a+bx)^2} dx}{a} \\
&= \frac{\cosh(c+dx)}{a(a+bx)} - \frac{d \int \frac{\sinh(c+dx)}{a+bx} dx}{a} + \frac{\cosh(c) \int \frac{\cosh(dx)}{x} dx}{a^2} - \frac{\left(b \cosh\left(c - \frac{ad}{b}\right) \right) \int \frac{\cosh\left(\frac{ad}{b}+dx\right)}{a+bx} dx}{a^2} \\
&= \frac{\cosh(c+dx)}{a(a+bx)} + \frac{\cosh(c) \operatorname{Chi}(dx)}{a^2} - \frac{\cosh\left(c - \frac{ad}{b}\right) \operatorname{Chi}\left(\frac{ad}{b}+dx\right)}{a^2} + \frac{\sinh(c) \operatorname{Shi}(dx)}{a^2} - \frac{\sinh\left(c - \frac{ad}{b}\right) \operatorname{Shi}\left(\frac{ad}{b}+dx\right)}{ab} \\
&= \frac{\cosh(c+dx)}{a(a+bx)} + \frac{\cosh(c) \operatorname{Chi}(dx)}{a^2} - \frac{\cosh\left(c - \frac{ad}{b}\right) \operatorname{Chi}\left(\frac{ad}{b}+dx\right)}{a^2} - \frac{d \operatorname{Chi}\left(\frac{ad}{b}+dx\right) \sinh\left(c - \frac{ad}{b}\right)}{ab}
\end{aligned}$$

Mathematica [A] time = 1.21, size = 139, normalized size = 0.93

$$\frac{\operatorname{Chi}\left(\frac{a}{b}+x\right) \left(ad \sinh\left(c - \frac{ad}{b}\right) + b \cosh\left(c - \frac{ad}{b}\right) \right) - \sinh\left(c - \frac{ad}{b}\right) \operatorname{Shi}\left(d\left(\frac{a}{b}+x\right)\right) - \frac{ad \cosh\left(c - \frac{ad}{b}\right) \operatorname{Shi}\left(d\left(\frac{a}{b}+x\right)\right)}{b} + \frac{a \sinh(c) \sinh(dx)}{a+bx} + \frac{a \cosh(c) \operatorname{Chi}(dx)}{a^2}}{a^2}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d*x]/(x*(a + b*x)^2),x]

[Out] ((a*Cosh[c]*Cosh[d*x])/(a + b*x) + Cosh[c]*CoshIntegral[d*x] - (CoshIntegral[d*(a/b + x)]*(b*Cosh[c - (a*d)/b] + a*d*Sinh[c - (a*d)/b]))/b + (a*Sinh[c]*Sinh[d*x])/(a + b*x) + Sinh[c]*SinhIntegral[d*x] - (a*d*Cosh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)])/b - Sinh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)]/a^2

fricas [A] time = 0.61, size = 270, normalized size = 1.80

$$2ab \cosh(dx + c) + \left((b^2x + ab) \operatorname{Ei}(dx) + (b^2x + ab) \operatorname{Ei}(-dx) \right) \cosh(c) - \left((a^2d + ab + (abd + b^2)x) \operatorname{Ei}\left(\frac{bdx+ad}{b}\right) - \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/x/(b*x+a)^2,x, algorithm="fricas")

[Out] 1/2*(2*a*b*cosh(d*x + c) + ((b^2*x + a*b)*Ei(d*x) + (b^2*x + a*b)*Ei(-d*x))*cosh(c) - ((a^2*d + a*b + (a*b*d + b^2)*x)*Ei((b*d*x + a*d)/b) - (a^2*d - a*b + (a*b*d - b^2)*x)*Ei(-(b*d*x + a*d)/b))*cosh(-(b*c - a*d)/b) + ((b^2*x + a*b)*Ei(d*x) - (b^2*x + a*b)*Ei(-d*x))*sinh(c) + ((a^2*d + a*b + (a*b*d + b^2)*x)*Ei((b*d*x + a*d)/b) + (a^2*d - a*b + (a*b*d - b^2)*x)*Ei(-(b*d*x + a*d)/b))*sinh(-(b*c - a*d)/b))/(a^2*b^2*x + a^3*b)

giac [B] time = 0.20, size = 1329, normalized size = 8.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/x/(b*x+a)^2,x, algorithm="giac")

[Out] -1/2*((b*x + a)*a*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^2*Ei(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^((b*c - a*d)/b) - a*b*c*d^2*Ei(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^((b*c - a*d)/b) + a^2*d^3*Ei(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^((b*c - a*d)/b) - (b*x + a)*a*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^2*Ei(-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^(-(b*c - a*d)/b) + a*b*c*d^2*Ei(-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^(-(b*c - a*d)/b) - a^2*d^3*Ei(-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^(-(b*c - a*d)/b) - (b*x + a)*b*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d*Ei(-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b + c)*e^(-c) + b^2*c*d*Ei(-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b + c)*e^(-c) - a*b*d^2*Ei(-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b + c)*e^(-c) - (b*x + a)*b*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d*Ei((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b - c)*e^c + b^2*c*

$d \cdot \text{Ei}\left(\frac{(b \cdot x + a) \cdot (b \cdot c / (b \cdot x + a) - a \cdot d / (b \cdot x + a) + d)}{b - c}\right) \cdot e^{-c} - a \cdot b \cdot d^2 \cdot \text{Ei}\left(\frac{(b \cdot x + a) \cdot (b \cdot c / (b \cdot x + a) - a \cdot d / (b \cdot x + a) + d)}{b - c}\right) \cdot e^{-c} + (b \cdot x + a) \cdot b \cdot \left(\frac{b \cdot c}{b \cdot x + a} - \frac{a \cdot d}{b \cdot x + a} + d\right) \cdot d \cdot \text{Ei}\left(\frac{(b \cdot x + a) \cdot (b \cdot c / (b \cdot x + a) - a \cdot d / (b \cdot x + a) + d) - b \cdot c + a \cdot d}{b}\right) \cdot e^{-\left(\frac{b \cdot c - a \cdot d}{b}\right)} - b^2 \cdot c \cdot d \cdot \text{Ei}\left(\frac{(b \cdot x + a) \cdot (b \cdot c / (b \cdot x + a) - a \cdot d / (b \cdot x + a) + d) - b \cdot c + a \cdot d}{b}\right) \cdot e^{-\left(\frac{b \cdot c - a \cdot d}{b}\right)} + a \cdot b \cdot d^2 \cdot \text{Ei}\left(\frac{(b \cdot x + a) \cdot (b \cdot c / (b \cdot x + a) - a \cdot d / (b \cdot x + a) + d) - b \cdot c + a \cdot d}{b}\right) \cdot e^{-\left(\frac{b \cdot c - a \cdot d}{b}\right)} + (b \cdot x + a) \cdot b \cdot \left(\frac{b \cdot c}{b \cdot x + a} - \frac{a \cdot d}{b \cdot x + a} + d\right) \cdot d \cdot \text{Ei}\left(-\frac{(b \cdot x + a) \cdot (b \cdot c / (b \cdot x + a) - a \cdot d / (b \cdot x + a) + d) - b \cdot c + a \cdot d}{b}\right) \cdot e^{-\left(-\frac{b \cdot c - a \cdot d}{b}\right)} - b^2 \cdot c \cdot d \cdot \text{Ei}\left(-\frac{(b \cdot x + a) \cdot (b \cdot c / (b \cdot x + a) - a \cdot d / (b \cdot x + a) + d) - b \cdot c + a \cdot d}{b}\right) \cdot e^{-\left(-\frac{b \cdot c - a \cdot d}{b}\right)} + a \cdot b \cdot d^2 \cdot \text{Ei}\left(-\frac{(b \cdot x + a) \cdot (b \cdot c / (b \cdot x + a) - a \cdot d / (b \cdot x + a) + d) - b \cdot c + a \cdot d}{b}\right) \cdot e^{-\left(-\frac{b \cdot c - a \cdot d}{b}\right)} - a \cdot b \cdot d^2 \cdot e^{-\left(\frac{(b \cdot x + a) \cdot (b \cdot c / (b \cdot x + a) - a \cdot d / (b \cdot x + a) + d)}{b}\right)} - a \cdot b \cdot d^2 \cdot e^{-\left(-\frac{(b \cdot x + a) \cdot (b \cdot c / (b \cdot x + a) - a \cdot d / (b \cdot x + a) + d)}{b}\right)} \cdot b^3 / \left(\left(\frac{(b \cdot x + a) \cdot a^2 \cdot b^4 \cdot (b \cdot c / (b \cdot x + a) - a \cdot d / (b \cdot x + a) + d) - a^2 \cdot b^5 \cdot c + a^3 \cdot b^4 \cdot d}{d}\right)\right)$

maple [A] time = 0.10, size = 254, normalized size = 1.69

$$\frac{e^{-dx-cd}}{2a((dx+c)b+da-cb)} - \frac{e^{\frac{da-cb}{b}} \text{Ei}\left(1, dx+c+\frac{da-cb}{b}\right) d}{2ab} + \frac{e^{\frac{da-cb}{b}} \text{Ei}\left(1, dx+c+\frac{da-cb}{b}\right)}{2a^2} - \frac{e^{-c} \text{Ei}(1, dx)}{2a^2} + \frac{e^{-\frac{da-cb}{b}} \text{Ei}\left(1, dx+c+\frac{da-cb}{b}\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cosh(d*x+c)/x/(b*x+a)^2,x)`

[Out] $\frac{1}{2} \cdot \exp(-d \cdot x - c) \cdot d / a / \left((d \cdot x + c) \cdot b + d \cdot a - c \cdot b \right) - \frac{1}{2} \cdot a / b \cdot \exp\left(\frac{a \cdot d - b \cdot c}{b}\right) \cdot \text{Ei}\left(1, d \cdot x + c + \frac{a \cdot d - b \cdot c}{b}\right) \cdot d + \frac{1}{2} \cdot a^2 \cdot \exp\left(\frac{a \cdot d - b \cdot c}{b}\right) \cdot \text{Ei}\left(1, d \cdot x + c + \frac{a \cdot d - b \cdot c}{b}\right) - \frac{1}{2} \cdot a^2 \cdot \exp(-c) \cdot \text{Ei}(1, d \cdot x) + \frac{1}{2} \cdot a^2 \cdot \exp\left(-\frac{a \cdot d - b \cdot c}{b}\right) \cdot \text{Ei}\left(1, -d \cdot x - c - \frac{a \cdot d - b \cdot c}{b}\right) + \frac{1}{2} \cdot a \cdot d / b \cdot \exp(d \cdot x + c) / (a \cdot d / b + d \cdot x) + \frac{1}{2} \cdot a \cdot d / b \cdot \exp\left(-\frac{a \cdot d - b \cdot c}{b}\right) \cdot \text{Ei}\left(1, -d \cdot x - c - \frac{a \cdot d - b \cdot c}{b}\right) - \frac{1}{2} \cdot a^2 \cdot \exp(c) \cdot \text{Ei}(1, -d \cdot x)$

maxima [A] time = 0.46, size = 227, normalized size = 1.51

$$-\frac{1}{2} d \left(\frac{e^{\left(-c + \frac{ad}{b}\right)} \text{E}_1\left(\frac{(bx+a)d}{b}\right)}{ab} - \frac{e^{\left(c - \frac{ad}{b}\right)} \text{E}_1\left(-\frac{(bx+a)d}{b}\right)}{ab} - \frac{b \left(\frac{e^{\left(-c + \frac{ad}{b}\right)} \text{E}_1\left(\frac{(bx+a)d}{b}\right)}{b} + \frac{e^{\left(c - \frac{ad}{b}\right)} \text{E}_1\left(-\frac{(bx+a)d}{b}\right)}{b} \right)}{a^2 d} - \frac{2 \cosh(dx+c) \log(\dots)}{a^2 d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(d*x+c)/x/(b*x+a)^2,x, algorithm="maxima")`

[Out] $-\frac{1}{2} \cdot d \cdot \left(e^{-c + a \cdot d / b} \cdot \exp_integral_e(1, (b \cdot x + a) \cdot d / b) / (a \cdot b) - e^{c - a \cdot d / b} \cdot \exp_integral_e(1, -(b \cdot x + a) \cdot d / b) / (a \cdot b) - b \cdot \left(e^{-c + a \cdot d / b} \cdot \exp_integral_e(1, (b \cdot x + a) \cdot d / b) / b + e^{c - a \cdot d / b} \cdot \exp_integral_e(1, -(b \cdot x + a) \cdot d / b) / b \right) / a^2 \right)$

$\frac{e^{(b*x + a)*d/b}}{b} + e^{(c - a*d/b)} * \text{exp_integral_e}(1, -(b*x + a)*d/b) / (a^2*d) - 2*\cosh(d*x + c)*\log(b*x + a)/(a^2*d) + 2*\cosh(d*x + c)*\log(x)/(a^2*d) - (\text{Ei}(-d*x)*e^{-c} + \text{Ei}(d*x)*e^c)/(a^2*d) + (1/(a*b*x + a^2) - \log(b*x + a)/a^2 + \log(x)/a^2)*\cosh(d*x + c)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c + dx)}{x(a + bx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(c + d*x)/(x*(a + b*x)^2), x)

[Out] int(cosh(c + d*x)/(x*(a + b*x)^2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(c + dx)}{x(a + bx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/x/(b*x+a)**2, x)

[Out] Integral(cosh(c + d*x)/(x*(a + b*x)**2), x)

3.32 $\int \frac{\cosh(c+dx)}{x^2(a+bx)^2} dx$

Optimal. Leaf size=186

$$-\frac{2b \cosh(c) \operatorname{Chi}(dx)}{a^3} + \frac{2b \cosh\left(c - \frac{ad}{b}\right) \operatorname{Chi}\left(xd + \frac{ad}{b}\right)}{a^3} - \frac{2b \sinh(c) \operatorname{Shi}(dx)}{a^3} + \frac{2b \sinh\left(c - \frac{ad}{b}\right) \operatorname{Shi}\left(xd + \frac{ad}{b}\right)}{a^3} + \frac{d \sinh(c)}{a^2}$$

[Out] $-2*b*\operatorname{Chi}(d*x)*\cosh(c)/a^3+2*b*\operatorname{Chi}(a*d/b+d*x)*\cosh(-c+a*d/b)/a^3-\cosh(d*x+c)/a^2/x-b*\cosh(d*x+c)/a^2/(b*x+a)+d*\cosh(c)*\operatorname{Shi}(d*x)/a^2+d*\cosh(-c+a*d/b)*\operatorname{Shi}(a*d/b+d*x)/a^2+d*\operatorname{Chi}(d*x)*\sinh(c)/a^2-2*b*\operatorname{Shi}(d*x)*\sinh(c)/a^3-d*\operatorname{Chi}(a*d/b+d*x)*\sinh(-c+a*d/b)/a^2-2*b*\operatorname{Shi}(a*d/b+d*x)*\sinh(-c+a*d/b)/a^3$

Rubi [A] time = 0.50, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {6742, 3297, 3303, 3298, 3301}

$$\frac{d \sinh\left(c - \frac{ad}{b}\right) \operatorname{Chi}\left(xd + \frac{ad}{b}\right)}{a^2} - \frac{2b \cosh(c) \operatorname{Chi}(dx)}{a^3} + \frac{2b \cosh\left(c - \frac{ad}{b}\right) \operatorname{Chi}\left(xd + \frac{ad}{b}\right)}{a^3} - \frac{2b \sinh(c) \operatorname{Shi}(dx)}{a^3} + \frac{2b \sinh(c)}{a^2}$$

Antiderivative was successfully verified.

[In] `Int[Cosh[c + d*x]/(x^2*(a + b*x)^2), x]`

[Out] $-(\operatorname{Cosh}[c + d*x]/(a^2*x)) - (b*\operatorname{Cosh}[c + d*x])/(a^2*(a + b*x)) - (2*b*\operatorname{Cosh}[c]*\operatorname{CoshIntegral}[d*x])/a^3 + (2*b*\operatorname{Cosh}[c - (a*d)/b]*\operatorname{CoshIntegral}[(a*d)/b + d*x])/a^3 + (d*\operatorname{CoshIntegral}[d*x]*\operatorname{Sinh}[c])/a^2 + (d*\operatorname{CoshIntegral}[(a*d)/b + d*x]*\operatorname{Sinh}[c - (a*d)/b])/a^2 + (d*\operatorname{Cosh}[c]*\operatorname{SinhIntegral}[d*x])/a^2 - (2*b*\operatorname{Sinh}[c]*\operatorname{SinhIntegral}[d*x])/a^3 + (d*\operatorname{Cosh}[c - (a*d)/b]*\operatorname{SinhIntegral}[(a*d)/b + d*x])/a^2 + (2*b*\operatorname{Sinh}[c - (a*d)/b]*\operatorname{SinhIntegral}[(a*d)/b + d*x])/a^3$

Rule 3297

`Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]`

Rule 3298

`Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)])/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]`

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
  := Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x]
  && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh(c+dx)}{x^2(a+bx)^2} dx &= \int \left(\frac{\cosh(c+dx)}{a^2 x^2} - \frac{2b \cosh(c+dx)}{a^3 x} + \frac{b^2 \cosh(c+dx)}{a^2(a+bx)^2} + \frac{2b^2 \cosh(c+dx)}{a^3(a+bx)} \right) dx \\
&= \frac{\int \frac{\cosh(c+dx)}{x^2} dx}{a^2} - \frac{(2b) \int \frac{\cosh(c+dx)}{x} dx}{a^3} + \frac{(2b^2) \int \frac{\cosh(c+dx)}{a+bx} dx}{a^3} + \frac{b^2 \int \frac{\cosh(c+dx)}{(a+bx)^2} dx}{a^2} \\
&= -\frac{\cosh(c+dx)}{a^2 x} - \frac{b \cosh(c+dx)}{a^2(a+bx)} + \frac{d \int \frac{\sinh(c+dx)}{x} dx}{a^2} + \frac{(bd) \int \frac{\sinh(c+dx)}{a+bx} dx}{a^2} - \frac{(2b \cosh(c))}{a^2} \\
&= -\frac{\cosh(c+dx)}{a^2 x} - \frac{b \cosh(c+dx)}{a^2(a+bx)} - \frac{2b \cosh(c) \text{Chi}(dx)}{a^3} + \frac{2b \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(\frac{ad}{b} + dx\right)}{a^3} \\
&= -\frac{\cosh(c+dx)}{a^2 x} - \frac{b \cosh(c+dx)}{a^2(a+bx)} - \frac{2b \cosh(c) \text{Chi}(dx)}{a^3} + \frac{2b \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(\frac{ad}{b} + dx\right)}{a^3}
\end{aligned}$$

Mathematica [A] time = 1.42, size = 183, normalized size = 0.98

$$\frac{ad \sinh\left(c - \frac{ad}{b}\right) \text{Chi}\left(d\left(\frac{a}{b} + x\right)\right) + 2b \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(d\left(\frac{a}{b} + x\right)\right) + 2b \sinh\left(c - \frac{ad}{b}\right) \text{Shi}\left(d\left(\frac{a}{b} + x\right)\right) + ad \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(\frac{ad}{b}\right)}{1}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d*x]/(x^2*(a + b*x)^2),x]

[Out]
$$\begin{aligned} & -((a*(a + 2*b*x)*\text{Cosh}[c]*\text{Cosh}[d*x])/(x*(a + b*x))) - 2*b*\text{Cosh}[c]*\text{CoshIntegral}[d*x] \\ & + 2*b*\text{Cosh}[c - (a*d)/b]*\text{CoshIntegral}[d*(a/b + x)] + a*d*\text{CoshIntegral}[d*x]*\text{Sinh}[c] \\ & + a*d*\text{CoshIntegral}[d*(a/b + x)]*\text{Sinh}[c - (a*d)/b] - (a*(a + 2*b*x)*\text{Sinh}[c]*\text{Sinh}[d*x])/(x*(a + b*x)) \\ & + a*d*\text{Cosh}[c]*\text{SinhIntegral}[d*x] - 2*b*\text{Sinh}[c]*\text{SinhIntegral}[d*x] \\ & + a*d*\text{Cosh}[c - (a*d)/b]*\text{SinhIntegral}[d*(a/b + x)] + 2*b*\text{Sinh}[c - (a*d)/b]*\text{SinhIntegral}[d*(a/b + x)] \end{aligned} / a^3$$

fricas [A] time = 0.50, size = 377, normalized size = 2.03

$$2(2abx + a^2) \cosh(dx + c) - \left((abd - 2b^2)x^2 + (a^2d - 2ab)x \right) \text{Ei}(dx) - \left((abd + 2b^2)x^2 + (a^2d + 2ab)x \right) \text{Ei}(-dx)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/x^2/(b*x+a)^2,x, algorithm="fricas")

[Out]
$$\begin{aligned} & -1/2*(2*(2*a*b*x + a^2)*\text{cosh}(d*x + c) - ((a*b*d - 2*b^2)*x^2 + (a^2*d - 2*a*b)*x)*\text{Ei}(d*x) \\ & - ((a*b*d + 2*b^2)*x^2 + (a^2*d + 2*a*b)*x)*\text{Ei}(-d*x))*\text{cosh}(c) \\ & - ((a*b*d + 2*b^2)*x^2 + (a^2*d + 2*a*b)*x)*\text{Ei}((b*d*x + a*d)/b) - ((a*b*d - 2*b^2)*x^2 \\ & + (a^2*d - 2*a*b)*x)*\text{Ei}(-(b*d*x + a*d)/b))*\text{cosh}(-(b*c - a*d)/b) \\ & - ((a*b*d - 2*b^2)*x^2 + (a^2*d - 2*a*b)*x)*\text{Ei}(d*x) + ((a*b*d + 2*b^2)*x^2 \\ & + (a^2*d + 2*a*b)*x)*\text{Ei}(-d*x))*\text{sinh}(c) + ((a*b*d + 2*b^2)*x^2 + (a^2*d + 2*a*b)*x)*\text{Ei}((b*d*x \\ & + a*d)/b) + ((a*b*d - 2*b^2)*x^2 + (a^2*d - 2*a*b)*x)*\text{Ei}(-(b*d*x + a*d)/b))*\text{sinh}(-(b*c - a*d)/b)) / (a^3*b*x^2 + a^4*x) \end{aligned}$$

giac [B] time = 0.22, size = 3353, normalized size = 18.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/x^2/(b*x+a)^2,x, algorithm="giac")

[Out]
$$\begin{aligned} & -1/2*((b*x + a)^2*a*(b*c/(b*x + a) - a*d/(b*x + a) + d)^2*d^2*\text{Ei}(-(b*x + a)*(b*c/(b*x + a) \\ & - a*d/(b*x + a) + d)/b + c)*e^{(-c)/b} - 2*(b*x + a)*a*(b*c/(b*x + a) - a*d/(b*x + a) + d)*c*d^2*\text{Ei}(-(b*x + a)*(b*c/(b*x + a) \\ & - a*d/(b*x + a) + d)/b + c)*e^{(-c)} + a*b*c^2*d^2*\text{Ei}(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b + c)*e^{(-c)} \\ & + (b*x + a)*a^2*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^3*\text{Ei}(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b + c)*e^{(-c)/b} \\ & - a^2*c*d^3*\text{Ei}(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b + c)*e^{(-c)} - (b*x + a)^2*a*(b*c/(b*x + a) - a*d/(b*x + a) + d)^2*d^2*\text{Ei}((b*x + a)*(b*c/(b*x + a) \\ & - a*d/(b*x + a) + d)/b - c)*e^{c/b} + 2*(b*x + a)*a*(b*c/(b*x + a) - a*d/(b*x + a) + d)*c*d^2*\text{Ei}((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b - c)*e^{c/b} - a*b*c^2*d^2*\text{Ei}((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b - c)*e^{c/b} \end{aligned}$$

$$\begin{aligned}
&)/b - c) * e^c - (b*x + a) * a^2 * (b*c/(b*x + a) - a*d/(b*x + a) + d) * d^3 * Ei((b*x + a) * (b*c/(b*x + a) - a*d/(b*x + a) + d) / b - c) * e^c / b + a^2 * c * d^3 * Ei((b*x + a) * (b*c/(b*x + a) - a*d/(b*x + a) + d) / b - c) * e^c - (b*x + a)^2 * a * (b*c/(b*x + a) - a*d/(b*x + a) + d)^2 * d^2 * Ei(((b*x + a) * (b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d) / b) * e^((b*c - a*d) / b) / b + 2 * (b*x + a) * a * (b*c/(b*x + a) - a*d/(b*x + a) + d) * c * d^2 * Ei(((b*x + a) * (b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d) / b) * e^((b*c - a*d) / b) - a * b * c^2 * d^2 * Ei(((b*x + a) * (b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d) / b) * e^((b*c - a*d) / b) - (b*x + a) * a^2 * (b*c/(b*x + a) - a*d/(b*x + a) + d) * d^3 * Ei(((b*x + a) * (b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d) / b) * e^((b*c - a*d) / b) / b + a^2 * c * d^3 * Ei(((b*x + a) * (b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d) / b) * e^((b*c - a*d) / b) + (b*x + a)^2 * a * (b*c/(b*x + a) - a*d/(b*x + a) + d)^2 * d^2 * Ei(-((b*x + a) * (b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d) / b) * e^(-(b*c - a*d) / b) / b - 2 * (b*x + a) * a * (b*c/(b*x + a) - a*d/(b*x + a) + d) * c * d^2 * Ei(-((b*x + a) * (b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d) / b) * e^(-(b*c - a*d) / b) + a * b * c^2 * d^2 * Ei(-((b*x + a) * (b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d) / b) * e^(-(b*c - a*d) / b) + (b*x + a) * a^2 * (b*c/(b*x + a) - a*d/(b*x + a) + d) * d^3 * Ei(-((b*x + a) * (b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d) / b) * e^(-(b*c - a*d) / b) / b - a^2 * c * d^3 * Ei(-((b*x + a) * (b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d) / b) * e^(-(b*c - a*d) / b) + 2 * (b*x + a)^2 * (b*c/(b*x + a) - a*d/(b*x + a) + d)^2 * d * Ei(-((b*x + a) * (b*c/(b*x + a) - a*d/(b*x + a) + d) / b + c) * e^(-c) - 4 * (b*x + a) * b * (b*c/(b*x + a) - a*d/(b*x + a) + d) * c * d * Ei(-((b*x + a) * (b*c/(b*x + a) - a*d/(b*x + a) + d) / b + c) * e^(-c) + 2 * b^2 * c^2 * d * Ei(-((b*x + a) * (b*c/(b*x + a) - a*d/(b*x + a) + d) / b + c) * e^(-c) + 2 * (b*x + a) * a * (b*c/(b*x + a) - a*d/(b*x + a) + d) * d^2 * Ei(-((b*x + a) * (b*c/(b*x + a) - a*d/(b*x + a) + d) / b + c) * e^(-c) - 2 * a * b * c * d^2 * Ei(-((b*x + a) * (b*c/(b*x + a) - a*d/(b*x + a) + d) / b + c) * e^(-c) + 2 * (b*x + a)^2 * (b*c/(b*x + a) - a*d/(b*x + a) + d)^2 * d * Ei(((b*x + a) * (b*c/(b*x + a) - a*d/(b*x + a) + d) / b - c) * e^c - 4 * (b*x + a) * b * (b*c/(b*x + a) - a*d/(b*x + a) + d) * c * d * Ei(((b*x + a) * (b*c/(b*x + a) - a*d/(b*x + a) + d) / b - c) * e^c + 2 * b^2 * c^2 * d * Ei(((b*x + a) * (b*c/(b*x + a) - a*d/(b*x + a) + d) / b - c) * e^c + 2 * (b*x + a) * a * (b*c/(b*x + a) - a*d/(b*x + a) + d) * d^2 * Ei(((b*x + a) * (b*c/(b*x + a) - a*d/(b*x + a) + d) / b - c) * e^c - 2 * a * b * c * d^2 * Ei(((b*x + a) * (b*c/(b*x + a) - a*d/(b*x + a) + d) / b - c) * e^c - 2 * (b*x + a)^2 * (b*c/(b*x + a) - a*d/(b*x + a) + d)^2 * d * Ei(((b*x + a) * (b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d) / b) * e^((b*c - a*d) / b) + 4 * (b*x + a) * b * (b*c/(b*x + a) - a*d/(b*x + a) + d) * c * d * Ei(((b*x + a) * (b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d) / b) * e^((b*c - a*d) / b) - 2 * b^2 * c^2 * d * Ei(((b*x + a) * (b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d) / b) * e^((b*c - a*d) / b) - 2 * (b*x + a) * a * (b*c/(b*x + a) - a*d/(b*x + a) + d) * d^2 * Ei(((b*x + a) * (b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d) / b) * e^((b*c - a*d) / b) + 2 * a * b * c * d^2 * Ei(((b*x + a) * (b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d) / b) * e^((b*c - a*d) / b) - 2 * (b*x + a)^2 * (b*c/(b*x + a) - a*d/(b*x + a) + d)^2 * d * Ei(-((b*x + a) * (b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d) / b) * e^(-(b*c - a*d) / b) + 4 * (b*x + a) * b * (b*c/(b*x + a) - a*d/(b*x + a) + d) * c * d * Ei(-((b*x + a) * (b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d) / b) * e^(-(b*c - a*d) / b) - 2 * b^
\end{aligned}$$

$2*c^2*d*Ei(-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*$
 $e^{-((b*c - a*d)/b) - 2*(b*x + a)*a*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^2*}$
 $Ei(-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^{-((b*c$
 $- a*d)/b) + 2*a*b*c*d^2*Ei(-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)$
 $- b*c + a*d)/b)*e^{-((b*c - a*d)/b) + 2*(b*x + a)*a*(b*c/(b*x + a) - a*d/(b$
 $*x + a) + d)*d^2*e^{((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) - 2*a*$
 $b*c*d^2*e^{((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) + a^2*d^3*e^{((b$
 $*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) + 2*(b*x + a)*a*(b*c/(b*x +$
 $a) - a*d/(b*x + a) + d)*d^2*e^{-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) +$
 $d)/b) - 2*a*b*c*d^2*e^{-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) +$
 $a^2*d^3*e^{-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b)}*b^2/(((b*x +$
 $a)^2*a^3*b*(b*c/(b*x + a) - a*d/(b*x + a) + d)^2 - 2*(b*x + a)*a^3*b^2*(b*c$
 $/(b*x + a) - a*d/(b*x + a) + d)*c + a^3*b^3*c^2 + (b*x + a)*a^4*b*(b*c/(b*x$
 $+ a) - a*d/(b*x + a) + d)*d - a^4*b^2*c*d)*d)$

maple [A] time = 0.11, size = 312, normalized size = 1.68

$$\frac{d e^{-dx-cb}}{a^2 (bdx + da)} - \frac{d e^{-dx-c}}{2a (bdx + da)x} + \frac{d e^{\frac{da-cb}{b}} Ei\left(1, dx + c + \frac{da-cb}{b}\right)}{2a^2} - \frac{e^{\frac{da-cb}{b}} Ei\left(1, dx + c + \frac{da-cb}{b}\right) b}{a^3} + \frac{d e^{-c} Ei(1, dx)}{2a^2} + e$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d*x+c)/x^2/(b*x+a)^2, x)

[Out] $-d*\exp(-d*x-c)/a^2/(b*d*x+a*d)*b-1/2*d*\exp(-d*x-c)/a/(b*d*x+a*d)/x+1/2*d/a^2*\exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)-1/a^3*\exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)*b+1/2*d/a^2*\exp(-c)*Ei(1,d*x)+1/a^3*\exp(-c)*Ei(1,d*x)*b-b/a^3*\exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)-1/2*d/a^2*\exp(d*x+c)/(a*d/b+d*x)-1/2*d/a^2*\exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)-1/2/a^2/x*\exp(d*x+c)-1/2*d/a^2*\exp(c)*Ei(1,-d*x)+1/a^3*b*\exp(c)*Ei(1,-d*x)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(dx + c)}{(bx + a)^2 x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/x^2/(b*x+a)^2, x, algorithm="maxima")

[Out] integrate(cosh(d*x + c)/((b*x + a)^2*x^2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c + dx)}{x^2 (a + bx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)/(x^2*(a + b*x)^2), x)
```

```
[Out] int(cosh(c + d*x)/(x^2*(a + b*x)^2), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x**2/(b*x+a)**2, x)
```

```
[Out] Timed out
```

$$3.33 \quad \int \frac{x^3 \cosh(c+dx)}{(a+bx)^3} dx$$

Optimal. Leaf size=264

$$\frac{a^3 d^2 \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{2b^6} - \frac{a^3 d^2 \sinh\left(c - \frac{ad}{b}\right) \text{Shi}\left(xd + \frac{ad}{b}\right)}{2b^6} + \frac{a^3 d \sinh(c+dx)}{2b^5(a+bx)} + \frac{a^3 \cosh(c+dx)}{2b^4(a+bx)^2} + \frac{3a^2 d \cosh(c+dx)}{2b^3(a+bx)^3}$$

[Out] $-3*a*\text{Chi}(a*d/b+d*x)*\cosh(-c+a*d/b)/b^4-1/2*a^3*d^2*\text{Chi}(a*d/b+d*x)*\cosh(-c+a*d/b)/b^6+1/2*a^3*\cosh(d*x+c)/b^4/(b*x+a)^2-3*a^2*\cosh(d*x+c)/b^4/(b*x+a)+3*a^2*d*\cosh(-c+a*d/b)*\text{Shi}(a*d/b+d*x)/b^5-3*a^2*d*\text{Chi}(a*d/b+d*x)*\sinh(-c+a*d/b)/b^5+3*a*\text{Shi}(a*d/b+d*x)*\sinh(-c+a*d/b)/b^4+1/2*a^3*d^2*\text{Shi}(a*d/b+d*x)*\sinh(-c+a*d/b)/b^6+\sinh(d*x+c)/b^3/d+1/2*a^3*d*\sinh(d*x+c)/b^5/(b*x+a)$

Rubi [A] time = 0.62, antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {6742, 2637, 3297, 3303, 3298, 3301}

$$\frac{a^3 d^2 \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{2b^6} + \frac{3a^2 d \sinh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{b^5} - \frac{a^3 d^2 \sinh\left(c - \frac{ad}{b}\right) \text{Shi}\left(xd + \frac{ad}{b}\right)}{2b^6} + \frac{3a^2 d \cosh(c+dx)}{2b^3(a+bx)^3}$$

Antiderivative was successfully verified.

[In] `Int[(x^3*Cosh[c + d*x])/(a + b*x)^3,x]`

[Out] $(a^3*\text{Cosh}[c + d*x])/(2*b^4*(a + b*x)^2) - (3*a^2*\text{Cosh}[c + d*x])/(b^4*(a + b*x)) - (3*a*\text{Cosh}[c - (a*d)/b]*\text{CoshIntegral}[(a*d)/b + d*x])/b^4 - (a^3*d^2*\text{Cosh}[c - (a*d)/b]*\text{CoshIntegral}[(a*d)/b + d*x])/(2*b^6) + (3*a^2*d*\text{CoshIntegral}[(a*d)/b + d*x]*\text{Sinh}[c - (a*d)/b])/b^5 + \text{Sinh}[c + d*x]/(b^3*d) + (a^3*d*\text{Sinh}[c + d*x])/(2*b^5*(a + b*x)) + (3*a^2*d*\text{Cosh}[c - (a*d)/b]*\text{SinhIntegral}[(a*d)/b + d*x])/b^5 - (3*a*\text{Sinh}[c - (a*d)/b]*\text{SinhIntegral}[(a*d)/b + d*x])/b^4 - (a^3*d^2*\text{Sinh}[c - (a*d)/b]*\text{SinhIntegral}[(a*d)/b + d*x])/(2*b^6)$

Rule 2637

`Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;`
`FreeQ[{c, d}, x]`

Rule 3297

`Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /;`
`FreeQ[{c, d, e, f}, x] && LtQ[m, -1]`

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f,
fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz},
x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)
]/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3 \cosh(c + dx)}{(a + bx)^3} dx &= \int \left(\frac{\cosh(c + dx)}{b^3} - \frac{a^3 \cosh(c + dx)}{b^3(a + bx)^3} + \frac{3a^2 \cosh(c + dx)}{b^3(a + bx)^2} - \frac{3a \cosh(c + dx)}{b^3(a + bx)} \right) dx \\
&= \frac{\int \cosh(c + dx) dx}{b^3} - \frac{(3a) \int \frac{\cosh(c+dx)}{a+bx} dx}{b^3} + \frac{(3a^2) \int \frac{\cosh(c+dx)}{(a+bx)^2} dx}{b^3} - \frac{a^3 \int \frac{\cosh(c+dx)}{(a+bx)^3} dx}{b^3} \\
&= \frac{a^3 \cosh(c + dx)}{2b^4(a + bx)^2} - \frac{3a^2 \cosh(c + dx)}{b^4(a + bx)} + \frac{\sinh(c + dx)}{b^3 d} + \frac{(3a^2 d) \int \frac{\sinh(c+dx)}{a+bx} dx}{b^4} - \frac{(a^3 d) \int \frac{\sinh(c+dx)}{(a+bx)^2} dx}{b^4} \\
&= \frac{a^3 \cosh(c + dx)}{2b^4(a + bx)^2} - \frac{3a^2 \cosh(c + dx)}{b^4(a + bx)} - \frac{3a \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(\frac{ad}{b} + dx\right)}{b^4} + \frac{\sinh(c + dx)}{b^3 d} + \frac{3a^2 d \text{Shi}\left(\frac{ad}{b} + dx\right)}{b^4} \\
&= \frac{a^3 \cosh(c + dx)}{2b^4(a + bx)^2} - \frac{3a^2 \cosh(c + dx)}{b^4(a + bx)} - \frac{3a \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(\frac{ad}{b} + dx\right)}{b^4} + \frac{3a^2 d \text{Chi}\left(\frac{ad}{b} + dx\right)}{b^4} \\
&= \frac{a^3 \cosh(c + dx)}{2b^4(a + bx)^2} - \frac{3a^2 \cosh(c + dx)}{b^4(a + bx)} - \frac{3a \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(\frac{ad}{b} + dx\right)}{b^4} - \frac{a^3 d^2 \cosh\left(c - \frac{ad}{b}\right)}{b^4} + \frac{3a^2 d \text{Shi}\left(\frac{ad}{b} + dx\right)}{b^4}
\end{aligned}$$

Mathematica [A] time = 1.05, size = 236, normalized size = 0.89

$$\frac{ad(a + bx)^2 \left(\text{Chi}\left(d\left(\frac{a}{b} + x\right)\right) \left((a^2 d^2 + 6b^2) \cosh\left(c - \frac{ad}{b}\right) - 6abd \sinh\left(c - \frac{ad}{b}\right) \right) + \text{Shi}\left(d\left(\frac{a}{b} + x\right)\right) \left((a^2 d^2 + 6b^2) \right) \right)}{b^6 d^2 (a + bx)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(x^3*Cosh[c + d*x])/(a + b*x)^3,x]

[Out]
$$\begin{aligned}
& -1/2*(b*Cosh[d*x]*(a^2*b*d*(5*a + 6*b*x)*Cosh[c] - (a + b*x)*(2*a*b^2 + a^3*d^2 + 2*b^3*x)*Sinh[c]) - b*((a + b*x)*(2*a*b^2 + a^3*d^2 + 2*b^3*x)*Cosh[c] - a^2*b*d*(5*a + 6*b*x)*Sinh[c])*Sinh[d*x] + a*d*(a + b*x)^2*(CoshIntegral[d*(a/b + x)]*((6*b^2 + a^2*d^2)*Cosh[c - (a*d)/b] - 6*a*b*d*Sinh[c - (a*d)/b]) + (-6*a*b*d*Cosh[c - (a*d)/b] + (6*b^2 + a^2*d^2)*Sinh[c - (a*d)/b])*SinhIntegral[d*(a/b + x)])/(b^6*d*(a + b*x)^2)
\end{aligned}$$

fricas [B] time = 0.87, size = 566, normalized size = 2.14

$$\frac{2(6a^2b^3dx + 5a^3b^2d) \cosh(dx + c) + \left((a^5d^3 - 6a^4bd^2 + 6a^3b^2d + (a^3b^2d^3 - 6a^2b^3d^2 + 6ab^4d)x^2 + 2(a^4bd^3 - 6a^3b^2d^2 + 6a^2b^3d - 6abd^2 + 6b^4d) \right)}{b^6 d^2 (a + bx)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*cosh(d*x+c)/(b*x+a)^3,x, algorithm="fricas")

[Out]
$$-1/4*(2*(6*a^2*b^3*d*x + 5*a^3*b^2*d)*\cosh(d*x + c) + ((a^5*d^3 - 6*a^4*b*d^2 + 6*a^3*b^2*d + (a^3*b^2*d^3 - 6*a^2*b^3*d^2 + 6*a*b^4*d)*x^2 + 2*(a^4*b*d^3 - 6*a^3*b^2*d^2 + 6*a^2*b^3*d)*x)*\text{Ei}((b*d*x + a*d)/b) + (a^5*d^3 + 6*a^4*b*d^2 + 6*a^3*b^2*d + (a^3*b^2*d^3 + 6*a^2*b^3*d^2 + 6*a*b^4*d)*x^2 + 2*(a^4*b*d^3 + 6*a^3*b^2*d^2 + 6*a^2*b^3*d)*x)*\text{Ei}(-(b*d*x + a*d)/b))*\cosh(-(b*c - a*d)/b) - 2*(a^4*b*d^2 + 2*b^5*x^2 + 2*a^2*b^3 + (a^3*b^2*d^2 + 4*a*b^4)*x)*\sinh(d*x + c) - ((a^5*d^3 - 6*a^4*b*d^2 + 6*a^3*b^2*d + (a^3*b^2*d^3 - 6*a^2*b^3*d^2 + 6*a*b^4*d)*x^2 + 2*(a^4*b*d^3 - 6*a^3*b^2*d^2 + 6*a^2*b^3*d)*x)*\text{Ei}((b*d*x + a*d)/b) - (a^5*d^3 + 6*a^4*b*d^2 + 6*a^3*b^2*d + (a^3*b^2*d^3 + 6*a^2*b^3*d^2 + 6*a*b^4*d)*x^2 + 2*(a^4*b*d^3 + 6*a^3*b^2*d^2 + 6*a^2*b^3*d)*x)*\text{Ei}(-(b*d*x + a*d)/b))*\sinh(-(b*c - a*d)/b))/(b^8*d*x^2 + 2*a*b^7*d*x + a^2*b^6*d)$$

giac [B] time = 0.15, size = 879, normalized size = 3.33

$$a^3 b^2 d^3 x^2 \text{Ei}\left(\frac{bdx+ad}{b}\right) e^{\left(c-\frac{ad}{b}\right)} + a^3 b^2 d^3 x^2 \text{Ei}\left(-\frac{bdx+ad}{b}\right) e^{\left(-c+\frac{ad}{b}\right)} + 2 a^4 b d^3 x \text{Ei}\left(\frac{bdx+ad}{b}\right) e^{\left(c-\frac{ad}{b}\right)} - 6 a^2 b^3 d^2 x^2 \text{Ei}\left(\frac{bdx+ad}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*cosh(d*x+c)/(b*x+a)^3,x, algorithm="giac")

[Out]
$$-1/4*(a^3*b^2*d^3*x^2*\text{Ei}((b*d*x + a*d)/b)*e^{(c - a*d/b)} + a^3*b^2*d^3*x^2*\text{Ei}(-(b*d*x + a*d)/b)*e^{(-c + a*d/b)} + 2*a^4*b*d^3*x*\text{Ei}((b*d*x + a*d)/b)*e^{(c - a*d/b)} - 6*a^2*b^3*d^2*x^2*\text{Ei}((b*d*x + a*d)/b)*e^{(c - a*d/b)} + 2*a^4*b*d^3*x*\text{Ei}(-(b*d*x + a*d)/b)*e^{(-c + a*d/b)} + 6*a^2*b^3*d^2*x^2*\text{Ei}(-(b*d*x + a*d)/b)*e^{(-c + a*d/b)} + a^5*d^3*\text{Ei}((b*d*x + a*d)/b)*e^{(c - a*d/b)} - 12*a^3*b^2*d^2*x*\text{Ei}((b*d*x + a*d)/b)*e^{(c - a*d/b)} + 6*a*b^4*d*x^2*\text{Ei}((b*d*x + a*d)/b)*e^{(c - a*d/b)} + a^5*d^3*\text{Ei}(-(b*d*x + a*d)/b)*e^{(-c + a*d/b)} + 12*a^3*b^2*d^2*x*\text{Ei}(-(b*d*x + a*d)/b)*e^{(-c + a*d/b)} + 6*a*b^4*d*x^2*\text{Ei}(-(b*d*x + a*d)/b)*e^{(-c + a*d/b)} - a^3*b^2*d^2*x*e^{(d*x + c)} + a^3*b^2*d^2*x*e^{(-d*x - c)} - 6*a^4*b*d^2*\text{Ei}((b*d*x + a*d)/b)*e^{(c - a*d/b)} + 12*a^2*b^3*d*x*\text{Ei}((b*d*x + a*d)/b)*e^{(c - a*d/b)} + 6*a^4*b*d^2*\text{Ei}(-(b*d*x + a*d)/b)*e^{(-c + a*d/b)} + 12*a^2*b^3*d*x*\text{Ei}(-(b*d*x + a*d)/b)*e^{(-c + a*d/b)} - a^4*b*d^2*e^{(d*x + c)} + 6*a^2*b^3*d*x*e^{(d*x + c)} - 2*b^5*x^2*e^{(d*x + c)} + a^4*b*d^2*e^{(-d*x - c)} + 6*a^2*b^3*d*x*e^{(-d*x - c)} + 2*b^5*x^2*e^{(-d*x - c)} + 6*a^3*b^2*d*\text{Ei}((b*d*x + a*d)/b)*e^{(c - a*d/b)} + 6*a^3*b^2*d*\text{Ei}(-(b*d*x + a*d)/b)*e^{(-c + a*d/b)} + 5*a^3*b^2*d*e^{(d*x + c)} - 4*a*b^4*x*e^{(d*x + c)} + 5*a^3*b^2*d*e^{(-d*x - c)} + 4*a*b^4*x*e^{(-d*x - c)} - 2*a^2*b^3*e^{(d*x + c)} + 2*a^2*b^3*e^{(-d*x - c)})/(b^8*d*x^2 + 2*a*b^7*d*x + a^2*b^6*d)$$

maple [B] time = 0.20, size = 571, normalized size = 2.16

$$\frac{d^2 e^{\frac{da-cb}{b}} \operatorname{Ei}\left(1, dx + c + \frac{da-cb}{b}\right) a^3}{4b^6} + \frac{3d e^{\frac{da-cb}{b}} \operatorname{Ei}\left(1, dx + c + \frac{da-cb}{b}\right) a^2}{2b^5} + \frac{3e^{\frac{da-cb}{b}} \operatorname{Ei}\left(1, dx + c + \frac{da-cb}{b}\right) a}{2b^4} - \frac{e^{-dx-c}}{2db^3} - \frac{1}{4b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*cosh(d*x+c)/(b*x+a)^3,x)`

[Out] $\frac{1}{4}d^2/b^6 \exp((a*d-b*c)/b) \operatorname{Ei}(1, d*x+c+(a*d-b*c)/b) a^3 + \frac{3}{2}d/b^5 \exp((a*d-b*c)/b) \operatorname{Ei}(1, d*x+c+(a*d-b*c)/b) a^2 + \frac{3}{2}d/b^4 \exp((a*d-b*c)/b) \operatorname{Ei}(1, d*x+c+(a*d-b*c)/b) a - \frac{1}{2}d \exp(-d*x-c)/b^3 - \frac{5}{4}d^2 \exp(-d*x-c)/b^4 / (b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2) a^3 - \frac{1}{4}d^3 \exp(-d*x-c)/b^5 / (b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2) a^4 - \frac{1}{4}d^3 \exp(-d*x-c)/b^4 / (b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2) a^3 x - \frac{3}{2}d^2 \exp(-d*x-c)/b^3 / (b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2) a^2 x - \frac{3}{2}d/b^5 \exp(-(a*d-b*c)/b) \operatorname{Ei}(1, -d*x-c-(a*d-b*c)/b) a^2 - \frac{3}{2}d/b^5 \exp(d*x+c)/(a*d/b+d*x) a^2 + \frac{1}{2}d/b^3 \exp(d*x+c) + \frac{3}{2}d/b^4 \exp(-(a*d-b*c)/b) \operatorname{Ei}(1, -d*x-c-(a*d-b*c)/b) a + \frac{1}{4}d^2/b^6 \exp(d*x+c)/(a*d/b+d*x) a^3 + \frac{1}{4}d^2/b^6 \exp(-(a*d-b*c)/b) \operatorname{Ei}(1, -d*x-c-(a*d-b*c)/b) a^3 + \frac{1}{4}d^2/b^6 \exp(d*x+c)/(a*d/b+d*x)^2 a^3$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{3}{2} a^2 d \int \frac{x e^{(dx+c)}}{b^5 d^2 x^4 + 4 a b^4 d^2 x^3 + 6 a^2 b^3 d^2 x^2 + 4 a^3 b^2 d^2 x + a^4 b d^2} dx - \frac{3}{2} a^2 d \int \frac{1}{b^5 d^2 x^4 e^{(dx+c)} + 4 a b^4 d^2 x^3 e^{(dx+c)} + 6 a^2 b^3 d^2 x^2 e^{(dx+c)} + 4 a^3 b^2 d^2 x e^{(dx+c)} + a^4 b d^2 e^{(dx+c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*cosh(d*x+c)/(b*x+a)^3,x, algorithm="maxima")`

[Out] $\frac{3}{2} a^2 d \operatorname{integrate}(x e^{(d*x+c)} / (b^5 d^2 x^4 + 4 a b^4 d^2 x^3 + 6 a^2 b^3 d^2 x^2 + 4 a^3 b^2 d^2 x + a^4 b d^2), x) - \frac{3}{2} a^2 d \operatorname{integrate}(x / (b^5 d^2 x^4 e^{(d*x+c)} + 4 a b^4 d^2 x^3 e^{(d*x+c)} + 6 a^2 b^3 d^2 x^2 e^{(d*x+c)} + 4 a^3 b^2 d^2 x e^{(d*x+c)} + a^4 b d^2 e^{(d*x+c)}), x) - 3 a b \operatorname{integrate}(x e^{(d*x+c)} / (b^5 d^2 x^4 + 4 a b^4 d^2 x^3 + 6 a^2 b^3 d^2 x^2 + 4 a^3 b^2 d^2 x + a^4 b d^2), x) - 3 a b \operatorname{integrate}(x / (b^5 d^2 x^4 e^{(d*x+c)} + 4 a b^4 d^2 x^3 e^{(d*x+c)} + 6 a^2 b^3 d^2 x^2 e^{(d*x+c)} + 4 a^3 b^2 d^2 x e^{(d*x+c)} + a^4 b d^2 e^{(d*x+c)}), x) + \frac{1}{2} * ((b*d*x^3*e^{(2*c)} - 3*a*x*e^{(2*c)}) * e^{(d*x)} - (b*d*x^3 + 3*a*x) * e^{(-d*x)}) / (b^4*d^2*x^3*e^c + 3*a*b^3*d^2*x^2*e^c + 3*a^2*b^2*d^2*x*e^c + a^3*b*d^2*e^c) - \frac{3}{2} * a^2 * e^{(-c+a*d/b)} * \operatorname{exp_integral_e}(4, (b*x+a)*d/b) / ((b*x+a)^3*b^2*d^2) - \frac{3}{2} * a^2 * e^{(c-a*d/b)} * \operatorname{exp_integral_e}(4, -(b*x+a)*d/b) / ((b*x+a)^3*b^2*d^2)$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3 \cosh(c + dx)}{(a + bx)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3*cosh(c + d*x))/(a + b*x)^3, x)

[Out] int((x^3*cosh(c + d*x))/(a + b*x)^3, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \cosh(c + dx)}{(a + bx)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*cosh(d*x+c)/(b*x+a)**3, x)

[Out] Integral(x**3*cosh(c + d*x)/(a + b*x)**3, x)

$$3.34 \quad \int \frac{x^2 \cosh(c+dx)}{(a+bx)^3} dx$$

Optimal. Leaf size=241

$$\frac{a^2 d^2 \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{2b^5} + \frac{a^2 d^2 \sinh\left(c - \frac{ad}{b}\right) \text{Shi}\left(xd + \frac{ad}{b}\right)}{2b^5} - \frac{a^2 d \sinh(c+dx)}{2b^4(a+bx)} - \frac{a^2 \cosh(c+dx)}{2b^3(a+bx)^2} - \frac{2ad \sinh(c+dx)}{2b^3(a+bx)^2}$$

[Out] Chi(a*d/b+d*x)*cosh(-c+a*d/b)/b^3+1/2*a^2*d^2*Chi(a*d/b+d*x)*cosh(-c+a*d/b)/b^5-1/2*a^2*cosh(d*x+c)/b^3/(b*x+a)^2+2*a*cosh(d*x+c)/b^3/(b*x+a)-2*a*d*cosh(-c+a*d/b)*Shi(a*d/b+d*x)/b^4+2*a*d*Chi(a*d/b+d*x)*sinh(-c+a*d/b)/b^4-Shi(a*d/b+d*x)*sinh(-c+a*d/b)/b^3-1/2*a^2*d^2*Shi(a*d/b+d*x)*sinh(-c+a*d/b)/b^5-1/2*a^2*d*sinh(d*x+c)/b^4/(b*x+a)

Rubi [A] time = 0.53, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {6742, 3297, 3303, 3298, 3301}

$$\frac{a^2 d^2 \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{2b^5} + \frac{a^2 d^2 \sinh\left(c - \frac{ad}{b}\right) \text{Shi}\left(xd + \frac{ad}{b}\right)}{2b^5} - \frac{a^2 d \sinh(c+dx)}{2b^4(a+bx)} - \frac{a^2 \cosh(c+dx)}{2b^3(a+bx)^2} - \frac{2ad \sinh(c+dx)}{2b^3(a+bx)^2}$$

Antiderivative was successfully verified.

[In] Int[(x^2*Cosh[c + d*x])/(a + b*x)^3,x]

[Out] -(a^2*Cosh[c + d*x])/(2*b^3*(a + b*x)^2) + (2*a*Cosh[c + d*x])/(b^3*(a + b*x)) + (Cosh[c - (a*d)/b]*CoshIntegral[(a*d)/b + d*x])/b^3 + (a^2*d^2*Cosh[c - (a*d)/b]*CoshIntegral[(a*d)/b + d*x])/(2*b^5) - (2*a*d*CoshIntegral[(a*d)/b + d*x]*Sinh[c - (a*d)/b])/b^4 - (a^2*d*Sinh[c + d*x])/(2*b^4*(a + b*x)) - (2*a*d*Cosh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/b^4 + (Sinh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/b^3 + (a^2*d^2*Sinh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/(2*b^5)

Rule 3297

Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]

Rule 3298

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)])/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f}

, fz}, x] && EqQ[d*e - c*f*fz*I, 0]

Rule 3301

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x_Symbol] := Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]

Rule 3303

Int[sin[(e_.) + (f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x_Symbol] := Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]

Rule 6742

Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

Rubi steps

$$\begin{aligned}
 \int \frac{x^2 \cosh(c + dx)}{(a + bx)^3} dx &= \int \left(\frac{a^2 \cosh(c + dx)}{b^2(a + bx)^3} - \frac{2a \cosh(c + dx)}{b^2(a + bx)^2} + \frac{\cosh(c + dx)}{b^2(a + bx)} \right) dx \\
 &= \frac{\int \frac{\cosh(c+dx)}{a+bx} dx}{b^2} - \frac{(2a) \int \frac{\cosh(c+dx)}{(a+bx)^2} dx}{b^2} + \frac{a^2 \int \frac{\cosh(c+dx)}{(a+bx)^3} dx}{b^2} \\
 &= -\frac{a^2 \cosh(c + dx)}{2b^3(a + bx)^2} + \frac{2a \cosh(c + dx)}{b^3(a + bx)} - \frac{(2ad) \int \frac{\sinh(c+dx)}{a+bx} dx}{b^3} + \frac{(a^2d) \int \frac{\sinh(c+dx)}{(a+bx)^2} dx}{2b^3} + \dots \\
 &= -\frac{a^2 \cosh(c + dx)}{2b^3(a + bx)^2} + \frac{2a \cosh(c + dx)}{b^3(a + bx)} + \frac{\cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(\frac{ad}{b} + dx\right)}{b^3} - \frac{a^2d \sinh(c + dx)}{2b^4(a + bx)} \\
 &= -\frac{a^2 \cosh(c + dx)}{2b^3(a + bx)^2} + \frac{2a \cosh(c + dx)}{b^3(a + bx)} + \frac{\cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(\frac{ad}{b} + dx\right)}{b^3} - \frac{2ad \text{Chi}\left(\frac{ad}{b} + dx\right)}{b^3} \\
 &= -\frac{a^2 \cosh(c + dx)}{2b^3(a + bx)^2} + \frac{2a \cosh(c + dx)}{b^3(a + bx)} + \frac{\cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(\frac{ad}{b} + dx\right)}{b^3} + \frac{a^2d^2 \cosh\left(c - \frac{ad}{b}\right)}{2b^3}
 \end{aligned}$$

Mathematica [A] time = 0.97, size = 153, normalized size = 0.63

$$\frac{\text{Chi}\left(d\left(\frac{a}{b} + x\right)\right)\left(\left(a^2d^2 + 2b^2\right)\cosh\left(c - \frac{ad}{b}\right) - 4abd\sinh\left(c - \frac{ad}{b}\right)\right) + \text{Shi}\left(d\left(\frac{a}{b} + x\right)\right)\left(\left(a^2d^2 + 2b^2\right)\sinh\left(c - \frac{ad}{b}\right) - 4abd\cosh\left(c - \frac{ad}{b}\right)\right)}{2b^5}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2*Cosh[c + d*x])/(a + b*x)^3,x]

[Out] (CoshIntegral[d*(a/b + x)]*((2*b^2 + a^2*d^2)*Cosh[c - (a*d)/b] - 4*a*b*d*Sinh[c - (a*d)/b]) - (a*b*(-(b*(3*a + 4*b*x)*Cosh[c + d*x]) + a*d*(a + b*x)*Sinh[c + d*x]))/(a + b*x)^2 + (-4*a*b*d*Cosh[c - (a*d)/b] + (2*b^2 + a^2*d^2)*Sinh[c - (a*d)/b])*SinhIntegral[d*(a/b + x)]/(2*b^5)

fricas [A] time = 0.61, size = 475, normalized size = 1.97

$$\frac{2\left(4ab^3x + 3a^2b^2\right)\cosh(dx + c) + \left(a^4d^2 - 4a^3bd + 2a^2b^2 + \left(a^2b^2d^2 - 4ab^3d + 2b^4\right)x^2 + 2\left(a^3bd^2 - 4a^2b^2d + 2ab^3\right)x\right)e^{c + dx}}{2b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*cosh(d*x+c)/(b*x+a)^3,x, algorithm="fricas")

[Out] 1/4*(2*(4*a*b^3*x + 3*a^2*b^2)*cosh(d*x + c) + ((a^4*d^2 - 4*a^3*b*d + 2*a^2*b^2 + (a^2*b^2*d^2 - 4*a*b^3*d + 2*b^4)*x^2 + 2*(a^3*b*d^2 - 4*a^2*b^2*d + 2*a*b^3)*x)*Ei((b*d*x + a*d)/b) + (a^4*d^2 + 4*a^3*b*d + 2*a^2*b^2 + (a^2*b^2*d^2 + 4*a*b^3*d + 2*b^4)*x^2 + 2*(a^3*b*d^2 + 4*a^2*b^2*d + 2*a*b^3)*x)*Ei(-(b*d*x + a*d)/b))*cosh(-(b*c - a*d)/b) - 2*(a^2*b^2*d*x + a^3*b*d)*sinh(d*x + c) - ((a^4*d^2 - 4*a^3*b*d + 2*a^2*b^2 + (a^2*b^2*d^2 - 4*a*b^3*d + 2*b^4)*x^2 + 2*(a^3*b*d^2 - 4*a^2*b^2*d + 2*a*b^3)*x)*Ei((b*d*x + a*d)/b) - (a^4*d^2 + 4*a^3*b*d + 2*a^2*b^2 + (a^2*b^2*d^2 + 4*a*b^3*d + 2*b^4)*x^2 + 2*(a^3*b*d^2 + 4*a^2*b^2*d + 2*a*b^3)*x)*Ei(-(b*d*x + a*d)/b))*sinh(-(b*c - a*d)/b))/(b^7*x^2 + 2*a*b^6*x + a^2*b^5)

giac [B] time = 0.13, size = 741, normalized size = 3.07

$$\frac{a^2b^2d^2x^2\text{Ei}\left(\frac{bdx+ad}{b}\right)e^{\left(c-\frac{ad}{b}\right)} + a^2b^2d^2x^2\text{Ei}\left(-\frac{bdx+ad}{b}\right)e^{\left(-c+\frac{ad}{b}\right)} + 2a^3bd^2x\text{Ei}\left(\frac{bdx+ad}{b}\right)e^{\left(c-\frac{ad}{b}\right)} - 4ab^3dx^2\text{Ei}\left(\frac{bdx+ad}{b}\right)e^{\left(c-\frac{ad}{b}\right)} - 2a^3bd^2x\text{Ei}\left(-\frac{bdx+ad}{b}\right)e^{\left(-c+\frac{ad}{b}\right)} + 4ab^3dx^2\text{Ei}\left(-\frac{bdx+ad}{b}\right)e^{\left(-c+\frac{ad}{b}\right)}}{b^7x^2 + 2ab^6x + a^2b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*cosh(d*x+c)/(b*x+a)^3,x, algorithm="giac")

[Out] $\frac{1}{4}*(a^2*b^2*d^2*x^2*Ei((b*d*x + a*d)/b)*e^{(c - a*d/b)} + a^2*b^2*d^2*x^2*Ei(- (b*d*x + a*d)/b)*e^{(-c + a*d/b)} + 2*a^3*b*d^2*x*Ei((b*d*x + a*d)/b)*e^{(c - a*d/b)} - 4*a*b^3*d*x^2*Ei((b*d*x + a*d)/b)*e^{(c - a*d/b)} + 2*a^3*b*d^2*x*Ei(- (b*d*x + a*d)/b)*e^{(-c + a*d/b)} + 4*a*b^3*d*x^2*Ei(- (b*d*x + a*d)/b)*e^{(-c + a*d/b)} + a^4*d^2*Ei((b*d*x + a*d)/b)*e^{(c - a*d/b)} - 8*a^2*b^2*d*x*Ei((b*d*x + a*d)/b)*e^{(c - a*d/b)} + 2*b^4*x^2*Ei((b*d*x + a*d)/b)*e^{(c - a*d/b)} + a^4*d^2*Ei(- (b*d*x + a*d)/b)*e^{(-c + a*d/b)} + 8*a^2*b^2*d*x*Ei(- (b*d*x + a*d)/b)*e^{(-c + a*d/b)} + 2*b^4*x^2*Ei(- (b*d*x + a*d)/b)*e^{(-c + a*d/b)} - a^2*b^2*d*x*e^{(d*x + c)} + a^2*b^2*d*x*e^{(-d*x - c)} - 4*a^3*b*d*Ei((b*d*x + a*d)/b)*e^{(c - a*d/b)} + 4*a*b^3*x*Ei((b*d*x + a*d)/b)*e^{(c - a*d/b)} + 4*a^3*b*d*Ei(- (b*d*x + a*d)/b)*e^{(-c + a*d/b)} + 4*a*b^3*x*Ei(- (b*d*x + a*d)/b)*e^{(-c + a*d/b)} - a^3*b*d*e^{(d*x + c)} + 4*a*b^3*x*e^{(d*x + c)} + a^3*b*d*e^{(-d*x - c)} + 4*a*b^3*x*e^{(-d*x - c)} + 2*a^2*b^2*Ei((b*d*x + a*d)/b)*e^{(c - a*d/b)} + 2*a^2*b^2*Ei(- (b*d*x + a*d)/b)*e^{(-c + a*d/b)} + 3*a^2*b^2*e^{(d*x + c)} + 3*a^2*b^2*e^{(-d*x - c)})/(b^7*x^2 + 2*a*b^6*x + a^2*b^5)$

maple [B] time = 0.13, size = 527, normalized size = 2.19

$$\frac{d^3 e^{-dx-c} a^2 x}{4b^3 (b^2 d^2 x^2 + 2ab d^2 x + a^2 d^2)} + \frac{d^3 e^{-dx-c} a^3}{4b^4 (b^2 d^2 x^2 + 2ab d^2 x + a^2 d^2)} + \frac{d^2 e^{-dx-c} a x}{b^2 (b^2 d^2 x^2 + 2ab d^2 x + a^2 d^2)} + \frac{3d^2 e^{-dx-c}}{4b^3 (b^2 d^2 x^2 + 2ab d^2 x + a^2 d^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^2*\cosh(dx+c)/(b*x+a)^3, x)$

[Out] $\frac{1}{4}*d^3*\exp(-d*x-c)/b^3/(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)*a^2*x+1/4*d^3*\exp(-d*x-c)/b^4/(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)*a^3+d^2*\exp(-d*x-c)/b^2/(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)*a*x+3/4*d^2*\exp(-d*x-c)/b^3/(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)*a^2-1/4*d^2/b^5*\exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)*a^2-d/b^4*\exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)*a-1/2/b^3*\exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)-1/2/b^3*\exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)-1/4*d^2/b^5*\exp(d*x+c)/(a*d/b+d*x)^2*a^2-1/4*d^2/b^5*\exp(d*x+c)/(a*d/b+d*x)*a^2-1/4*d^2/b^5*\exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)*a^2+d/b^4*\exp(d*x+c)/(a*d/b+d*x)*a+d/b^4*\exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)*a$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{3}{2}ad \int \frac{x e^{(dx+c)}}{b^4 d^2 x^4 + 4ab^3 d^2 x^3 + 6a^2 b^2 d^2 x^2 + 4a^3 b d^2 x + a^4 d^2} dx + \frac{3}{2}ad \int \frac{x e^{(dx+c)}}{b^4 d^2 x^4 e^{(dx+c)} + 4ab^3 d^2 x^3 e^{(dx+c)} + 6a^2 b^2 d^2 x^2 e^{(dx+c)} + 4a^3 b d^2 x e^{(dx+c)} + a^4 d^2 e^{(dx+c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x^2*\cosh(dx+c)/(b*x+a)^3, x, \text{algorithm}=\text{"maxima"})$

```
[Out] -3/2*a*d*integrate(x*e^(d*x + c)/(b^4*d^2*x^4 + 4*a*b^3*d^2*x^3 + 6*a^2*b^2*d^2*x^2 + 4*a^3*b*d^2*x + a^4*d^2), x) + 3/2*a*d*integrate(x/(b^4*d^2*x^4*e^(d*x + c) + 4*a*b^3*d^2*x^3*e^(d*x + c) + 6*a^2*b^2*d^2*x^2*e^(d*x + c) + 4*a^3*b*d^2*x*e^(d*x + c) + a^4*d^2*e^(d*x + c)), x) + b*integrate(x*e^(d*x + c)/(b^4*d^2*x^4 + 4*a*b^3*d^2*x^3 + 6*a^2*b^2*d^2*x^2 + 4*a^3*b*d^2*x + a^4*d^2), x) + b*integrate(x/(b^4*d^2*x^4*e^(d*x + c) + 4*a*b^3*d^2*x^3*e^(d*x + c) + 6*a^2*b^2*d^2*x^2*e^(d*x + c) + 4*a^3*b*d^2*x*e^(d*x + c) + a^4*d^2*e^(d*x + c)), x) + 1/2*((d*x^2*e^(2*c) + x*e^(2*c))*e^(d*x) - (d*x^2 - x)*e^(-d*x))/(b^3*d^2*x^3*e^c + 3*a*b^2*d^2*x^2*e^c + 3*a^2*b*d^2*x*e^c + a^3*d^2*e^c) + 1/2*a*e^(-c + a*d/b)*exp_integral_e(4, (b*x + a)*d/b)/((b*x + a)^3*b*d^2) + 1/2*a*e^(c - a*d/b)*exp_integral_e(4, -(b*x + a)*d/b)/((b*x + a)^3*b*d^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2 \cosh(c + dx)}{(a + bx)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*cosh(c + d*x))/(a + b*x)^3, x)
```

```
[Out] int((x^2*cosh(c + d*x))/(a + b*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \cosh(c + dx)}{(a + bx)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cosh(d*x+c)/(b*x+a)**3, x)
```

```
[Out] Integral(x**2*cosh(c + d*x)/(a + b*x)**3, x)
```


$$3.35 \quad \int \frac{x \cosh(c+dx)}{(a+bx)^3} dx$$

Optimal. Leaf size=178

$$\frac{ad^2 \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{2b^4} - \frac{ad^2 \sinh\left(c - \frac{ad}{b}\right) \text{Shi}\left(xd + \frac{ad}{b}\right)}{2b^4} + \frac{d \sinh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{b^3} + \frac{d \cosh\left(c - \frac{ad}{b}\right) \text{Shi}\left(xd + \frac{ad}{b}\right)}{b^3}$$

[Out] $-1/2*a*d^2*Chi(a*d/b+d*x)*cosh(-c+a*d/b)/b^4+1/2*a*cosh(d*x+c)/b^2/(b*x+a)^2-cosh(d*x+c)/b^2/(b*x+a)+d*cosh(-c+a*d/b)*Shi(a*d/b+d*x)/b^3-d*Chi(a*d/b+d*x)*sinh(-c+a*d/b)/b^3+1/2*a*d^2*Shi(a*d/b+d*x)*sinh(-c+a*d/b)/b^4+1/2*a*d*sinh(d*x+c)/b^3/(b*x+a)$

Rubi [A] time = 0.37, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6742, 3297, 3303, 3298, 3301}

$$\frac{ad^2 \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{2b^4} + \frac{d \sinh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{b^3} - \frac{ad^2 \sinh\left(c - \frac{ad}{b}\right) \text{Shi}\left(xd + \frac{ad}{b}\right)}{2b^4} + \frac{d \cosh\left(c - \frac{ad}{b}\right) \text{Shi}\left(xd + \frac{ad}{b}\right)}{b^3}$$

Antiderivative was successfully verified.

[In] Int[(x*Cosh[c + d*x])/(a + b*x)^3,x]

[Out] $(a*\text{Cosh}[c + d*x])/(2*b^2*(a + b*x)^2) - \text{Cosh}[c + d*x]/(b^2*(a + b*x)) - (a*d^2*\text{Cosh}[c - (a*d)/b]*\text{CoshIntegral}[(a*d)/b + d*x])/(2*b^4) + (d*\text{CoshIntegral}[(a*d)/b + d*x]*\text{Sinh}[c - (a*d)/b])/b^3 + (a*d*\text{Sinh}[c + d*x])/(2*b^3*(a + b*x)) + (d*\text{Cosh}[c - (a*d)/b]*\text{SinhIntegral}[(a*d)/b + d*x])/b^3 - (a*d^2*\text{Sinh}[c - (a*d)/b]*\text{SinhIntegral}[(a*d)/b + d*x])/(2*b^4)$

Rule 3297

Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]

Rule 3298

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)])/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x]
&& EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x]
+ Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /;
FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]
```

Rule 6742

```
Int[u_, x_Symbol]
:> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x \cosh(c + dx)}{(a + bx)^3} dx &= \int \left(-\frac{a \cosh(c + dx)}{b(a + bx)^3} + \frac{\cosh(c + dx)}{b(a + bx)^2} \right) dx \\
&= \frac{\int \frac{\cosh(c+dx)}{(a+bx)^2} dx}{b} - \frac{a \int \frac{\cosh(c+dx)}{(a+bx)^3} dx}{b} \\
&= \frac{a \cosh(c + dx)}{2b^2(a + bx)^2} - \frac{\cosh(c + dx)}{b^2(a + bx)} + \frac{d \int \frac{\sinh(c+dx)}{a+bx} dx}{b^2} - \frac{(ad) \int \frac{\sinh(c+dx)}{(a+bx)^2} dx}{2b^2} \\
&= \frac{a \cosh(c + dx)}{2b^2(a + bx)^2} - \frac{\cosh(c + dx)}{b^2(a + bx)} + \frac{ad \sinh(c + dx)}{2b^3(a + bx)} - \frac{(ad^2) \int \frac{\cosh(c+dx)}{a+bx} dx}{2b^3} + \frac{\left(d \cosh \left(c - \frac{ad}{b} \right) \right)}{2b^3} \\
&= \frac{a \cosh(c + dx)}{2b^2(a + bx)^2} - \frac{\cosh(c + dx)}{b^2(a + bx)} + \frac{d \operatorname{Chi} \left(\frac{ad}{b} + dx \right) \sinh \left(c - \frac{ad}{b} \right)}{b^3} + \frac{ad \sinh(c + dx)}{2b^3(a + bx)} + \frac{d \cosh \left(c - \frac{ad}{b} \right)}{2b^3} \\
&= \frac{a \cosh(c + dx)}{2b^2(a + bx)^2} - \frac{\cosh(c + dx)}{b^2(a + bx)} - \frac{ad^2 \cosh \left(c - \frac{ad}{b} \right) \operatorname{Chi} \left(\frac{ad}{b} + dx \right)}{2b^4} + \frac{d \operatorname{Chi} \left(\frac{ad}{b} + dx \right) \sinh \left(c - \frac{ad}{b} \right)}{b^3}
\end{aligned}$$

Mathematica [A] time = 0.60, size = 158, normalized size = 0.89

$$\frac{d(a + bx)^2 \left(\operatorname{Chi} \left(d \left(\frac{a}{b} + x \right) \right) \left(ad \cosh \left(c - \frac{ad}{b} \right) - 2b \sinh \left(c - \frac{ad}{b} \right) \right) + \operatorname{Shi} \left(d \left(\frac{a}{b} + x \right) \right) \left(ad \sinh \left(c - \frac{ad}{b} \right) - 2b \cosh \left(c - \frac{ad}{b} \right) \right) \right)}{2b^4}$$

Antiderivative was successfully verified.

[In] Integrate[(x*Cosh[c + d*x])/(a + b*x)^3,x]

[Out]
$$-1/2*(b*\text{Cosh}[d*x]*(b*(a + 2*b*x)*\text{Cosh}[c] - a*d*(a + b*x)*\text{Sinh}[c]) - b*(a*d*(a + b*x)*\text{Cosh}[c] - b*(a + 2*b*x)*\text{Sinh}[c]))*\text{Sinh}[d*x] + d*(a + b*x)^2*(\text{CoshIntegral}[d*(a/b + x)]*(a*d*\text{Cosh}[c - (a*d)/b] - 2*b*\text{Sinh}[c - (a*d)/b])) + (-2*b*\text{Cosh}[c - (a*d)/b] + a*d*\text{Sinh}[c - (a*d)/b])*\text{SinhIntegral}[d*(a/b + x)))/(b^4*(a + b*x)^2)$$

fricas [B] time = 0.53, size = 373, normalized size = 2.10

$$2(2b^3x + ab^2) \cosh(dx + c) + \left((a^3d^2 - 2a^2bd + (ab^2d^2 - 2b^3d)x^2 + 2(a^2bd^2 - 2ab^2d)x \right) \text{Ei}\left(\frac{bdx+ad}{b}\right) + (a^3d^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cosh(d*x+c)/(b*x+a)^3,x, algorithm="fricas")

[Out]
$$-1/4*(2*(2*b^3*x + a*b^2)*\cosh(d*x + c) + ((a^3*d^2 - 2*a^2*b*d + (a*b^2*d^2 - 2*b^3*d)*x^2 + 2*(a^2*b*d^2 - 2*a*b^2*d)*x)*\text{Ei}((b*d*x + a*d)/b) + (a^3*d^2 + 2*a^2*b*d + (a*b^2*d^2 + 2*b^3*d)*x^2 + 2*(a^2*b*d^2 + 2*a*b^2*d)*x)*\text{Ei}(-(b*d*x + a*d)/b))*\cosh(-(b*c - a*d)/b) - 2*(a*b^2*d*x + a^2*b*d)*\sinh(d*x + c) - ((a^3*d^2 - 2*a^2*b*d + (a*b^2*d^2 - 2*b^3*d)*x^2 + 2*(a^2*b*d^2 - 2*a*b^2*d)*x)*\text{Ei}((b*d*x + a*d)/b) - (a^3*d^2 + 2*a^2*b*d + (a*b^2*d^2 + 2*b^3*d)*x^2 + 2*(a^2*b*d^2 + 2*a*b^2*d)*x)*\text{Ei}(-(b*d*x + a*d)/b))*\sinh(-(b*c - a*d)/b))/(b^6*x^2 + 2*a*b^5*x + a^2*b^4)$$

giac [B] time = 0.14, size = 529, normalized size = 2.97

$$ab^2d^2x^2\text{Ei}\left(\frac{bdx+ad}{b}\right)e^{\left(c-\frac{ad}{b}\right)} + ab^2d^2x^2\text{Ei}\left(-\frac{bdx+ad}{b}\right)e^{\left(-c+\frac{ad}{b}\right)} + 2a^2bd^2x\text{Ei}\left(\frac{bdx+ad}{b}\right)e^{\left(c-\frac{ad}{b}\right)} - 2b^3dx^2\text{Ei}\left(\frac{bdx+ad}{b}\right)e^{\left(c-\frac{ad}{b}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cosh(d*x+c)/(b*x+a)^3,x, algorithm="giac")

[Out]
$$-1/4*(a*b^2*d^2*x^2*\text{Ei}((b*d*x + a*d)/b)*e^{(c - a*d/b)} + a*b^2*d^2*x^2*\text{Ei}(-(b*d*x + a*d)/b)*e^{(-c + a*d/b)} + 2*a^2*b*d^2*x*\text{Ei}((b*d*x + a*d)/b)*e^{(c - a*d/b)} - 2*b^3*d*x^2*\text{Ei}((b*d*x + a*d)/b)*e^{(c - a*d/b)} + 2*a^2*b*d^2*x*\text{Ei}(-(b*d*x + a*d)/b)*e^{(-c + a*d/b)} + 2*b^3*d*x^2*\text{Ei}(-(b*d*x + a*d)/b)*e^{(-c + a*d/b)} + a^3*d^2*\text{Ei}((b*d*x + a*d)/b)*e^{(c - a*d/b)} - 4*a*b^2*d*x*\text{Ei}((b*d*x + a*d)/b)*e^{(c - a*d/b)} + a^3*d^2*\text{Ei}(-(b*d*x + a*d)/b)*e^{(-c + a*d/b)} + 4*a*b^2*d*x*\text{Ei}(-(b*d*x + a*d)/b)*e^{(-c + a*d/b)} - a*b^2*d*x*e^{(d*x + c)} + a*b^2$$

$d*x*e^{(-d*x - c)} - 2*a^2*b*d*Ei((b*d*x + a*d)/b)*e^{(c - a*d/b)} + 2*a^2*b*d*Ei(-(b*d*x + a*d)/b)*e^{(-c + a*d/b)} - a^2*b*d*e^{(d*x + c)} + 2*b^3*x*e^{(d*x + c)} + a^2*b*d*e^{(-d*x - c)} + 2*b^3*x*e^{(-d*x - c)} + a*b^2*e^{(d*x + c)} + a*b^2*e^{(-d*x - c)} / (b^6*x^2 + 2*a*b^5*x + a^2*b^4)$

maple [B] time = 0.11, size = 435, normalized size = 2.44

$$\frac{d^3 e^{-dx-c} ax}{4b^2 (b^2 d^2 x^2 + 2ab d^2 x + a^2 d^2)} - \frac{d^3 e^{-dx-c} a^2}{4b^3 (b^2 d^2 x^2 + 2ab d^2 x + a^2 d^2)} - \frac{d^2 e^{-dx-c} x}{2b (b^2 d^2 x^2 + 2ab d^2 x + a^2 d^2)} - \frac{d^2 e^{-dx-c}}{4b^2 (b^2 d^2 x^2 + 2ab d^2 x + a^2 d^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*cosh(d*x+c)/(b*x+a)^3,x)`

[Out] $-1/4*d^3*exp(-d*x-c)/b^2/(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)*a*x-1/4*d^3*exp(-d*x-c)/b^3/(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)*a^2-1/2*d^2*exp(-d*x-c)/b/(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)*x-1/4*d^2*exp(-d*x-c)/b^2/(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)*a+1/4*d^2/b^4*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)*a+1/2*d/b^3*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)+1/4*d^2/b^4*exp(d*x+c)/(a*d/b+d*x)^2*a+1/4*d^2/b^4*exp(d*x+c)/(a*d/b+d*x)*a+1/4*d^2/b^4*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)*a-1/2*d/b^3*exp(d*x+c)/(a*d/b+d*x)-1/2*d/b^3*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$b \int \frac{x e^{(dx+c)}}{b^4 dx^4 + 4 ab^3 dx^3 + 6 a^2 b^2 dx^2 + 4 a^3 b dx + a^4 d} dx - b \int \frac{x}{b^4 dx^4 e^{(dx+c)} + 4 ab^3 dx^3 e^{(dx+c)} + 6 a^2 b^2 dx^2 e^{(dx+c)} + 4 a^3 b dx e^{(dx+c)} + a^4 d e^{(dx+c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*cosh(d*x+c)/(b*x+a)^3,x, algorithm="maxima")`

[Out] $b*\text{integrate}(x*e^{(d*x + c)}/(b^4*d*x^4 + 4*a*b^3*d*x^3 + 6*a^2*b^2*d*x^2 + 4*a^3*b*d*x + a^4*d), x) - b*\text{integrate}(x/(b^4*d*x^4*e^{(d*x + c)} + 4*a*b^3*d*x^3*e^{(d*x + c)} + 6*a^2*b^2*d*x^2*e^{(d*x + c)} + 4*a^3*b*d*x*e^{(d*x + c)} + a^4*d*e^{(d*x + c)}), x) + 1/2*(x*e^{(d*x + 2*c)} - x*e^{(-d*x)})/(b^3*d*x^3*e^c + 3*a*b^2*d*x^2*e^c + 3*a^2*b*d*x*e^c + a^3*d*e^c) - 1/2*a*e^{(-c + a*d/b)}*exp_integral_e(4, (b*x + a)*d/b)/((b*x + a)^3*b*d) + 1/2*a*e^{(c - a*d/b)}*exp_integral_e(4, -(b*x + a)*d/b)/((b*x + a)^3*b*d)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x \cosh(c + d x)}{(a + b x)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*cosh(c + d*x))/(a + b*x)^3,x)
```

```
[Out] int((x*cosh(c + d*x))/(a + b*x)^3, x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(d*x+c)/(b*x+a)**3,x)
```

```
[Out] Timed out
```

$$3.36 \quad \int \frac{\cosh(c+dx)}{(a+bx)^3} dx$$

Optimal. Leaf size=104

$$\frac{d^2 \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{2b^3} + \frac{d^2 \sinh\left(c - \frac{ad}{b}\right) \text{Shi}\left(xd + \frac{ad}{b}\right)}{2b^3} - \frac{d \sinh(c+dx)}{2b^2(a+bx)} - \frac{\cosh(c+dx)}{2b(a+bx)^2}$$

[Out] $1/2*d^2*Chi(a*d/b+d*x)*cosh(-c+a*d/b)/b^3-1/2*cosh(d*x+c)/b/(b*x+a)^2-1/2*d^2*Shi(a*d/b+d*x)*sinh(-c+a*d/b)/b^3-1/2*d*sinh(d*x+c)/b^2/(b*x+a)$

Rubi [A] time = 0.15, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3297, 3303, 3298, 3301}

$$\frac{d^2 \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{2b^3} + \frac{d^2 \sinh\left(c - \frac{ad}{b}\right) \text{Shi}\left(xd + \frac{ad}{b}\right)}{2b^3} - \frac{d \sinh(c+dx)}{2b^2(a+bx)} - \frac{\cosh(c+dx)}{2b(a+bx)^2}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]/(a + b*x)^3, x]

[Out] $-Cosh[c + d*x]/(2*b*(a + b*x)^2) + (d^2*Cosh[c - (a*d)/b]*CoshIntegral[(a*d)/b + d*x])/(2*b^3) - (d*Sinh[c + d*x])/(2*b^2*(a + b*x)) + (d^2*Sinh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/(2*b^3)$

Rule 3297

Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]

Rule 3298

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)])/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]

Rule 3301

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)])/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{\cosh(c+dx)}{(a+bx)^3} dx &= -\frac{\cosh(c+dx)}{2b(a+bx)^2} + \frac{d \int \frac{\sinh(c+dx)}{(a+bx)^2} dx}{2b} \\ &= -\frac{\cosh(c+dx)}{2b(a+bx)^2} - \frac{d \sinh(c+dx)}{2b^2(a+bx)} + \frac{d^2 \int \frac{\cosh(c+dx)}{a+bx} dx}{2b^2} \\ &= -\frac{\cosh(c+dx)}{2b(a+bx)^2} - \frac{d \sinh(c+dx)}{2b^2(a+bx)} + \frac{\left(d^2 \cosh\left(c - \frac{ad}{b}\right)\right) \int \frac{\cosh\left(\frac{ad}{b} + dx\right)}{a+bx} dx}{2b^2} + \frac{\left(d^2 \sinh\left(c - \frac{ad}{b}\right)\right) \text{Shi}\left(d\left(\frac{a}{b} + x\right)\right)}{2b^2} \\ &= -\frac{\cosh(c+dx)}{2b(a+bx)^2} + \frac{d^2 \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(\frac{ad}{b} + dx\right)}{2b^3} - \frac{d \sinh(c+dx)}{2b^2(a+bx)} + \frac{d^2 \sinh\left(c - \frac{ad}{b}\right) \text{Shi}\left(d\left(\frac{a}{b} + x\right)\right)}{2b^3} \end{aligned}$$

Mathematica [A] time = 0.51, size = 88, normalized size = 0.85

$$\frac{d^2 \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(d\left(\frac{a}{b} + x\right)\right) + d^2 \sinh\left(c - \frac{ad}{b}\right) \text{Shi}\left(d\left(\frac{a}{b} + x\right)\right) - \frac{b(d(a+bx) \sinh(c+dx) + b \cosh(c+dx))}{(a+bx)^2}}{2b^3}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d*x]/(a + b*x)^3, x]

[Out] (d^2*Cosh[c - (a*d)/b]*CoshIntegral[d*(a/b + x)] - (b*(b*Cosh[c + d*x] + d*(a + b*x)*Sinh[c + d*x]))/(a + b*x)^2 + d^2*Sinh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)]/(2*b^3)

fricas [B] time = 0.48, size = 253, normalized size = 2.43

$$\frac{2b^2 \cosh(dx + c) - \left(\left(b^2 d^2 x^2 + 2abd^2 x + a^2 d^2\right) \text{Ei}\left(\frac{bdx+ad}{b}\right) + \left(b^2 d^2 x^2 + 2abd^2 x + a^2 d^2\right) \text{Ei}\left(-\frac{bdx+ad}{b}\right)\right) \cosh\left(-\right)}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/(b*x+a)^3,x, algorithm="fricas")

[Out]
$$-1/4*(2*b^2*cosh(d*x + c) - ((b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*Ei((b*d*x + a*d)/b) + (b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*Ei(-(b*d*x + a*d)/b))*cosh(-(b*c - a*d)/b) + 2*(b^2*d*x + a*b*d)*sinh(d*x + c) + ((b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*Ei((b*d*x + a*d)/b) - (b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*Ei(-(b*d*x + a*d)/b))*sinh(-(b*c - a*d)/b))/(b^5*x^2 + 2*a*b^4*x + a^2*b^3)$$

giac [B] time = 0.13, size = 298, normalized size = 2.87

$$\frac{b^2 d^2 x^2 \operatorname{Ei}\left(\frac{bdx+ad}{b}\right) e^{\left(c-\frac{ad}{b}\right)} + b^2 d^2 x^2 \operatorname{Ei}\left(-\frac{bdx+ad}{b}\right) e^{\left(-c+\frac{ad}{b}\right)} + 2 ab d^2 x \operatorname{Ei}\left(\frac{bdx+ad}{b}\right) e^{\left(c-\frac{ad}{b}\right)} + 2 ab d^2 x \operatorname{Ei}\left(-\frac{bdx+ad}{b}\right) e^{\left(-c+\frac{ad}{b}\right)}}{4(b^5 x^2 + 2 a b^4 x + a^2 b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/(b*x+a)^3,x, algorithm="giac")

[Out]
$$1/4*(b^2*d^2*x^2*Ei((b*d*x + a*d)/b)*e^{(c - a*d/b)} + b^2*d^2*x^2*Ei(-(b*d*x + a*d)/b)*e^{(-c + a*d/b)} + 2*a*b*d^2*x*Ei((b*d*x + a*d)/b)*e^{(c - a*d/b)} + 2*a*b*d^2*x*Ei(-(b*d*x + a*d)/b)*e^{(-c + a*d/b)} + a^2*d^2*Ei((b*d*x + a*d)/b)*e^{(c - a*d/b)} + a^2*d^2*Ei(-(b*d*x + a*d)/b)*e^{(-c + a*d/b)} - b^2*d*x*e^{(d*x + c)} + b^2*d*x*e^{(-d*x - c)} - a*b*d*e^{(d*x + c)} + a*b*d*e^{(-d*x - c)} - b^2*e^{(d*x + c)} - b^2*e^{(-d*x - c)})/(b^5*x^2 + 2*a*b^4*x + a^2*b^3)$$

maple [B] time = 0.09, size = 276, normalized size = 2.65

$$\frac{d^3 e^{-dx-c} x}{4b(b^2 d^2 x^2 + 2ab d^2 x + a^2 d^2)} + \frac{d^3 e^{-dx-c} a}{4b^2(b^2 d^2 x^2 + 2ab d^2 x + a^2 d^2)} - \frac{d^2 e^{-dx-c}}{4b(b^2 d^2 x^2 + 2ab d^2 x + a^2 d^2)} - \frac{d^2 e^{\frac{da-cb}{b}} \operatorname{Ei}\left(1, dx + \frac{da-cb}{b}\right)}{4b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d*x+c)/(b*x+a)^3,x)

[Out]
$$1/4*d^3*exp(-d*x-c)/b/(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)*x+1/4*d^3*exp(-d*x-c)/b^2/(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)*a-1/4*d^2*exp(-d*x-c)/b/(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)-1/4*d^2/b^3*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)-1/4*d^2/b^3*exp(d*x+c)/(a*d/b+d*x)^2-1/4*d^2/b^3*exp(d*x+c)/(a*d/b+d*x)-1/4*d^2/b^3*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)$$

maxima [A] time = 0.36, size = 95, normalized size = 0.91

$$\frac{d \left(\frac{e^{\left(-c + \frac{ad}{b}\right)} E_2\left(\frac{(bx+a)d}{b}\right)}{(bx+a)b} - \frac{e^{\left(c - \frac{ad}{b}\right)} E_2\left(-\frac{(bx+a)d}{b}\right)}{(bx+a)b} \right)}{4b} - \frac{\cosh(dx + c)}{2(bx + a)^2 b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/(b*x+a)^3,x, algorithm="maxima")

[Out] 1/4*d*(e^(-c + a*d/b)*exp_integral_e(2, (b*x + a)*d/b)/((b*x + a)*b) - e^(c - a*d/b)*exp_integral_e(2, -(b*x + a)*d/b)/((b*x + a)*b))/b - 1/2*cosh(d*x + c)/((b*x + a)^2*b)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c + dx)}{(a + bx)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(c + d*x)/(a + b*x)^3,x)

[Out] int(cosh(c + d*x)/(a + b*x)^3, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/(b*x+a)**3,x)

[Out] Timed out

$$3.37 \quad \int \frac{\cosh(c+dx)}{x(a+bx)^3} dx$$

Optimal. Leaf size=262

$$\frac{\cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right) - \sinh\left(c - \frac{ad}{b}\right) \text{Shi}\left(xd + \frac{ad}{b}\right)}{a^3} + \frac{\cosh(c) \text{Chi}(dx) - \sinh(c) \text{Shi}(dx)}{a^3} + \frac{d \sinh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right) - d \cosh\left(c - \frac{ad}{b}\right) \text{Shi}\left(xd + \frac{ad}{b}\right)}{a^2 b}$$

[Out] Chi(d*x)*cosh(c)/a^3-Chi(a*d/b+d*x)*cosh(-c+a*d/b)/a^3-1/2*d^2*Chi(a*d/b+d*x)*cosh(-c+a*d/b)/a/b^2+1/2*cosh(d*x+c)/a/(b*x+a)^2+cosh(d*x+c)/a^2/(b*x+a)-d*cosh(-c+a*d/b)*Shi(a*d/b+d*x)/a^2/b+Shi(d*x)*sinh(c)/a^3+d*Chi(a*d/b+d*x)*sinh(-c+a*d/b)/a^2/b+Shi(a*d/b+d*x)*sinh(-c+a*d/b)/a^3+1/2*d^2*Shi(a*d/b+d*x)*sinh(-c+a*d/b)/a/b^2+1/2*d*sinh(d*x+c)/a/b/(b*x+a)

Rubi [A] time = 0.56, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {6742, 3303, 3298, 3301, 3297}

$$\frac{d \sinh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right) - d \cosh\left(c - \frac{ad}{b}\right) \text{Shi}\left(xd + \frac{ad}{b}\right)}{a^2 b} + \frac{\cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right) - \sinh\left(c - \frac{ad}{b}\right) \text{Shi}\left(xd + \frac{ad}{b}\right)}{a^3} + \frac{d \cosh\left(c - \frac{ad}{b}\right) \text{Shi}\left(xd + \frac{ad}{b}\right) - d \sinh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{a^2 b}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]/(x*(a + b*x)^3), x]

[Out] Cosh[c + d*x]/(2*a*(a + b*x)^2) + Cosh[c + d*x]/(a^2*(a + b*x)) + (Cosh[c]*CoshIntegral[d*x])/a^3 - (Cosh[c - (a*d)/b]*CoshIntegral[(a*d)/b + d*x])/a^3 - (d^2*Cosh[c - (a*d)/b]*CoshIntegral[(a*d)/b + d*x])/(2*a*b^2) - (d*CoshIntegral[(a*d)/b + d*x]*Sinh[c - (a*d)/b])/(a^2*b) + (d*Sinh[c + d*x])/(2*a*b*(a + b*x)) + (Sinh[c]*SinhIntegral[d*x])/a^3 - (d*Cosh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/(a^2*b) - (Sinh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/a^3 - (d^2*Sinh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/(2*a*b^2)

Rule 3297

Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]

Rule 3298

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)])/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f}

, fz}, x] && EqQ[d*e - c*f*fz*I, 0]

Rule 3301

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]

Rule 3303

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]

Rule 6742

Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

Rubi steps

$$\begin{aligned}
 \int \frac{\cosh(c+dx)}{x(a+bx)^3} dx &= \int \left(\frac{\cosh(c+dx)}{a^3 x} - \frac{b \cosh(c+dx)}{a(a+bx)^3} - \frac{b \cosh(c+dx)}{a^2(a+bx)^2} - \frac{b \cosh(c+dx)}{a^3(a+bx)} \right) dx \\
 &= \frac{\int \frac{\cosh(c+dx)}{x} dx}{a^3} - \frac{b \int \frac{\cosh(c+dx)}{a+bx} dx}{a^3} - \frac{b \int \frac{\cosh(c+dx)}{(a+bx)^2} dx}{a^2} - \frac{b \int \frac{\cosh(c+dx)}{(a+bx)^3} dx}{a} \\
 &= \frac{\cosh(c+dx)}{2a(a+bx)^2} + \frac{\cosh(c+dx)}{a^2(a+bx)} - \frac{d \int \frac{\sinh(c+dx)}{a+bx} dx}{a^2} - \frac{d \int \frac{\sinh(c+dx)}{(a+bx)^2} dx}{2a} + \frac{\cosh(c) \int \frac{\cosh(dx)}{x} dx}{a^3} \\
 &= \frac{\cosh(c+dx)}{2a(a+bx)^2} + \frac{\cosh(c+dx)}{a^2(a+bx)} + \frac{\cosh(c)\text{Chi}(dx)}{a^3} - \frac{\cosh\left(c - \frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b} + dx\right)}{a^3} + \frac{d \sinh(c)}{2ab(a+bx)} \\
 &= \frac{\cosh(c+dx)}{2a(a+bx)^2} + \frac{\cosh(c+dx)}{a^2(a+bx)} + \frac{\cosh(c)\text{Chi}(dx)}{a^3} - \frac{\cosh\left(c - \frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b} + dx\right)}{a^3} - \frac{d \text{Chi}\left(\frac{ad}{b} + dx\right)}{2ab(a+bx)} \\
 &= \frac{\cosh(c+dx)}{2a(a+bx)^2} + \frac{\cosh(c+dx)}{a^2(a+bx)} + \frac{\cosh(c)\text{Chi}(dx)}{a^3} - \frac{\cosh\left(c - \frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b} + dx\right)}{a^3} - \frac{d^2 \cos\left(\frac{ad}{b} + dx\right)}{2ab(a+bx)}
 \end{aligned}$$

Mathematica [A] time = 1.38, size = 451, normalized size = 1.72

$$a^4 d^2 \sinh\left(c - \frac{ad}{b}\right) \text{Shi}\left(d\left(\frac{a}{b} + x\right)\right) + 2a^3 b d^2 x \sinh\left(c - \frac{ad}{b}\right) \text{Shi}\left(d\left(\frac{a}{b} + x\right)\right) + 2a^3 b d \cosh\left(c - \frac{ad}{b}\right) \text{Shi}\left(d\left(\frac{a}{b} + x\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d*x]/(x*(a + b*x)^3), x]

[Out]
$$\begin{aligned} & -1/2*(-3*a^2*b^2*Cosh[c + d*x] - 2*a*b^3*x*Cosh[c + d*x] - 2*b^2*(a + b*x)^2*Cosh[c]*CoshIntegral[d*x] + (a + b*x)^2*CoshIntegral[d*(a/b + x)]*((2*b^2 + a^2*d^2)*Cosh[c - (a*d)/b] + 2*a*b*d*Sinh[c - (a*d)/b]) - a^3*b*d*Sinh[c + d*x] - a^2*b^2*d*x*Sinh[c + d*x] - 2*a^2*b^2*Sinh[c]*SinhIntegral[d*x] - 4*a*b^3*x*Sinh[c]*SinhIntegral[d*x] - 2*b^4*x^2*Sinh[c]*SinhIntegral[d*x] \\ & + 2*a^3*b*d*Cosh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)] + 4*a^2*b^2*d*x*Cosh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)] + 2*a*b^3*d*x^2*Cosh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)] + 2*a^2*b^2*Sinh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)] + a^4*d^2*Sinh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)] + 4*a*b^3*x*Sinh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)] + 2*a^3*b*d^2*x*Sinh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)] + 2*b^4*x^2*Sinh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)] + a^2*b^2*d^2*x^2*Sinh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)]/(a^3*b^2*(a + b*x)^2) \end{aligned}$$

fricas [B] time = 0.48, size = 601, normalized size = 2.29

$$2\left(2ab^3x + 3a^2b^2\right) \cosh(dx + c) + 2\left(\left(b^4x^2 + 2ab^3x + a^2b^2\right) \text{Ei}(dx) + \left(b^4x^2 + 2ab^3x + a^2b^2\right) \text{Ei}(-dx)\right) \cosh(c) -$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/x/(b*x+a)^3,x, algorithm="fricas")

[Out]
$$\begin{aligned} & 1/4*(2*(2*a*b^3*x + 3*a^2*b^2)*\cosh(d*x + c) + 2*((b^4*x^2 + 2*a*b^3*x + a^2*b^2)*\text{Ei}(d*x) + (b^4*x^2 + 2*a*b^3*x + a^2*b^2)*\text{Ei}(-d*x))*\cosh(c) - ((a^4*d^2 + 2*a^3*b*d + 2*a^2*b^2 + (a^2*b^2*d^2 + 2*a*b^3*d + 2*b^4)*x^2 + 2*(a^3*b*d^2 + 2*a^2*b^2*d + 2*a*b^3)*x)*\text{Ei}((b*d*x + a*d)/b) + (a^4*d^2 - 2*a^3*b*d + 2*a^2*b^2 + (a^2*b^2*d^2 - 2*a*b^3*d + 2*b^4)*x^2 + 2*(a^3*b*d^2 - 2*a^2*b^2*d + 2*a*b^3)*x)*\text{Ei}(-(b*d*x + a*d)/b))*\cosh(-(b*c - a*d)/b) + 2*(a^2*b^2*d*x + a^3*b*d)*\sinh(d*x + c) + 2*((b^4*x^2 + 2*a*b^3*x + a^2*b^2)*\text{Ei}(d*x) - (b^4*x^2 + 2*a*b^3*x + a^2*b^2)*\text{Ei}(-d*x))*\sinh(c) + ((a^4*d^2 + 2*a^3*b*d + 2*a^2*b^2 + (a^2*b^2*d^2 + 2*a*b^3*d + 2*b^4)*x^2 + 2*(a^3*b*d^2 + 2*a^2*b^2*d + 2*a*b^3)*x)*\text{Ei}((b*d*x + a*d)/b) - (a^4*d^2 - 2*a^3*b*d + 2*a^2*b^2 + (a^2*b^2*d^2 - 2*a*b^3*d + 2*b^4)*x^2 + 2*(a^3*b*d^2 - 2*a^2*b^2*d + 2*a*b^3)*x)*\text{Ei}(-(b*d*x + a*d)/b))*\sinh(-(b*c - a*d)/b))/(a^3*b^4*x^2 + 2*a^4*b^3*x + a^5*b^2) \end{aligned}$$

giac [B] time = 0.14, size = 837, normalized size = 3.19

$$\frac{a^2 b^2 d^2 x^2 \operatorname{Ei}\left(\frac{bdx+ad}{b}\right) e^{\left(c-\frac{ad}{b}\right)} + a^2 b^2 d^2 x^2 \operatorname{Ei}\left(-\frac{bdx+ad}{b}\right) e^{\left(-c+\frac{ad}{b}\right)} + 2 a^3 b d^2 x \operatorname{Ei}\left(\frac{bdx+ad}{b}\right) e^{\left(c-\frac{ad}{b}\right)} + 2 a b^3 d x^2 \operatorname{Ei}\left(\frac{bdx+ad}{b}\right)}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/x/(b*x+a)^3,x, algorithm="giac")

[Out]
$$\begin{aligned} & -1/4*(a^2*b^2*d^2*x^2*\operatorname{Ei}((b*d*x + a*d)/b)*e^{(c - a*d/b)} + a^2*b^2*d^2*x^2*\operatorname{Ei} \\ & i(-(b*d*x + a*d)/b)*e^{(-c + a*d/b)} + 2*a^3*b*d^2*x*\operatorname{Ei}((b*d*x + a*d)/b)*e^{(c - a*d/b)} + 2*a^3*b*d^2*x \\ & * \operatorname{Ei}(-(b*d*x + a*d)/b)*e^{(-c + a*d/b)} + 2*a^3*b*d^2*x*\operatorname{Ei}((b*d*x + a*d)/b)*e^{(c - a*d/b)} + 2*a^3*b*d^2*x \\ & * \operatorname{Ei}(-(b*d*x + a*d)/b)*e^{(-c + a*d/b)} - 2*a*b^3*d*x^2*\operatorname{Ei}(-(b*d*x + a*d)/b)*e^{(-c + a*d/b)} - 2*b^4*x^2*\operatorname{Ei}(-d*x)*e^{(-c)} + a^4*d^2*\operatorname{Ei}((b*d*x + a*d)/b)*e^{(c - a*d/b)} + 4*a^2*b^2*d*x*\operatorname{Ei}((b*d*x + a*d)/b)*e^{(c - a*d/b)} + 2*b^4*x^2*\operatorname{Ei}((b*d*x + a*d)/b)*e^{(c - a*d/b)} - 2*b^4*x^2*\operatorname{Ei}(d*x)*e^c + a^4*d^2*\operatorname{Ei}(-(b*d*x + a*d)/b)*e^{(-c + a*d/b)} - 4*a^2*b^2*d*x*\operatorname{Ei}(-(b*d*x + a*d)/b)*e^{(-c + a*d/b)} + 2*b^4*x^2*\operatorname{Ei}(-(b*d*x + a*d)/b)*e^{(-c + a*d/b)} - a^2*b^2*d*x*e^{(d*x + c)} + a^2*b^2*d*x*e^{(-d*x - c)} - 4*a*b^3*x*\operatorname{Ei}(-d*x)*e^{(-c)} + 2*a^3*b*d*\operatorname{Ei}((b*d*x + a*d)/b)*e^{(c - a*d/b)} + 4*a*b^3*x*\operatorname{Ei}((b*d*x + a*d)/b)*e^{(c - a*d/b)} - 4*a*b^3*x*\operatorname{Ei}(d*x)*e^c - 2*a^3*b*d*\operatorname{Ei}(-(b*d*x + a*d)/b)*e^{(-c + a*d/b)} + 4*a*b^3*x*\operatorname{Ei}(-(b*d*x + a*d)/b)*e^{(-c + a*d/b)} - a^3*b*d*e^{(d*x + c)} - 2*a*b^3*x*e^{(d*x + c)} + a^3*b*d*e^{(-d*x - c)} - 2*a*b^3*x*e^{(-d*x - c)} - 2*a^2*b^2*\operatorname{Ei}(-d*x)*e^{(-c)} + 2*a^2*b^2*\operatorname{Ei}((b*d*x + a*d)/b)*e^{(c - a*d/b)} - 2*a^2*b^2*\operatorname{Ei}(d*x)*e^c + 2*a^2*b^2*\operatorname{Ei}(-(b*d*x + a*d)/b)*e^{(-c + a*d/b)} - 3*a^2*b^2*e^{(d*x + c)} - 3*a^2*b^2*e^{(-d*x - c)})/(a^3*b^4*x^2 + 2*a^4*b^3*x + a^5*b^2) \end{aligned}$$

maple [A] time = 0.11, size = 488, normalized size = 1.86

$$\frac{e^{-dx-c} d \left((dx+c) abd + a^2 d^2 - cdba - 2b^2 (dx+c) - 3bda + 2c b^2 \right)}{4a^2 b \left(b^2 (dx+c)^2 + 2(dx+c) abd - 2(dx+c) b^2 c + a^2 d^2 - 2cdba + c^2 b^2 \right)} + \frac{e^{\frac{da-cb}{b}} \operatorname{Ei}\left(1, dx+c+\frac{da-cb}{b}\right) d^2}{4a b^2} - e^{\frac{da-cb}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d*x+c)/x/(b*x+a)^3,x)

[Out]
$$\begin{aligned} & -1/4*\exp(-d*x-c)*d*((d*x+c)*a*b*d+a^2*d^2-c*d*b*a-2*b^2*(d*x+c)-3*b*d*a+2*c \\ & *b^2)/a^2/b/(b^2*(d*x+c)^2+2*(d*x+c)*a*b*d-2*(d*x+c)*b^2*c+a^2*d^2-2*c*d*b* \\ & a+c^2*b^2)+1/4/a/b^2*\exp((a*d-b*c)/b)*\operatorname{Ei}\left(1, d*x+c+(a*d-b*c)/b\right)*d^2-1/2/a^2/b \\ & * \exp((a*d-b*c)/b)*\operatorname{Ei}\left(1, d*x+c+(a*d-b*c)/b\right)*d+1/2/a^3*\exp((a*d-b*c)/b)*\operatorname{Ei}\left(1, d \\ & *x+c+(a*d-b*c)/b\right)-1/2/a^3*\exp(-c)*\operatorname{Ei}\left(1, d*x\right)+1/2/a^3*\exp(-(a*d-b*c)/b)*\operatorname{Ei}\left(1, \\ & -d*x-c-(a*d-b*c)/b\right)+1/4/a*d^2/b^2*\exp(d*x+c)/(a*d/b+d*x)^2+1/4/a*d^2/b^2*\exp \end{aligned}$$

$p(d*x+c)/(a*d/b+d*x)+1/4/a*d^2/b^2*\exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)+1/2/b/a^2*d*\exp(d*x+c)/(a*d/b+d*x)+1/2/b/a^2*d*\exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)-1/2/a^3*\exp(c)*Ei(1,-d*x)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(dx + c)}{(bx + a)^3 x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/x/(b*x+a)^3,x, algorithm="maxima")

[Out] integrate(cosh(d*x + c)/((b*x + a)^3*x), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(c + dx)}{x(a + bx)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(c + d*x)/(x*(a + b*x)^3),x)

[Out] int(cosh(c + d*x)/(x*(a + b*x)^3), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(c + dx)}{x(a + bx)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/x/(b*x+a)**3,x)

[Out] Integral(cosh(c + d*x)/(x*(a + b*x)**3), x)

$$3.38 \quad \int \frac{\cosh(c+dx)}{x^2(a+bx)^3} dx$$

Optimal. Leaf size=298

$$\frac{3b \cosh(c) \operatorname{Chi}(dx)}{a^4} + \frac{3b \cosh\left(c - \frac{ad}{b}\right) \operatorname{Chi}\left(xd + \frac{ad}{b}\right)}{a^4} - \frac{3b \sinh(c) \operatorname{Shi}(dx)}{a^4} + \frac{3b \sinh\left(c - \frac{ad}{b}\right) \operatorname{Shi}\left(xd + \frac{ad}{b}\right)}{a^4} + \frac{2d \sinh(c) \operatorname{Chi}(dx)}{a^3}$$

[Out] $-3*b*\operatorname{Chi}(d*x)*\cosh(c)/a^4+3*b*\operatorname{Chi}(a*d/b+d*x)*\cosh(-c+a*d/b)/a^4+1/2*d^2*\operatorname{Chi}(a*d/b+d*x)*\cosh(-c+a*d/b)/a^2/b-\cosh(d*x+c)/a^3/x-1/2*b*\cosh(d*x+c)/a^2/(b*x+a)^2-2*b*\cosh(d*x+c)/a^3/(b*x+a)+d*\cosh(c)*\operatorname{Shi}(d*x)/a^3+2*d*\cosh(-c+a*d/b)*\operatorname{Shi}(a*d/b+d*x)/a^3+d*\operatorname{Chi}(d*x)*\sinh(c)/a^3-3*b*\operatorname{Shi}(d*x)*\sinh(c)/a^4-2*d*\operatorname{Chi}(a*d/b+d*x)*\sinh(-c+a*d/b)/a^3-3*b*\operatorname{Shi}(a*d/b+d*x)*\sinh(-c+a*d/b)/a^4-1/2*d^2*\operatorname{Shi}(a*d/b+d*x)*\sinh(-c+a*d/b)/a^2/b-1/2*d*\sinh(d*x+c)/a^2/(b*x+a)$

Rubi [A] time = 0.69, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {6742, 3297, 3303, 3298, 3301}

$$\frac{d^2 \cosh\left(c - \frac{ad}{b}\right) \operatorname{Chi}\left(xd + \frac{ad}{b}\right)}{2a^2b} + \frac{2d \sinh\left(c - \frac{ad}{b}\right) \operatorname{Chi}\left(xd + \frac{ad}{b}\right)}{a^3} - \frac{3b \cosh(c) \operatorname{Chi}(dx)}{a^4} + \frac{3b \cosh\left(c - \frac{ad}{b}\right) \operatorname{Chi}\left(xd + \frac{ad}{b}\right)}{a^4}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]/(x^2*(a + b*x)^3), x]

[Out] $-(\operatorname{Cosh}[c + d*x]/(a^3*x)) - (b*\operatorname{Cosh}[c + d*x])/(2*a^2*(a + b*x)^2) - (2*b*\operatorname{Cosh}[c + d*x])/(a^3*(a + b*x)) - (3*b*\operatorname{Cosh}[c]*\operatorname{CoshIntegral}[d*x])/a^4 + (3*b*\operatorname{Cosh}[c - (a*d)/b]*\operatorname{CoshIntegral}[(a*d)/b + d*x])/a^4 + (d^2*\operatorname{Cosh}[c - (a*d)/b]*\operatorname{CoshIntegral}[(a*d)/b + d*x])/(2*a^2*b) + (d*\operatorname{CoshIntegral}[d*x]*\operatorname{Sinh}[c])/a^3 + (2*d*\operatorname{CoshIntegral}[(a*d)/b + d*x]*\operatorname{Sinh}[c - (a*d)/b])/a^3 - (d*\operatorname{Sinh}[c + d*x])/(2*a^2*(a + b*x)) + (d*\operatorname{Cosh}[c]*\operatorname{SinhIntegral}[d*x])/a^3 - (3*b*\operatorname{Sinh}[c]*\operatorname{SinhIntegral}[d*x])/a^4 + (2*d*\operatorname{Cosh}[c - (a*d)/b]*\operatorname{SinhIntegral}[(a*d)/b + d*x])/a^3 + (3*b*\operatorname{Sinh}[c - (a*d)/b]*\operatorname{SinhIntegral}[(a*d)/b + d*x])/a^4 + (d^2*\operatorname{Sinh}[c - (a*d)/b]*\operatorname{SinhIntegral}[(a*d)/b + d*x])/(2*a^2*b)$

Rule 3297

Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x]
&& EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x]
&& EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*e - c*f)/d],
Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d],
Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh(c+dx)}{x^2(a+bx)^3} dx &= \int \left(\frac{\cosh(c+dx)}{a^3 x^2} - \frac{3b \cosh(c+dx)}{a^4 x} + \frac{b^2 \cosh(c+dx)}{a^2(a+bx)^3} + \frac{2b^2 \cosh(c+dx)}{a^3(a+bx)^2} + \frac{3b^2 \cosh(c+dx)}{a^4(a+bx)} \right) dx \\
&= \frac{\int \frac{\cosh(c+dx)}{x^2} dx}{a^3} - \frac{(3b) \int \frac{\cosh(c+dx)}{x} dx}{a^4} + \frac{(3b^2) \int \frac{\cosh(c+dx)}{a+bx} dx}{a^4} + \frac{(2b^2) \int \frac{\cosh(c+dx)}{(a+bx)^2} dx}{a^3} + \frac{b^2 \int \frac{\cosh(c+dx)}{a+bx} dx}{a^4} \\
&= -\frac{\cosh(c+dx)}{a^3 x} - \frac{b \cosh(c+dx)}{2a^2(a+bx)^2} - \frac{2b \cosh(c+dx)}{a^3(a+bx)} + \frac{d \int \frac{\sinh(c+dx)}{x} dx}{a^3} + \frac{(2bd) \int \frac{\sinh(c+dx)}{a+bx} dx}{a^3} \\
&= -\frac{\cosh(c+dx)}{a^3 x} - \frac{b \cosh(c+dx)}{2a^2(a+bx)^2} - \frac{2b \cosh(c+dx)}{a^3(a+bx)} - \frac{3b \cosh(c) \text{Chi}(dx)}{a^4} + \frac{3b \cosh\left(c - \frac{ad}{b}\right) \text{Shi}\left(d\left(\frac{a}{b} + x\right)\right)}{a^4} \\
&= -\frac{\cosh(c+dx)}{a^3 x} - \frac{b \cosh(c+dx)}{2a^2(a+bx)^2} - \frac{2b \cosh(c+dx)}{a^3(a+bx)} - \frac{3b \cosh(c) \text{Chi}(dx)}{a^4} + \frac{3b \cosh\left(c - \frac{ad}{b}\right) \text{Shi}\left(d\left(\frac{a}{b} + x\right)\right)}{a^4} \\
&= -\frac{\cosh(c+dx)}{a^3 x} - \frac{b \cosh(c+dx)}{2a^2(a+bx)^2} - \frac{2b \cosh(c+dx)}{a^3(a+bx)} - \frac{3b \cosh(c) \text{Chi}(dx)}{a^4} + \frac{3b \cosh\left(c - \frac{ad}{b}\right) \text{Shi}\left(d\left(\frac{a}{b} + x\right)\right)}{a^4}
\end{aligned}$$

Mathematica [A] time = 1.57, size = 541, normalized size = 1.82

$$a^4 d^2 x \sinh\left(c - \frac{ad}{b}\right) \text{Shi}\left(d\left(\frac{a}{b} + x\right)\right) + 2a^3 b d^2 x^2 \sinh\left(c - \frac{ad}{b}\right) \text{Shi}\left(d\left(\frac{a}{b} + x\right)\right) + 2a^3 b d x \cosh(c) \text{Shi}(dx) + 4a^3 b d x$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d*x]/(x^2*(a + b*x)^3), x]

[Out] (-2*a^3*b*Cosh[c + d*x] - 9*a^2*b^2*x*Cosh[c + d*x] - 6*a*b^3*x^2*Cosh[c + d*x] + 2*b*x*(a + b*x)^2*CoshIntegral[d*x]*(-3*b*Cosh[c] + a*d*Sinh[c]) + x*(a + b*x)^2*CoshIntegral[d*(a/b + x)]*((6*b^2 + a^2*d^2)*Cosh[c - (a*d)/b] + 4*a*b*d*Sinh[c - (a*d)/b]) - a^3*b*d*x*Sinh[c + d*x] - a^2*b^2*d*x^2*Sinh[c + d*x] + 2*a^3*b*d*x*Cosh[c]*SinhIntegral[d*x] + 4*a^2*b^2*d*x^2*Cosh[c]*SinhIntegral[d*x] + 2*a*b^3*d*x^3*Cosh[c]*SinhIntegral[d*x] - 6*a^2*b^2*x*Sinh[c]*SinhIntegral[d*x] - 12*a*b^3*x^2*Sinh[c]*SinhIntegral[d*x] - 6*b^4*x^3*Sinh[c]*SinhIntegral[d*x] + 4*a^3*b*d*x*Cosh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)] + 8*a^2*b^2*d*x^2*Cosh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)] + 4*a*b^3*d*x^3*Cosh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)] + 6*a^2*b^2*x*Sinh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)] + a^4*d^2*x*Sinh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)] + 12*a*b^3*x^2*Sinh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)] + 2*a^3*b*d^2*x^2*Sinh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)] + 6*

$$b^4 x^3 \operatorname{Sinh}[c - (a*d)/b] \operatorname{SinhIntegral}[d*(a/b + x)] + a^2 b^2 d^2 x^3 \operatorname{Sinh}[c - (a*d)/b] \operatorname{SinhIntegral}[d*(a/b + x)] / (2 a^4 b x (a + b x)^2)$$

fricas [B] time = 0.54, size = 762, normalized size = 2.56

$$2 \left(6 a b^3 x^2 + 9 a^2 b^2 x + 2 a^3 b \right) \cosh(dx + c) - 2 \left((a b^3 d - 3 b^4) x^3 + 2 (a^2 b^2 d - 3 a b^3) x^2 + (a^3 b d - 3 a^2 b^2) x \right) \operatorname{Ei}(d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/x^2/(b*x+a)^3,x, algorithm="fricas")

[Out]
$$-1/4 * (2 * (6 * a * b^3 * x^2 + 9 * a^2 * b^2 * x + 2 * a^3 * b) * \cosh(d * x + c) - 2 * ((a * b^3 * d - 3 * b^4) * x^3 + 2 * (a^2 * b^2 * d - 3 * a * b^3) * x^2 + (a^3 * b * d - 3 * a^2 * b^2) * x) * \operatorname{Ei}(d * x) - ((a * b^3 * d + 3 * b^4) * x^3 + 2 * (a^2 * b^2 * d + 3 * a * b^3) * x^2 + (a^3 * b * d + 3 * a^2 * b^2) * x) * \operatorname{Ei}(-d * x)) * \cosh(c) - ((a^2 * b^2 * d^2 + 4 * a * b^3 * d + 6 * b^4) * x^3 + 2 * (a^3 * b * d^2 + 4 * a^2 * b^2 * d + 6 * a * b^3) * x^2 + (a^4 * d^2 + 4 * a^3 * b * d + 6 * a^2 * b^2) * x) * \operatorname{Ei}((b * d * x + a * d) / b) + ((a^2 * b^2 * d^2 - 4 * a * b^3 * d + 6 * b^4) * x^3 + 2 * (a^3 * b * d^2 - 4 * a^2 * b^2 * d + 6 * a * b^3) * x^2 + (a^4 * d^2 - 4 * a^3 * b * d + 6 * a^2 * b^2) * x) * \operatorname{Ei}(-(b * d * x + a * d) / b) * \cosh(-(b * c - a * d) / b) + 2 * (a^2 * b^2 * d * x^2 + a^3 * b * d * x) * \sinh(d * x + c) - 2 * ((a * b^3 * d - 3 * b^4) * x^3 + 2 * (a^2 * b^2 * d - 3 * a * b^3) * x^2 + (a^3 * b * d - 3 * a^2 * b^2) * x) * \operatorname{Ei}(d * x) + ((a * b^3 * d + 3 * b^4) * x^3 + 2 * (a^2 * b^2 * d + 3 * a * b^3) * x^2 + (a^3 * b * d + 3 * a^2 * b^2) * x) * \operatorname{Ei}(-d * x)) * \sinh(c) + (((a^2 * b^2 * d^2 + 4 * a * b^3 * d + 6 * b^4) * x^3 + 2 * (a^3 * b * d^2 + 4 * a^2 * b^2 * d + 6 * a * b^3) * x^2 + (a^4 * d^2 + 4 * a^3 * b * d + 6 * a^2 * b^2) * x) * \operatorname{Ei}((b * d * x + a * d) / b) - ((a^2 * b^2 * d^2 - 4 * a * b^3 * d + 6 * b^4) * x^3 + 2 * (a^3 * b * d^2 - 4 * a^2 * b^2 * d + 6 * a * b^3) * x^2 + (a^4 * d^2 - 4 * a^3 * b * d + 6 * a^2 * b^2) * x) * \operatorname{Ei}(-(b * d * x + a * d) / b)) * \sinh(-(b * c - a * d) / b)) / (a^4 * b^3 * x^3 + 2 * a^5 * b^2 * x^2 + a^6 * b * x)$$

giac [B] time = 0.15, size = 1006, normalized size = 3.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/x^2/(b*x+a)^3,x, algorithm="giac")

[Out]
$$1/4 * (a^2 * b^2 * d^2 * x^3 * \operatorname{Ei}((b * d * x + a * d) / b) * e^{(c - a * d) / b} + a^2 * b^2 * d^2 * x^3 * \operatorname{Ei}(-(b * d * x + a * d) / b) * e^{-(c + a * d) / b} - 2 * a * b^3 * d * x^3 * \operatorname{Ei}(-d * x) * e^{-(c)} + 2 * a^3 * b * d^2 * x^2 * \operatorname{Ei}((b * d * x + a * d) / b) * e^{(c - a * d) / b} + 4 * a * b^3 * d * x^3 * \operatorname{Ei}((b * d * x + a * d) / b) * e^{(c - a * d) / b} + 2 * a * b^3 * d * x^3 * \operatorname{Ei}(d * x) * e^c + 2 * a^3 * b * d^2 * x^2 * \operatorname{Ei}(-(b * d * x + a * d) / b) * e^{-(c + a * d) / b} - 4 * a * b^3 * d * x^3 * \operatorname{Ei}(-(b * d * x + a * d) / b) * e^{-(c + a * d) / b}) - 4 * a^2 * b^2 * d * x^2 * \operatorname{Ei}(-d * x) * e^{-(c)} - 6 * b^4 * x^3 * \operatorname{Ei}(-d * x) * e^{-(c)} + a^4 * d^2 * x * \operatorname{Ei}((b * d * x + a * d) / b) * e^{(c - a * d) / b} + 8 * a^2 * b^2 * d * x^2 * \operatorname{Ei}((b * d * x + a * d) / b) * e^{(c - a * d) / b} + 6 * b^4 * x^3 * \operatorname{Ei}((b * d * x + a * d) / b) * e^{(c - a * d) / b} + 4 * a^2 * b^2 * d * x^2$$

$$\begin{aligned}
 & *Ei(d*x)*e^c - 6*b^4*x^3*Ei(d*x)*e^c + a^4*d^2*x*Ei(-(b*d*x + a*d)/b)*e^{(-c + a*d/b)} - 8*a^2*b^2*d*x^2*Ei(-(b*d*x + a*d)/b)*e^{(-c + a*d/b)} + 6*b^4*x^3 \\
 & *Ei(-(b*d*x + a*d)/b)*e^{(-c + a*d/b)} - a^2*b^2*d*x^2*e^{(d*x + c)} + a^2*b^2*d*x^2*e^{(-d*x - c)} - 2*a^3*b*d*x*Ei(-d*x)*e^{(-c)} - 12*a*b^3*x^2*Ei(-d*x)*e^{(-c)} \\
 & + 4*a^3*b*d*x*Ei((b*d*x + a*d)/b)*e^{(c - a*d/b)} + 12*a*b^3*x^2*Ei((b*d*x + a*d)/b)*e^{(c - a*d/b)} + 2*a^3*b*d*x*Ei(d*x)*e^c - 12*a*b^3*x^2*Ei(d*x) \\
 & *e^c - 4*a^3*b*d*x*Ei(-(b*d*x + a*d)/b)*e^{(-c + a*d/b)} + 12*a*b^3*x^2*Ei(-(b*d*x + a*d)/b)*e^{(-c + a*d/b)} - a^3*b*d*x*e^{(d*x + c)} - 6*a*b^3*x^2*e^{(d*x + c)} \\
 & + a^3*b*d*x*e^{(-d*x - c)} - 6*a*b^3*x^2*e^{(-d*x - c)} - 6*a^2*b^2*x*Ei(-d*x)*e^{(-c)} + 6*a^2*b^2*x*Ei((b*d*x + a*d)/b)*e^{(c - a*d/b)} - 6*a^2*b^2*x* \\
 & Ei(d*x)*e^c + 6*a^2*b^2*x*Ei(-(b*d*x + a*d)/b)*e^{(-c + a*d/b)} - 9*a^2*b^2*x*e^{(d*x + c)} - 9*a^2*b^2*x*e^{(-d*x - c)} - 2*a^3*b*e^{(d*x + c)} - 2*a^3*b*e^{(-d*x - c)} \\
 &)/(a^4*b^3*x^3 + 2*a^5*b^2*x^2 + a^6*b*x)
 \end{aligned}$$

maple [B] time = 0.13, size = 643, normalized size = 2.16

$$\frac{e^{-dx-c} x d^3 b}{4a^2 (b^2 d^2 x^2 + 2ab d^2 x + a^2 d^2)} + \frac{e^{-dx-c} d^3}{4a (b^2 d^2 x^2 + 2ab d^2 x + a^2 d^2)} - \frac{3e^{-dx-c} x d^2 b^2}{2a^3 (b^2 d^2 x^2 + 2ab d^2 x + a^2 d^2)} - \frac{9e^{-dx-c}}{4a^2 (b^2 d^2 x^2 + 2ab d^2 x + a^2 d^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d*x+c)/x^2/(b*x+a)^3,x)

[Out] $\begin{aligned}
 & 1/4*\exp(-d*x-c)/a^2*x*d^3/(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)*b+1/4*\exp(-d*x-c)/a*d^3/(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)-3/2*\exp(-d*x-c)/a^3*x*d^2/(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)*b^2-9/4*\exp(-d*x-c)/a^2*d^2/(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)*b-1/2*\exp(-d*x-c)/a/x*d^2/(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)-1/4/b/a^2*d^2*\exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)+d/a^3*\exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)-3/2*b/a^4*\exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)+1/2*d/a^3*\exp(-c)*Ei(1,d*x)+3/2/a^4*\exp(-c)*Ei(1,d*x)*b-3/2*b/a^4*\exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)-1/4/a^2*d^2/b*\exp(d*x+c)/(a*d/b+d*x)^2-1/4/a^2*d^2/b*\exp(d*x+c)/(a*d/b+d*x)-1/4/a^2*d^2/b*\exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)-d/a^3*\exp(d*x+c)/(a*d/b+d*x)-d/a^3*\exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)-1/2/a^3/x*\exp(d*x+c)-1/2*d/a^3*\exp(c)*Ei(1,-d*x)+3/2/a^4*b*\exp(c)*Ei(1,-d*x)
 \end{aligned}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(dx + c)}{(bx + a)^3 x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/x^2/(b*x+a)^3,x, algorithm="maxima")

[Out] integrate(cosh(d*x + c)/((b*x + a)^3*x^2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(c + dx)}{x^2 (a + bx)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(c + d*x)/(x^2*(a + b*x)^3), x)

[Out] int(cosh(c + d*x)/(x^2*(a + b*x)^3), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/x**2/(b*x+a)**3,x)

[Out] Timed out

$$3.39 \quad \int \frac{\cosh(c+dx)}{x^3(a+bx)^3} dx$$

Optimal. Leaf size=377

$$\frac{6b^2 \cosh(c)\text{Chi}(dx)}{a^5} - \frac{6b^2 \cosh\left(c - \frac{ad}{b}\right)\text{Chi}\left(xd + \frac{ad}{b}\right)}{a^5} + \frac{6b^2 \sinh(c)\text{Shi}(dx)}{a^5} - \frac{6b^2 \sinh\left(c - \frac{ad}{b}\right)\text{Shi}\left(xd + \frac{ad}{b}\right)}{a^5} + \dots$$

[Out] $6*b^2*\text{Chi}(d*x)*\cosh(c)/a^5+1/2*d^2*\text{Chi}(d*x)*\cosh(c)/a^3-6*b^2*\text{Chi}(a*d/b+d*x)*\cosh(-c+a*d/b)/a^5-1/2*d^2*\text{Chi}(a*d/b+d*x)*\cosh(-c+a*d/b)/a^3-1/2*\cosh(d*x+c)/a^3/x^2+3*b*\cosh(d*x+c)/a^4/x+1/2*b^2*\cosh(d*x+c)/a^3/(b*x+a)^2+3*b^2*\cosh(d*x+c)/a^4/(b*x+a)-3*b*d*\cosh(c)*\text{Shi}(d*x)/a^4-3*b*d*\cosh(-c+a*d/b)*\text{Shi}(a*d/b+d*x)/a^4-3*b*d*\text{Chi}(d*x)*\sinh(c)/a^4+6*b^2*\text{Shi}(d*x)*\sinh(c)/a^5+1/2*d^2*\text{Shi}(d*x)*\sinh(c)/a^3+3*b*d*\text{Chi}(a*d/b+d*x)*\sinh(-c+a*d/b)/a^4+6*b^2*\text{Shi}(a*d/b+d*x)*\sinh(-c+a*d/b)/a^5+1/2*d^2*\text{Shi}(a*d/b+d*x)*\sinh(-c+a*d/b)/a^3-1/2*d*\sinh(d*x+c)/a^3/x+1/2*b*d*\sinh(d*x+c)/a^3/(b*x+a)$

Rubi [A] time = 0.82, antiderivative size = 377, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {6742, 3297, 3303, 3298, 3301}

$$\frac{6b^2 \cosh(c)\text{Chi}(dx)}{a^5} - \frac{6b^2 \cosh\left(c - \frac{ad}{b}\right)\text{Chi}\left(xd + \frac{ad}{b}\right)}{a^5} + \frac{6b^2 \sinh(c)\text{Shi}(dx)}{a^5} - \frac{6b^2 \sinh\left(c - \frac{ad}{b}\right)\text{Shi}\left(xd + \frac{ad}{b}\right)}{a^5} + \dots$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]/(x^3*(a + b*x)^3), x]

[Out] $-\text{Cosh}[c + d*x]/(2*a^3*x^2) + (3*b*\text{Cosh}[c + d*x])/(a^4*x) + (b^2*\text{Cosh}[c + d*x])/(2*a^3*(a + b*x)^2) + (3*b^2*\text{Cosh}[c + d*x])/(a^4*(a + b*x)) + (6*b^2*\text{Cosh}[c]*\text{CoshIntegral}[d*x])/a^5 + (d^2*\text{Cosh}[c]*\text{CoshIntegral}[d*x])/(2*a^3) - (6*b^2*\text{Cosh}[c - (a*d)/b]*\text{CoshIntegral}[(a*d)/b + d*x])/a^5 - (d^2*\text{Cosh}[c - (a*d)/b]*\text{CoshIntegral}[(a*d)/b + d*x])/(2*a^3) - (3*b*d*\text{CoshIntegral}[d*x]*\text{Sinh}[c])/a^4 - (3*b*d*\text{CoshIntegral}[(a*d)/b + d*x]*\text{Sinh}[c - (a*d)/b])/a^4 - (d*\text{Sinh}[c + d*x])/(2*a^3*x) + (b*d*\text{Sinh}[c + d*x])/(2*a^3*(a + b*x)) - (3*b*d*\text{Cosh}[c]*\text{SinhIntegral}[d*x])/a^4 + (6*b^2*\text{Sinh}[c]*\text{SinhIntegral}[d*x])/a^5 + (d^2*\text{Sinh}[c]*\text{SinhIntegral}[d*x])/(2*a^3) - (3*b*d*\text{Cosh}[c - (a*d)/b]*\text{SinhIntegral}[(a*d)/b + d*x])/a^4 - (6*b^2*\text{Sinh}[c - (a*d)/b]*\text{SinhIntegral}[(a*d)/b + d*x])/a^5 - (d^2*\text{Sinh}[c - (a*d)/b]*\text{SinhIntegral}[(a*d)/b + d*x])/(2*a^3)$

Rule 3297

Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1

]

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f,
fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz},
x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh(c+dx)}{x^3(a+bx)^3} dx &= \int \left(\frac{\cosh(c+dx)}{a^3x^3} - \frac{3b \cosh(c+dx)}{a^4x^2} + \frac{6b^2 \cosh(c+dx)}{a^5x} - \frac{b^3 \cosh(c+dx)}{a^3(a+bx)^3} - \frac{3b^3 \cosh(c+dx)}{a^4(a+bx)^2} \right) dx \\
&= \frac{\int \frac{\cosh(c+dx)}{x^3} dx}{a^3} - \frac{(3b) \int \frac{\cosh(c+dx)}{x^2} dx}{a^4} + \frac{(6b^2) \int \frac{\cosh(c+dx)}{x} dx}{a^5} - \frac{(6b^3) \int \frac{\cosh(c+dx)}{a+bx} dx}{a^5} - \frac{(3b^3) \int \frac{\cosh(c+dx)}{(a+bx)^2} dx}{a^4} \\
&= -\frac{\cosh(c+dx)}{2a^3x^2} + \frac{3b \cosh(c+dx)}{a^4x} + \frac{b^2 \cosh(c+dx)}{2a^3(a+bx)^2} + \frac{3b^2 \cosh(c+dx)}{a^4(a+bx)} + \frac{d \int \frac{\sinh(c+dx)}{x^2} dx}{2a^3} \\
&= -\frac{\cosh(c+dx)}{2a^3x^2} + \frac{3b \cosh(c+dx)}{a^4x} + \frac{b^2 \cosh(c+dx)}{2a^3(a+bx)^2} + \frac{3b^2 \cosh(c+dx)}{a^4(a+bx)} + \frac{6b^2 \cosh(c)C}{a^5} \\
&= -\frac{\cosh(c+dx)}{2a^3x^2} + \frac{3b \cosh(c+dx)}{a^4x} + \frac{b^2 \cosh(c+dx)}{2a^3(a+bx)^2} + \frac{3b^2 \cosh(c+dx)}{a^4(a+bx)} + \frac{6b^2 \cosh(c)C}{a^5} \\
&= -\frac{\cosh(c+dx)}{2a^3x^2} + \frac{3b \cosh(c+dx)}{a^4x} + \frac{b^2 \cosh(c+dx)}{2a^3(a+bx)^2} + \frac{3b^2 \cosh(c+dx)}{a^4(a+bx)} + \frac{6b^2 \cosh(c)C}{a^5}
\end{aligned}$$

Mathematica [A] time = 1.87, size = 627, normalized size = 1.66

$$a^4 d^2 x^2 \sinh\left(c - \frac{ad}{b}\right) \text{Shi}\left(d\left(\frac{a}{b} + x\right)\right) - a^4 d^2 x^2 \sinh(c) \text{Shi}(dx) + a^4 dx \sinh(c+dx) + a^4 \cosh(c+dx) - 2a^3 b d^2 x$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d*x]/(x^3*(a + b*x)^3), x]

[Out] -1/2*(a^4*Cosh[c + d*x] - 4*a^3*b*x*Cosh[c + d*x] - 18*a^2*b^2*x^2*Cosh[c + d*x] - 12*a*b^3*x^3*Cosh[c + d*x] - x^2*(a + b*x)^2*CoshIntegral[d*x]*((12*b^2 + a^2*d^2)*Cosh[c] - 6*a*b*d*Sinh[c]) + x^2*(a + b*x)^2*CoshIntegral[d*(a/b + x)]*((12*b^2 + a^2*d^2)*Cosh[c - (a*d)/b] + 6*a*b*d*Sinh[c - (a*d)/b]) + a^4*d*x*Sinh[c + d*x] + a^3*b*d*x^2*Sinh[c + d*x] + 6*a^3*b*d*x^2*Cosh[c]*SinhIntegral[d*x] + 12*a^2*b^2*d*x^3*Cosh[c]*SinhIntegral[d*x] + 6*a*b^3*d*x^4*Cosh[c]*SinhIntegral[d*x] - 12*a^2*b^2*x^2*Sinh[c]*SinhIntegral[d*x] - a^4*d^2*x^2*Sinh[c]*SinhIntegral[d*x] - 24*a*b^3*x^3*Sinh[c]*SinhIntegral[d*x] - 2*a^3*b*d^2*x^3*Sinh[c]*SinhIntegral[d*x] - 12*b^4*x^4*Sinh[c]*SinhIntegral[d*x] - a^2*b^2*d^2*x^4*Sinh[c]*SinhIntegral[d*x] + 6*a^3*b*d*x^2*Cosh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)] + 12*a^2*b^2*d*x^3*Cosh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)] + 6*a*b^3*d*x^4*Cosh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)] + 12*a^2*b^2*x^2*Sinh[c - (a*d)/b]*SinhIntegral[d*(a/b +

$$x)] + a^4 d^2 x^2 \operatorname{Sinh}[c - (a*d)/b] \operatorname{SinhIntegral}[d*(a/b + x)] + 24 a^3 b^3 x^3 \operatorname{Sinh}[c - (a*d)/b] \operatorname{SinhIntegral}[d*(a/b + x)] + 2 a^3 b^3 d^2 x^3 \operatorname{Sinh}[c - (a*d)/b] \operatorname{SinhIntegral}[d*(a/b + x)] + 12 b^4 x^4 \operatorname{Sinh}[c - (a*d)/b] \operatorname{SinhIntegral}[d*(a/b + x)] + a^2 b^2 d^2 x^4 \operatorname{Sinh}[c - (a*d)/b] \operatorname{SinhIntegral}[d*(a/b + x)] / (a^5 x^2 (a + b*x)^2)$$

fricas [B] time = 0.56, size = 892, normalized size = 2.37

$$2 \left(12 ab^3 x^3 + 18 a^2 b^2 x^2 + 4 a^3 b x - a^4 \right) \cosh(dx + c) + \left((a^2 b^2 d^2 - 6 ab^3 d + 12 b^4) x^4 + 2 (a^3 b d^2 - 6 a^2 b^2 d + 12 ab^3) x^3 + a^4 d^2 - 6 a^3 b d + 12 a^2 b^2 \right) \operatorname{Ei}(dx) + \left((a^2 b^2 d^2 + 6 a^3 b^3 d + 12 b^4) x^4 + 2 (a^3 b^3 d^2 + 6 a^2 b^2 d + 12 a^3 b^3) x^3 + (a^4 d^2 + 6 a^3 b^3 d + 12 a^2 b^2) x^2 \right) \operatorname{Ei}(-dx) \cosh(c) - \left((a^2 b^2 d^2 + 6 a^3 b^3 d + 12 b^4) x^4 + 2 (a^3 b^3 d^2 + 6 a^2 b^2 d + 12 a^3 b^3) x^3 + (a^4 d^2 + 6 a^3 b^3 d + 12 a^2 b^2) x^2 \right) \operatorname{Ei}((b*d*x + a*d)/b) + \left((a^2 b^2 d^2 - 6 a^3 b^3 d + 12 b^4) x^4 + 2 (a^3 b^3 d^2 - 6 a^2 b^2 d + 12 a^3 b^3) x^3 + (a^4 d^2 - 6 a^3 b^3 d + 12 a^2 b^2) x^2 \right) \operatorname{Ei}(-(b*d*x + a*d)/b) \cosh(-(b*c - a*d)/b) - 2 (a^3 b^3 d^2 x^2 + a^4 d^2 x) \sinh(dx + c) + \left((a^2 b^2 d^2 - 6 a^3 b^3 d + 12 b^4) x^4 + 2 (a^3 b^3 d^2 - 6 a^2 b^2 d + 12 a^3 b^3) x^3 + (a^4 d^2 - 6 a^3 b^3 d + 12 a^2 b^2) x^2 \right) \operatorname{Ei}(dx) - \left((a^2 b^2 d^2 + 6 a^3 b^3 d + 12 b^4) x^4 + 2 (a^3 b^3 d^2 + 6 a^2 b^2 d + 12 a^3 b^3) x^3 + (a^4 d^2 + 6 a^3 b^3 d + 12 a^2 b^2) x^2 \right) \operatorname{Ei}(-dx) \sinh(c) + \left((a^2 b^2 d^2 + 6 a^3 b^3 d + 12 b^4) x^4 + 2 (a^3 b^3 d^2 + 6 a^2 b^2 d + 12 a^3 b^3) x^3 + (a^4 d^2 + 6 a^3 b^3 d + 12 a^2 b^2) x^2 \right) \operatorname{Ei}((b*d*x + a*d)/b) - \left((a^2 b^2 d^2 - 6 a^3 b^3 d + 12 b^4) x^4 + 2 (a^3 b^3 d^2 - 6 a^2 b^2 d + 12 a^3 b^3) x^3 + (a^4 d^2 - 6 a^3 b^3 d + 12 a^2 b^2) x^2 \right) \operatorname{Ei}(-(b*d*x + a*d)/b) \sinh(-(b*c - a*d)/b) / (a^5 b^2 x^4 + 2 a^6 b^3 x^3 + a^7 x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/x^3/(b*x+a)^3,x, algorithm="fricas")

[Out] 1/4*(2*(12*a*b^3*x^3 + 18*a^2*b^2*x^2 + 4*a^3*b*x - a^4)*cosh(d*x + c) + ((a^2*b^2*d^2 - 6*a*b^3*d + 12*b^4)*x^4 + 2*(a^3*b*d^2 - 6*a^2*b^2*d + 12*a*b^3)*x^3 + (a^4*d^2 - 6*a^3*b*d + 12*a^2*b^2)*x^2)*Ei(d*x) + ((a^2*b^2*d^2 + 6*a*b^3*d + 12*b^4)*x^4 + 2*(a^3*b*d^2 + 6*a^2*b^2*d + 12*a*b^3)*x^3 + (a^4*d^2 + 6*a^3*b*d + 12*a^2*b^2)*x^2)*Ei(-d*x))cosh(c) - (((a^2*b^2*d^2 + 6*a*b^3*d + 12*b^4)*x^4 + 2*(a^3*b*d^2 + 6*a^2*b^2*d + 12*a*b^3)*x^3 + (a^4*d^2 + 6*a^3*b*d + 12*a^2*b^2)*x^2)*Ei((b*d*x + a*d)/b) + ((a^2*b^2*d^2 - 6*a*b^3*d + 12*b^4)*x^4 + 2*(a^3*b*d^2 - 6*a^2*b^2*d + 12*a*b^3)*x^3 + (a^4*d^2 - 6*a^3*b*d + 12*a^2*b^2)*x^2)*Ei(-(b*d*x + a*d)/b))*cosh(-(b*c - a*d)/b) - 2*(a^3*b*d*x^2 + a^4*d*x)*sinh(d*x + c) + (((a^2*b^2*d^2 - 6*a*b^3*d + 12*b^4)*x^4 + 2*(a^3*b*d^2 - 6*a^2*b^2*d + 12*a*b^3)*x^3 + (a^4*d^2 - 6*a^3*b*d + 12*a^2*b^2)*x^2)*Ei(d*x) - ((a^2*b^2*d^2 + 6*a*b^3*d + 12*b^4)*x^4 + 2*(a^3*b*d^2 + 6*a^2*b^2*d + 12*a*b^3)*x^3 + (a^4*d^2 + 6*a^3*b*d + 12*a^2*b^2)*x^2)*Ei(-d*x))*sinh(c) + (((a^2*b^2*d^2 + 6*a*b^3*d + 12*b^4)*x^4 + 2*(a^3*b*d^2 + 6*a^2*b^2*d + 12*a*b^3)*x^3 + (a^4*d^2 + 6*a^3*b*d + 12*a^2*b^2)*x^2)*Ei((b*d*x + a*d)/b) - ((a^2*b^2*d^2 - 6*a*b^3*d + 12*b^4)*x^4 + 2*(a^3*b*d^2 - 6*a^2*b^2*d + 12*a*b^3)*x^3 + (a^4*d^2 - 6*a^3*b*d + 12*a^2*b^2)*x^2)*Ei(-(b*d*x + a*d)/b))*sinh(-(b*c - a*d)/b))/(a^5*b^2*x^4 + 2*a^6*b^3*x^3 + a^7*x^2)

giac [B] time = 0.14, size = 1169, normalized size = 3.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/x^3/(b*x+a)^3,x, algorithm="giac")

[Out] 1/4*(a^2*b^2*d^2*x^4*Ei(-d*x)*e^(-c) - a^2*b^2*d^2*x^4*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + a^2*b^2*d^2*x^4*Ei(d*x)*e^c - a^2*b^2*d^2*x^4*Ei(-(b*d*x +

$a*d)/b)*e^{(-c + a*d/b)} + 2*a^3*b*d^2*x^3*Ei(-d*x)*e^{(-c)} + 6*a*b^3*d*x^4*Ei(-d*x)*e^{(-c)} - 2*a^3*b*d^2*x^3*Ei((b*d*x + a*d)/b)*e^{(c - a*d/b)} - 6*a*b^3*d*x^4*Ei((b*d*x + a*d)/b)*e^{(c - a*d/b)} + 2*a^3*b*d^2*x^3*Ei(d*x)*e^c - 6*a*b^3*d*x^4*Ei(d*x)*e^c - 2*a^3*b*d^2*x^3*Ei(-(b*d*x + a*d)/b)*e^{(-c + a*d/b)} + 6*a*b^3*d*x^4*Ei(-(b*d*x + a*d)/b)*e^{(-c + a*d/b)} + a^4*d^2*x^2*Ei(-d*x)*e^{(-c)} + 12*a^2*b^2*d*x^3*Ei(-d*x)*e^{(-c)} + 12*b^4*x^4*Ei(-d*x)*e^{(-c)} - a^4*d^2*x^2*Ei((b*d*x + a*d)/b)*e^{(c - a*d/b)} - 12*a^2*b^2*d*x^3*Ei((b*d*x + a*d)/b)*e^{(c - a*d/b)} - 12*b^4*x^4*Ei((b*d*x + a*d)/b)*e^{(c - a*d/b)} + a^4*d^2*x^2*Ei(d*x)*e^c - 12*a^2*b^2*d*x^3*Ei(d*x)*e^c + 12*b^4*x^4*Ei(d*x)*e^c - a^4*d^2*x^2*Ei(-(b*d*x + a*d)/b)*e^{(-c + a*d/b)} + 12*a^2*b^2*d*x^3*Ei(-(b*d*x + a*d)/b)*e^{(-c + a*d/b)} - 12*b^4*x^4*Ei(-(b*d*x + a*d)/b)*e^{(-c + a*d/b)} + 6*a^3*b*d*x^2*Ei(-d*x)*e^{(-c)} + 24*a*b^3*x^3*Ei(-d*x)*e^{(-c)} - 6*a^3*b*d*x^2*Ei((b*d*x + a*d)/b)*e^{(c - a*d/b)} - 24*a*b^3*x^3*Ei((b*d*x + a*d)/b)*e^{(c - a*d/b)} - 6*a^3*b*d*x^2*Ei(d*x)*e^c + 24*a*b^3*x^3*Ei(d*x)*e^c + 6*a^3*b*d*x^2*Ei(-(b*d*x + a*d)/b)*e^{(-c + a*d/b)} - 24*a*b^3*x^3*Ei(-(b*d*x + a*d)/b)*e^{(-c + a*d/b)} - a^3*b*d*x^2*e^{(d*x + c)} + 12*a*b^3*x^3*e^{(d*x + c)} + a^3*b*d*x^2*e^{(-d*x - c)} + 12*a*b^3*x^3*e^{(-d*x - c)} + 12*a^2*b^2*x^2*Ei(-d*x)*e^{(-c)} - 12*a^2*b^2*x^2*Ei((b*d*x + a*d)/b)*e^{(c - a*d/b)} + 12*a^2*b^2*x^2*Ei(d*x)*e^c - 12*a^2*b^2*x^2*Ei(-(b*d*x + a*d)/b)*e^{(-c + a*d/b)} - a^4*d*x*e^{(d*x + c)} + 18*a^2*b^2*x^2*e^{(d*x + c)} + a^4*d*x*e^{(-d*x - c)} + 18*a^2*b^2*x^2*e^{(-d*x - c)} + 4*a^3*b*x*e^{(d*x + c)} + 4*a^3*b*x*e^{(-d*x - c)} - a^4*e^{(d*x + c)} - a^4*e^{(-d*x - c)})/(a^5*b^2*x^4 + 2*a^6*b*x^3 + a^7*x^2)$

maple [B] time = 0.14, size = 760, normalized size = 2.02

$$\frac{d^3 e^{-dx-cb}}{4a^2 (b^2 d^2 x^2 + 2ab d^2 x + a^2 d^2)} + \frac{3d^2 e^{-dx-c} x b^3}{a^4 (b^2 d^2 x^2 + 2ab d^2 x + a^2 d^2)} + \frac{d^3 e^{-dx-c}}{4ax (b^2 d^2 x^2 + 2ab d^2 x + a^2 d^2)} + \frac{9d^2 e^{-c}}{2a^3 (b^2 d^2 x^2 + 2ab d^2 x + a^2 d^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d*x+c)/x^3/(b*x+a)^3,x)

[Out] $1/4*d^3*exp(-d*x-c)/a^2/(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)*b+3*d^2*exp(-d*x-c)/a^4*x/(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)*b^3+1/4*d^3*exp(-d*x-c)/a/x/(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)+9/2*d^2*exp(-d*x-c)/a^3/(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)*b^2+d^2*exp(-d*x-c)/a^2/x/(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)*b-1/4*d^2*exp(-d*x-c)/a/x^2/(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)+1/4*d^2/a^3*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)-3/2*d/a^4*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)*b+3/a^5*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)*b^2-1/4*d^2/a^3*exp(-c)*Ei(1,d*x)-3/2*d/a^4*exp(-c)*Ei(1,d*x)*b-3/a^5*exp(-c)*Ei(1,d*x)*b^2+3*b^2/a^5*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)+1/4*d^2/a^3*exp(d*x+c)/(a*d/b+d*x)^2+1/4*d^2/a^3*exp(d*x+c)/(a*d/b+d*x)+1/4*d^2/a^3*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)+3/2*d/a^4*b*exp(d*x+c)/(a*d/b+d*x)+3/2*d$

$$\frac{1}{a^4 b} \exp\left(-\frac{a d - b c}{b}\right) \operatorname{Ei}\left(1, -d x - c - \frac{a d - b c}{b}\right) + \frac{3}{2} \frac{1}{a^4 b} \frac{1}{x} \exp(d x + c) + \frac{3}{2} \frac{d}{a^4 b} \exp(c) \operatorname{Ei}\left(1, -d x\right) - \frac{1}{4} \frac{1}{a^3} \frac{1}{x^2} \exp(d x + c) - \frac{1}{4} \frac{d}{a^3} \frac{1}{x} \exp(d x + c) - \frac{1}{4} \frac{d^2}{a^3} \exp(c) \operatorname{Ei}\left(1, -d x\right) - \frac{3}{a^5 b^2} \exp(c) \operatorname{Ei}\left(1, -d x\right)$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(dx + c)}{(bx + a)^3 x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/x^3/(b*x+a)^3,x, algorithm="maxima")

[Out] integrate(cosh(d*x + c)/((b*x + a)^3*x^3), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(c + dx)}{x^3 (a + bx)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(c + d*x)/(x^3*(a + b*x)^3), x)

[Out] int(cosh(c + d*x)/(x^3*(a + b*x)^3), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/x**3/(b*x+a)**3,x)

[Out] Timed out

3.40 $\int x^3 (a + bx^2) \cosh(c + dx) dx$

Optimal. Leaf size=139

$$-\frac{6a \cosh(c + dx)}{d^4} + \frac{6ax \sinh(c + dx)}{d^3} - \frac{3ax^2 \cosh(c + dx)}{d^2} + \frac{ax^3 \sinh(c + dx)}{d} - \frac{120b \cosh(c + dx)}{d^6} + \frac{120bx \sinh(c + dx)}{d^5}$$

[Out] $-120*b*\cosh(d*x+c)/d^6-6*a*\cosh(d*x+c)/d^4-60*b*x^2*\cosh(d*x+c)/d^4-3*a*x^2*\cosh(d*x+c)/d^2-5*b*x^4*\cosh(d*x+c)/d^2+120*b*x*\sinh(d*x+c)/d^5+6*a*x*\sinh(d*x+c)/d^3+20*b*x^3*\sinh(d*x+c)/d^3+a*x^3*\sinh(d*x+c)/d+b*x^5*\sinh(d*x+c)/d$

Rubi [A] time = 0.26, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {5287, 3296, 2638}

$$-\frac{3ax^2 \cosh(c + dx)}{d^2} + \frac{6ax \sinh(c + dx)}{d^3} - \frac{6a \cosh(c + dx)}{d^4} + \frac{ax^3 \sinh(c + dx)}{d} + \frac{20bx^3 \sinh(c + dx)}{d^3} - \frac{5bx^4 \cosh(c + dx)}{d^2}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^3*(a + b*x^2)*\text{Cosh}[c + d*x], x]$

[Out] $(-120*b*\text{Cosh}[c + d*x])/d^6 - (6*a*\text{Cosh}[c + d*x])/d^4 - (60*b*x^2*\text{Cosh}[c + d*x])/d^4 - (3*a*x^2*\text{Cosh}[c + d*x])/d^2 - (5*b*x^4*\text{Cosh}[c + d*x])/d^2 + (120*b*x*\text{Sinh}[c + d*x])/d^5 + (6*a*x*\text{Sinh}[c + d*x])/d^3 + (20*b*x^3*\text{Sinh}[c + d*x])/d^3 + (a*x^3*\text{Sinh}[c + d*x])/d + (b*x^5*\text{Sinh}[c + d*x])/d$

Rule 2638

$\text{Int}[\sin[(c_.) + (d_.)*(x_.)], x_Symbol] \text{ :> } -\text{Simp}[\cos[c + d*x]/d, x] \text{ /; FreeQ}[\{c, d\}, x]$

Rule 3296

$\text{Int}[((c_.) + (d_.)*(x_.))^{(m_.)}*\sin[(e_.) + (f_.)*(x_.)], x_Symbol] \text{ :> } -\text{Simp}[(c + d*x)^m*\cos[e + f*x]/f, x] + \text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{(m-1)}*\cos[e + f*x], x], x] \text{ /; FreeQ}\{c, d, e, f\}, x \ \&\& \text{ GtQ}[m, 0]$

Rule 5287

$\text{Int}[\text{Cosh}[(c_.) + (d_.)*(x_.)]*((e_.)*(x_.))^{(m_.)}*((a_.) + (b_.)*(x_.)^{(n_.)})^{(p_.)}, x_Symbol] \text{ :> } \text{Int}[\text{ExpandIntegrand}[\text{Cosh}[c + d*x], (e*x)^m*(a + b*x^n)^p, x], x] \text{ /; FreeQ}\{a, b, c, d, e, m, n\}, x \ \&\& \text{ IGtQ}[p, 0]$

Rubi steps

$$\begin{aligned}
\int x^3 (a + bx^2) \cosh(c + dx) dx &= \int (ax^3 \cosh(c + dx) + bx^5 \cosh(c + dx)) dx \\
&= a \int x^3 \cosh(c + dx) dx + b \int x^5 \cosh(c + dx) dx \\
&= \frac{ax^3 \sinh(c + dx)}{d} + \frac{bx^5 \sinh(c + dx)}{d} - \frac{(3a) \int x^2 \sinh(c + dx) dx}{d} - \frac{(5b) \int x^4 \sinh(c + dx) dx}{d} \\
&= -\frac{3ax^2 \cosh(c + dx)}{d^2} - \frac{5bx^4 \cosh(c + dx)}{d^2} + \frac{ax^3 \sinh(c + dx)}{d} + \frac{bx^5 \sinh(c + dx)}{d} \\
&= -\frac{3ax^2 \cosh(c + dx)}{d^2} - \frac{5bx^4 \cosh(c + dx)}{d^2} + \frac{6ax \sinh(c + dx)}{d^3} + \frac{20bx^3 \sinh(c + dx)}{d^3} \\
&= -\frac{6a \cosh(c + dx)}{d^4} - \frac{60bx^2 \cosh(c + dx)}{d^4} - \frac{3ax^2 \cosh(c + dx)}{d^2} - \frac{5bx^4 \cosh(c + dx)}{d^2} \\
&= -\frac{6a \cosh(c + dx)}{d^4} - \frac{60bx^2 \cosh(c + dx)}{d^4} - \frac{3ax^2 \cosh(c + dx)}{d^2} - \frac{5bx^4 \cosh(c + dx)}{d^2} \\
&= -\frac{120b \cosh(c + dx)}{d^6} - \frac{6a \cosh(c + dx)}{d^4} - \frac{60bx^2 \cosh(c + dx)}{d^4} - \frac{3ax^2 \cosh(c + dx)}{d^2}
\end{aligned}$$

Mathematica [A] time = 0.17, size = 92, normalized size = 0.66

$$\frac{dx \left(ad^2 (d^2 x^2 + 6) + b (d^4 x^4 + 20d^2 x^2 + 120) \right) \sinh(c + dx) - \left(3ad^2 (d^2 x^2 + 2) + 5b (d^4 x^4 + 12d^2 x^2 + 24) \right) \cosh(c + dx)}{d^6}$$

Antiderivative was successfully verified.

[In] Integrate[x^3*(a + b*x^2)*Cosh[c + d*x], x]

[Out] (-((3*a*d^2*(2 + d^2*x^2) + 5*b*(24 + 12*d^2*x^2 + d^4*x^4))*Cosh[c + d*x]) + d*x*(a*d^2*(6 + d^2*x^2) + b*(120 + 20*d^2*x^2 + d^4*x^4))*Sinh[c + d*x])/d^6

fricas [A] time = 0.53, size = 95, normalized size = 0.68

$$\frac{(5bd^4x^4 + 6ad^2 + 3(ad^4 + 20bd^2)x^2 + 120b) \cosh(dx + c) - (bd^5x^5 + (ad^5 + 20bd^3)x^3 + 6(ad^3 + 20bd)x) \sinh(dx + c)}{d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(b*x^2+a)*cosh(d*x+c), x, algorithm="fricas")

[Out] -((5*b*d^4*x^4 + 6*a*d^2 + 3*(a*d^4 + 20*b*d^2)*x^2 + 120*b)*cosh(d*x + c) - (b*d^5*x^5 + (a*d^5 + 20*b*d^3)*x^3 + 6*(a*d^3 + 20*b*d)*x)*sinh(d*x + c))/d^6

giac [A] time = 0.12, size = 174, normalized size = 1.25

$$\frac{(bd^5x^5 + ad^5x^3 - 5bd^4x^4 - 3ad^4x^2 + 20bd^3x^3 + 6ad^3x - 60bd^2x^2 - 6ad^2 + 120bdx - 120b)e^{(dx+c)}}{2d^6} - \frac{(bd^5x^5 +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(b*x^2+a)*cosh(d*x+c), x, algorithm="giac")

[Out] 1/2*(b*d^5*x^5 + a*d^5*x^3 - 5*b*d^4*x^4 - 3*a*d^4*x^2 + 20*b*d^3*x^3 + 6*a*d^3*x - 60*b*d^2*x^2 - 6*a*d^2 + 120*b*d*x - 120*b)*e^(d*x + c)/d^6 - 1/2*(b*d^5*x^5 + a*d^5*x^3 + 5*b*d^4*x^4 + 3*a*d^4*x^2 + 20*b*d^3*x^3 + 6*a*d^3*x + 60*b*d^2*x^2 + 6*a*d^2 + 120*b*d*x + 120*b)*e^(-d*x - c)/d^6

maple [B] time = 0.04, size = 447, normalized size = 3.22

$$\frac{b((dx+c)^5 \sinh(dx+c) - 5(dx+c)^4 \cosh(dx+c) + 20(dx+c)^3 \sinh(dx+c) - 60(dx+c)^2 \cosh(dx+c) + 120(dx+c) \sinh(dx+c) - 120 \cosh(dx+c))}{d^2} - \frac{5bc((dx+c)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(b*x^2+a)*cosh(d*x+c), x)

[Out] 1/d^4*(1/d^2*b*((d*x+c)^5*sinh(d*x+c)-5*(d*x+c)^4*cosh(d*x+c)+20*(d*x+c)^3*sinh(d*x+c)-60*(d*x+c)^2*cosh(d*x+c)+120*(d*x+c)*sinh(d*x+c)-120*cosh(d*x+c))-5/d^2*b*c*((d*x+c)^4*sinh(d*x+c)-4*(d*x+c)^3*cosh(d*x+c)+12*(d*x+c)^2*sinh(d*x+c)-24*(d*x+c)*cosh(d*x+c)+24*sinh(d*x+c))+10/d^2*b*c^2*((d*x+c)^3*sinh(d*x+c)-3*(d*x+c)^2*cosh(d*x+c)+6*(d*x+c)*sinh(d*x+c)-6*cosh(d*x+c))-10/d^2*b*c^3*((d*x+c)^2*sinh(d*x+c)-2*(d*x+c)*cosh(d*x+c)+2*sinh(d*x+c))+5/d^2*b*c^4*((d*x+c)*sinh(d*x+c)-cosh(d*x+c))-1/d^2*b*c^5*sinh(d*x+c)+a*((d*x+c)^3*sinh(d*x+c)-3*(d*x+c)^2*cosh(d*x+c)+6*(d*x+c)*sinh(d*x+c)-6*cosh(d*x+c))-3*a*c*((d*x+c)^2*sinh(d*x+c)-2*(d*x+c)*cosh(d*x+c)+2*sinh(d*x+c))+3*a*c^2*((d*x+c)*sinh(d*x+c)-cosh(d*x+c))-a*c^3*sinh(d*x+c))

maxima [A] time = 0.32, size = 250, normalized size = 1.80

$$-\frac{1}{24}d \left(\frac{3(d^4x^4e^c - 4d^3x^3e^c + 12d^2x^2e^c - 24dxe^c + 24e^c)ae^{(dx)}}{d^5} + \frac{3(d^4x^4 + 4d^3x^3 + 12d^2x^2 + 24dx + 24)ae^{(-d}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(b*x^2+a)*cosh(d*x+c), x, algorithm="maxima")

[Out] -1/24*d*(3*(d^4*x^4*e^c - 4*d^3*x^3*e^c + 12*d^2*x^2*e^c - 24*d*x*e^c + 24*e^c)*a*e^(d*x)/d^5 + 3*(d^4*x^4 + 4*d^3*x^3 + 12*d^2*x^2 + 24*d*x + 24)*a*e

$$\begin{aligned} & \frac{(-dx - c)/d^5 + 2*(d^6*x^6*e^c - 6*d^5*x^5*e^c + 30*d^4*x^4*e^c - 120*d^3*x^3*e^c + 360*d^2*x^2*e^c - 720*d*x*e^c + 720*e^c)*b*e^{(d*x)}/d^7 + 2*(d^6*x^6 + 6*d^5*x^5 + 30*d^4*x^4 + 120*d^3*x^3 + 360*d^2*x^2 + 720*d*x + 720)*b*e^{(-d*x - c)}/d^7 + 1/12*(2*b*x^6 + 3*a*x^4)*\cosh(d*x + c)}{d^3} \end{aligned}$$

mupad [B] time = 0.99, size = 116, normalized size = 0.83

$$\frac{x^3 \sinh(c + dx) (ad^2 + 20b)}{d^3} - \frac{3x^2 \cosh(c + dx) (ad^2 + 20b)}{d^4} - \frac{6 \cosh(c + dx) (ad^2 + 20b)}{d^6} + \frac{6x \sinh(c + dx)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*cosh(c + d*x)*(a + b*x^2),x)`

[Out] $(x^3*\sinh(c + d*x)*(20*b + a*d^2))/d^3 - (3*x^2*\cosh(c + d*x)*(20*b + a*d^2))/d^4 - (6*\cosh(c + d*x)*(20*b + a*d^2))/d^6 + (6*x*\sinh(c + d*x)*(20*b + a*d^2))/d^5 - (5*b*x^4*\cosh(c + d*x))/d^2 + (b*x^5*\sinh(c + d*x))/d$

sympy [A] time = 3.42, size = 168, normalized size = 1.21

$$\left\{ \begin{aligned} & \frac{ax^3 \sinh(c+dx)}{d} - \frac{3ax^2 \cosh(c+dx)}{d^2} + \frac{6ax \sinh(c+dx)}{d^3} - \frac{6a \cosh(c+dx)}{d^4} + \frac{bx^5 \sinh(c+dx)}{d} - \frac{5bx^4 \cosh(c+dx)}{d^2} + \frac{20bx^3 \sinh(c+dx)}{d^3} - \frac{60bx^2 \cosh(c+dx)}{d^4} \\ & \left(\frac{ax^4}{4} + \frac{bx^6}{6} \right) \cosh(c) \end{aligned} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*(b*x**2+a)*cosh(d*x+c),x)`

[Out] `Piecewise((a*x**3*sinh(c + d*x)/d - 3*a*x**2*cosh(c + d*x)/d**2 + 6*a*x*sinh(c + d*x)/d**3 - 6*a*cosh(c + d*x)/d**4 + b*x**5*sinh(c + d*x)/d - 5*b*x**4*cosh(c + d*x)/d**2 + 20*b*x**3*sinh(c + d*x)/d**3 - 60*b*x**2*cosh(c + d*x)/d**4 + 120*b*x*sinh(c + d*x)/d**5 - 120*b*cosh(c + d*x)/d**6, Ne(d, 0)), ((a*x**4/4 + b*x**6/6)*cosh(c), True))`

3.41 $\int x^2 (a + bx^2) \cosh(c + dx) dx$

Optimal. Leaf size=109

$$\frac{2a \sinh(c + dx)}{d^3} - \frac{2ax \cosh(c + dx)}{d^2} + \frac{ax^2 \sinh(c + dx)}{d} + \frac{24b \sinh(c + dx)}{d^5} - \frac{24bx \cosh(c + dx)}{d^4} + \frac{12bx^2 \sinh(c + dx)}{d^3}$$

[Out] $-24*b*x*cosh(d*x+c)/d^4-2*a*x*cosh(d*x+c)/d^2-4*b*x^3*cosh(d*x+c)/d^2+24*b*\sinh(d*x+c)/d^5+2*a*\sinh(d*x+c)/d^3+12*b*x^2*\sinh(d*x+c)/d^3+a*x^2*\sinh(d*x+c)/d+b*x^4*\sinh(d*x+c)/d$

Rubi [A] time = 0.19, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {5287, 3296, 2637}

$$\frac{2a \sinh(c + dx)}{d^3} - \frac{2ax \cosh(c + dx)}{d^2} + \frac{ax^2 \sinh(c + dx)}{d} + \frac{12bx^2 \sinh(c + dx)}{d^3} - \frac{4bx^3 \cosh(c + dx)}{d^2} + \frac{24b \sinh(c + dx)}{d^5}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^2*(a + b*x^2)*\text{Cosh}[c + d*x], x]$

[Out] $(-24*b*x*\text{Cosh}[c + d*x])/d^4 - (2*a*x*\text{Cosh}[c + d*x])/d^2 - (4*b*x^3*\text{Cosh}[c + d*x])/d^2 + (24*b*\text{Sinh}[c + d*x])/d^5 + (2*a*\text{Sinh}[c + d*x])/d^3 + (12*b*x^2*\text{Sinh}[c + d*x])/d^3 + (a*x^2*\text{Sinh}[c + d*x])/d + (b*x^4*\text{Sinh}[c + d*x])/d$

Rule 2637

$\text{Int}[\sin[\text{Pi}/2 + (c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow \text{Simp}[\sin[c + d*x]/d, x] /;$
FreeQ[{c, d}, x]

Rule 3296

$\text{Int}[((c_.) + (d_.)*(x_.))^{(m_.)}*\sin[(e_.) + (f_.)*(x_.)], x_Symbol] \rightarrow -\text{Simp}[(c + d*x)^m*\cos[e + f*x]/f, x] + \text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{(m-1)}*\cos[e + f*x], x], x] /;$ FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 5287

$\text{Int}[\text{Cosh}[(c_.) + (d_.)*(x_.)]*((e_.)*(x_.))^{(m_.)}*((a_.) + (b_.)*(x_.)^{(n_.)})^{(p_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[\text{Cosh}[c + d*x], (e*x)^m*(a + b*x^n)^p, x], x] /;$ FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]

Rubi steps

$$\begin{aligned}
\int x^2 (a + bx^2) \cosh(c + dx) dx &= \int (ax^2 \cosh(c + dx) + bx^4 \cosh(c + dx)) dx \\
&= a \int x^2 \cosh(c + dx) dx + b \int x^4 \cosh(c + dx) dx \\
&= \frac{ax^2 \sinh(c + dx)}{d} + \frac{bx^4 \sinh(c + dx)}{d} - \frac{(2a) \int x \sinh(c + dx) dx}{d} - \frac{(4b) \int x^3 \sinh(c + dx) dx}{d} \\
&= -\frac{2ax \cosh(c + dx)}{d^2} - \frac{4bx^3 \cosh(c + dx)}{d^2} + \frac{ax^2 \sinh(c + dx)}{d} + \frac{bx^4 \sinh(c + dx)}{d} \\
&= -\frac{2ax \cosh(c + dx)}{d^2} - \frac{4bx^3 \cosh(c + dx)}{d^2} + \frac{2a \sinh(c + dx)}{d^3} + \frac{12bx^2 \sinh(c + dx)}{d^3} \\
&= -\frac{24bx \cosh(c + dx)}{d^4} - \frac{2ax \cosh(c + dx)}{d^2} - \frac{4bx^3 \cosh(c + dx)}{d^2} + \frac{2a \sinh(c + dx)}{d^3} \\
&= -\frac{24bx \cosh(c + dx)}{d^4} - \frac{2ax \cosh(c + dx)}{d^2} - \frac{4bx^3 \cosh(c + dx)}{d^2} + \frac{24b \sinh(c + dx)}{d^5}
\end{aligned}$$

Mathematica [A] time = 0.14, size = 74, normalized size = 0.68

$$\frac{(ad^2(d^2x^2 + 2) + b(d^4x^4 + 12d^2x^2 + 24)) \sinh(c + dx) - 2dx(ad^2 + 2b(d^2x^2 + 6)) \cosh(c + dx)}{d^5}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*(a + b*x^2)*Cosh[c + d*x], x]

[Out] (-2*d*x*(a*d^2 + 2*b*(6 + d^2*x^2))*Cosh[c + d*x] + (a*d^2*(2 + d^2*x^2) + b*(24 + 12*d^2*x^2 + d^4*x^4))*Sinh[c + d*x])/d^5

fricas [A] time = 0.47, size = 78, normalized size = 0.72

$$\frac{2(2bd^3x^3 + (ad^3 + 12bd)x) \cosh(dx + c) - (bd^4x^4 + 2ad^2 + (ad^4 + 12bd^2)x^2 + 24b) \sinh(dx + c)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(b*x^2+a)*cosh(d*x+c), x, algorithm="fricas")

[Out] -(2*(2*b*d^3*x^3 + (a*d^3 + 12*b*d)*x)*cosh(d*x + c) - (b*d^4*x^4 + 2*a*d^2 + (a*d^4 + 12*b*d^2)*x^2 + 24*b)*sinh(d*x + c))/d^5

giac [A] time = 0.12, size = 138, normalized size = 1.27

$$\frac{(bd^4x^4 + ad^4x^2 - 4bd^3x^3 - 2ad^3x + 12bd^2x^2 + 2ad^2 - 24bdx + 24b)e^{(dx+c)} - (bd^4x^4 + ad^4x^2 + 4bd^3x^3 + 2ad^3x)}{2d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(b*x^2+a)*cosh(d*x+c),x, algorithm="giac")

[Out] $\frac{1}{2}*(b*d^4*x^4 + a*d^4*x^2 - 4*b*d^3*x^3 - 2*a*d^3*x + 12*b*d^2*x^2 + 2*a*d^2 - 24*b*d*x + 24*b)*e^{(d*x + c)}/d^5 - \frac{1}{2}*(b*d^4*x^4 + a*d^4*x^2 + 4*b*d^3*x^3 + 2*a*d^3*x + 12*b*d^2*x^2 + 2*a*d^2 + 24*b*d*x + 24*b)*e^{(-d*x - c)}/d^5$

maple [B] time = 0.04, size = 298, normalized size = 2.73

$$\frac{b((dx+c)^4 \sinh(dx+c) - 4(dx+c)^3 \cosh(dx+c) + 12(dx+c)^2 \sinh(dx+c) - 24(dx+c) \cosh(dx+c) + 24 \sinh(dx+c))}{d^2} - \frac{4bc((dx+c)^3 \sinh(dx+c) - 3(dx+c)^2 \cosh(dx+c) + 3c \sinh(dx+c) - 3c^2 \cosh(dx+c))}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(b*x^2+a)*cosh(d*x+c),x)

[Out] $\frac{1}{d^3}*(\frac{1}{d^2}*b*((d*x+c)^4*\sinh(d*x+c) - 4*(d*x+c)^3*\cosh(d*x+c) + 12*(d*x+c)^2*\sinh(d*x+c) - 24*(d*x+c)*\cosh(d*x+c) + 24*\sinh(d*x+c)) - \frac{4}{d^2}*b*c*((d*x+c)^3*\sinh(d*x+c) - 3*(d*x+c)^2*\cosh(d*x+c) + 6*(d*x+c)*\sinh(d*x+c) - 6*\cosh(d*x+c)) + \frac{6}{d^2}*b*c^2*((d*x+c)^2*\sinh(d*x+c) - 2*(d*x+c)*\cosh(d*x+c) + 2*\sinh(d*x+c)) - \frac{4}{d^2}*b*c^3*((d*x+c)*\sinh(d*x+c) - \cosh(d*x+c)) + \frac{1}{d^2}*b*c^4*\sinh(d*x+c) + a*((d*x+c)^2*\sinh(d*x+c) - 2*(d*x+c)*\cosh(d*x+c) + 2*\sinh(d*x+c)) - 2*a*c*((d*x+c)*\sinh(d*x+c) - \cosh(d*x+c)) + a*c^2*\sinh(d*x+c))$

maxima [A] time = 0.33, size = 214, normalized size = 1.96

$$-\frac{1}{30}d \left(\frac{5(d^3x^3e^c - 3d^2x^2e^c + 6dxe^c - 6e^c)ae^{(dx)}}{d^4} + \frac{5(d^3x^3 + 3d^2x^2 + 6dx + 6)ae^{(-dx-c)}}{d^4} + \frac{3(d^5x^5e^c - 5d^4x^4e^c + 10d^3x^3e^c - 10d^2x^2e^c + 6dxe^c - 6e^c)ae^{(dx)}}{d^5} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(b*x^2+a)*cosh(d*x+c),x, algorithm="maxima")

[Out] $-1/30*d*(5*(d^3*x^3*e^c - 3*d^2*x^2*e^c + 6*d*x*e^c - 6*e^c)*a*e^{(d*x)}/d^4 + 5*(d^3*x^3 + 3*d^2*x^2 + 6*d*x + 6)*a*e^{(-d*x - c)}/d^4 + 3*(d^5*x^5*e^c - 5*d^4*x^4*e^c + 20*d^3*x^3*e^c - 60*d^2*x^2*e^c + 120*d*x*e^c - 120*e^c)*b*e^{(d*x)}/d^6 + 3*(d^5*x^5 + 5*d^4*x^4 + 20*d^3*x^3 + 60*d^2*x^2 + 120*d*x + 120)*b*e^{(-d*x - c)}/d^6 + 1/15*(3*b*x^5 + 5*a*x^3)*\cosh(d*x + c)$

mupad [B] time = 0.93, size = 93, normalized size = 0.85

$$\frac{2 \sinh(c + dx) (a d^2 + 12 b)}{d^5} + \frac{x^2 \sinh(c + dx) (a d^2 + 12 b)}{d^3} - \frac{2 x \cosh(c + dx) (a d^2 + 12 b)}{d^4} - \frac{4 b x^3 \cosh(c + dx)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cosh(c + d*x)*(a + b*x^2),x)
```

```
[Out] (2*sinh(c + d*x)*(12*b + a*d^2))/d^5 + (x^2*sinh(c + d*x)*(12*b + a*d^2))/d^3 - (2*x*cosh(c + d*x)*(12*b + a*d^2))/d^4 - (4*b*x^3*cosh(c + d*x))/d^2 + (b*x^4*sinh(c + d*x))/d
```

sympy [A] time = 1.96, size = 134, normalized size = 1.23

$$\left\{ \begin{array}{l} \frac{ax^2 \sinh(c+dx)}{d} - \frac{2ax \cosh(c+dx)}{d^2} + \frac{2a \sinh(c+dx)}{d^3} + \frac{bx^4 \sinh(c+dx)}{d} - \frac{4bx^3 \cosh(c+dx)}{d^2} + \frac{12bx^2 \sinh(c+dx)}{d^3} - \frac{24bx \cosh(c+dx)}{d^4} + \frac{24b \sinh(c+dx)}{d^5} \\ \left(\frac{ax^3}{3} + \frac{bx^5}{5} \right) \cosh(c) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)*cosh(d*x+c),x)
```

```
[Out] Piecewise((a*x**2*sinh(c + d*x)/d - 2*a*x*cosh(c + d*x)/d**2 + 2*a*sinh(c + d*x)/d**3 + b*x**4*sinh(c + d*x)/d - 4*b*x**3*cosh(c + d*x)/d**2 + 12*b*x**2*sinh(c + d*x)/d**3 - 24*b*x*cosh(c + d*x)/d**4 + 24*b*sinh(c + d*x)/d**5, Ne(d, 0)), ((a*x**3/3 + b*x**5/5)*cosh(c), True))
```

3.42 $\int x(a + bx^2) \cosh(c + dx) dx$

Optimal. Leaf size=79

$$-\frac{a \cosh(c + dx)}{d^2} + \frac{ax \sinh(c + dx)}{d} - \frac{6b \cosh(c + dx)}{d^4} + \frac{6bx \sinh(c + dx)}{d^3} - \frac{3bx^2 \cosh(c + dx)}{d^2} + \frac{bx^3 \sinh(c + dx)}{d}$$

[Out] $-6*b*\cosh(d*x+c)/d^4-a*\cosh(d*x+c)/d^2-3*b*x^2*\cosh(d*x+c)/d^2+6*b*x*\sinh(d*x+c)/d^3+a*x*\sinh(d*x+c)/d+b*x^3*\sinh(d*x+c)/d$

Rubi [A] time = 0.12, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5287, 3296, 2638}

$$-\frac{a \cosh(c + dx)}{d^2} + \frac{ax \sinh(c + dx)}{d} - \frac{3bx^2 \cosh(c + dx)}{d^2} + \frac{6bx \sinh(c + dx)}{d^3} - \frac{6b \cosh(c + dx)}{d^4} + \frac{bx^3 \sinh(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x*(a + b*x^2)*\text{Cosh}[c + d*x], x]$

[Out] $(-6*b*\text{Cosh}[c + d*x])/d^4 - (a*\text{Cosh}[c + d*x])/d^2 - (3*b*x^2*\text{Cosh}[c + d*x])/d^2 + (6*b*x*\text{Sinh}[c + d*x])/d^3 + (a*x*\text{Sinh}[c + d*x])/d + (b*x^3*\text{Sinh}[c + d*x])/d$

Rule 2638

$\text{Int}[\sin[(c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow -\text{Simp}[\cos[c + d*x]/d, x] \text{ ; FreeQ}[\{c, d\}, x]$

Rule 3296

$\text{Int}[(c_. + (d_.)*(x_.))^{(m_.)}*\sin[(e_.) + (f_.)*(x_.)], x_Symbol] \rightarrow -\text{Simp}[(c + d*x)^m*\cos[e + f*x]/f, x] + \text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{(m-1)}*\cos[e + f*x], x], x] \text{ ; FreeQ}[\{c, d, e, f\}, x] \ \&\& \ \text{GtQ}[m, 0]$

Rule 5287

$\text{Int}[\text{Cosh}[(c_.) + (d_.)*(x_.)]*((e_.)*(x_.))^{(m_.)}*((a_.) + (b_.)*(x_.)^{(n_.)})^{(p_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[\text{Cosh}[c + d*x], (e*x)^m*(a + b*x^n)^p, x], x] \text{ ; FreeQ}[\{a, b, c, d, e, m, n\}, x] \ \&\& \ \text{IGtQ}[p, 0]$

Rubi steps

$$\begin{aligned}
\int x(a + bx^2) \cosh(c + dx) dx &= \int (ax \cosh(c + dx) + bx^3 \cosh(c + dx)) dx \\
&= a \int x \cosh(c + dx) dx + b \int x^3 \cosh(c + dx) dx \\
&= \frac{ax \sinh(c + dx)}{d} + \frac{bx^3 \sinh(c + dx)}{d} - \frac{a \int \sinh(c + dx) dx}{d} - \frac{(3b) \int x^2 \sinh(c + dx) dx}{d} \\
&= -\frac{a \cosh(c + dx)}{d^2} - \frac{3bx^2 \cosh(c + dx)}{d^2} + \frac{ax \sinh(c + dx)}{d} + \frac{bx^3 \sinh(c + dx)}{d} + \frac{3bx^2 \sinh(c + dx)}{d^2} \\
&= -\frac{a \cosh(c + dx)}{d^2} - \frac{3bx^2 \cosh(c + dx)}{d^2} + \frac{6bx \sinh(c + dx)}{d^3} + \frac{ax \sinh(c + dx)}{d} + \frac{3bx^2 \sinh(c + dx)}{d^2} \\
&= -\frac{6b \cosh(c + dx)}{d^4} - \frac{a \cosh(c + dx)}{d^2} - \frac{3bx^2 \cosh(c + dx)}{d^2} + \frac{6bx \sinh(c + dx)}{d^3} + \frac{ax \sinh(c + dx)}{d}
\end{aligned}$$

Mathematica [A] time = 0.12, size = 57, normalized size = 0.72

$$\frac{dx(ad^2 + b(d^2x^2 + 6)) \sinh(c + dx) - (ad^2 + 3b(d^2x^2 + 2)) \cosh(c + dx)}{d^4}$$

Antiderivative was successfully verified.

[In] Integrate[x*(a + b*x^2)*Cosh[c + d*x],x]

[Out] (-((a*d^2 + 3*b*(2 + d^2*x^2))*Cosh[c + d*x]) + d*x*(a*d^2 + b*(6 + d^2*x^2))*Sinh[c + d*x])/d^4

fricas [A] time = 0.42, size = 60, normalized size = 0.76

$$\frac{(3bd^2x^2 + ad^2 + 6b) \cosh(dx + c) - (bd^3x^3 + (ad^3 + 6bd)x) \sinh(dx + c)}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x^2+a)*cosh(d*x+c),x, algorithm="fricas")

[Out] -((3*b*d^2*x^2 + a*d^2 + 6*b)*cosh(d*x + c) - (b*d^3*x^3 + (a*d^3 + 6*b*d)*x)*sinh(d*x + c))/d^4

giac [A] time = 0.11, size = 101, normalized size = 1.28

$$\frac{(bd^3x^3 + ad^3x - 3bd^2x^2 - ad^2 + 6bdx - 6b)e^{(dx+c)}}{2d^4} - \frac{(bd^3x^3 + ad^3x + 3bd^2x^2 + ad^2 + 6bdx + 6b)e^{(-dx-c)}}{2d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x^2+a)*cosh(d*x+c),x, algorithm="giac")

[Out] $\frac{1}{2}*(b*d^3*x^3 + a*d^3*x - 3*b*d^2*x^2 - a*d^2 + 6*b*d*x - 6*b)*e^{(d*x + c)}/d^4 - \frac{1}{2}*(b*d^3*x^3 + a*d^3*x + 3*b*d^2*x^2 + a*d^2 + 6*b*d*x + 6*b)*e^{(-d*x - c)}/d^4$

maple [B] time = 0.04, size = 183, normalized size = 2.32

$$\frac{b((dx+c)^3 \sinh(dx+c) - 3(dx+c)^2 \cosh(dx+c) + 6(dx+c) \sinh(dx+c) - 6 \cosh(dx+c))}{d^2} - \frac{3bc((dx+c)^2 \sinh(dx+c) - 2(dx+c) \cosh(dx+c) + 2 \sinh(dx+c))}{d^2} + \frac{d^2}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(b*x^2+a)*cosh(d*x+c),x)

[Out] $\frac{1}{d^2}*(\frac{1}{d^2}*b*((d*x+c)^3*\sinh(d*x+c) - 3*(d*x+c)^2*\cosh(d*x+c) + 6*(d*x+c)*\sinh(d*x+c) - 6*\cosh(d*x+c)) - \frac{3}{d^2}*b*c*((d*x+c)^2*\sinh(d*x+c) - 2*(d*x+c)*\cosh(d*x+c) + 2*\sinh(d*x+c)) + \frac{3}{d^2}*b*c^2*((d*x+c)*\sinh(d*x+c) - \cosh(d*x+c)) + a*((d*x+c)*\sinh(d*x+c) - \cosh(d*x+c)) - \frac{1}{d^2}*b*c^3*\sinh(d*x+c) - a*c*\sinh(d*x+c))$

maxima [B] time = 0.33, size = 213, normalized size = 2.70

$$\frac{(bx^2 + a)^2 \cosh(dx + c)}{4b} - \left(\frac{a^2 e^{(dx+c)}}{d} + \frac{a^2 e^{(-dx-c)}}{d} + \frac{2(d^2 x^2 e^c - 2 dx e^c + 2 e^c) a b e^{(dx)}}{d^3} + \frac{2(d^2 x^2 + 2 dx + 2) a b e^{(-dx-c)}}{d^3} + \frac{(d^4 x^4 e^c - 4 d^3 x^3 e^c + 12 d^2 x^2 e^c - 24 d x e^c + 24 e^c) b^2 e^{(dx)}}{d^5} + \frac{(d^4 x^4 + 4 d^3 x^3 + 12 d^2 x^2 + 24 d x + 24) b^2 e^{(-dx-c)}}{d^5} \right) \frac{d}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x^2+a)*cosh(d*x+c),x, algorithm="maxima")

[Out] $\frac{1}{4}*(b*x^2 + a)^2*\cosh(d*x + c)/b - \frac{1}{8}*(a^2*e^{(d*x + c)}/d + a^2*e^{(-d*x - c)}/d + 2*(d^2*x^2*e^c - 2*d*x*e^c + 2*e^c)*a*b*e^{(d*x)}/d^3 + 2*(d^2*x^2 + 2*d*x + 2)*a*b*e^{(-d*x - c)}/d^3 + (d^4*x^4*e^c - 4*d^3*x^3*e^c + 12*d^2*x^2*e^c - 24*d*x*e^c + 24*e^c)*b^2*e^{(d*x)}/d^5 + (d^4*x^4 + 4*d^3*x^3 + 12*d^2*x^2 + 24*d*x + 24)*b^2*e^{(-d*x - c)}/d^5)*d/b$

mupad [B] time = 0.98, size = 70, normalized size = 0.89

$$\frac{x \sinh(c + dx) (ad^2 + 6b)}{d^3} - \frac{\cosh(c + dx) (ad^2 + 6b)}{d^4} - \frac{3bx^2 \cosh(c + dx)}{d^2} + \frac{bx^3 \sinh(c + dx)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*cosh(c + d*x)*(a + b*x^2),x)

[Out] $(x*\sinh(c + d*x)*(6*b + a*d^2))/d^3 - (\cosh(c + d*x)*(6*b + a*d^2))/d^4 - (3*b*x^2*\cosh(c + d*x))/d^2 + (b*x^3*\sinh(c + d*x))/d$

sympy [A] time = 0.99, size = 99, normalized size = 1.25

$$\begin{cases} \frac{ax \sinh(c+dx)}{d} - \frac{a \cosh(c+dx)}{d^2} + \frac{bx^3 \sinh(c+dx)}{d} - \frac{3bx^2 \cosh(c+dx)}{d^2} + \frac{6bx \sinh(c+dx)}{d^3} - \frac{6b \cosh(c+dx)}{d^4} & \text{for } d \neq 0 \\ \left(\frac{ax^2}{2} + \frac{bx^4}{4}\right) \cosh(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x**2+a)*cosh(d*x+c),x)

[Out] Piecewise((a*x*sinh(c + d*x)/d - a*cosh(c + d*x)/d**2 + b*x**3*sinh(c + d*x)/d - 3*b*x**2*cosh(c + d*x)/d**2 + 6*b*x*sinh(c + d*x)/d**3 - 6*b*cosh(c + d*x)/d**4, Ne(d, 0)), ((a*x**2/2 + b*x**4/4)*cosh(c), True))

3.43 $\int (a + bx^2) \cosh(c + dx) dx$

Optimal. Leaf size=51

$$\frac{a \sinh(c + dx)}{d} + \frac{2b \sinh(c + dx)}{d^3} - \frac{2bx \cosh(c + dx)}{d^2} + \frac{bx^2 \sinh(c + dx)}{d}$$

[Out] $-2*b*x*cosh(d*x+c)/d^2+2*b*sinh(d*x+c)/d^3+a*sinh(d*x+c)/d+b*x^2*sinh(d*x+c)/d$

Rubi [A] time = 0.07, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {5277, 2637, 3296}

$$\frac{a \sinh(c + dx)}{d} + \frac{2b \sinh(c + dx)}{d^3} - \frac{2bx \cosh(c + dx)}{d^2} + \frac{bx^2 \sinh(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^2)*Cosh[c + d*x], x]

[Out] $(-2*b*x*Cosh[c + d*x])/d^2 + (2*b*Sinh[c + d*x])/d^3 + (a*Sinh[c + d*x])/d + (b*x^2*Sinh[c + d*x])/d$

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3296

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 5277

Int[Cosh[(c_.) + (d_.)*(x_)]*((a_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] := Int[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[p, 0]

Rubi steps

$$\begin{aligned}
\int (a + bx^2) \cosh(c + dx) dx &= \int (a \cosh(c + dx) + bx^2 \cosh(c + dx)) dx \\
&= a \int \cosh(c + dx) dx + b \int x^2 \cosh(c + dx) dx \\
&= \frac{a \sinh(c + dx)}{d} + \frac{bx^2 \sinh(c + dx)}{d} - \frac{(2b) \int x \sinh(c + dx) dx}{d} \\
&= -\frac{2bx \cosh(c + dx)}{d^2} + \frac{a \sinh(c + dx)}{d} + \frac{bx^2 \sinh(c + dx)}{d} + \frac{(2b) \int \cosh(c + dx) dx}{d^2} \\
&= -\frac{2bx \cosh(c + dx)}{d^2} + \frac{2b \sinh(c + dx)}{d^3} + \frac{a \sinh(c + dx)}{d} + \frac{bx^2 \sinh(c + dx)}{d}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 40, normalized size = 0.78

$$\frac{(ad^2 + b(d^2x^2 + 2)) \sinh(c + dx) - 2bdx \cosh(c + dx)}{d^3}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^2)*Cosh[c + d*x], x]

[Out] (-2*b*d*x*Cosh[c + d*x] + (a*d^2 + b*(2 + d^2*x^2))*Sinh[c + d*x])/d^3

fricas [A] time = 0.44, size = 42, normalized size = 0.82

$$-\frac{2bdx \cosh(dx + c) - (bd^2x^2 + ad^2 + 2b) \sinh(dx + c)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)*cosh(d*x+c), x, algorithm="fricas")

[Out] -(2*b*d*x*cosh(d*x + c) - (b*d^2*x^2 + a*d^2 + 2*b)*sinh(d*x + c))/d^3

giac [A] time = 0.15, size = 70, normalized size = 1.37

$$\frac{(bd^2x^2 + ad^2 - 2bdx + 2b)e^{(dx+c)}}{2d^3} - \frac{(bd^2x^2 + ad^2 + 2bdx + 2b)e^{(-dx-c)}}{2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)*cosh(d*x+c), x, algorithm="giac")

[Out] 1/2*(b*d^2*x^2 + a*d^2 - 2*b*d*x + 2*b)*e^(d*x + c)/d^3 - 1/2*(b*d^2*x^2 + a*d^2 + 2*b*d*x + 2*b)*e^(-d*x - c)/d^3

maple [A] time = 0.04, size = 97, normalized size = 1.90

$$\frac{b((dx+c)^2 \sinh(dx+c) - 2(dx+c) \cosh(dx+c) + 2 \sinh(dx+c))}{d^2} - \frac{2bc((dx+c) \sinh(dx+c) - \cosh(dx+c))}{d^2} + \frac{bc^2 \sinh(dx+c)}{d^2} + a \sinh(dx+c)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^2+a)*cosh(d*x+c),x)

[Out] 1/d*(1/d^2*b*((d*x+c)^2*sinh(d*x+c)-2*(d*x+c)*cosh(d*x+c)+2*sinh(d*x+c))-2/d^2*b*c*((d*x+c)*sinh(d*x+c)-cosh(d*x+c))+1/d^2*b*c^2*sinh(d*x+c)+a*sinh(d*x+c))

maxima [A] time = 0.33, size = 86, normalized size = 1.69

$$\frac{ae^{(dx+c)}}{2d} - \frac{ae^{-(dx+c)}}{2d} + \frac{(d^2x^2e^c - 2dxe^c + 2e^c)be^{(dx)}}{2d^3} - \frac{(d^2x^2 + 2dx + 2)be^{-(dx-c)}}{2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)*cosh(d*x+c),x, algorithm="maxima")

[Out] 1/2*a*e^(d*x + c)/d - 1/2*a*e^(-d*x - c)/d + 1/2*(d^2*x^2*e^c - 2*d*x*e^c + 2*e^c)*b*e^(d*x)/d^3 - 1/2*(d^2*x^2 + 2*d*x + 2)*b*e^(-d*x - c)/d^3

mupad [B] time = 0.93, size = 47, normalized size = 0.92

$$\frac{\sinh(c + dx) (a d^2 + 2b)}{d^3} - \frac{2bx \cosh(c + dx)}{d^2} + \frac{bx^2 \sinh(c + dx)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(c + d*x)*(a + b*x^2),x)

[Out] (sinh(c + d*x)*(2*b + a*d^2))/d^3 - (2*b*x*cosh(c + d*x))/d^2 + (b*x^2*sinh(c + d*x))/d

sympy [A] time = 0.48, size = 65, normalized size = 1.27

$$\begin{cases} \frac{a \sinh(c+dx)}{d} + \frac{bx^2 \sinh(c+dx)}{d} - \frac{2bx \cosh(c+dx)}{d^2} + \frac{2b \sinh(c+dx)}{d^3} & \text{for } d \neq 0 \\ \left(ax + \frac{bx^3}{3}\right) \cosh(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*cosh(d*x+c),x)
```

```
[Out] Piecewise((a*sinh(c + d*x)/d + b*x**2*sinh(c + d*x)/d - 2*b*x*cosh(c + d*x)/d**2 + 2*b*sinh(c + d*x)/d**3, Ne(d, 0)), ((a*x + b*x**3/3)*cosh(c), True)
```

$$3.44 \quad \int \frac{(a+bx^2) \cosh(c+dx)}{x} dx$$

Optimal. Leaf size=41

$$a \cosh(c) \text{Chi}(dx) + a \sinh(c) \text{Shi}(dx) - \frac{b \cosh(c+dx)}{d^2} + \frac{bx \sinh(c+dx)}{d}$$

[Out] a*Chi(d*x)*cosh(c)-b*cosh(d*x+c)/d^2+a*Shi(d*x)*sinh(c)+b*x*sinh(d*x+c)/d

Rubi [A] time = 0.10, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {5287, 3303, 3298, 3301, 3296, 2638}

$$a \cosh(c) \text{Chi}(dx) + a \sinh(c) \text{Shi}(dx) - \frac{b \cosh(c+dx)}{d^2} + \frac{bx \sinh(c+dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[((a + b*x^2)*Cosh[c + d*x])/x,x]

[Out] -((b*Cosh[c + d*x])/d^2) + a*Cosh[c]*CoshIntegral[d*x] + (b*x*Sinh[c + d*x])/d + a*Sinh[c]*SinhIntegral[d*x]

Rule 2638

Int[sin[(c_.) + (d_.)*(x_.)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3296

Int[((c_.) + (d_.)*(x_.))^(m_.)*sin[(e_.) + (f_.)*(x_.)], x_Symbol] := -Simp[((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m-1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 3298

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x_Symbol] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]

Rule 3301

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x_Symbol] := Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

Rule 5287

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] := Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{(a + bx^2) \cosh(c + dx)}{x} dx &= \int \left(\frac{a \cosh(c + dx)}{x} + bx \cosh(c + dx) \right) dx \\ &= a \int \frac{\cosh(c + dx)}{x} dx + b \int x \cosh(c + dx) dx \\ &= \frac{bx \sinh(c + dx)}{d} - \frac{b \int \sinh(c + dx) dx}{d} + (a \cosh(c)) \int \frac{\cosh(dx)}{x} dx + (a \sinh(c)) \int \frac{\sinh(dx)}{x} dx \\ &= -\frac{b \cosh(c + dx)}{d^2} + a \cosh(c) \text{Chi}(dx) + \frac{bx \sinh(c + dx)}{d} + a \sinh(c) \text{Shi}(dx) \end{aligned}$$

Mathematica [A] time = 0.12, size = 55, normalized size = 1.34

$$a \cosh(c) \text{Chi}(dx) + a \sinh(c) \text{Shi}(dx) + \frac{b \cosh(dx)(dx \sinh(c) - \cosh(c))}{d^2} + \frac{b \sinh(dx)(dx \cosh(c) - \sinh(c))}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*Cosh[c + d*x])/x,x]
```

```
[Out] a*Cosh[c]*CoshIntegral[d*x] + (b*Cosh[d*x]*(-Cosh[c] + d*x*Sinh[c]))/d^2 +
(b*(d*x*Cosh[c] - Sinh[c])*Sinh[d*x])/d^2 + a*Sinh[c]*SinhIntegral[d*x]
```

fricas [A] time = 0.54, size = 73, normalized size = 1.78

$$\frac{2 b dx \sinh(dx + c) - 2 b \cosh(dx + c) + (ad^2 \text{Ei}(dx) + ad^2 \text{Ei}(-dx)) \cosh(c) + (ad^2 \text{Ei}(dx) - ad^2 \text{Ei}(-dx)) \sinh(c)}{2 d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)*cosh(d*x+c)/x,x, algorithm="fricas")

[Out] $1/2*(2*b*d*x*\sinh(d*x + c) - 2*b*\cosh(d*x + c) + (a*d^2*Ei(d*x) + a*d^2*Ei(-d*x))*\cosh(c) + (a*d^2*Ei(d*x) - a*d^2*Ei(-d*x))*\sinh(c))/d^2$

giac [A] time = 0.12, size = 76, normalized size = 1.85

$$\frac{ad^2Ei(-dx)e^{(-c)} + ad^2Ei(dx)e^c + bdx e^{(dx+c)} - bdx e^{(-dx-c)} - be^{(dx+c)} - be^{(-dx-c)}}{2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)*cosh(d*x+c)/x,x, algorithm="giac")

[Out] $1/2*(a*d^2*Ei(-d*x)*e^{(-c)} + a*d^2*Ei(d*x)*e^c + b*d*x*e^{(d*x + c)} - b*d*x*e^{(-d*x - c)} - b*e^{(d*x + c)} - b*e^{(-d*x - c)})/d^2$

maple [A] time = 0.10, size = 81, normalized size = 1.98

$$-\frac{a e^{-c} Ei(1, dx)}{2} - \frac{b e^{-dx-c} x}{2d} - \frac{b e^{-dx-c}}{2d^2} - \frac{a e^c Ei(1, -dx)}{2} + \frac{b e^{dx+c} x}{2d} - \frac{b e^{dx+c}}{2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^2+a)*cosh(d*x+c)/x,x)

[Out] $-1/2*a*\exp(-c)*Ei(1,d*x)-1/2/d*b*\exp(-d*x-c)*x-1/2/d^2*b*\exp(-d*x-c)-1/2*a*\exp(c)*Ei(1,-d*x)+1/2/d*b*\exp(d*x+c)*x-1/2/d^2*b*\exp(d*x+c)$

maxima [B] time = 0.39, size = 122, normalized size = 2.98

$$-\frac{1}{4} \left(b \left(\frac{(d^2 x^2 e^c - 2 dx e^c + 2 e^c) e^{(dx)}}{d^3} + \frac{(d^2 x^2 + 2 dx + 2) e^{(-dx-c)}}{d^3} \right) \right) + \frac{2 a \cosh(dx + c) \log(x^2)}{d} - \frac{2 (Ei(-dx) e^{(-c)})}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)*cosh(d*x+c)/x,x, algorithm="maxima")

[Out] $-1/4*(b*((d^2*x^2*e^c - 2*d*x*e^c + 2*e^c)*e^{(d*x)}/d^3 + (d^2*x^2 + 2*d*x + 2)*e^{(-d*x - c)}/d^3) + 2*a*cosh(d*x + c)*log(x^2)/d - 2*(Ei(-d*x)*e^{(-c)} + Ei(d*x)*e^c)*a/d*d + 1/2*(b*x^2 + a*log(x^2))*cosh(d*x + c)$

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$a \operatorname{coshint}(dx) \cosh(c) + a \operatorname{sinhint}(dx) \sinh(c) - \frac{b (\cosh(c + dx) - dx \sinh(c + dx))}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)*(a + b*x^2))/x,x)
```

```
[Out] a*coshint(d*x)*cosh(c) + a*sinhint(d*x)*sinh(c) - (b*(cosh(c + d*x) - d*x*sinh(c + d*x)))/d^2
```

sympy [A] time = 3.41, size = 49, normalized size = 1.20

$$a \sinh(c) \operatorname{Shi}(dx) + a \cosh(c) \operatorname{Chi}(dx) + b \left\{ \begin{array}{ll} \frac{x \sinh(c+dx)}{d} - \frac{\cosh(c+dx)}{d^2} & \text{for } d \neq 0 \\ \frac{x^2 \cosh(c)}{2} & \text{otherwise} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*cosh(d*x+c)/x,x)
```

```
[Out] a*sinh(c)*Shi(d*x) + a*cosh(c)*Chi(d*x) + b*Piecewise((x*sinh(c + d*x)/d - cosh(c + d*x)/d**2, Ne(d, 0)), (x**2*cosh(c)/2, True))
```

$$3.45 \quad \int \frac{(a+bx^2) \cosh(c+dx)}{x^2} dx$$

Optimal. Leaf size=42

$$ad \sinh(c)\text{Chi}(dx) + ad \cosh(c)\text{Shi}(dx) - \frac{a \cosh(c+dx)}{x} + \frac{b \sinh(c+dx)}{d}$$

[Out] $-a*\cosh(d*x+c)/x+a*d*\cosh(c)*\text{Shi}(d*x)+a*d*\text{Chi}(d*x)*\sinh(c)+b*\sinh(d*x+c)/d$

Rubi [A] time = 0.11, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {5287, 2637, 3297, 3303, 3298, 3301}

$$ad \sinh(c)\text{Chi}(dx) + ad \cosh(c)\text{Shi}(dx) - \frac{a \cosh(c+dx)}{x} + \frac{b \sinh(c+dx)}{d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + b*x^2)*\text{Cosh}[c + d*x])/x^2, x]$

[Out] $-((a*\text{Cosh}[c + d*x])/x) + a*d*\text{CoshIntegral}[d*x]*\text{Sinh}[c] + (b*\text{Sinh}[c + d*x])/d + a*d*\text{Cosh}[c]*\text{SinhIntegral}[d*x]$

Rule 2637

$\text{Int}[\sin[\text{Pi}/2 + (c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow \text{Simp}[\sin[c + d*x]/d, x] /;$
FreeQ[{c, d}, x]

Rule 3297

$\text{Int}[(c_. + (d_.)*(x_.))^{(m)}*\sin[(e_.) + (f_.)*(x_.)], x_Symbol] \rightarrow \text{Simp}[(c + d*x)^{(m+1)}*\sin[e + f*x]/(d*(m+1)), x] - \text{Dist}[f/(d*(m+1)), \text{Int}[(c + d*x)^{(m+1)}*\cos[e + f*x], x], x] /;$ FreeQ[{c, d, e, f}, x] && LtQ[m, -1]

Rule 3298

$\text{Int}[\sin[(e_.) + (\text{Complex}[0, fz_])*(f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x_Symbol] \rightarrow \text{Simp}[(I*\text{SinhIntegral}[(c*f*fz)/d + f*fz*x])/d, x] /;$ FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]

Rule 3301

$\text{Int}[\sin[(e_.) + (\text{Complex}[0, fz_])*(f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x_Symbol] \rightarrow \text{Simp}[\text{CoshIntegral}[(c*f*fz)/d + f*fz*x]/d, x] /;$ FreeQ[{c, d, e, f, fz}

}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]

Rule 3303

Int[sin[(e_.) + (f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x_Symbol] :> Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]

Rule 5287

Int[Cosh[(c_.) + (d_.)*(x_.)]*((e_.)*(x_.))^(m_.)*((a_.) + (b_.)*(x_.)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{(a + bx^2) \cosh(c + dx)}{x^2} dx &= \int \left(b \cosh(c + dx) + \frac{a \cosh(c + dx)}{x^2} \right) dx \\
 &= a \int \frac{\cosh(c + dx)}{x^2} dx + b \int \cosh(c + dx) dx \\
 &= -\frac{a \cosh(c + dx)}{x} + \frac{b \sinh(c + dx)}{d} + (ad) \int \frac{\sinh(c + dx)}{x} dx \\
 &= -\frac{a \cosh(c + dx)}{x} + \frac{b \sinh(c + dx)}{d} + (ad \cosh(c)) \int \frac{\sinh(dx)}{x} dx + (ad \sinh(c)) \int \frac{1}{x} dx \\
 &= -\frac{a \cosh(c + dx)}{x} + ad \operatorname{Chi}(dx) \sinh(c) + \frac{b \sinh(c + dx)}{d} + ad \cosh(c) \operatorname{Shi}(dx)
 \end{aligned}$$

Mathematica [A] time = 0.09, size = 42, normalized size = 1.00

$$ad \sinh(c) \operatorname{Chi}(dx) + ad \cosh(c) \operatorname{Shi}(dx) - \frac{a \cosh(c + dx)}{x} + \frac{b \sinh(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x^2)*Cosh[c + d*x])/x^2,x]

[Out] -((a*Cosh[c + d*x])/x) + a*d*CoshIntegral[d*x]*Sinh[c] + (b*Sinh[c + d*x])/d + a*d*Cosh[c]*SinhIntegral[d*x]

fricas [A] time = 0.58, size = 82, normalized size = 1.95

$$\frac{2 ad \cosh(dx + c) - 2 bx \sinh(dx + c) - (ad^2 x \operatorname{Ei}(dx) - ad^2 x \operatorname{Ei}(-dx)) \cosh(c) - (ad^2 x \operatorname{Ei}(dx) + ad^2 x \operatorname{Ei}(-dx)) \sinh(c)}{2 dx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)*cosh(d*x+c)/x^2,x, algorithm="fricas")

[Out] $-1/2*(2*a*d*\cosh(d*x + c) - 2*b*x*\sinh(d*x + c) - (a*d^2*x*Ei(d*x) - a*d^2*x*Ei(-d*x))*\cosh(c) - (a*d^2*x*Ei(d*x) + a*d^2*x*Ei(-d*x))*\sinh(c))/(d*x)$

giac [A] time = 0.12, size = 80, normalized size = 1.90

$$\frac{ad^2x\text{Ei}(-dx)e^{(-c)} - ad^2x\text{Ei}(dx)e^c + ade^{(dx+c)} - bxe^{(dx+c)} + ade^{(-dx-c)} + bxe^{(-dx-c)}}{2dx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)*cosh(d*x+c)/x^2,x, algorithm="giac")

[Out] $-1/2*(a*d^2*x*Ei(-d*x)*e^{(-c)} - a*d^2*x*Ei(d*x)*e^c + a*d*e^{(d*x + c)} - b*x*e^{(d*x + c)} + a*d*e^{(-d*x - c)} + b*x*e^{(-d*x - c)})/(d*x)$

maple [A] time = 0.11, size = 81, normalized size = 1.93

$$-\frac{ae^{-dx-c}}{2x} + \frac{dae^{-c}\text{Ei}(1,dx)}{2} - \frac{be^{-dx-c}}{2d} - \frac{ae^{dx+c}}{2x} - \frac{dae^c\text{Ei}(1,-dx)}{2} + \frac{be^{dx+c}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^2+a)*cosh(d*x+c)/x^2,x)

[Out] $-1/2*a*\exp(-d*x-c)/x+1/2*d*a*\exp(-c)*Ei(1,d*x)-1/2/d*b*\exp(-d*x-c)-1/2*a/x*\exp(d*x+c)-1/2*d*a*\exp(c)*Ei(1,-d*x)+1/2/d*b*\exp(d*x+c)$

maxima [A] time = 0.38, size = 80, normalized size = 1.90

$$-\frac{1}{2}\left(a\text{Ei}(-dx)e^{(-c)} - a\text{Ei}(dx)e^c + \frac{(dxe^c - e^c)be^{(dx)}}{d^2} + \frac{(dx+1)be^{(-dx-c)}}{d^2}\right)d + \left(bx - \frac{a}{x}\right)\cosh(dx+c)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)*cosh(d*x+c)/x^2,x, algorithm="maxima")

[Out] $-1/2*(a*Ei(-d*x)*e^{(-c)} - a*Ei(d*x)*e^c + (d*x*e^c - e^c)*b*e^{(d*x)}/d^2 + (d*x + 1)*b*e^{(-d*x - c)}/d^2)*d + (b*x - a/x)*\cosh(d*x + c)$

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cosh(c+dx)(bx^2+a)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((cosh(c + d*x)*(a + b*x^2))/x^2, x)`

[Out] `int((cosh(c + d*x)*(a + b*x^2))/x^2, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^2) \cosh(c + dx)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)*cosh(d*x+c)/x**2, x)`

[Out] `Integral((a + b*x**2)*cosh(c + d*x)/x**2, x)`

$$3.46 \quad \int \frac{(a+bx^2) \cosh(c+dx)}{x^3} dx$$

Optimal. Leaf size=74

$$\frac{1}{2}ad^2 \cosh(c)\text{Chi}(dx) + \frac{1}{2}ad^2 \sinh(c)\text{Shi}(dx) - \frac{a \cosh(c+dx)}{2x^2} - \frac{ad \sinh(c+dx)}{2x} + b \cosh(c)\text{Chi}(dx) + b \sinh(c)\text{Shi}(dx)$$

[Out] b*Chi(d*x)*cosh(c)+1/2*a*d^2*Chi(d*x)*cosh(c)-1/2*a*cosh(d*x+c)/x^2+b*Shi(d*x)*sinh(c)+1/2*a*d^2*Shi(d*x)*sinh(c)-1/2*a*d*sinh(d*x+c)/x

Rubi [A] time = 0.18, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {5287, 3297, 3303, 3298, 3301}

$$\frac{1}{2}ad^2 \cosh(c)\text{Chi}(dx) + \frac{1}{2}ad^2 \sinh(c)\text{Shi}(dx) - \frac{a \cosh(c+dx)}{2x^2} - \frac{ad \sinh(c+dx)}{2x} + b \cosh(c)\text{Chi}(dx) + b \sinh(c)\text{Shi}(dx)$$

Antiderivative was successfully verified.

[In] Int[((a + b*x^2)*Cosh[c + d*x])/x^3, x]

[Out] -(a*Cosh[c + d*x])/(2*x^2) + b*Cosh[c]*CoshIntegral[d*x] + (a*d^2*Cosh[c]*CoshIntegral[d*x])/2 - (a*d*Sinh[c + d*x])/(2*x) + b*Sinh[c]*SinhIntegral[d*x] + (a*d^2*Sinh[c]*SinhIntegral[d*x])/2

Rule 3297

Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]

Rule 3298

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)])/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]

Rule 3301

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)])/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

Rule 5287

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] := Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + bx^2) \cosh(c + dx)}{x^3} dx &= \int \left(\frac{a \cosh(c + dx)}{x^3} + \frac{b \cosh(c + dx)}{x} \right) dx \\
&= a \int \frac{\cosh(c + dx)}{x^3} dx + b \int \frac{\cosh(c + dx)}{x} dx \\
&= -\frac{a \cosh(c + dx)}{2x^2} + \frac{1}{2}(ad) \int \frac{\sinh(c + dx)}{x^2} dx + (b \cosh(c)) \int \frac{\cosh(dx)}{x} dx + (b \sinh(c)) \int \frac{\sinh(dx)}{x} dx \\
&= -\frac{a \cosh(c + dx)}{2x^2} + b \cosh(c) \text{Chi}(dx) - \frac{ad \sinh(c + dx)}{2x} + b \sinh(c) \text{Shi}(dx) + \frac{1}{2} \left(\frac{ad \cosh(c + dx)}{x} + \frac{ad \sinh(c + dx)}{x} \right) \\
&= -\frac{a \cosh(c + dx)}{2x^2} + b \cosh(c) \text{Chi}(dx) - \frac{ad \sinh(c + dx)}{2x} + b \sinh(c) \text{Shi}(dx) + \frac{1}{2} \left(\frac{ad \cosh(c + dx)}{x} + \frac{ad \sinh(c + dx)}{x} \right) \\
&= -\frac{a \cosh(c + dx)}{2x^2} + b \cosh(c) \text{Chi}(dx) + \frac{1}{2} ad^2 \cosh(c) \text{Chi}(dx) - \frac{ad \sinh(c + dx)}{2x} + \frac{1}{2} \left(\frac{ad \cosh(c + dx)}{x} + \frac{ad \sinh(c + dx)}{x} \right)
\end{aligned}$$

Mathematica [A] time = 0.16, size = 80, normalized size = 1.08

$$\frac{1}{2} ad^2 (\cosh(c) \text{Chi}(dx) + \sinh(c) \text{Shi}(dx)) - \frac{a \cosh(dx) (dx \sinh(c) + \cosh(c))}{2x^2} - \frac{a \sinh(dx) (dx \cosh(c) + \sinh(c))}{2x^2} + b \cosh(c) \int \frac{\cosh(dx)}{x} dx + b \sinh(c) \int \frac{\sinh(dx)}{x} dx$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*Cosh[c + d*x])/x^3,x]
```

```
[Out] b*Cosh[c]*CoshIntegral[d*x] - (a*Cosh[d*x]*(Cosh[c] + d*x*Sinh[c]))/(2*x^2)
- (a*(d*x*Cosh[c] + Sinh[c])*Sinh[d*x])/(2*x^2) + b*Sinh[c]*SinhIntegral[d
*x] + (a*d^2*(Cosh[c]*CoshIntegral[d*x] + Sinh[c]*SinhIntegral[d*x]))/2
```

fricas [A] time = 0.50, size = 107, normalized size = 1.45

$$\frac{2 adx \sinh(dx + c) + 2 a \cosh(dx + c) - ((ad^2 + 2b)x^2 \text{Ei}(dx) + (ad^2 + 2b)x^2 \text{Ei}(-dx)) \cosh(c) - ((ad^2 + 2b)x^2 \text{Ei}(dx) + (ad^2 + 2b)x^2 \text{Ei}(-dx)) \sinh(c)}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)*cosh(d*x+c)/x^3,x, algorithm="fricas")

[Out] $-1/4*(2*a*d*x*\sinh(d*x + c) + 2*a*\cosh(d*x + c) - ((a*d^2 + 2*b)*x^2*Ei(d*x) + (a*d^2 + 2*b)*x^2*Ei(-d*x))*\cosh(c) - ((a*d^2 + 2*b)*x^2*Ei(d*x) - (a*d^2 + 2*b)*x^2*Ei(-d*x))*\sinh(c))/x^2$

giac [A] time = 0.12, size = 109, normalized size = 1.47

$$\frac{ad^2x^2Ei(-dx)e^{(-c)} + ad^2x^2Ei(dx)e^c + 2bx^2Ei(-dx)e^{(-c)} + 2bx^2Ei(dx)e^c - adxe^{(dx+c)} + adxe^{(-dx-c)} - ae^{(dx+c)}}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)*cosh(d*x+c)/x^3,x, algorithm="giac")

[Out] $1/4*(a*d^2*x^2*Ei(-d*x)*e^{(-c)} + a*d^2*x^2*Ei(d*x)*e^c + 2*b*x^2*Ei(-d*x)*e^{(-c)} + 2*b*x^2*Ei(d*x)*e^c - a*d*x*e^{(d*x + c)} + a*d*x*e^{(-d*x - c)} - a*e^{(d*x + c)} - a*e^{(-d*x - c)})/x^2$

maple [A] time = 0.11, size = 110, normalized size = 1.49

$$\frac{da e^{-dx-c}}{4x} - \frac{a e^{-dx-c}}{4x^2} - \frac{d^2 a e^{-c} Ei(1, dx)}{4} - \frac{b e^{-c} Ei(1, dx)}{2} - \frac{a e^{dx+c}}{4x^2} - \frac{da e^{dx+c}}{4x} - \frac{d^2 a e^c Ei(1, -dx)}{4} - \frac{b e^c Ei(1, -dx)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^2+a)*cosh(d*x+c)/x^3,x)

[Out] $1/4*d*a*\exp(-d*x-c)/x - 1/4*a*\exp(-d*x-c)/x^2 - 1/4*d^2*a*\exp(-c)*Ei(1, d*x) - 1/2*b*\exp(-c)*Ei(1, d*x) - 1/4*a/x^2*\exp(d*x+c) - 1/4*d*a/x*\exp(d*x+c) - 1/4*d^2*a*\exp(c)*Ei(1, -d*x) - 1/2*b*\exp(c)*Ei(1, -d*x)$

maxima [A] time = 0.41, size = 90, normalized size = 1.22

$$\frac{1}{4} \left((de^{(-c)}\Gamma(-1, dx) + de^c\Gamma(-1, -dx))a - \frac{2b \cosh(dx + c) \log(x^2)}{d} + \frac{2(Ei(-dx)e^{(-c)} + Ei(dx)e^c)b}{d} \right) d + \frac{1}{2} (b \log$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)*cosh(d*x+c)/x^3,x, algorithm="maxima")

[Out] $1/4*((d*e^{(-c)}*\gamma(-1, d*x) + d*e^c*\gamma(-1, -d*x))*a - 2*b*\cosh(d*x + c)*\log(x^2)/d + 2*(Ei(-d*x)*e^{(-c)} + Ei(d*x)*e^c)*b/d)*d + 1/2*(b*\log(x^2) - a/x^2)*\cosh(d*x + c)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c + dx) (bx^2 + a)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cosh(c + d*x)*(a + b*x^2))/x^3,x)

[Out] int((cosh(c + d*x)*(a + b*x^2))/x^3, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^2) \cosh(c + dx)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**2+a)*cosh(d*x+c)/x**3,x)

[Out] Integral((a + b*x**2)*cosh(c + d*x)/x**3, x)

$$3.47 \quad \int \frac{(a+bx^2) \cosh(c+dx)}{x^4} dx$$

Optimal. Leaf size=105

$$\frac{1}{6}ad^3 \sinh(c)\text{Chi}(dx) + \frac{1}{6}ad^3 \cosh(c)\text{Shi}(dx) - \frac{ad^2 \cosh(c+dx)}{6x} - \frac{a \cosh(c+dx)}{3x^3} - \frac{ad \sinh(c+dx)}{6x^2} + bd \sinh(c)\text{Chi}(dx)$$

[Out] $-1/3*a*\cosh(d*x+c)/x^3 - b*\cosh(d*x+c)/x - 1/6*a*d^2*\cosh(d*x+c)/x + b*d*\cosh(c)*\text{Shi}(d*x) + 1/6*a*d^3*\cosh(c)*\text{Shi}(d*x) + b*d*\text{Chi}(d*x)*\sinh(c) + 1/6*a*d^3*\text{Chi}(d*x)*\sinh(c) - 1/6*a*d*\sinh(d*x+c)/x^2$

Rubi [A] time = 0.23, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {5287, 3297, 3303, 3298, 3301}

$$\frac{1}{6}ad^3 \sinh(c)\text{Chi}(dx) + \frac{1}{6}ad^3 \cosh(c)\text{Shi}(dx) - \frac{ad^2 \cosh(c+dx)}{6x} - \frac{ad \sinh(c+dx)}{6x^2} - \frac{a \cosh(c+dx)}{3x^3} + bd \sinh(c)\text{Chi}(dx)$$

Antiderivative was successfully verified.

[In] Int[((a + b*x^2)*Cosh[c + d*x])/x^4, x]

[Out] $-(a*\text{Cosh}[c + d*x])/(3*x^3) - (b*\text{Cosh}[c + d*x])/x - (a*d^2*\text{Cosh}[c + d*x])/(6*x) + b*d*\text{CoshIntegral}[d*x]*\text{Sinh}[c] + (a*d^3*\text{CoshIntegral}[d*x]*\text{Sinh}[c])/6 - (a*d*\text{Sinh}[c + d*x])/(6*x^2) + b*d*\text{Cosh}[c]*\text{SinhIntegral}[d*x] + (a*d^3*\text{Cosh}[c]*\text{SinhIntegral}[d*x])/6$

Rule 3297

Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]

Rule 3298

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)])/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]

Rule 3301

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)])/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

Rule 5287

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] := Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + bx^2) \cosh(c + dx)}{x^4} dx &= \int \left(\frac{a \cosh(c + dx)}{x^4} + \frac{b \cosh(c + dx)}{x^2} \right) dx \\
&= a \int \frac{\cosh(c + dx)}{x^4} dx + b \int \frac{\cosh(c + dx)}{x^2} dx \\
&= -\frac{a \cosh(c + dx)}{3x^3} - \frac{b \cosh(c + dx)}{x} + \frac{1}{3}(ad) \int \frac{\sinh(c + dx)}{x^3} dx + (bd) \int \frac{\sinh(c + dx)}{x} dx \\
&= -\frac{a \cosh(c + dx)}{3x^3} - \frac{b \cosh(c + dx)}{x} - \frac{ad \sinh(c + dx)}{6x^2} + \frac{1}{6}(ad^2) \int \frac{\cosh(c + dx)}{x^2} dx \\
&= -\frac{a \cosh(c + dx)}{3x^3} - \frac{b \cosh(c + dx)}{x} - \frac{ad^2 \cosh(c + dx)}{6x} + bd \operatorname{Chi}(dx) \sinh(c) - \frac{ad}{6} \operatorname{Shi}(dx) \\
&= -\frac{a \cosh(c + dx)}{3x^3} - \frac{b \cosh(c + dx)}{x} - \frac{ad^2 \cosh(c + dx)}{6x} + bd \operatorname{Chi}(dx) \sinh(c) - \frac{ad}{6} \operatorname{Shi}(dx) \\
&= -\frac{a \cosh(c + dx)}{3x^3} - \frac{b \cosh(c + dx)}{x} - \frac{ad^2 \cosh(c + dx)}{6x} + bd \operatorname{Chi}(dx) \sinh(c) + \frac{1}{6} ad \operatorname{Shi}(dx)
\end{aligned}$$

Mathematica [A] time = 0.24, size = 95, normalized size = 0.90

$$\frac{-dx^3 \sinh(c) (ad^2 + 6b) \operatorname{Chi}(dx) - dx^3 \cosh(c) (ad^2 + 6b) \operatorname{Shi}(dx) + ad^2 x^2 \cosh(c + dx) + adx \sinh(c + dx) + 2ad \operatorname{Shi}(dx)}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*Cosh[c + d*x])/x^4,x]
```

```
[Out] -1/6*(2*a*Cosh[c + d*x] + 6*b*x^2*Cosh[c + d*x] + a*d^2*x^2*Cosh[c + d*x] -
d*(6*b + a*d^2)*x^3*CoshIntegral[d*x]*Sinh[c] + a*d*x*Sinh[c + d*x] - d*(6
*b + a*d^2)*x^3*Cosh[c]*SinhIntegral[d*x])/x^3
```


fricas [A] time = 0.51, size = 127, normalized size = 1.21

$$\frac{2 adx \sinh(dx + c) + 2 \left((ad^2 + 6b)x^2 + 2a \right) \cosh(dx + c) - \left((ad^3 + 6bd)x^3 \operatorname{Ei}(dx) - (ad^3 + 6bd)x^3 \operatorname{Ei}(-dx) \right)}{12 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)*cosh(d*x+c)/x^4,x, algorithm="fricas")

[Out] $-1/12*(2*a*d*x*\sinh(d*x + c) + 2*((a*d^2 + 6*b)*x^2 + 2*a)*\cosh(d*x + c) - ((a*d^3 + 6*b*d)*x^3*\operatorname{Ei}(d*x) - (a*d^3 + 6*b*d)*x^3*\operatorname{Ei}(-d*x))*\cosh(c) - ((a*d^3 + 6*b*d)*x^3*\operatorname{Ei}(d*x) + (a*d^3 + 6*b*d)*x^3*\operatorname{Ei}(-d*x))*\sinh(c))/x^3$

giac [A] time = 0.12, size = 170, normalized size = 1.62

$$\frac{ad^3 x^3 \operatorname{Ei}(-dx) e^{-c} - ad^3 x^3 \operatorname{Ei}(dx) e^c + 6 b dx^3 \operatorname{Ei}(-dx) e^{-c} - 6 b dx^3 \operatorname{Ei}(dx) e^c + ad^2 x^2 e^{(dx+c)} + ad^2 x^2 e^{(-dx-c)} + a dx}{12 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)*cosh(d*x+c)/x^4,x, algorithm="giac")

[Out] $-1/12*(a*d^3*x^3*\operatorname{Ei}(-d*x)*e^{-c} - a*d^3*x^3*\operatorname{Ei}(d*x)*e^c + 6*b*d*x^3*\operatorname{Ei}(-d*x)*e^{-c} - 6*b*d*x^3*\operatorname{Ei}(d*x)*e^c + a*d^2*x^2*e^{(d*x + c)} + a*d^2*x^2*e^{(-d*x - c)} + a*d*x*e^{(d*x + c)} + 6*b*x^2*e^{(d*x + c)} - a*d*x*e^{(-d*x - c)} + 6*b*x^2*e^{(-d*x - c)} + 2*a*e^{(d*x + c)} + 2*a*e^{(-d*x - c)})/x^3$

maple [A] time = 0.12, size = 172, normalized size = 1.64

$$\frac{d^2 a e^{-dx-c}}{12x} + \frac{da e^{-dx-c}}{12x^2} - \frac{a e^{-dx-c}}{6x^3} + \frac{d^3 a e^{-c} \operatorname{Ei}(1, dx)}{12} - \frac{b e^{-dx-c}}{2x} + \frac{db e^{-c} \operatorname{Ei}(1, dx)}{2} - \frac{a e^{dx+c}}{6x^3} - \frac{da e^{dx+c}}{12x^2} - \frac{d^2 a e^{dx+c}}{12x} - \frac{d^3 a e^{dx+c}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^2+a)*cosh(d*x+c)/x^4,x)

[Out] $-1/12*d^2*a*\exp(-d*x-c)/x+1/12*d*a*\exp(-d*x-c)/x^2-1/6*a*\exp(-d*x-c)/x^3+1/12*d^3*a*\exp(-c)*\operatorname{Ei}(1,d*x)-1/2*b*\exp(-d*x-c)/x+1/2*d*b*\exp(-c)*\operatorname{Ei}(1,d*x)-1/6*a/x^3*\exp(d*x+c)-1/12*d*a/x^2*\exp(d*x+c)-1/12*d^2*a/x*\exp(d*x+c)-1/12*d^3*a*\exp(c)*\operatorname{Ei}(1,-d*x)-1/2*b/x*\exp(d*x+c)-1/2*d*b*\exp(c)*\operatorname{Ei}(1,-d*x)$

maxima [A] time = 0.41, size = 73, normalized size = 0.70

$$\frac{1}{6} \left(ad^2 e^{(-c)} \Gamma(-2, dx) - ad^2 e^c \Gamma(-2, -dx) - 3 b \operatorname{Ei}(-dx) e^{(-c)} + 3 b \operatorname{Ei}(dx) e^c \right) d - \frac{(3 bx^2 + a) \cosh(dx + c)}{3 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)*cosh(d*x+c)/x^4,x, algorithm="maxima")

[Out] $\frac{1}{6}(a*d^2*e^{-c}*\gamma(-2, d*x) - a*d^2*e^c*\gamma(-2, -d*x) - 3*b*Ei(-d*x)*e^{-c} + 3*b*Ei(d*x)*e^c)*d - \frac{1}{3}(3*b*x^2 + a)*\cosh(d*x + c)/x^3$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c + dx) (bx^2 + a)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cosh(c + d*x)*(a + b*x^2))/x^4,x)

[Out] int((cosh(c + d*x)*(a + b*x^2))/x^4, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^2) \cosh(c + dx)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**2+a)*cosh(d*x+c)/x**4,x)

[Out] Integral((a + b*x**2)*cosh(c + d*x)/x**4, x)

$$3.48 \quad \int \frac{(a+bx^2) \cosh(c+dx)}{x^5} dx$$

Optimal. Leaf size=149

$$\frac{1}{24}ad^4 \cosh(c)\text{Chi}(dx) + \frac{1}{24}ad^4 \sinh(c)\text{Shi}(dx) - \frac{ad^3 \sinh(c+dx)}{24x} - \frac{ad^2 \cosh(c+dx)}{24x^2} - \frac{a \cosh(c+dx)}{4x^4} - \frac{ad \sinh(c+dx)}{12x^3}$$

[Out] $1/2*b*d^2*\text{Chi}(d*x)*\cosh(c)+1/24*a*d^4*\text{Chi}(d*x)*\cosh(c)-1/4*a*\cosh(d*x+c)/x^4-1/2*b*\cosh(d*x+c)/x^2-1/24*a*d^2*\cosh(d*x+c)/x^2+1/2*b*d^2*\text{Shi}(d*x)*\sinh(c)+1/24*a*d^4*\text{Shi}(d*x)*\sinh(c)-1/12*a*d*\sinh(d*x+c)/x^3-1/2*b*d*\sinh(d*x+c)/x-1/24*a*d^3*\sinh(d*x+c)/x$

Rubi [A] time = 0.29, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {5287, 3297, 3303, 3298, 3301}

$$\frac{1}{24}ad^4 \cosh(c)\text{Chi}(dx) + \frac{1}{24}ad^4 \sinh(c)\text{Shi}(dx) - \frac{ad^2 \cosh(c+dx)}{24x^2} - \frac{ad^3 \sinh(c+dx)}{24x} - \frac{ad \sinh(c+dx)}{12x^3} - \frac{a \cosh(c+dx)}{4x^4}$$

Antiderivative was successfully verified.

[In] Int[((a + b*x^2)*Cosh[c + d*x])/x^5,x]

[Out] $-(a*\text{Cosh}[c + d*x])/(4*x^4) - (b*\text{Cosh}[c + d*x])/(2*x^2) - (a*d^2*\text{Cosh}[c + d*x])/(24*x^2) + (b*d^2*\text{Cosh}[c]*\text{CoshIntegral}[d*x])/2 + (a*d^4*\text{Cosh}[c]*\text{CoshIntegral}[d*x])/24 - (a*d*\text{Sinh}[c + d*x])/(12*x^3) - (b*d*\text{Sinh}[c + d*x])/(2*x) - (a*d^3*\text{Sinh}[c + d*x])/(24*x) + (b*d^2*\text{Sinh}[c]*\text{SinhIntegral}[d*x])/2 + (a*d^4*\text{Sinh}[c]*\text{SinhIntegral}[d*x])/24$

Rule 3297

Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]

Rule 3298

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)])/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]

Rule 3301

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)])/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}

}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]

Rule 3303

Int[sin[(e_.) + (f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x_Symbol] :> Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]

Rule 5287

Int[Cosh[(c_.) + (d_.)*(x_.)]*((e_.)*(x_.))^(m_.)*((a_.) + (b_.)*(x_.)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{(a + bx^2) \cosh(c + dx)}{x^5} dx &= \int \left(\frac{a \cosh(c + dx)}{x^5} + \frac{b \cosh(c + dx)}{x^3} \right) dx \\
 &= a \int \frac{\cosh(c + dx)}{x^5} dx + b \int \frac{\cosh(c + dx)}{x^3} dx \\
 &= -\frac{a \cosh(c + dx)}{4x^4} - \frac{b \cosh(c + dx)}{2x^2} + \frac{1}{4}(ad) \int \frac{\sinh(c + dx)}{x^4} dx + \frac{1}{2}(bd) \int \frac{\sinh(c + dx)}{x^3} dx \\
 &= -\frac{a \cosh(c + dx)}{4x^4} - \frac{b \cosh(c + dx)}{2x^2} - \frac{ad \sinh(c + dx)}{12x^3} - \frac{bd \sinh(c + dx)}{2x} + \frac{1}{12}(ad^2) \int \frac{\cosh(c + dx)}{x^3} dx \\
 &= -\frac{a \cosh(c + dx)}{4x^4} - \frac{b \cosh(c + dx)}{2x^2} - \frac{ad^2 \cosh(c + dx)}{24x^2} - \frac{ad \sinh(c + dx)}{12x^3} - \frac{bd \sinh(c + dx)}{2x} \\
 &= -\frac{a \cosh(c + dx)}{4x^4} - \frac{b \cosh(c + dx)}{2x^2} - \frac{ad^2 \cosh(c + dx)}{24x^2} + \frac{1}{2}bd^2 \cosh(c) \text{Chi}(dx) - \frac{1}{2}bd^2 \sinh(c) \text{Shi}(dx) \\
 &= -\frac{a \cosh(c + dx)}{4x^4} - \frac{b \cosh(c + dx)}{2x^2} - \frac{ad^2 \cosh(c + dx)}{24x^2} + \frac{1}{2}bd^2 \cosh(c) \text{Chi}(dx) - \frac{1}{2}bd^2 \sinh(c) \text{Shi}(dx) \\
 &= -\frac{a \cosh(c + dx)}{4x^4} - \frac{b \cosh(c + dx)}{2x^2} - \frac{ad^2 \cosh(c + dx)}{24x^2} + \frac{1}{2}bd^2 \cosh(c) \text{Chi}(dx) + \frac{1}{2}bd^2 \sinh(c) \text{Shi}(dx)
 \end{aligned}$$

Mathematica [A] time = 0.29, size = 127, normalized size = 0.85

$$\frac{-d^2 x^4 \cosh(c) (ad^2 + 12b) \text{Chi}(dx) - d^2 x^4 \sinh(c) (ad^2 + 12b) \text{Shi}(dx) + ad^3 x^3 \sinh(c + dx) + ad^2 x^2 \cosh(c + dx)}{24x^4}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x^2)*Cosh[c + d*x])/x^5,x]

[Out]
$$-1/24*(6*a*Cosh[c + d*x] + 12*b*x^2*Cosh[c + d*x] + a*d^2*x^2*Cosh[c + d*x] - d^2*(12*b + a*d^2)*x^4*Cosh[c]*CoshIntegral[d*x] + 2*a*d*x*Sinh[c + d*x] + 12*b*d*x^3*Sinh[c + d*x] + a*d^3*x^3*Sinh[c + d*x] - d^2*(12*b + a*d^2)*x^4*Sinh[c]*SinhIntegral[d*x])/x^4$$

fricas [A] time = 0.48, size = 152, normalized size = 1.02

$$\frac{2\left(\left(ad^2 + 12b\right)x^2 + 6a\right)\cosh(dx + c) - \left(\left(ad^4 + 12bd^2\right)x^4\text{Ei}(dx) + \left(ad^4 + 12bd^2\right)x^4\text{Ei}(-dx)\right)\cosh(c) + 2\left(\left(ad^4 + 12bd^2\right)x^4\text{Ei}(dx) + \left(ad^4 + 12bd^2\right)x^4\text{Ei}(-dx)\right)\sinh(c)}{48x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)*cosh(d*x+c)/x^5,x, algorithm="fricas")

[Out]
$$-1/48*(2*((a*d^2 + 12*b)*x^2 + 6*a)*\cosh(d*x + c) - ((a*d^4 + 12*b*d^2)*x^4*\text{Ei}(d*x) + (a*d^4 + 12*b*d^2)*x^4*\text{Ei}(-d*x))*\cosh(c) + 2*((a*d^3 + 12*b*d)*x^3 + 2*a*d*x)*\sinh(d*x + c) - ((a*d^4 + 12*b*d^2)*x^4*\text{Ei}(d*x) - (a*d^4 + 12*b*d^2)*x^4*\text{Ei}(-d*x))*\sinh(c))/x^4$$

giac [A] time = 0.14, size = 237, normalized size = 1.59

$$\frac{ad^4x^4\text{Ei}(-dx)e^{(-c)} + ad^4x^4\text{Ei}(dx)e^c + 12bd^2x^4\text{Ei}(-dx)e^{(-c)} + 12bd^2x^4\text{Ei}(dx)e^c - ad^3x^3e^{(dx+c)} + ad^3x^3e^{(-dx-c)}}{48x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)*cosh(d*x+c)/x^5,x, algorithm="giac")

[Out]
$$1/48*(a*d^4*x^4*\text{Ei}(-d*x)*e^{(-c)} + a*d^4*x^4*\text{Ei}(d*x)*e^c + 12*b*d^2*x^4*\text{Ei}(-d*x)*e^{(-c)} + 12*b*d^2*x^4*\text{Ei}(d*x)*e^c - a*d^3*x^3*e^{(d*x + c)} + a*d^3*x^3*e^{(-d*x - c)} - a*d^2*x^2*e^{(d*x + c)} - 12*b*d*x^3*e^{(d*x + c)} - a*d^2*x^2*e^{(-d*x - c)} + 12*b*d*x^3*e^{(-d*x - c)} - 2*a*d*x*e^{(d*x + c)} - 12*b*x^2*e^{(d*x + c)} + 2*a*d*x*e^{(-d*x - c)} - 12*b*x^2*e^{(-d*x - c)} - 6*a*e^{(d*x + c)} - 6*a*e^{(-d*x - c)})/x^4$$

maple [A] time = 0.12, size = 238, normalized size = 1.60

$$\frac{db e^{-dx-c}}{4x} - \frac{b e^{-dx-c}}{4x^2} - \frac{d^2 b e^{-c} \text{Ei}(1, dx)}{4} + \frac{d^3 a e^{-dx-c}}{48x} - \frac{d^2 a e^{-dx-c}}{48x^2} + \frac{d a e^{-dx-c}}{24x^3} - \frac{a e^{-dx-c}}{8x^4} - \frac{d^4 a e^{-c} \text{Ei}(1, dx)}{48} - \frac{b e^{dx+c}}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^2+a)*cosh(d*x+c)/x^5,x)

[Out]
$$1/4*d*b*\exp(-d*x-c)/x - 1/4*b*\exp(-d*x-c)/x^2 - 1/4*d^2*b*\exp(-c)*\text{Ei}(1, d*x) + 1/4*8*d^3*a*\exp(-d*x-c)/x - 1/48*d^2*a*\exp(-d*x-c)/x^2 + 1/24*d*a*\exp(-d*x-c)/x^3 - 1/48*d^4*a*\exp(-c)*\text{Ei}(1, d*x) - b*\exp(dx+c)/x^2$$

$/8*a*\exp(-d*x-c)/x^4-1/48*d^4*a*\exp(-c)*\text{Ei}(1,d*x)-1/4*b/x^2*\exp(d*x+c)-1/4*d*b/x*\exp(d*x+c)-1/4*d^2*b*\exp(c)*\text{Ei}(1,-d*x)-1/8*a/x^4*\exp(d*x+c)-1/24*d*a/x^3*\exp(d*x+c)-1/48*d^2*a/x^2*\exp(d*x+c)-1/48*d^3*a/x*\exp(d*x+c)-1/48*d^4*a*\exp(c)*\text{Ei}(1,-d*x)$

maxima [A] time = 0.41, size = 76, normalized size = 0.51

$$\frac{1}{8} \left(ad^3 e^{(-c)} \Gamma(-3, dx) + ad^3 e^c \Gamma(-3, -dx) + 2 bde^{(-c)} \Gamma(-1, dx) + 2 bde^c \Gamma(-1, -dx) \right) d - \frac{(2bx^2 + a) \cosh(dx + c)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)*cosh(d*x+c)/x^5,x, algorithm="maxima")

[Out] $1/8*(a*d^3*e^{(-c)}*\text{gamma}(-3, d*x) + a*d^3*e^c*\text{gamma}(-3, -d*x) + 2*b*d*e^{(-c)}*\text{gamma}(-1, d*x) + 2*b*d*e^c*\text{gamma}(-1, -d*x))*d - 1/4*(2*b*x^2 + a)*\cosh(d*x + c)/x^4$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c + dx) (bx^2 + a)}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cosh(c + d*x)*(a + b*x^2))/x^5,x)

[Out] int((cosh(c + d*x)*(a + b*x^2))/x^5, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**2+a)*cosh(d*x+c)/x**5,x)

[Out] Timed out

3.49 $\int x^2 (a + bx^2)^2 \cosh(c + dx) dx$

Optimal. Leaf size=234

$$\frac{2a^2 \sinh(c + dx)}{d^3} - \frac{2a^2 x \cosh(c + dx)}{d^2} + \frac{a^2 x^2 \sinh(c + dx)}{d} + \frac{48ab \sinh(c + dx)}{d^5} - \frac{48abx \cosh(c + dx)}{d^4} + \frac{24abx^2 \sinh(c + dx)}{d^3}$$

[Out] $-720*b^2*x*\cosh(d*x+c)/d^6-48*a*b*x*\cosh(d*x+c)/d^4-2*a^2*x*\cosh(d*x+c)/d^2-120*b^2*x^3*\cosh(d*x+c)/d^4-8*a*b*x^3*\cosh(d*x+c)/d^2-6*b^2*x^5*\cosh(d*x+c)/d^2+720*b^2*\sinh(d*x+c)/d^7+48*a*b*\sinh(d*x+c)/d^5+2*a^2*\sinh(d*x+c)/d^3+360*b^2*x^2*\sinh(d*x+c)/d^5+24*a*b*x^2*\sinh(d*x+c)/d^3+a^2*x^2*\sinh(d*x+c)/d+30*b^2*x^4*\sinh(d*x+c)/d^3+2*a*b*x^4*\sinh(d*x+c)/d+b^2*x^6*\sinh(d*x+c)/d$

Rubi [A] time = 0.39, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {5287, 3296, 2637}

$$\frac{2a^2 \sinh(c + dx)}{d^3} - \frac{2a^2 x \cosh(c + dx)}{d^2} + \frac{a^2 x^2 \sinh(c + dx)}{d} + \frac{24abx^2 \sinh(c + dx)}{d^3} - \frac{8abx^3 \cosh(c + dx)}{d^2} + \frac{48ab \sinh(c + dx)}{d^5}$$

Antiderivative was successfully verified.

[In] Int[x^2*(a + b*x^2)^2*Cosh[c + d*x], x]

[Out] $(-720*b^2*x*Cosh[c + d*x])/d^6 - (48*a*b*x*Cosh[c + d*x])/d^4 - (2*a^2*x*Cosh[c + d*x])/d^2 - (120*b^2*x^3*Cosh[c + d*x])/d^4 - (8*a*b*x^3*Cosh[c + d*x])/d^2 - (6*b^2*x^5*Cosh[c + d*x])/d^2 + (720*b^2*Sinh[c + d*x])/d^7 + (48*a*b*Sinh[c + d*x])/d^5 + (2*a^2*Sinh[c + d*x])/d^3 + (360*b^2*x^2*Sinh[c + d*x])/d^5 + (24*a*b*x^2*Sinh[c + d*x])/d^3 + (a^2*x^2*Sinh[c + d*x])/d + (30*b^2*x^4*Sinh[c + d*x])/d^3 + (2*a*b*x^4*Sinh[c + d*x])/d + (b^2*x^6*Sinh[c + d*x])/d$

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3296

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 5287

Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]

Rubi steps

$$\begin{aligned}
 \int x^2 (a + bx^2)^2 \cosh(c + dx) dx &= \int (a^2 x^2 \cosh(c + dx) + 2abx^4 \cosh(c + dx) + b^2 x^6 \cosh(c + dx)) dx \\
 &= a^2 \int x^2 \cosh(c + dx) dx + (2ab) \int x^4 \cosh(c + dx) dx + b^2 \int x^6 \cosh(c + dx) dx \\
 &= \frac{a^2 x^2 \sinh(c + dx)}{d} + \frac{2abx^4 \sinh(c + dx)}{d} + \frac{b^2 x^6 \sinh(c + dx)}{d} - \frac{(2a^2) \int x \sinh(c + dx) dx}{d} \\
 &= -\frac{2a^2 x \cosh(c + dx)}{d^2} - \frac{8abx^3 \cosh(c + dx)}{d^2} - \frac{6b^2 x^5 \cosh(c + dx)}{d^2} + \frac{a^2 x^2 \sinh(c + dx)}{d} \\
 &= -\frac{2a^2 x \cosh(c + dx)}{d^2} - \frac{8abx^3 \cosh(c + dx)}{d^2} - \frac{6b^2 x^5 \cosh(c + dx)}{d^2} + \frac{2a^2 \sinh(c + dx)}{d^3} \\
 &= -\frac{48abx \cosh(c + dx)}{d^4} - \frac{2a^2 x \cosh(c + dx)}{d^2} - \frac{120b^2 x^3 \cosh(c + dx)}{d^4} - \frac{8abx^3 \cosh(c + dx)}{d^4} \\
 &= -\frac{48abx \cosh(c + dx)}{d^4} - \frac{2a^2 x \cosh(c + dx)}{d^2} - \frac{120b^2 x^3 \cosh(c + dx)}{d^4} - \frac{8abx^3 \cosh(c + dx)}{d^4} \\
 &= -\frac{720b^2 x \cosh(c + dx)}{d^6} - \frac{48abx \cosh(c + dx)}{d^4} - \frac{2a^2 x \cosh(c + dx)}{d^2} - \frac{120b^2 x^3 \cosh(c + dx)}{d^4} \\
 &= -\frac{720b^2 x \cosh(c + dx)}{d^6} - \frac{48abx \cosh(c + dx)}{d^4} - \frac{2a^2 x \cosh(c + dx)}{d^2} - \frac{120b^2 x^3 \cosh(c + dx)}{d^4}
 \end{aligned}$$

Mathematica [A] time = 0.35, size = 138, normalized size = 0.59

$$\frac{(a^2 d^4 (d^2 x^2 + 2) + 2abd^2 (d^4 x^4 + 12d^2 x^2 + 24) + b^2 (d^6 x^6 + 30d^4 x^4 + 360d^2 x^2 + 720)) \sinh(c + dx) - 2dx (a^2 d^4 (d^2 x^2 + 2) + 2abd^2 (d^4 x^4 + 12d^2 x^2 + 24) + b^2 (d^6 x^6 + 30d^4 x^4 + 360d^2 x^2 + 720)) \cosh(c + dx)}{d^7}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*(a + b*x^2)^2*Cosh[c + d*x], x]

[Out] (-2*d*x*(a^2*d^4 + 4*a*b*d^2*(6 + d^2*x^2) + 3*b^2*(120 + 20*d^2*x^2 + d^4*x^4))*Cosh[c + d*x] + (a^2*d^4*(2 + d^2*x^2) + 2*a*b*d^2*(24 + 12*d^2*x^2 + d^4*x^4) + b^2*(720 + 360*d^2*x^2 + 30*d^4*x^4 + d^6*x^6))*Sinh[c + d*x])/d^7

fricas [A] time = 0.62, size = 155, normalized size = 0.66

$$\frac{2(3b^2d^5x^5 + 4(abd^5 + 15b^2d^3)x^3 + (a^2d^5 + 24abd^3 + 360b^2d)x) \cosh(dx + c) - (b^2d^6x^6 + 2a^2d^4 + 2(abd^6 + 15b^2d^4)x^4 + 48abd^2 + (a^2d^6 + 24a^2bd^4 + 360b^2d^2)x^2 + 720b^2) \sinh(dx + c)}{d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(b*x^2+a)^2*cosh(d*x+c), x, algorithm="fricas")

[Out] $-(2*(3*b^2*d^5*x^5 + 4*(a*b*d^5 + 15*b^2*d^3)*x^3 + (a^2*d^5 + 24*a*b*d^3 + 360*b^2*d)*x)*\cosh(d*x + c) - (b^2*d^6*x^6 + 2*a^2*d^4 + 2*(a*b*d^6 + 15*b^2*d^4)*x^4 + 48*a*b*d^2 + (a^2*d^6 + 24*a*b*d^4 + 360*b^2*d^2)*x^2 + 720*b^2)*\sinh(d*x + c))/d^7$

giac [A] time = 0.14, size = 304, normalized size = 1.30

$$\frac{(b^2d^6x^6 + 2abd^6x^4 - 6b^2d^5x^5 + a^2d^6x^2 - 8abd^5x^3 + 30b^2d^4x^4 - 2a^2d^5x + 24abd^4x^2 - 120b^2d^3x^3 + 2a^2d^4 - 4a^2d^5x + 48abd^3x^2 - 120b^2d^2x + 720b^2) \cosh(dx + c) - (b^2d^6x^6 + 2abd^6x^4 - 6b^2d^5x^5 + a^2d^6x^2 - 8abd^5x^3 + 30b^2d^4x^4 - 2a^2d^5x + 24abd^4x^2 - 120b^2d^3x^3 + 2a^2d^4 - 4a^2d^5x + 48abd^3x^2 - 120b^2d^2x + 720b^2) \sinh(dx + c)}{2d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(b*x^2+a)^2*cosh(d*x+c), x, algorithm="giac")

[Out] $1/2*(b^2*d^6*x^6 + 2*a*b*d^6*x^4 - 6*b^2*d^5*x^5 + a^2*d^6*x^2 - 8*a*b*d^5*x^3 + 30*b^2*d^4*x^4 - 2*a^2*d^5*x + 24*a*b*d^4*x^2 - 120*b^2*d^3*x^3 + 2*a^2*d^4 - 48*a*b*d^3*x + 360*b^2*d^2*x^2 + 48*a*b*d^2 - 720*b^2*d*x + 720*b^2)*e^(d*x + c)/d^7 - 1/2*(b^2*d^6*x^6 + 2*a*b*d^6*x^4 + 6*b^2*d^5*x^5 + a^2*d^6*x^2 + 8*a*b*d^5*x^3 + 30*b^2*d^4*x^4 + 2*a^2*d^5*x + 24*a*b*d^4*x^2 + 120*b^2*d^3*x^3 + 2*a^2*d^4 + 48*a*b*d^3*x + 360*b^2*d^2*x^2 + 48*a*b*d^2 + 720*b^2*d*x + 720*b^2)*e^(-d*x - c)/d^7$

maple [B] time = 0.05, size = 738, normalized size = 3.15

$$\frac{b^2((dx+c)^6 \sinh(dx+c) - 6(dx+c)^5 \cosh(dx+c) + 30(dx+c)^4 \sinh(dx+c) - 120(dx+c)^3 \cosh(dx+c) + 360(dx+c)^2 \sinh(dx+c) - 720(dx+c) \cosh(dx+c) + 720) \cosh(dx+c) - (b^2((dx+c)^6 \sinh(dx+c) - 6(dx+c)^5 \cosh(dx+c) + 30(dx+c)^4 \sinh(dx+c) - 120(dx+c)^3 \cosh(dx+c) + 360(dx+c)^2 \sinh(dx+c) - 720(dx+c) \cosh(dx+c) + 720) \sinh(dx+c)}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(b*x^2+a)^2*cosh(d*x+c), x)

[Out] $1/d^3*(1/d^4*b^2*((d*x+c)^6*\sinh(d*x+c)-6*(d*x+c)^5*\cosh(d*x+c)+30*(d*x+c)^4*\sinh(d*x+c)-120*(d*x+c)^3*\cosh(d*x+c)+360*(d*x+c)^2*\sinh(d*x+c)-720*(d*x+c)*\cosh(d*x+c)+720*\sinh(d*x+c))+15/d^4*b^2*c^4*((d*x+c)^2*\sinh(d*x+c)-2*(d*x+c)*\cosh(d*x+c)+2*\sinh(d*x+c))+2/d^2*b*a*((d*x+c)^4*\sinh(d*x+c)-4*(d*x+c)^3*\cosh(d*x+c)+12*(d*x+c)^2*\sinh(d*x+c)-24*(d*x+c)*\cosh(d*x+c)+24*\sinh(d*x+c))-20/d^4*b^2*c^3*((d*x+c)^3*\sinh(d*x+c)-3*(d*x+c)^2*\cosh(d*x+c)+6*(d*x+c)*$

$$\begin{aligned} & \sinh(dx+c) - 6 \cosh(dx+c) - 6/d^4 b^2 c^* ((dx+c)^5 \sinh(dx+c) - 5(dx+c)^4 \cosh(dx+c) \\ & + 20(dx+c)^3 \sinh(dx+c) - 60(dx+c)^2 \cosh(dx+c) + 120(dx+c) \sinh(dx+c) - 120 \cosh(dx+c) \\ & + 15/d^4 b^2 c^2 * ((dx+c)^4 \sinh(dx+c) - 4(dx+c)^3 \cosh(dx+c) + 12(dx+c)^2 \sinh(dx+c) \\ & - 24(dx+c) \cosh(dx+c) + 24 \sinh(dx+c)) + 1/d^4 b^2 c^6 \sinh(dx+c) - 2a^2 c^* ((dx+c) \sinh(dx+c) - \cosh(dx+c)) \\ & + a^2 ((dx+c)^2 \sinh(dx+c) - 2(dx+c) \cosh(dx+c) + 2 \sinh(dx+c)) + 2/d^2 b^* c^4 a^* \sinh(dx+c) \\ & - 6/d^4 b^2 c^5 * ((dx+c) \sinh(dx+c) - \cosh(dx+c)) + a^2 c^2 \sinh(dx+c) - 8/d^2 b^* c^3 a^* ((dx+c) \sinh(dx+c) - \cosh(dx+c)) \\ & - 8/d^2 b^* a^* c^* ((dx+c)^3 \sinh(dx+c) - 3(dx+c)^2 \cosh(dx+c) + 6(dx+c) \sinh(dx+c) - 6 \cosh(dx+c)) \\ & + 12/d^2 b^* c^2 a^* ((dx+c)^2 \sinh(dx+c) - 2(dx+c) \cosh(dx+c) + 2 \sinh(dx+c)) \end{aligned}$$

maxima [A] time = 0.34, size = 383, normalized size = 1.64

$$-\frac{1}{210} d \left(\frac{35(d^3 x^3 e^c - 3d^2 x^2 e^c + 6dx e^c - 6e^c) a^2 e^{(dx)}}{d^4} + \frac{35(d^3 x^3 + 3d^2 x^2 + 6dx + 6) a^2 e^{(-dx-c)}}{d^4} + \frac{42(d^5 x^5 e^c - 5d^4 x^4 e^c + 20d^3 x^3 e^c - 60d^2 x^2 e^c + 120d x e^c - 120e^c) a^* b^* e^{(dx)}}{d^6} + \frac{42(d^5 x^5 + 5d^4 x^4 + 20d^3 x^3 + 60d^2 x^2 + 120d x + 120) a^* b^* e^{(-dx-c)}}{d^6} + \frac{15(d^7 x^7 e^c - 7d^6 x^6 e^c + 42d^5 x^5 e^c - 210d^4 x^4 e^c + 840d^3 x^3 e^c - 2520d^2 x^2 e^c + 5040d x e^c - 5040e^c) b^2 e^{(dx)}}{d^8} + \frac{15(d^7 x^7 + 7d^6 x^6 + 42d^5 x^5 + 210d^4 x^4 + 840d^3 x^3 + 2520d^2 x^2 + 5040d x + 5040) b^2 e^{(-dx-c)}}{d^8} + \frac{1}{105} (15b^2 x^7 + 42a^* b^* x^5 + 35a^2 x^3) \cosh(dx+c) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(b*x^2+a)^2*cosh(dx+c),x, algorithm="maxima")

[Out]
$$-1/210*d*(35*(d^3*x^3*e^c - 3*d^2*x^2*e^c + 6*d*x*e^c - 6*e^c)*a^2*e^{(dx)}/d^4 + 35*(d^3*x^3 + 3*d^2*x^2 + 6*d*x + 6)*a^2*e^{(-dx-c)}/d^4 + 42*(d^5*x^5*e^c - 5*d^4*x^4*e^c + 20*d^3*x^3*e^c - 60*d^2*x^2*e^c + 120*d*x*e^c - 120*e^c)*a*b*e^{(dx)}/d^6 + 42*(d^5*x^5 + 5*d^4*x^4 + 20*d^3*x^3 + 60*d^2*x^2 + 120*d*x + 120)*a*b*e^{(-dx-c)}/d^6 + 15*(d^7*x^7*e^c - 7*d^6*x^6*e^c + 42*d^5*x^5*e^c - 210*d^4*x^4*e^c + 840*d^3*x^3*e^c - 2520*d^2*x^2*e^c + 5040*d*x*e^c - 5040*e^c)*b^2*e^{(dx)}/d^8 + 15*(d^7*x^7 + 7*d^6*x^6 + 42*d^5*x^5 + 210*d^4*x^4 + 840*d^3*x^3 + 2520*d^2*x^2 + 5040*d*x + 5040)*b^2*e^{(-dx-c)}/d^8) + 1/105*(15*b^2*x^7 + 42*a*b*x^5 + 35*a^2*x^3)*cosh(dx+c)$$

mupad [B] time = 1.03, size = 182, normalized size = 0.78

$$\frac{2 \sinh(c+dx) (a^2 d^4 + 24 a b d^2 + 360 b^2)}{d^7} - \frac{6 b^2 x^5 \cosh(c+dx)}{d^2} + \frac{b^2 x^6 \sinh(c+dx)}{d} - \frac{2 x \cosh(c+dx) (a^2 d^4 + 24 a b d^2 + 360 b^2)}{d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*cosh(c+dx)*(a+b*x^2)^2,x)

[Out]
$$(2*\sinh(c+dx)*(360*b^2 + a^2*d^4 + 24*a*b*d^2))/d^7 - (6*b^2*x^5*\cosh(c+dx))/d^2 + (b^2*x^6*\sinh(c+dx))/d - (2*x*\cosh(c+dx)*(360*b^2 + a^2*d^4 + 24*a*b*d^2))/d^6 + (x^2*\sinh(c+dx)*(360*b^2 + a^2*d^4 + 24*a*b*d^2))/d^5 - (8*x^3*\cosh(c+dx)*(15*b^2 + a*b*d^2))/d^4 + (2*x^4*\sinh(c+dx)*(15*b^2 + a*b*d^2))/d^3$$

sympy [A] time = 6.02, size = 286, normalized size = 1.22

$$\left\{ \begin{array}{l} \frac{a^2 x^2 \sinh(c+dx)}{d} - \frac{2a^2 x \cosh(c+dx)}{d^2} + \frac{2a^2 \sinh(c+dx)}{d^3} + \frac{2abx^4 \sinh(c+dx)}{d} - \frac{8abx^3 \cosh(c+dx)}{d^2} + \frac{24abx^2 \sinh(c+dx)}{d^3} - \frac{48abx \cosh(c+dx)}{d^4} \\ \left(\frac{a^2 x^3}{3} + \frac{2abx^5}{5} + \frac{b^2 x^7}{7} \right) \cosh(c) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(b*x**2+a)**2*cosh(d*x+c),x)

[Out] Piecewise((a**2*x**2*sinh(c + d*x)/d - 2*a**2*x*cosh(c + d*x)/d**2 + 2*a**2*sinh(c + d*x)/d**3 + 2*a*b*x**4*sinh(c + d*x)/d - 8*a*b*x**3*cosh(c + d*x)/d**2 + 24*a*b*x**2*sinh(c + d*x)/d**3 - 48*a*b*x*cosh(c + d*x)/d**4 + 48*a*b*sinh(c + d*x)/d**5 + b**2*x**6*sinh(c + d*x)/d - 6*b**2*x**5*cosh(c + d*x)/d**2 + 30*b**2*x**4*sinh(c + d*x)/d**3 - 120*b**2*x**3*cosh(c + d*x)/d**4 + 360*b**2*x**2*sinh(c + d*x)/d**5 - 720*b**2*x*cosh(c + d*x)/d**6 + 720*b**2*sinh(c + d*x)/d**7, Ne(d, 0)), ((a**2*x**3/3 + 2*a*b*x**5/5 + b**2*x**7/7)*cosh(c), True))

3.50 $\int x (a + bx^2)^2 \cosh(c + dx) dx$

Optimal. Leaf size=184

$$-\frac{a^2 \cosh(c + dx)}{d^2} + \frac{a^2 x \sinh(c + dx)}{d} - \frac{12ab \cosh(c + dx)}{d^4} + \frac{12abx \sinh(c + dx)}{d^3} - \frac{6abx^2 \cosh(c + dx)}{d^2} + \frac{2abx^3 \sinh(c + dx)}{d}$$

[Out] $-120*b^2*\cosh(d*x+c)/d^6-12*a*b*\cosh(d*x+c)/d^4-a^2*\cosh(d*x+c)/d^2-60*b^2*x^2*\cosh(d*x+c)/d^4-6*a*b*x^2*\cosh(d*x+c)/d^2-5*b^2*x^4*\cosh(d*x+c)/d^2+120*b^2*x*\sinh(d*x+c)/d^5+12*a*b*x*\sinh(d*x+c)/d^3+a^2*x*\sinh(d*x+c)/d+20*b^2*x^3*\sinh(d*x+c)/d^3+2*a*b*x^3*\sinh(d*x+c)/d+b^2*x^5*\sinh(d*x+c)/d$

Rubi [A] time = 0.28, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {5287, 3296, 2638}

$$-\frac{a^2 \cosh(c + dx)}{d^2} + \frac{a^2 x \sinh(c + dx)}{d} - \frac{6abx^2 \cosh(c + dx)}{d^2} + \frac{12abx \sinh(c + dx)}{d^3} - \frac{12ab \cosh(c + dx)}{d^4} + \frac{2abx^3 \sinh(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] `Int[x*(a + b*x^2)^2*Cosh[c + d*x], x]`

[Out] $(-120*b^2*Cosh[c + d*x])/d^6 - (12*a*b*Cosh[c + d*x])/d^4 - (a^2*Cosh[c + d*x])/d^2 - (60*b^2*x^2*Cosh[c + d*x])/d^4 - (6*a*b*x^2*Cosh[c + d*x])/d^2 - (5*b^2*x^4*Cosh[c + d*x])/d^2 + (120*b^2*x*\sinh[c + d*x])/d^5 + (12*a*b*x*\sinh[c + d*x])/d^3 + (a^2*x*\sinh[c + d*x])/d + (20*b^2*x^3*\sinh[c + d*x])/d^3 + (2*a*b*x^3*\sinh[c + d*x])/d + (b^2*x^5*\sinh[c + d*x])/d$

Rule 2638

`Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]`

Rule 3296

`Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[((c + d*x)^m*cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]`

Rule 5287

`Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] := Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]`

Rubi steps

$$\begin{aligned}
\int x(a+bx^2)^2 \cosh(c+dx) dx &= \int (a^2x \cosh(c+dx) + 2abx^3 \cosh(c+dx) + b^2x^5 \cosh(c+dx)) dx \\
&= a^2 \int x \cosh(c+dx) dx + (2ab) \int x^3 \cosh(c+dx) dx + b^2 \int x^5 \cosh(c+dx) dx \\
&= \frac{a^2x \sinh(c+dx)}{d} + \frac{2abx^3 \sinh(c+dx)}{d} + \frac{b^2x^5 \sinh(c+dx)}{d} - \frac{a^2 \int \sinh(c+dx)}{d} \\
&= -\frac{a^2 \cosh(c+dx)}{d^2} - \frac{6abx^2 \cosh(c+dx)}{d^2} - \frac{5b^2x^4 \cosh(c+dx)}{d^2} + \frac{a^2x \sinh(c+dx)}{d} \\
&= -\frac{a^2 \cosh(c+dx)}{d^2} - \frac{6abx^2 \cosh(c+dx)}{d^2} - \frac{5b^2x^4 \cosh(c+dx)}{d^2} + \frac{12abx \sinh(c+dx)}{d^3} \\
&= -\frac{12ab \cosh(c+dx)}{d^4} - \frac{a^2 \cosh(c+dx)}{d^2} - \frac{60b^2x^2 \cosh(c+dx)}{d^4} - \frac{6abx^2 \cosh(c+dx)}{d^2} \\
&= -\frac{12ab \cosh(c+dx)}{d^4} - \frac{a^2 \cosh(c+dx)}{d^2} - \frac{60b^2x^2 \cosh(c+dx)}{d^4} - \frac{6abx^2 \cosh(c+dx)}{d^2} \\
&= -\frac{120b^2 \cosh(c+dx)}{d^6} - \frac{12ab \cosh(c+dx)}{d^4} - \frac{a^2 \cosh(c+dx)}{d^2} - \frac{60b^2x^2 \cosh(c+dx)}{d^4}
\end{aligned}$$

Mathematica [A] time = 0.25, size = 113, normalized size = 0.61

$$\frac{dx \left(a^2 d^4 + 2abd^2 (d^2 x^2 + 6) + b^2 (d^4 x^4 + 20d^2 x^2 + 120) \right) \sinh(c+dx) - \left(a^2 d^4 + 6abd^2 (d^2 x^2 + 2) + 5b^2 (d^4 x^4 + 20d^2 x^2 + 120) \right) \cosh(c+dx)}{d^6}$$

Antiderivative was successfully verified.

[In] Integrate[x*(a + b*x^2)^2*Cosh[c + d*x], x]

[Out] (-((a^2*d^4 + 6*a*b*d^2*(2 + d^2*x^2) + 5*b^2*(24 + 12*d^2*x^2 + d^4*x^4))*Cosh[c + d*x]) + d*x*(a^2*d^4 + 2*a*b*d^2*(6 + d^2*x^2) + b^2*(120 + 20*d^2*x^2 + d^4*x^4))*Sinh[c + d*x])/d^6

fricas [A] time = 0.61, size = 126, normalized size = 0.68

$$\frac{\left(5b^2d^4x^4 + a^2d^4 + 12abd^2 + 6(abd^4 + 10b^2d^2)x^2 + 120b^2 \right) \cosh(dx+c) - \left(b^2d^5x^5 + 2(abd^5 + 10b^2d^3)x^3 + 120b^2d \right) \sinh(dx+c)}{d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x^2+a)^2*cosh(d*x+c), x, algorithm="fricas")

[Out] $-\left(\left(5b^2d^4x^4 + a^2d^4 + 12a*b*d^2 + 6(a*b*d^4 + 10b^2*d^2)\right)x^2 + 120b^2\right)*\cosh(dx + c) - \left(b^2d^5x^5 + 2(a*b*d^5 + 10b^2*d^3)x^3 + (a^2d^5 + 12a*b*d^3 + 120b^2*d)x\right)*\sinh(dx + c)\right)/d^6$

giac [A] time = 0.14, size = 239, normalized size = 1.30

$$\frac{(b^2d^5x^5 + 2abd^5x^3 - 5b^2d^4x^4 + a^2d^5x - 6abd^4x^2 + 20b^2d^3x^3 - a^2d^4 + 12abd^3x - 60b^2d^2x^2 - 12abd^2 + 120b^2d)}{2d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(b*x^2+a)^2*cosh(dx+c),x, algorithm="giac")`

[Out] $1/2*(b^2d^5x^5 + 2a*b*d^5x^3 - 5b^2d^4x^4 + a^2d^5x - 6a*b*d^4x^2 + 20b^2d^3x^3 - a^2d^4 + 12a*b*d^3x - 60b^2d^2x^2 - 12a*b*d^2 + 120b^2d*x - 120b^2)*e^{(dx + c)}/d^6 - 1/2*(b^2d^5x^5 + 2a*b*d^5x^3 + 5b^2d^4x^4 + a^2d^5x + 6a*b*d^4x^2 + 20b^2d^3x^3 + a^2d^4 + 12a*b*d^3x + 60b^2d^2x^2 + 12a*b*d^2 + 120b^2d*x + 120b^2)*e^{-(dx - c)}/d^6$

maple [B] time = 0.04, size = 513, normalized size = 2.79

$$\frac{b^2((dx+c)^5 \sinh(dx+c) - 5(dx+c)^4 \cosh(dx+c) + 20(dx+c)^3 \sinh(dx+c) - 60(dx+c)^2 \cosh(dx+c) + 120(dx+c) \sinh(dx+c) - 120 \cosh(dx+c))}{d^4} - \frac{5b^2c((dx+c)^5 \sinh(dx+c) - 5(dx+c)^4 \cosh(dx+c) + 20(dx+c)^3 \sinh(dx+c) - 60(dx+c)^2 \cosh(dx+c) + 120(dx+c) \sinh(dx+c) - 120 \cosh(dx+c))}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(b*x^2+a)^2*cosh(dx+c),x)`

[Out] $1/d^2*(1/d^4*b^2*((dx+c)^5*\sinh(dx+c)-5*(dx+c)^4*\cosh(dx+c)+20*(dx+c)^3*\sinh(dx+c)-60*(dx+c)^2*\cosh(dx+c)+120*(dx+c)*\sinh(dx+c)-120*\cosh(dx+c))-5/d^4*b^2*c*((dx+c)^4*\sinh(dx+c)-4*(dx+c)^3*\cosh(dx+c)+12*(dx+c)^2*\sinh(dx+c)-24*(dx+c)*\cosh(dx+c)+24*\sinh(dx+c))+10/d^4*b^2*c^2*((dx+c)^3*\sinh(dx+c)-3*(dx+c)^2*\cosh(dx+c)+6*(dx+c)*\sinh(dx+c)-6*\cosh(dx+c))+2/d^2*b*a*((dx+c)^3*\sinh(dx+c)-3*(dx+c)^2*\cosh(dx+c)+6*(dx+c)*\sinh(dx+c)-6*\cosh(dx+c))-10/d^4*b^2*c^3*((dx+c)^2*\sinh(dx+c)-2*(dx+c)*\cosh(dx+c)+2*\sinh(dx+c))-6/d^2*b*c*a*((dx+c)^2*\sinh(dx+c)-2*(dx+c)*\cosh(dx+c)+2*\sinh(dx+c))+5/d^4*b^2*c^4*((dx+c)*\sinh(dx+c)-\cosh(dx+c))+6/d^2*b*c^2*a*((dx+c)*\sinh(dx+c)-\cosh(dx+c))+a^2*((dx+c)*\sinh(dx+c)-\cosh(dx+c))-1/d^4*b^2*c^5*\sinh(dx+c)-2/d^2*b*c^3*a*\sinh(dx+c)-c*a^2*\sinh(dx+c))$

maxima [A] time = 0.33, size = 353, normalized size = 1.92

$$\frac{(bx^2 + a)^3 \cosh(dx + c)}{6b} - \left(\frac{a^3 e^{(dx+c)}}{d} + \frac{a^3 e^{-(dx-c)}}{d} + \frac{3(d^2 x^2 e^c - 2 dx e^c + 2 e^c) a^2 b e^{(dx)}}{d^3} + \frac{3(d^2 x^2 + 2 dx + 2) a^2 b e^{-(dx-c)}}{d^3} + \frac{3(d^4 x^4 e^c - 4 d^3 x^3 e^c + 6 d^2 x^2 e^c - 4 d x e^c + 2 e^c) a^2 b e^{(dx)}}{d^5} + \frac{3(d^4 x^4 e^{-c} - 4 d^3 x^3 e^{-c} + 6 d^2 x^2 e^{-c} - 4 d x e^{-c} + 2 e^{-c}) a^2 b e^{-(dx-c)}}{d^5} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x^2+a)^2*cosh(d*x+c),x, algorithm="maxima")

[Out] $\frac{1}{6}(b^2x^5 + 2abx^3 + a^2x) \cosh(dx+c) - \frac{1}{12}(a^3e^{dx+c})/d + a^3e^{-(dx-c)}/d + 3(d^2x^2e^c - 2dxe^c + 2e^c)a^2b^2e^{dx}/d^3 + 3(d^2x^2 + 2dx + 2)a^2b^2e^{-(dx-c)}/d^3 + 3(d^4x^4e^c - 4d^3x^3e^c + 12d^2x^2e^c - 24dxe^c + 24e^c)a^2b^2e^{dx}/d^5 + 3(d^4x^4 + 4d^3x^3 + 12d^2x^2 + 24dx + 24)a^2b^2e^{-(dx-c)}/d^5 + (d^6x^6e^c - 6d^5x^5e^c + 30d^4x^4e^c - 120d^3x^3e^c + 360d^2x^2e^c - 720dxe^c + 720e^c)b^3e^{dx}/d^7 + (d^6x^6 + 6d^5x^5 + 30d^4x^4 + 120d^3x^3 + 360d^2x^2 + 720dx + 720)b^3e^{-(dx-c)}/d^7)/d/b$

mupad [B] time = 0.98, size = 148, normalized size = 0.80

$$\frac{b^2 x^5 \sinh(c + dx)}{d} - \frac{5 b^2 x^4 \cosh(c + dx)}{d^2} - \frac{\cosh(c + dx) (a^2 d^4 + 12 a b d^2 + 120 b^2)}{d^6} + \frac{x \sinh(c + dx) (a^2 d^4 + 12 a b d^2 + 120 b^2)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*cosh(c + d*x)*(a + b*x^2)^2,x)

[Out] $(b^2x^5 \sinh(c + dx))/d - (5b^2x^4 \cosh(c + dx))/d^2 - (\cosh(c + dx) * (120b^2 + a^2d^4 + 12a*b*d^2))/d^6 + (x \sinh(c + dx) * (120b^2 + a^2d^4 + 12a*b*d^2))/d^5 - (6x^2 \cosh(c + dx) * (10b^2 + a*b*d^2))/d^4 + (2x^3 \sinh(c + dx) * (10b^2 + a*b*d^2))/d^3$

sympy [A] time = 3.58, size = 226, normalized size = 1.23

$$\left\{ \begin{array}{l} \frac{a^2 x \sinh(c+dx)}{d} - \frac{a^2 \cosh(c+dx)}{d^2} + \frac{2abx^3 \sinh(c+dx)}{d} - \frac{6abx^2 \cosh(c+dx)}{d^2} + \frac{12abx \sinh(c+dx)}{d^3} - \frac{12ab \cosh(c+dx)}{d^4} + \frac{b^2 x^5 \sinh(c+dx)}{d} \\ \left(\frac{a^2 x^2}{2} + \frac{abx^4}{2} + \frac{b^2 x^6}{6} \right) \cosh(c) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x**2+a)**2*cosh(d*x+c),x)

[Out] Piecewise((a**2*x*sinh(c + d*x)/d - a**2*cosh(c + d*x)/d**2 + 2*a*b*x**3*sinh(c + d*x)/d - 6*a*b*x**2*cosh(c + d*x)/d**2 + 12*a*b*x*sinh(c + d*x)/d**3 - 12*a*b*cosh(c + d*x)/d**4 + b**2*x**5*sinh(c + d*x)/d - 5*b**2*x**4*cosh(c + d*x)/d**2 + 20*b**2*x**3*sinh(c + d*x)/d**3 - 60*b**2*x**2*cosh(c + d*x)/d**4 + 120*b**2*x*sinh(c + d*x)/d**5 - 120*b**2*cosh(c + d*x)/d**6, Ne(d, 0)), ((a**2*x**2/2 + a*b*x**4/2 + b**2*x**6/6)*cosh(c), True))

3.51 $\int (a + bx^2)^2 \cosh(c + dx) dx$

Optimal. Leaf size=136

$$\frac{a^2 \sinh(c + dx)}{d} + \frac{4ab \sinh(c + dx)}{d^3} - \frac{4abx \cosh(c + dx)}{d^2} + \frac{2abx^2 \sinh(c + dx)}{d} + \frac{24b^2 \sinh(c + dx)}{d^5} - \frac{24b^2 x \cosh(c + dx)}{d^4}$$

[Out] $-24*b^2*x*cosh(d*x+c)/d^4-4*a*b*x*cosh(d*x+c)/d^2-4*b^2*x^3*cosh(d*x+c)/d^2+24*b^2*sinh(d*x+c)/d^5+4*a*b*sinh(d*x+c)/d^3+a^2*sinh(d*x+c)/d+12*b^2*x^2*sinh(d*x+c)/d^3+2*a*b*x^2*sinh(d*x+c)/d+b^2*x^4*sinh(d*x+c)/d$

Rubi [A] time = 0.18, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {5277, 2637, 3296}

$$\frac{a^2 \sinh(c + dx)}{d} + \frac{4ab \sinh(c + dx)}{d^3} - \frac{4abx \cosh(c + dx)}{d^2} + \frac{2abx^2 \sinh(c + dx)}{d} + \frac{12b^2 x^2 \sinh(c + dx)}{d^3} - \frac{4b^2 x^3 \cosh(c + dx)}{d^2}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + b*x^2)^2*\text{Cosh}[c + d*x], x]$

[Out] $(-24*b^2*x*\text{Cosh}[c + d*x])/d^4 - (4*a*b*x*\text{Cosh}[c + d*x])/d^2 - (4*b^2*x^3*\text{Cosh}[c + d*x])/d^2 + (24*b^2*\text{Sinh}[c + d*x])/d^5 + (4*a*b*\text{Sinh}[c + d*x])/d^3 + (a^2*\text{Sinh}[c + d*x])/d + (12*b^2*x^2*\text{Sinh}[c + d*x])/d^3 + (2*a*b*x^2*\text{Sinh}[c + d*x])/d + (b^2*x^4*\text{Sinh}[c + d*x])/d$

Rule 2637

$\text{Int}[\sin[\text{Pi}/2 + (c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow \text{Simp}[\text{Sin}[c + d*x]/d, x] /;$
 $\text{FreeQ}[\{c, d\}, x]$

Rule 3296

$\text{Int}[((c_.) + (d_.)*(x_.))^{(m_.)}*\sin[(e_.) + (f_.)*(x_.)], x_Symbol] \rightarrow -\text{Simp}[(c + d*x)^m*\text{Cos}[e + f*x]/f, x] + \text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{(m-1)}*\text{Cos}[e + f*x], x], x] /;$
 $\text{FreeQ}[\{c, d, e, f\}, x] \ \&\& \ \text{GtQ}[m, 0]$

Rule 5277

$\text{Int}[\text{Cosh}[(c_.) + (d_.)*(x_.)]*((a_.) + (b_.)*(x_.)^{(n_.)})^{(p_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[\text{Cosh}[c + d*x], (a + b*x^n)^p, x], x] /;$
 $\text{FreeQ}[\{a, b, c, d, n\}, x] \ \&\& \ \text{IGtQ}[p, 0]$

Rubi steps

$$\begin{aligned}
\int (a + bx^2)^2 \cosh(c + dx) dx &= \int (a^2 \cosh(c + dx) + 2abx^2 \cosh(c + dx) + b^2x^4 \cosh(c + dx)) dx \\
&= a^2 \int \cosh(c + dx) dx + (2ab) \int x^2 \cosh(c + dx) dx + b^2 \int x^4 \cosh(c + dx) dx \\
&= \frac{a^2 \sinh(c + dx)}{d} + \frac{2abx^2 \sinh(c + dx)}{d} + \frac{b^2x^4 \sinh(c + dx)}{d} - \frac{(4ab) \int x \sinh(c + dx) dx}{d} \\
&= -\frac{4abx \cosh(c + dx)}{d^2} - \frac{4b^2x^3 \cosh(c + dx)}{d^2} + \frac{a^2 \sinh(c + dx)}{d} + \frac{2abx^2 \sinh(c + dx)}{d} \\
&= -\frac{4abx \cosh(c + dx)}{d^2} - \frac{4b^2x^3 \cosh(c + dx)}{d^2} + \frac{4ab \sinh(c + dx)}{d^3} + \frac{a^2 \sinh(c + dx)}{d} \\
&= -\frac{24b^2x \cosh(c + dx)}{d^4} - \frac{4abx \cosh(c + dx)}{d^2} - \frac{4b^2x^3 \cosh(c + dx)}{d^2} + \frac{4ab \sinh(c + dx)}{d^3} \\
&= -\frac{24b^2x \cosh(c + dx)}{d^4} - \frac{4abx \cosh(c + dx)}{d^2} - \frac{4b^2x^3 \cosh(c + dx)}{d^2} + \frac{24b^2 \sinh(c + dx)}{d^5}
\end{aligned}$$

Mathematica [A] time = 0.20, size = 85, normalized size = 0.62

$$\frac{(a^2d^4 + 2abd^2(d^2x^2 + 2) + b^2(d^4x^4 + 12d^2x^2 + 24)) \sinh(c + dx) - 4bdx(ad^2 + b(d^2x^2 + 6)) \cosh(c + dx)}{d^5}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^2)^2*Cosh[c + d*x], x]

[Out] (-4*b*d*x*(a*d^2 + b*(6 + d^2*x^2))*Cosh[c + d*x] + (a^2*d^4 + 2*a*b*d^2*(2 + d^2*x^2) + b^2*(24 + 12*d^2*x^2 + d^4*x^4))*Sinh[c + d*x])/d^5

fricas [A] time = 0.44, size = 98, normalized size = 0.72

$$\frac{4(b^2d^3x^3 + (abd^3 + 6b^2d)x) \cosh(dx + c) - (b^2d^4x^4 + a^2d^4 + 4abd^2 + 2(abd^4 + 6b^2d^2)x^2 + 24b^2) \sinh(dx + c)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^2*cosh(d*x+c), x, algorithm="fricas")

[Out] -(4*(b^2*d^3*x^3 + (a*b*d^3 + 6*b^2*d)*x)*cosh(d*x + c) - (b^2*d^4*x^4 + a^2*d^4 + 4*a*b*d^2 + 2*(a*b*d^4 + 6*b^2*d^2)*x^2 + 24*b^2)*sinh(d*x + c))/d^5

giac [A] time = 0.14, size = 180, normalized size = 1.32

$$\frac{(b^2d^4x^4 + 2abd^4x^2 - 4b^2d^3x^3 + a^2d^4 - 4abd^3x + 12b^2d^2x^2 + 4abd^2 - 24b^2dx + 24b^2)e^{(dx+c)} (b^2d^4x^4 + 2abd^4x^2 - 4b^2d^3x^3 + a^2d^4 - 4abd^3x + 12b^2d^2x^2 + 4abd^2 - 24b^2dx + 24b^2)}{2d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^2*cosh(d*x+c),x, algorithm="giac")

[Out] $\frac{1}{2}*(b^2*d^4*x^4 + 2*a*b*d^4*x^2 - 4*b^2*d^3*x^3 + a^2*d^4 - 4*a*b*d^3*x + 12*b^2*d^2*x^2 + 4*a*b*d^2 - 24*b^2*d*x + 24*b^2)*e^{(d*x + c)}/d^5 - \frac{1}{2}*(b^2*d^4*x^4 + 2*a*b*d^4*x^2 + 4*b^2*d^3*x^3 + a^2*d^4 + 4*a*b*d^3*x + 12*b^2*d^2*x^2 + 4*a*b*d^2 + 24*b^2*d*x + 24*b^2)*e^{(-d*x - c)}/d^5$

maple [B] time = 0.05, size = 332, normalized size = 2.44

$$\frac{b^2((dx+c)^4 \sinh(dx+c) - 4(dx+c)^3 \cosh(dx+c) + 12(dx+c)^2 \sinh(dx+c) - 24(dx+c) \cosh(dx+c) + 24 \sinh(dx+c))}{d^4} - \frac{4b^2c((dx+c)^3 \sinh(dx+c) - 3(dx+c)^2 \cosh(dx+c) + 6(dx+c) \sinh(dx+c) - 6 \cosh(dx+c) + 6/d^4 * b^2 * c^2 * ((dx+c)^2 \sinh(dx+c) - 2(dx+c) \cosh(dx+c) + 2 \sinh(dx+c)) + 2/d^2 * b * a * ((dx+c)^2 \sinh(dx+c) - 2(dx+c) \cosh(dx+c) + 2 \sinh(dx+c)) - 4/d^4 * b^2 * c^3 * ((dx+c) \sinh(dx+c) - \cosh(dx+c)) - 4/d^2 * b * c * a * ((dx+c) \sinh(dx+c) - \cosh(dx+c)) + 1/d^4 * b^2 * c^4 \sinh(dx+c) + 2/d^2 * b * c^2 * a \sinh(dx+c) + a^2 \sinh(dx+c))}{2d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^2+a)^2*cosh(d*x+c),x)

[Out] $\frac{1}{d}*(\frac{1}{d^4}*b^2*((d*x+c)^4*\sinh(d*x+c)-4*(d*x+c)^3*\cosh(d*x+c)+12*(d*x+c)^2*\sinh(d*x+c)-24*(d*x+c)*\cosh(d*x+c)+24*\sinh(d*x+c))-4/d^4*b^2*c*((d*x+c)^3*\sinh(d*x+c)-3*(d*x+c)^2*\cosh(d*x+c)+6*(d*x+c)*\sinh(d*x+c)-6*\cosh(d*x+c))+6/d^4*b^2*c^2*((d*x+c)^2*\sinh(d*x+c)-2*(d*x+c)*\cosh(d*x+c)+2*\sinh(d*x+c))+2/d^2*b*a*((d*x+c)^2*\sinh(d*x+c)-2*(d*x+c)*\cosh(d*x+c)+2*\sinh(d*x+c))-4/d^4*b^2*c^3*((d*x+c)*\sinh(d*x+c)-\cosh(d*x+c))-4/d^2*b*c*a*((d*x+c)*\sinh(d*x+c)-\cosh(d*x+c))+1/d^4*b^2*c^4*\sinh(d*x+c)+2/d^2*b*c^2*a*\sinh(d*x+c)+a^2*\sinh(d*x+c))$

maxima [A] time = 0.34, size = 189, normalized size = 1.39

$$\frac{a^2 e^{(dx+c)}}{2d} - \frac{a^2 e^{(-dx-c)}}{2d} + \frac{(d^2 x^2 e^c - 2 dx e^c + 2 e^c) a b e^{(dx)}}{d^3} - \frac{(d^2 x^2 + 2 dx + 2) a b e^{(-dx-c)}}{d^3} + \frac{(d^4 x^4 e^c - 4 d^3 x^3 e^c + 12 d^2 x^2 e^c - 24 d x e^c + 24 e^c) b^2 e^{(dx)}}{2 d^5} - \frac{(d^4 x^4 e^c + 4 d^3 x^3 e^c + 12 d^2 x^2 e^c + 24 d x e^c + 24 e^c) b^2 e^{(-dx-c)}}{2 d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^2*cosh(d*x+c),x, algorithm="maxima")

[Out] $\frac{1}{2}*a^2*e^{(d*x + c)}/d - \frac{1}{2}*a^2*e^{(-d*x - c)}/d + (d^2*x^2*e^c - 2*d*x*e^c + 2*e^c)*a*b*e^{(d*x)}/d^3 - (d^2*x^2 + 2*d*x + 2)*a*b*e^{(-d*x - c)}/d^3 + \frac{1}{2}*(d^4*x^4*e^c - 4*d^3*x^3*e^c + 12*d^2*x^2*e^c - 24*d*x*e^c + 24*e^c)*b^2*e^{(d*x)}/d^5 - \frac{1}{2}*(d^4*x^4 + 4*d^3*x^3 + 12*d^2*x^2 + 24*d*x + 24)*b^2*e^{(-d*x - c)}/d^5$

mupad [B] time = 0.12, size = 114, normalized size = 0.84

$$\frac{\sinh(c + dx) (a^2 d^4 + 4 a b d^2 + 24 b^2)}{d^5} - \frac{4 b^2 x^3 \cosh(c + dx)}{d^2} + \frac{b^2 x^4 \sinh(c + dx)}{d} - \frac{4 x \cosh(c + dx) (6 b^2 + a b)}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)*(a + b*x^2)^2,x)
```

```
[Out] (sinh(c + d*x)*(24*b^2 + a^2*d^4 + 4*a*b*d^2))/d^5 - (4*b^2*x^3*cosh(c + d*x))/d^2 + (b^2*x^4*sinh(c + d*x))/d - (4*x*cosh(c + d*x)*(6*b^2 + a*b*d^2))/d^4 + (2*x^2*sinh(c + d*x)*(6*b^2 + a*b*d^2))/d^3
```

sympy [A] time = 2.10, size = 172, normalized size = 1.26

$$\left\{ \begin{array}{l} \frac{a^2 \sinh(c+dx)}{d} + \frac{2abx^2 \sinh(c+dx)}{d} - \frac{4abx \cosh(c+dx)}{d^2} + \frac{4ab \sinh(c+dx)}{d^3} + \frac{b^2 x^4 \sinh(c+dx)}{d} - \frac{4b^2 x^3 \cosh(c+dx)}{d^2} + \frac{12b^2 x^2 \sinh(c+dx)}{d^3} \\ \left(a^2 x + \frac{2abx^3}{3} + \frac{b^2 x^5}{5} \right) \cosh(c) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*cosh(d*x+c),x)
```

```
[Out] Piecewise((a**2*sinh(c + d*x)/d + 2*a*b*x**2*sinh(c + d*x)/d - 4*a*b*x*cosh(c + d*x)/d**2 + 4*a*b*sinh(c + d*x)/d**3 + b**2*x**4*sinh(c + d*x)/d - 4*b**2*x**3*cosh(c + d*x)/d**2 + 12*b**2*x**2*sinh(c + d*x)/d**3 - 24*b**2*x*cosh(c + d*x)/d**4 + 24*b**2*sinh(c + d*x)/d**5, Ne(d, 0)), ((a**2*x + 2*a*b*x**3/3 + b**2*x**5/5)*cosh(c), True))
```

$$3.52 \quad \int \frac{(a+bx^2)^2 \cosh(c+dx)}{x} dx$$

Optimal. Leaf size=110

$$a^2 \cosh(c)\text{Chi}(dx)+a^2 \sinh(c)\text{Shi}(dx)-\frac{2ab \cosh(c+dx)}{d^2}+\frac{2abx \sinh(c+dx)}{d}-\frac{6b^2 \cosh(c+dx)}{d^4}+\frac{6b^2x \sinh(c+dx)}{d^3}$$

[Out] $a^2 \text{Chi}(d*x) * \cosh(c) - 6*b^2 * \cosh(d*x+c) / d^4 - 2*a*b * \cosh(d*x+c) / d^2 - 3*b^2 * x^2 * \cosh(d*x+c) / d^2 + a^2 * \text{Shi}(d*x) * \sinh(c) + 6*b^2 * x * \sinh(d*x+c) / d^3 + 2*a*b * x * \sinh(d*x+c) / d + b^2 * x^3 * \sinh(d*x+c) / d$

Rubi [A] time = 0.19, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {5287, 3303, 3298, 3301, 3296, 2638}

$$a^2 \cosh(c)\text{Chi}(dx)+a^2 \sinh(c)\text{Shi}(dx)-\frac{2ab \cosh(c+dx)}{d^2}+\frac{2abx \sinh(c+dx)}{d}-\frac{3b^2x^2 \cosh(c+dx)}{d^2}+\frac{6b^2x \sinh(c+dx)}{d^3}$$

Antiderivative was successfully verified.

[In] Int[((a + b*x^2)^2*Cosh[c + d*x])/x,x]

[Out] $(-6*b^2 * \text{Cosh}[c + d*x]) / d^4 - (2*a*b * \text{Cosh}[c + d*x]) / d^2 - (3*b^2 * x^2 * \text{Cosh}[c + d*x]) / d^2 + a^2 * \text{Cosh}[c] * \text{CoshIntegral}[d*x] + (6*b^2 * x * \text{Sinh}[c + d*x]) / d^3 + (2*a*b * x * \text{Sinh}[c + d*x]) / d + (b^2 * x^3 * \text{Sinh}[c + d*x]) / d + a^2 * \text{Sinh}[c] * \text{SinhIntegral}[d*x]$

Rule 2638

Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3296

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[((c + d*x)^m * Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m-1) * Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 3298

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[(I * SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x]
&& EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[
Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

Rule 5287

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_.))^
(p_.), x_Symbol]
:> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p, x], x] /;
FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + bx^2)^2 \cosh(c + dx)}{x} dx &= \int \left(\frac{a^2 \cosh(c + dx)}{x} + 2abx \cosh(c + dx) + b^2 x^3 \cosh(c + dx) \right) dx \\
&= a^2 \int \frac{\cosh(c + dx)}{x} dx + (2ab) \int x \cosh(c + dx) dx + b^2 \int x^3 \cosh(c + dx) dx \\
&= \frac{2abx \sinh(c + dx)}{d} + \frac{b^2 x^3 \sinh(c + dx)}{d} - \frac{(2ab) \int \sinh(c + dx) dx}{d} - \frac{(3b^2) \int x^2 \cosh(c + dx) dx}{d} \\
&= -\frac{2ab \cosh(c + dx)}{d^2} - \frac{3b^2 x^2 \cosh(c + dx)}{d^2} + a^2 \cosh(c) \text{Chi}(dx) + \frac{2abx \sinh(c + dx)}{d} \\
&= -\frac{2ab \cosh(c + dx)}{d^2} - \frac{3b^2 x^2 \cosh(c + dx)}{d^2} + a^2 \cosh(c) \text{Chi}(dx) + \frac{6b^2 x \sinh(c + dx)}{d^3} \\
&= -\frac{6b^2 \cosh(c + dx)}{d^4} - \frac{2ab \cosh(c + dx)}{d^2} - \frac{3b^2 x^2 \cosh(c + dx)}{d^2} + a^2 \cosh(c) \text{Chi}(dx)
\end{aligned}$$

Mathematica [A] time = 0.39, size = 82, normalized size = 0.75

$$a^2 \cosh(c) \text{Chi}(dx) + a^2 \sinh(c) \text{Shi}(dx) - \frac{b(2ad^2 + 3b(d^2x^2 + 2)) \cosh(c + dx)}{d^4} + \frac{bx(2ad^2 + b(d^2x^2 + 6)) \sinh(c + dx)}{d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*Cosh[c + d*x])/x,x]
```

[Out] $-\left(\left(b*(2*a*d^2 + 3*b*(2 + d^2*x^2))*\text{Cosh}[c + d*x]\right)/d^4\right) + a^2*\text{Cosh}[c]*\text{CoshIntegral}[d*x] + \left(b*x*(2*a*d^2 + b*(6 + d^2*x^2))*\text{Sinh}[c + d*x]\right)/d^3 + a^2*\text{Sinh}[c]*\text{SinhIntegral}[d*x]$

fricas [A] time = 0.59, size = 130, normalized size = 1.18

$$\frac{2\left(3b^2d^2x^2 + 2abd^2 + 6b^2\right)\cosh(dx + c) - \left(a^2d^4\text{Ei}(dx) + a^2d^4\text{Ei}(-dx)\right)\cosh(c) - 2\left(b^2d^3x^3 + 2\left(abd^3 + 3b^2d\right)x\right)\sinh(dx + c) - \left(a^2d^4\text{Ei}(dx) - a^2d^4\text{Ei}(-dx)\right)\sinh(c)}{2d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2+a)^2*cosh(d*x+c)/x,x, algorithm="fricas")`

[Out] $-1/2*(2*(3*b^2*d^2*x^2 + 2*a*b*d^2 + 6*b^2)*\cosh(d*x + c) - (a^2*d^4*\text{Ei}(d*x) + a^2*d^4*\text{Ei}(-d*x))*\cosh(c) - 2*(b^2*d^3*x^3 + 2*(a*b*d^3 + 3*b^2*d)*x)*\sinh(d*x + c) - (a^2*d^4*\text{Ei}(d*x) - a^2*d^4*\text{Ei}(-d*x))*\sinh(c))/d^4$

giac [B] time = 0.13, size = 222, normalized size = 2.02

$$\frac{b^2d^3x^3e^{(dx+c)} - b^2d^3x^3e^{(-dx-c)} + a^2d^4\text{Ei}(-dx)e^{(-c)} + a^2d^4\text{Ei}(dx)e^c + 2abd^3xe^{(dx+c)} - 3b^2d^2x^2e^{(dx+c)} - 2abd^3xe^{(-dx-c)} - a^2d^4\text{Ei}(1, dx)}{2d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2+a)^2*cosh(d*x+c)/x,x, algorithm="giac")`

[Out] $1/2*(b^2*d^3*x^3*e^{(d*x + c)} - b^2*d^3*x^3*e^{(-d*x - c)} + a^2*d^4*\text{Ei}(-d*x)*e^{(-c)} + a^2*d^4*\text{Ei}(d*x)*e^c + 2*a*b*d^3*x*e^{(d*x + c)} - 3*b^2*d^2*x^2*e^{(d*x + c)} - 2*a*b*d^3*x*e^{(-d*x - c)} - 3*b^2*d^2*x^2*e^{(-d*x - c)} - 2*a*b*d^2*x*e^{(d*x + c)} + 6*b^2*d*x*e^{(d*x + c)} - 2*a*b*d^2*x*e^{(-d*x - c)} - 6*b^2*d*x*e^{(-d*x - c)} - 6*b^2*x*e^{(d*x + c)} - 6*b^2*x*e^{(-d*x - c)})/d^4$

maple [B] time = 0.15, size = 226, normalized size = 2.05

$$-\frac{3b^2e^{dx+c}x^2}{2d^2} + \frac{3b^2e^{dx+c}x}{d^3} + \frac{b^2e^{dx+c}x^3}{2d} - \frac{3b^2e^{-dx-c}x}{d^3} - \frac{b^2e^{-dx-c}x^3}{2d} - \frac{3b^2e^{-dx-c}x^2}{2d^2} - \frac{abe^{-dx-c}}{d^2} - \frac{abe^{dx+c}}{d^2} - \frac{a^2e^{-c}\text{Ei}(1, dx)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^2*cosh(d*x+c)/x,x)`

[Out] $-3/2/d^2*b^2*\exp(d*x+c)*x^2+3/d^3*b^2*\exp(d*x+c)*x+1/2/d*b^2*\exp(d*x+c)*x^3-3/d^3*b^2*\exp(-d*x-c)*x-1/2/d*b^2*\exp(-d*x-c)*x^3-3/2/d^2*b^2*\exp(-d*x-c)*x^2-a/d^2*b*\exp(-d*x-c)-a/d^2*b*\exp(d*x+c)-1/2*a^2*\exp(-c)*\text{Ei}(1, d*x)-1/2*a^2*\exp(c)*\text{Ei}(1, -d*x)-3/d^4*b^2*\exp(d*x+c)-3/d^4*b^2*\exp(-d*x-c)-a/d*b*\exp(-d*x-c)*x+a/d*b*\exp(d*x+c)*x$

maxima [B] time = 0.40, size = 235, normalized size = 2.14

$$-\frac{1}{8} \left(4ab \left(\frac{(d^2x^2e^c - 2dxe^c + 2e^c)e^{(dx)}}{d^3} + \frac{(d^2x^2 + 2dx + 2)e^{(-dx-c)}}{d^3} \right) + b^2 \left(\frac{(d^4x^4e^c - 4d^3x^3e^c + 12d^2x^2e^c - 24dxe^c + 24e^c)e^{(dx)}}{d^5} + \frac{(d^4x^4 + 4d^3x^3 + 12d^2x^2 + 24dx + 24)e^{(-dx-c)}}{d^5} + 4a^2 \cosh(dx+c) \log(x^2)/d - 4(\text{Ei}(-dx)e^{-c} + \text{Ei}(dx)e^c) a^2/d * d + 1/4(b^2x^4 + 4abx^2 + 2a^2 \log(x^2)) \cosh(dx+c) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^2*cosh(d*x+c)/x,x, algorithm="maxima")

[Out] -1/8*(4*a*b*((d^2*x^2*e^c - 2*d*x*e^c + 2*e^c)*e^(d*x)/d^3 + (d^2*x^2 + 2*d*x + 2)*e^(-d*x - c)/d^3) + b^2*((d^4*x^4*e^c - 4*d^3*x^3*e^c + 12*d^2*x^2*e^c - 24*d*x*e^c + 24*e^c)*e^(d*x)/d^5 + (d^4*x^4 + 4*d^3*x^3 + 12*d^2*x^2 + 24*d*x + 24)*e^(-d*x - c)/d^5) + 4*a^2*cosh(d*x + c)*log(x^2)/d - 4*(Ei(-d*x)*e^(-c) + Ei(d*x)*e^c)*a^2/d*d + 1/4*(b^2*x^4 + 4*a*b*x^2 + 2*a^2*log(x^2))*cosh(d*x + c)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c + dx) (bx^2 + a)^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cosh(c + d*x)*(a + b*x^2)^2)/x,x)

[Out] int((cosh(c + d*x)*(a + b*x^2)^2)/x, x)

sympy [A] time = 4.77, size = 121, normalized size = 1.10

$$a^2 \sinh(c) \text{Shi}(dx) + a^2 \cosh(c) \text{Chi}(dx) + 2ab \left(\begin{cases} \frac{x \sinh(c+dx)}{d} - \frac{\cosh(c+dx)}{d^2} & \text{for } d \neq 0 \\ \frac{x^2 \cosh(c)}{2} & \text{otherwise} \end{cases} \right) + b^2 \left(\begin{cases} \frac{x^3 \sinh(c+dx)}{d} - \frac{3x^2 \cosh(c+dx)}{d^2} & \text{for } d \neq 0 \\ \frac{x^4 \cosh(c)}{4} & \text{otherwise} \end{cases} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**2+a)**2*cosh(d*x+c)/x,x)

[Out] a**2*sinh(c)*Shi(d*x) + a**2*cosh(c)*Chi(d*x) + 2*a*b*Piecewise((x*sinh(c + d*x)/d - cosh(c + d*x)/d**2, Ne(d, 0)), (x**2*cosh(c)/2, True)) + b**2*Piecewise((x**3*sinh(c + d*x)/d - 3*x**2*cosh(c + d*x)/d**2 + 6*x*sinh(c + d*x)/d**3 - 6*cosh(c + d*x)/d**4, Ne(d, 0)), (x**4*cosh(c)/4, True))

$$3.53 \quad \int \frac{(a+bx^2)^2 \cosh(c+dx)}{x^2} dx$$

Optimal. Leaf size=95

$$a^2 d \sinh(c) \operatorname{Chi}(dx) + a^2 d \cosh(c) \operatorname{Shi}(dx) - \frac{a^2 \cosh(c+dx)}{x} + \frac{2ab \sinh(c+dx)}{d} + \frac{2b^2 \sinh(c+dx)}{d^3} - \frac{2b^2 x \cosh(c+dx)}{d^2}$$

[Out] $-a^2 \cosh(dx+c)/x - 2b^2 x \cosh(dx+c)/d^2 + a^2 d \cosh(c) \operatorname{Shi}(dx) + a^2 d \operatorname{Chi}(dx) \sinh(c) + 2b^2 \sinh(dx+c)/d^3 + 2ab \sinh(dx+c)/d + b^2 x^2 \sinh(dx+c)/d$

Rubi [A] time = 0.18, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {5287, 2637, 3297, 3303, 3298, 3301, 3296}

$$a^2 d \sinh(c) \operatorname{Chi}(dx) + a^2 d \cosh(c) \operatorname{Shi}(dx) - \frac{a^2 \cosh(c+dx)}{x} + \frac{2ab \sinh(c+dx)}{d} + \frac{2b^2 \sinh(c+dx)}{d^3} - \frac{2b^2 x \cosh(c+dx)}{d^2}$$

Antiderivative was successfully verified.

[In] `Int[((a + b*x^2)^2*Cosh[c + d*x])/x^2,x]`

[Out] `-((a^2*Cosh[c + d*x])/x) - (2*b^2*x*Cosh[c + d*x])/d^2 + a^2*d*CoshIntegral[d*x]*Sinh[c] + (2*b^2*Sinh[c + d*x])/d^3 + (2*a*b*Sinh[c + d*x])/d + (b^2*x^2*Sinh[c + d*x])/d + a^2*d*Cosh[c]*SinhIntegral[d*x]`

Rule 2637

`Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;`
`FreeQ[{c, d}, x]`

Rule 3296

`Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m-1)*Cos[e + f*x], x], x] /;`
`FreeQ[{c, d, e, f}, x] && GtQ[m, 0]`

Rule 3297

`Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((c + d*x)^(m+1)*Sin[e + f*x])/(d*(m+1)), x] - Dist[f/(d*(m+1)), Int[(c + d*x)^(m+1)*Cos[e + f*x], x], x] /;`
`FreeQ[{c, d, e, f}, x] && LtQ[m, -1]`

Rule 3298


```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x]
&& EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x]
&& EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*e - c*f)/d],
Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d],
Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

Rule 5287

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_.),
x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{(a + bx^2)^2 \cosh(c + dx)}{x^2} dx &= \int \left(2ab \cosh(c + dx) + \frac{a^2 \cosh(c + dx)}{x^2} + b^2 x^2 \cosh(c + dx) \right) dx \\ &= a^2 \int \frac{\cosh(c + dx)}{x^2} dx + (2ab) \int \cosh(c + dx) dx + b^2 \int x^2 \cosh(c + dx) dx \\ &= -\frac{a^2 \cosh(c + dx)}{x} + \frac{2ab \sinh(c + dx)}{d} + \frac{b^2 x^2 \sinh(c + dx)}{d} - \frac{(2b^2) \int x \sinh(c + dx) dx}{d} \\ &= -\frac{a^2 \cosh(c + dx)}{x} - \frac{2b^2 x \cosh(c + dx)}{d^2} + \frac{2ab \sinh(c + dx)}{d} + \frac{b^2 x^2 \sinh(c + dx)}{d} \\ &= -\frac{a^2 \cosh(c + dx)}{x} - \frac{2b^2 x \cosh(c + dx)}{d^2} + a^2 d \operatorname{Chi}(dx) \sinh(c) + \frac{2b^2 \sinh(c + dx)}{d^3} \end{aligned}$$

Mathematica [A] time = 0.28, size = 95, normalized size = 1.00

$$a^2 d \sinh(c) \operatorname{Chi}(dx) + a^2 d \cosh(c) \operatorname{Shi}(dx) - \frac{a^2 \cosh(c + dx)}{x} + \frac{2ab \sinh(c + dx)}{d} + \frac{2b^2 \sinh(c + dx)}{d^3} - \frac{2b^2 x \cosh(c + dx)}{d^2}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x^2)^2*Cosh[c + d*x])/x^2,x]

[Out] -((a^2*Cosh[c + d*x])/x) - (2*b^2*x*Cosh[c + d*x])/d^2 + a^2*d*CoshIntegral[d*x]*Sinh[c] + (2*b^2*Sinh[c + d*x])/d^3 + (2*a*b*Sinh[c + d*x])/d + (b^2*x^2*Sinh[c + d*x])/d + a^2*d*Cosh[c]*SinhIntegral[d*x]

fricas [A] time = 0.76, size = 127, normalized size = 1.34

$$\frac{2(a^2d^3 + 2b^2dx^2) \cosh(dx + c) - (a^2d^4xEi(dx) - a^2d^4xEi(-dx)) \cosh(c) - 2(b^2d^2x^3 + 2(abd^2 + b^2)x) \sinh(dx + c)}{2d^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^2*cosh(d*x+c)/x^2,x, algorithm="fricas")

[Out] -1/2*(2*(a^2*d^3 + 2*b^2*d*x^2)*cosh(d*x + c) - (a^2*d^4*x*Ei(d*x) - a^2*d^4*x*Ei(-d*x))*cosh(c) - 2*(b^2*d^2*x^3 + 2*(a*b*d^2 + b^2)*x)*sinh(d*x + c) - (a^2*d^4*x*Ei(d*x) + a^2*d^4*x*Ei(-d*x))*sinh(c))/(d^3*x)

giac [B] time = 0.13, size = 197, normalized size = 2.07

$$\frac{a^2d^4xEi(-dx)e^{(-c)} - a^2d^4xEi(dx)e^c - b^2d^2x^3e^{(dx+c)} + b^2d^2x^3e^{(-dx-c)} + a^2d^3e^{(dx+c)} - 2abd^2xe^{(dx+c)} + 2b^2dx^2e^{(dx+c)}}{2d^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^2*cosh(d*x+c)/x^2,x, algorithm="giac")

[Out] -1/2*(a^2*d^4*x*Ei(-d*x)*e^(-c) - a^2*d^4*x*Ei(d*x)*e^c - b^2*d^2*x^3*e^(d*x + c) + b^2*d^2*x^3*e^(-d*x - c) + a^2*d^3*e^(d*x + c) - 2*a*b*d^2*x*e^(d*x + c) + 2*b^2*d*x^2*e^(d*x + c) + a^2*d^3*e^(-d*x - c) + 2*a*b*d^2*x*e^(-d*x - c) + 2*b^2*d*x^2*e^(-d*x - c) - 2*b^2*x*e^(d*x + c) + 2*b^2*x*e^(-d*x - c))/(d^3*x)

maple [A] time = 0.16, size = 190, normalized size = 2.00

$$\frac{b^2e^{-dx-c}x}{d^2} - \frac{b^2e^{-dx-c}x^2}{2d} - \frac{abe^{-dx-c}}{d} - \frac{a^2e^{-dx-c}}{2x} + \frac{da^2e^{-c}Ei(1,dx)}{2} - \frac{b^2e^{-dx-c}}{d^3} + \frac{b^2e^{dx+c}x^2}{2d} - \frac{b^2e^{dx+c}x}{d^2} + \frac{abe^{dx+c}}{d} - \frac{a^2e^{dx+c}}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^2+a)^2*cosh(d*x+c)/x^2,x)

[Out] -1/d^2*b^2*exp(-d*x-c)*x-1/2/d*b^2*exp(-d*x-c)*x^2-a/d*b*exp(-d*x-c)-1/2*a^2*exp(-d*x-c)/x+1/2*d*a^2*exp(-c)*Ei(1,d*x)-1/d^3*b^2*exp(-d*x-c)+1/2/d*b^2

*exp(d*x+c)*x^2-1/d^2*b^2*exp(d*x+c)*x+a/d*b*exp(d*x+c)-1/2*a^2/x*exp(d*x+c)
)-1/2*d*a^2*exp(c)*Ei(1,-d*x)+1/d^3*b^2*exp(d*x+c)

maxima [A] time = 0.50, size = 179, normalized size = 1.88

$$-\frac{1}{6} \left(3 a^2 \operatorname{Ei}(-dx) e^{(-c)} - 3 a^2 \operatorname{Ei}(dx) e^c + \frac{6 (dx e^c - e^c) a b e^{(dx)}}{d^2} + \frac{6 (dx + 1) a b e^{(-dx-c)}}{d^2} + \frac{(d^3 x^3 e^c - 3 d^2 x^2 e^c + 6 dx e^c)}{d^4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^2*cosh(d*x+c)/x^2,x, algorithm="maxima")

[Out] -1/6*(3*a^2*Ei(-d*x)*e^(-c) - 3*a^2*Ei(d*x)*e^c + 6*(d*x*e^c - e^c)*a*b*e^(
d*x)/d^2 + 6*(d*x + 1)*a*b*e^(-d*x - c)/d^2 + (d^3*x^3*e^c - 3*d^2*x^2*e^c
+ 6*d*x*e^c - 6*e^c)*b^2*e^(d*x)/d^4 + (d^3*x^3 + 3*d^2*x^2 + 6*d*x + 6)*b^
2*e^(-d*x - c)/d^4)*d + 1/3*(b^2*x^3 + 6*a*b*x - 3*a^2/x)*cosh(d*x + c)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c + dx) (bx^2 + a)^2}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cosh(c + d*x)*(a + b*x^2)^2)/x^2,x)

[Out] int((cosh(c + d*x)*(a + b*x^2)^2)/x^2, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^2)^2 \cosh(c + dx)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**2+a)**2*cosh(d*x+c)/x**2,x)

[Out] Integral((a + b*x**2)**2*cosh(c + d*x)/x**2, x)

$$3.54 \quad \int \frac{(a+bx^2)^2 \cosh(c+dx)}{x^3} dx$$

Optimal. Leaf size=114

$$\frac{1}{2}a^2d^2 \cosh(c)\text{Chi}(dx) + \frac{1}{2}a^2d^2 \sinh(c)\text{Shi}(dx) - \frac{a^2 \cosh(c+dx)}{2x^2} - \frac{a^2d \sinh(c+dx)}{2x} + 2ab \cosh(c)\text{Chi}(dx) + 2ab \sinh(c)\text{Shi}(dx)$$

[Out] 2*a*b*Chi(d*x)*cosh(c)+1/2*a^2*d^2*Chi(d*x)*cosh(c)-b^2*cosh(d*x+c)/d^2-1/2*a^2*cosh(d*x+c)/x^2+2*a*b*Shi(d*x)*sinh(c)+1/2*a^2*d^2*Shi(d*x)*sinh(c)-1/2*a^2*d*sinh(d*x+c)/x+b^2*x*sinh(d*x+c)/d

Rubi [A] time = 0.23, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {5287, 3297, 3303, 3298, 3301, 3296, 2638}

$$\frac{1}{2}a^2d^2 \cosh(c)\text{Chi}(dx) + \frac{1}{2}a^2d^2 \sinh(c)\text{Shi}(dx) - \frac{a^2 \cosh(c+dx)}{2x^2} - \frac{a^2d \sinh(c+dx)}{2x} + 2ab \cosh(c)\text{Chi}(dx) + 2ab \sinh(c)\text{Shi}(dx)$$

Antiderivative was successfully verified.

[In] Int[((a + b*x^2)^2*Cosh[c + d*x])/x^3,x]

[Out] -((b^2*Cosh[c + d*x])/d^2) - (a^2*Cosh[c + d*x])/(2*x^2) + 2*a*b*Cosh[c]*CoshIntegral[d*x] + (a^2*d^2*Cosh[c]*CoshIntegral[d*x])/2 - (a^2*d*Sinh[c + d*x])/(2*x) + (b^2*x*Sinh[c + d*x])/d + 2*a*b*Sinh[c]*SinhIntegral[d*x] + (a^2*d^2*Sinh[c]*SinhIntegral[d*x])/2

Rule 2638

Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3296

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m-1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 3297

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((c + d*x)^(m+1)*Sin[e + f*x])/(d*(m+1)), x] - Dist[f/(d*(m+1)), Int[(c + d*x)^(m+1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f,
fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz},
x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)
]/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

Rule 5287

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + bx^2)^2 \cosh(c + dx)}{x^3} dx &= \int \left(\frac{a^2 \cosh(c + dx)}{x^3} + \frac{2ab \cosh(c + dx)}{x} + b^2 x \cosh(c + dx) \right) dx \\
&= a^2 \int \frac{\cosh(c + dx)}{x^3} dx + (2ab) \int \frac{\cosh(c + dx)}{x} dx + b^2 \int x \cosh(c + dx) dx \\
&= -\frac{a^2 \cosh(c + dx)}{2x^2} + \frac{b^2 x \sinh(c + dx)}{d} - \frac{b^2 \int \sinh(c + dx) dx}{d} + \frac{1}{2} (a^2 d) \int \frac{\sinh(c + dx)}{x} dx \\
&= -\frac{b^2 \cosh(c + dx)}{d^2} - \frac{a^2 \cosh(c + dx)}{2x^2} + 2ab \cosh(c) \text{Chi}(dx) - \frac{a^2 d \sinh(c + dx)}{2x} \\
&= -\frac{b^2 \cosh(c + dx)}{d^2} - \frac{a^2 \cosh(c + dx)}{2x^2} + 2ab \cosh(c) \text{Chi}(dx) - \frac{a^2 d \sinh(c + dx)}{2x} \\
&= -\frac{b^2 \cosh(c + dx)}{d^2} - \frac{a^2 \cosh(c + dx)}{2x^2} + 2ab \cosh(c) \text{Chi}(dx) + \frac{1}{2} a^2 d^2 \cosh(c) \text{Chi}(dx)
\end{aligned}$$

Mathematica [A] time = 0.40, size = 97, normalized size = 0.85

$$\frac{1}{2} \left(-\frac{a^2 \cosh(c + dx)}{x^2} - \frac{a^2 d \sinh(c + dx)}{x} + a \cosh(c) (ad^2 + 4b) \operatorname{Chi}(dx) + a \sinh(c) (ad^2 + 4b) \operatorname{Shi}(dx) - \frac{2b^2 \cos}{2} \right)$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x^2)^2*Cosh[c + d*x])/x^3,x]

[Out] ((-2*b^2*Cosh[c + d*x])/d^2 - (a^2*Cosh[c + d*x])/x^2 + a*(4*b + a*d^2)*Cos h[c]*CoshIntegral[d*x] - (a^2*d*Sinh[c + d*x])/x + (2*b^2*x*Sinh[c + d*x])/d + a*(4*b + a*d^2)*Sinh[c]*SinhIntegral[d*x])/2

fricas [A] time = 0.89, size = 164, normalized size = 1.44

$$\frac{2(a^2 d^2 + 2b^2 x^2) \cosh(dx + c) - ((a^2 d^4 + 4abd^2)x^2 \operatorname{Ei}(dx) + (a^2 d^4 + 4abd^2)x^2 \operatorname{Ei}(-dx)) \cosh(c) + 2(a^2 d^3 x - 2bd^2 x^2) \sinh(dx + c)}{4d^2 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^2*cosh(d*x+c)/x^3,x, algorithm="fricas")

[Out] -1/4*(2*(a^2*d^2 + 2*b^2*x^2)*cosh(d*x + c) - ((a^2*d^4 + 4*a*b*d^2)*x^2*Ei(d*x) + (a^2*d^4 + 4*a*b*d^2)*x^2*Ei(-d*x))*cosh(c) + 2*(a^2*d^3*x - 2*b^2*d*x^3)*sinh(d*x + c) - ((a^2*d^4 + 4*a*b*d^2)*x^2*Ei(d*x) - (a^2*d^4 + 4*a*b*d^2)*x^2*Ei(-d*x))*sinh(c))/(d^2*x^2)

giac [A] time = 0.13, size = 206, normalized size = 1.81

$$\frac{a^2 d^4 x^2 \operatorname{Ei}(-dx) e^{(-c)} + a^2 d^4 x^2 \operatorname{Ei}(dx) e^c + 4abd^2 x^2 \operatorname{Ei}(-dx) e^{(-c)} + 4abd^2 x^2 \operatorname{Ei}(dx) e^c - a^2 d^3 x e^{(dx+c)} + 2b^2 dx^3 e^{(dx+c)}}{4d^2 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^2*cosh(d*x+c)/x^3,x, algorithm="giac")

[Out] 1/4*(a^2*d^4*x^2*Ei(-d*x)*e^(-c) + a^2*d^4*x^2*Ei(d*x)*e^c + 4*a*b*d^2*x^2*Ei(-d*x)*e^(-c) + 4*a*b*d^2*x^2*Ei(d*x)*e^c - a^2*d^3*x*e^(d*x + c) + 2*b^2*d*x^3*e^(d*x + c) + a^2*d^3*x*e^(-d*x - c) - 2*b^2*d*x^3*e^(-d*x - c) - a^2*d^2*x^2*e^(d*x + c) - 2*b^2*x^2*e^(d*x + c) - a^2*d^2*x^2*e^(-d*x - c) - 2*b^2*x^2*e^(-d*x - c))/(d^2*x^2)

maple [A] time = 0.17, size = 188, normalized size = 1.65

$$-\frac{b^2 e^{-dx-c} x}{2d} - ab e^{-c} \operatorname{Ei}(1, dx) + \frac{d a^2 e^{-dx-c}}{4x} - \frac{a^2 e^{-dx-c}}{4x^2} - \frac{d^2 a^2 e^{-c} \operatorname{Ei}(1, dx)}{4} - \frac{b^2 e^{-dx-c}}{2d^2} + \frac{b^2 e^{dx+c} x}{2d} - \frac{d^2 a^2 e^c \operatorname{Ei}(1, -dx)}{4} - a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^2*cosh(d*x+c)/x^3,x)`

[Out] $-1/2/d*b^2*\exp(-d*x-c)*x-a*b*\exp(-c)*\text{Ei}(1,d*x)+1/4*d*a^2*\exp(-d*x-c)/x-1/4*a^2*\exp(-d*x-c)/x^2-1/4*d^2*a^2*\exp(-c)*\text{Ei}(1,d*x)-1/2/d^2*b^2*\exp(-d*x-c)+1/2/d*b^2*\exp(d*x+c)*x-1/4*d^2*a^2*\exp(c)*\text{Ei}(1,-d*x)-a*b*\exp(c)*\text{Ei}(1,-d*x)-1/4*a^2/x^2*\exp(d*x+c)-1/4*d*a^2/x*\exp(d*x+c)-1/2/d^2*b^2*\exp(d*x+c)$

maxima [A] time = 0.43, size = 165, normalized size = 1.45

$$\frac{1}{4} \left((de^{(-c)}\Gamma(-1, dx) + de^c\Gamma(-1, -dx))a^2 - b^2 \left(\frac{(d^2x^2e^c - 2dxe^c + 2e^c)e^{(dx)}}{d^3} + \frac{(d^2x^2 + 2dx + 2)e^{(-dx-c)}}{d^3} \right) - \frac{4ab \cosh(d*x+c)}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2+a)^2*cosh(d*x+c)/x^3,x, algorithm="maxima")`

[Out] $1/4*((d*e^{(-c)}*\text{gamma}(-1, d*x) + d*e^c*\text{gamma}(-1, -d*x))*a^2 - b^2*((d^2*x^2*e^c - 2*d*x*e^c + 2*e^c)*e^{(d*x)}/d^3 + (d^2*x^2 + 2*d*x + 2)*e^{(-d*x - c)}/d^3) - 4*a*b*\cosh(d*x + c)*\log(x^2)/d + 4*(\text{Ei}(-d*x)*e^{(-c)} + \text{Ei}(d*x)*e^c)*a*b/d)*d + 1/2*(b^2*x^2 + 2*a*b*\log(x^2) - a^2/x^2)*\cosh(d*x + c)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c + dx) (bx^2 + a)^2}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((cosh(c + d*x)*(a + b*x^2)^2)/x^3,x)`

[Out] `int((cosh(c + d*x)*(a + b*x^2)^2)/x^3, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^2)^2 \cosh(c + dx)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**2*cosh(d*x+c)/x**3,x)`

[Out] `Integral((a + b*x**2)**2*cosh(c + d*x)/x**3, x)`

$$3.55 \quad \int \frac{(a+bx^2)^2 \cosh(c+dx)}{x^4} dx$$

Optimal. Leaf size=133

$$\frac{1}{6}a^2d^3 \sinh(c)\text{Chi}(dx) + \frac{1}{6}a^2d^3 \cosh(c)\text{Shi}(dx) - \frac{a^2d^2 \cosh(c+dx)}{6x} - \frac{a^2 \cosh(c+dx)}{3x^3} - \frac{a^2d \sinh(c+dx)}{6x^2} + 2abd \sinh$$

[Out] $-1/3*a^2*\cosh(d*x+c)/x^3-2*a*b*\cosh(d*x+c)/x-1/6*a^2*d^2*\cosh(d*x+c)/x+2*a*b*d*\cosh(c)*\text{Shi}(d*x)+1/6*a^2*d^3*\cosh(c)*\text{Shi}(d*x)+2*a*b*d*\text{Chi}(d*x)*\sinh(c)+1/6*a^2*d^3*\text{Chi}(d*x)*\sinh(c)+b^2*\sinh(d*x+c)/d-1/6*a^2*d*\sinh(d*x+c)/x^2$

Rubi [A] time = 0.26, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {5287, 2637, 3297, 3303, 3298, 3301}

$$\frac{1}{6}a^2d^3 \sinh(c)\text{Chi}(dx) + \frac{1}{6}a^2d^3 \cosh(c)\text{Shi}(dx) - \frac{a^2d^2 \cosh(c+dx)}{6x} - \frac{a^2d \sinh(c+dx)}{6x^2} - \frac{a^2 \cosh(c+dx)}{3x^3} + 2abd \sinh$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + b*x^2)^2*\text{Cosh}[c + d*x])/x^4, x]$

[Out] $-(a^2*\text{Cosh}[c + d*x])/(3*x^3) - (2*a*b*\text{Cosh}[c + d*x])/x - (a^2*d^2*\text{Cosh}[c + d*x])/(6*x) + 2*a*b*d*\text{CoshIntegral}[d*x]*\text{Sinh}[c] + (a^2*d^3*\text{CoshIntegral}[d*x]*\text{Sinh}[c])/6 + (b^2*\text{Sinh}[c + d*x])/d - (a^2*d*\text{Sinh}[c + d*x])/(6*x^2) + 2*a*b*d*\text{Cosh}[c]*\text{SinhIntegral}[d*x] + (a^2*d^3*\text{Cosh}[c]*\text{SinhIntegral}[d*x])/6$

Rule 2637

$\text{Int}[\sin[\text{Pi}/2 + (c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow \text{Simp}[\sin[c + d*x]/d, x] /;$
FreeQ[{c, d}, x]

Rule 3297

$\text{Int}[(c_.) + (d_.)*(x_.)]^{(m_.)}*\sin[(e_.) + (f_.)*(x_.)], x_Symbol] \rightarrow \text{Simp}[(c + d*x)^{(m+1)}*\sin[e + f*x]/(d*(m+1)), x] - \text{Dist}[f/(d*(m+1)), \text{Int}[(c + d*x)^{(m+1)}*\cos[e + f*x], x], x] /;$ FreeQ[{c, d, e, f}, x] && LtQ[m, -1]

Rule 3298

$\text{Int}[\sin[(e_.) + (\text{Complex}[0, fz_])*(f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x_Symbol] \rightarrow \text{Simp}[(I*\text{SinhIntegral}[(c*f*fz)/d + f*fz*x])/d, x] /;$ FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x]
&& EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x]
+ Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /;
FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]
```

Rule 5287

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_))^(n_.)^(p_.),
x_Symbol]
:> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p, x], x] /;
FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + bx^2)^2 \cosh(c + dx)}{x^4} dx &= \int \left(b^2 \cosh(c + dx) + \frac{a^2 \cosh(c + dx)}{x^4} + \frac{2ab \cosh(c + dx)}{x^2} \right) dx \\
&= a^2 \int \frac{\cosh(c + dx)}{x^4} dx + (2ab) \int \frac{\cosh(c + dx)}{x^2} dx + b^2 \int \cosh(c + dx) dx \\
&= -\frac{a^2 \cosh(c + dx)}{3x^3} - \frac{2ab \cosh(c + dx)}{x} + \frac{b^2 \sinh(c + dx)}{d} + \frac{1}{3} (a^2 d) \int \frac{\sinh(c + dx)}{x^3} dx \\
&= -\frac{a^2 \cosh(c + dx)}{3x^3} - \frac{2ab \cosh(c + dx)}{x} + \frac{b^2 \sinh(c + dx)}{d} - \frac{a^2 d \sinh(c + dx)}{6x^2} + \frac{a^2 d^2 \cosh(c + dx)}{6x} + 2abd \operatorname{Chi}(dx) \sinh(c) \\
&= -\frac{a^2 \cosh(c + dx)}{3x^3} - \frac{2ab \cosh(c + dx)}{x} - \frac{a^2 d^2 \cosh(c + dx)}{6x} + 2abd \operatorname{Chi}(dx) \sinh(c) \\
&= -\frac{a^2 \cosh(c + dx)}{3x^3} - \frac{2ab \cosh(c + dx)}{x} - \frac{a^2 d^2 \cosh(c + dx)}{6x} + 2abd \operatorname{Chi}(dx) \sinh(c)
\end{aligned}$$

Mathematica [A] time = 0.44, size = 114, normalized size = 0.86

$$\frac{1}{6} \left(-\frac{a^2 d^2 \cosh(c + dx)}{x} - \frac{2a^2 \cosh(c + dx)}{x^3} - \frac{a^2 d \sinh(c + dx)}{x^2} + ad \sinh(c) (ad^2 + 12b) \operatorname{Chi}(dx) + ad \cosh(c) \right)$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x^2)^2*Cosh[c + d*x])/x^4,x]

[Out] ((-2*a^2*Cosh[c + d*x])/x^3 - (12*a*b*Cosh[c + d*x])/x - (a^2*d^2*Cosh[c + d*x])/x + a*d*(12*b + a*d^2)*CoshIntegral[d*x]*Sinh[c] + (6*b^2*Sinh[c + d*x])/d - (a^2*d*Sinh[c + d*x])/x^2 + a*d*(12*b + a*d^2)*Cosh[c]*SinhIntegral[d*x])/6

fricas [A] time = 0.48, size = 171, normalized size = 1.29

$$\frac{2(2a^2d + (a^2d^3 + 12abd)x^2) \cosh(dx + c) - ((a^2d^4 + 12abd^2)x^3 \operatorname{Ei}(dx) - (a^2d^4 + 12abd^2)x^3 \operatorname{Ei}(-dx)) \cosh(c)}{12dx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^2*cosh(d*x+c)/x^4,x, algorithm="fricas")

[Out] -1/12*(2*(2*a^2*d + (a^2*d^3 + 12*a*b*d)*x^2)*cosh(d*x + c) - ((a^2*d^4 + 12*a*b*d^2)*x^3*Ei(d*x) - (a^2*d^4 + 12*a*b*d^2)*x^3*Ei(-d*x))*cosh(c) + 2*(a^2*d^2*x - 6*b^2*x^3)*sinh(d*x + c) - ((a^2*d^4 + 12*a*b*d^2)*x^3*Ei(d*x) + (a^2*d^4 + 12*a*b*d^2)*x^3*Ei(-d*x))*sinh(c))/(d*x^3)

giac [A] time = 0.12, size = 236, normalized size = 1.77

$$\frac{a^2d^4x^3\operatorname{Ei}(-dx)e^{(-c)} - a^2d^4x^3\operatorname{Ei}(dx)e^c + 12abd^2x^3\operatorname{Ei}(-dx)e^{(-c)} - 12abd^2x^3\operatorname{Ei}(dx)e^c + a^2d^3x^2e^{(dx+c)} + a^2d^3x^2e^{(-c)}}{12dx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^2*cosh(d*x+c)/x^4,x, algorithm="giac")

[Out] -1/12*(a^2*d^4*x^3*Ei(-d*x)*e^(-c) - a^2*d^4*x^3*Ei(d*x)*e^c + 12*a*b*d^2*x^3*Ei(-d*x)*e^(-c) - 12*a*b*d^2*x^3*Ei(d*x)*e^c + a^2*d^3*x^2*e^(d*x + c) + a^2*d^3*x^2*e^(-d*x - c) + a^2*d^2*x*e^(d*x + c) + 12*a*b*d*x^2*e^(d*x + c) - 6*b^2*x^3*e^(d*x + c) - a^2*d^2*x*e^(-d*x - c) + 12*a*b*d*x^2*e^(-d*x - c) + 6*b^2*x^3*e^(-d*x - c) + 2*a^2*d*e^(d*x + c) + 2*a^2*d*e^(-d*x - c))/(d*x^3)

maple [A] time = 0.18, size = 222, normalized size = 1.67

$$\frac{d^3a^2e^{-c}\operatorname{Ei}(1,dx)}{12} - \frac{d^2a^2e^{-dx-c}}{12x} + \frac{da^2e^{-dx-c}}{12x^2} - \frac{a^2e^{-dx-c}}{6x^3} - \frac{b^2e^{-dx-c}}{2d} - \frac{abe^{-dx-c}}{x} + dab e^{-c}\operatorname{Ei}(1,dx) - \frac{a^2e^{dx+c}}{6x^3} - \frac{da^2e^{dx+c}}{12x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^2+a)^2*cosh(d*x+c)/x^4,x)

[Out] $1/12*d^3*a^2*\exp(-c)*\text{Ei}(1,d*x)-1/12*d^2*a^2*\exp(-d*x-c)/x+1/12*d*a^2*\exp(-d*x-c)/x^2-1/6*a^2*\exp(-d*x-c)/x^3-1/2/d*b^2*\exp(-d*x-c)-a*b*\exp(-d*x-c)/x+d*a*b*\exp(-c)*\text{Ei}(1,d*x)-1/6*a^2/x^3*\exp(d*x+c)-1/12*d*a^2/x^2*\exp(d*x+c)-1/12*d^3*a^2*\exp(c)*\text{Ei}(1,-d*x)-1/12*d^2*a^2/x*\exp(d*x+c)-a*b/x*\exp(d*x+c)-d*a*b*\exp(c)*\text{Ei}(1,-d*x)+1/2/d*b^2*\exp(d*x+c)$

maxima [A] time = 0.42, size = 135, normalized size = 1.02

$$\frac{1}{6} \left(a^2 d^2 e^{(-c)} \Gamma(-2, dx) - a^2 d^2 e^c \Gamma(-2, -dx) - 6 ab \text{Ei}(-dx) e^{(-c)} + 6 ab \text{Ei}(dx) e^c - \frac{3(dx e^c - e^c) b^2 e^{(dx)}}{d^2} - \frac{3(dx + 1)}{d^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2+a)^2*cosh(d*x+c)/x^4,x, algorithm="maxima")`

[Out] $1/6*(a^2*d^2*e^{(-c)}*\text{gamma}(-2, d*x) - a^2*d^2*e^c*\text{gamma}(-2, -d*x) - 6*a*b*\text{Ei}(-d*x)*e^{(-c)} + 6*a*b*\text{Ei}(d*x)*e^c - 3*(d*x*e^c - e^c)*b^2*e^{(d*x)}/d^2 - 3*(d*x + 1)*b^2*e^{(-d*x - c)}/d^2)*d + 1/3*(3*b^2*x - (6*a*b*x^2 + a^2)/x^3)*\text{cosh}(d*x + c)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c + dx) (bx^2 + a)^2}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((cosh(c + d*x)*(a + b*x^2)^2)/x^4,x)`

[Out] `int((cosh(c + d*x)*(a + b*x^2)^2)/x^4, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^2)^2 \cosh(c + dx)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**2*cosh(d*x+c)/x**4,x)`

[Out] `Integral((a + b*x**2)**2*cosh(c + d*x)/x**4, x)`

$$3.56 \quad \int \frac{(a+bx^2)^2 \cosh(c+dx)}{x^5} dx$$

Optimal. Leaf size=175

$$\frac{1}{24}a^2d^4 \cosh(c)\text{Chi}(dx) + \frac{1}{24}a^2d^4 \sinh(c)\text{Shi}(dx) - \frac{a^2d^3 \sinh(c+dx)}{24x} - \frac{a^2d^2 \cosh(c+dx)}{24x^2} - \frac{a^2 \cosh(c+dx)}{4x^4} - \frac{a^2d \sinh(c+dx)}{12x^3}$$

[Out] $b^2 \text{Chi}(d*x) \cosh(c) + a*b*d^2 \text{Chi}(d*x) \cosh(c) + 1/24*a^2*d^4 \text{Chi}(d*x) \cosh(c) - 1/4*a^2 \cosh(d*x+c)/x^4 - a*b \cosh(d*x+c)/x^2 - 1/24*a^2*d^2 \cosh(d*x+c)/x^2 + b^2 \text{Shi}(d*x) \sinh(c) + a*b*d^2 \text{Shi}(d*x) \sinh(c) + 1/24*a^2*d^4 \text{Shi}(d*x) \sinh(c) - 1/12*a^2*d \sinh(d*x+c)/x^3 - a*b*d \sinh(d*x+c)/x - 1/24*a^2*d^3 \sinh(d*x+c)/x$

Rubi [A] time = 0.36, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {5287, 3297, 3303, 3298, 3301}

$$\frac{1}{24}a^2d^4 \cosh(c)\text{Chi}(dx) + \frac{1}{24}a^2d^4 \sinh(c)\text{Shi}(dx) - \frac{a^2d^2 \cosh(c+dx)}{24x^2} - \frac{a^2d^3 \sinh(c+dx)}{24x} - \frac{a^2d \sinh(c+dx)}{12x^3} - \frac{a^2 \cosh(c+dx)}{4x^4}$$

Antiderivative was successfully verified.

[In] Int[((a + b*x^2)^2*Cosh[c + d*x])/x^5, x]

[Out] $-(a^2 \text{Cosh}[c + d*x])/(4*x^4) - (a*b \text{Cosh}[c + d*x])/x^2 - (a^2*d^2 \text{Cosh}[c + d*x])/(24*x^2) + b^2 \text{Cosh}[c] \text{CoshIntegral}[d*x] + a*b*d^2 \text{Cosh}[c] \text{CoshIntegral}[d*x] + (a^2*d^4 \text{Cosh}[c] \text{CoshIntegral}[d*x])/24 - (a^2*d \text{Sinh}[c + d*x])/(12*x^3) - (a*b*d \text{Sinh}[c + d*x])/x - (a^2*d^3 \text{Sinh}[c + d*x])/(24*x) + b^2 \text{Sinh}[c] \text{SinhIntegral}[d*x] + a*b*d^2 \text{Sinh}[c] \text{SinhIntegral}[d*x] + (a^2*d^4 \text{Sinh}[c] \text{SinhIntegral}[d*x])/24$

Rule 3297

Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]

Rule 3298

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)])/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x]
&& EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[
Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

Rule 5287

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_.),
x_Symbol]
:> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p, x], x] /;
FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + bx^2)^2 \cosh(c + dx)}{x^5} dx &= \int \left(\frac{a^2 \cosh(c + dx)}{x^5} + \frac{2ab \cosh(c + dx)}{x^3} + \frac{b^2 \cosh(c + dx)}{x} \right) dx \\
&= a^2 \int \frac{\cosh(c + dx)}{x^5} dx + (2ab) \int \frac{\cosh(c + dx)}{x^3} dx + b^2 \int \frac{\cosh(c + dx)}{x} dx \\
&= -\frac{a^2 \cosh(c + dx)}{4x^4} - \frac{ab \cosh(c + dx)}{x^2} + \frac{1}{4} (a^2 d) \int \frac{\sinh(c + dx)}{x^4} dx + (abd) \int \frac{\sinh(c + dx)}{x} dx \\
&= -\frac{a^2 \cosh(c + dx)}{4x^4} - \frac{ab \cosh(c + dx)}{x^2} + b^2 \cosh(c) \text{Chi}(dx) - \frac{a^2 d \sinh(c + dx)}{12x^3} \\
&= -\frac{a^2 \cosh(c + dx)}{4x^4} - \frac{ab \cosh(c + dx)}{x^2} - \frac{a^2 d^2 \cosh(c + dx)}{24x^2} + b^2 \cosh(c) \text{Chi}(dx) \\
&= -\frac{a^2 \cosh(c + dx)}{4x^4} - \frac{ab \cosh(c + dx)}{x^2} - \frac{a^2 d^2 \cosh(c + dx)}{24x^2} + b^2 \cosh(c) \text{Chi}(dx) \\
&= -\frac{a^2 \cosh(c + dx)}{4x^4} - \frac{ab \cosh(c + dx)}{x^2} - \frac{a^2 d^2 \cosh(c + dx)}{24x^2} + b^2 \cosh(c) \text{Chi}(dx) \\
&= -\frac{a^2 \cosh(c + dx)}{4x^4} - \frac{ab \cosh(c + dx)}{x^2} - \frac{a^2 d^2 \cosh(c + dx)}{24x^2} + b^2 \cosh(c) \text{Chi}(dx)
\end{aligned}$$

Mathematica [A] time = 0.49, size = 124, normalized size = 0.71

$$\frac{x^4 \cosh(c) (a^2 d^4 + 24abd^2 + 24b^2) \text{Chi}(dx) + x^4 \sinh(c) (a^2 d^4 + 24abd^2 + 24b^2) \text{Shi}(dx) - a (dx (ad^2 x^2 + 2a + 24x^4))}{24x^4}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x^2)^2*Cosh[c + d*x])/x^5,x]

[Out] ((24*b^2 + 24*a*b*d^2 + a^2*d^4)*x^4*Cosh[c]*CoshIntegral[d*x] - a*((6*a + 24*b*x^2 + a*d^2*x^2)*Cosh[c + d*x] + d*x*(2*a + 24*b*x^2 + a*d^2*x^2)*Sinh[c + d*x]) + (24*b^2 + 24*a*b*d^2 + a^2*d^4)*x^4*Sinh[c]*SinhIntegral[d*x])/(24*x^4)

fricas [A] time = 0.55, size = 194, normalized size = 1.11

$$\frac{2\left(\left(a^2d^2 + 24ab\right)x^2 + 6a^2\right)\cosh(dx + c) - \left(\left(a^2d^4 + 24abd^2 + 24b^2\right)x^4\text{Ei}(dx) + \left(a^2d^4 + 24abd^2 + 24b^2\right)x^4\text{Ei}(-dx)\right)}{24x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^2*cosh(d*x+c)/x^5,x, algorithm="fricas")

[Out] -1/48*(2*((a^2*d^2 + 24*a*b)*x^2 + 6*a^2)*cosh(d*x + c) - ((a^2*d^4 + 24*a*b*d^2 + 24*b^2)*x^4*Ei(d*x) + (a^2*d^4 + 24*a*b*d^2 + 24*b^2)*x^4*Ei(-d*x))*cosh(c) + 2*(2*a^2*d*x + (a^2*d^3 + 24*a*b*d)*x^3)*sinh(d*x + c) - ((a^2*d^4 + 24*a*b*d^2 + 24*b^2)*x^4*Ei(d*x) - (a^2*d^4 + 24*a*b*d^2 + 24*b^2)*x^4*Ei(-d*x))*sinh(c))/x^4

giac [A] time = 0.14, size = 294, normalized size = 1.68

$$\frac{a^2d^4x^4\text{Ei}(-dx)e^{(-c)} + a^2d^4x^4\text{Ei}(dx)e^c + 24abd^2x^4\text{Ei}(-dx)e^{(-c)} + 24abd^2x^4\text{Ei}(dx)e^c - a^2d^3x^3e^{(dx+c)} + a^2d^3x^3e^{(-dx-c)}}{48x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^2*cosh(d*x+c)/x^5,x, algorithm="giac")

[Out] 1/48*(a^2*d^4*x^4*Ei(-d*x)*e^(-c) + a^2*d^4*x^4*Ei(d*x)*e^c + 24*a*b*d^2*x^4*Ei(-d*x)*e^(-c) + 24*a*b*d^2*x^4*Ei(d*x)*e^c - a^2*d^3*x^3*e^(d*x + c) + a^2*d^3*x^3*e^(-d*x - c) + 24*b^2*x^4*Ei(-d*x)*e^(-c) + 24*b^2*x^4*Ei(d*x)*e^c - a^2*d^2*x^2*e^(d*x + c) - 24*a*b*d*x^3*e^(d*x + c) - a^2*d^2*x^2*e^(-d*x - c) + 24*a*b*d*x^3*e^(-d*x - c) - 2*a^2*d*x*e^(d*x + c) - 24*a*b*x^2*e^(d*x + c) + 2*a^2*d*x*e^(-d*x - c) - 24*a*b*x^2*e^(-d*x - c) - 6*a^2*e^(d*x + c) - 6*a^2*e^(-d*x - c))/x^4

maple [A] time = 0.20, size = 291, normalized size = 1.66

$$\frac{b^2e^{-c}\text{Ei}(1,dx)}{2} + \frac{d^3a^2e^{-dx-c}}{48x} - \frac{d^2a^2e^{-dx-c}}{48x^2} + \frac{da^2e^{-dx-c}}{24x^3} - \frac{a^2e^{-dx-c}}{8x^4} + \frac{dabe^{-dx-c}}{2x} - \frac{abe^{-dx-c}}{2x^2} - \frac{d^2abe^{-c}\text{Ei}(1,dx)}{2} - \frac{d^4a^2e^{-c}\text{Ei}(1,dx)}{24x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^2+a)^2*cosh(d*x+c)/x^5,x)

[Out] $-1/2*b^2*exp(-c)*Ei(1,d*x)+1/48*d^3*a^2*exp(-d*x-c)/x-1/48*d^2*a^2*exp(-d*x-c)/x^2+1/24*d*a^2*exp(-d*x-c)/x^3-1/8*a^2*exp(-d*x-c)/x^4+1/2*d*a*b*exp(-d*x-c)/x-1/2*a*b*exp(-d*x-c)/x^2-1/2*d^2*a*b*exp(-c)*Ei(1,d*x)-1/48*d^4*a^2*exp(-c)*Ei(1,d*x)-1/2*a*b/x^2*exp(d*x+c)-1/2*d*a*b/x*exp(d*x+c)-1/2*d^2*a*b*exp(c)*Ei(1,-d*x)-1/8*a^2/x^4*exp(d*x+c)-1/24*d*a^2/x^3*exp(d*x+c)-1/48*d^2*a^2/x^2*exp(d*x+c)-1/48*d^3*a^2/x*exp(d*x+c)-1/2*b^2*exp(c)*Ei(1,-d*x)-1/48*d^4*a^2*exp(c)*Ei(1,-d*x)$

maxima [A] time = 0.45, size = 139, normalized size = 0.79

$$\frac{1}{8} \left((d^3 e^{(-c)} \Gamma(-3, dx) + d^3 e^c \Gamma(-3, -dx)) a^2 + 4 (d e^{(-c)} \Gamma(-1, dx) + d e^c \Gamma(-1, -dx)) ab - \frac{4 b^2 \cosh(dx + c) \log(x^2)}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^2*cosh(d*x+c)/x^5,x, algorithm="maxima")

[Out] $1/8*((d^3*e^{(-c)}*\gamma(-3, d*x) + d^3*e^c*\gamma(-3, -d*x))*a^2 + 4*(d*e^{(-c)}*\gamma(-1, d*x) + d*e^c*\gamma(-1, -d*x))*a*b - 4*b^2*\cosh(d*x + c)*\log(x^2)/d + 4*(Ei(-d*x)*e^{(-c)} + Ei(d*x)*e^c)*b^2/d)*d + 1/4*(2*b^2*\log(x^2) - (4*a*b*x^2 + a^2)/x^4)*\cosh(d*x + c)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c + dx) (bx^2 + a)^2}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cosh(c + d*x)*(a + b*x^2)^2)/x^5,x)

[Out] int((cosh(c + d*x)*(a + b*x^2)^2)/x^5, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^2)^2 \cosh(c + dx)}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**2+a)**2*cosh(d*x+c)/x**5,x)

[Out] Integral((a + b*x**2)**2*cosh(c + d*x)/x**5, x)

$$3.57 \quad \int \frac{x^4 \cosh(c+dx)}{a+bx^2} dx$$

Optimal. Leaf size=273

$$\frac{(-a)^{3/2} \cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}} + c\right) \text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}} - dx\right)}{2b^{5/2}} - \frac{(-a)^{3/2} \cosh\left(c - \frac{\sqrt{-a}d}{\sqrt{b}}\right) \text{Chi}\left(xd + \frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b^{5/2}} - \frac{(-a)^{3/2} \sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}} + c\right) \text{Shi}\left(xd + \frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b^{5/2}}$$

[Out] $-2*x*\cosh(d*x+c)/b/d^2-1/2*(-a)^{(3/2)}*Chi(d*x+d*(-a)^{(1/2)}/b^{(1/2)})*\cosh(c-d*(-a)^{(1/2)}/b^{(1/2)})/b^{(5/2)}+1/2*(-a)^{(3/2)}*Chi(-d*x+d*(-a)^{(1/2)}/b^{(1/2)})*\cosh(c+d*(-a)^{(1/2)}/b^{(1/2)})/b^{(5/2)}+2*\sinh(d*x+c)/b/d^3-a*\sinh(d*x+c)/b^2/d+x^2*\sinh(d*x+c)/b/d-1/2*(-a)^{(3/2)}*Shi(d*x+d*(-a)^{(1/2)}/b^{(1/2)})*\sinh(c-d*(-a)^{(1/2)}/b^{(1/2)})/b^{(5/2)}+1/2*(-a)^{(3/2)}*Shi(d*x-d*(-a)^{(1/2)}/b^{(1/2)})*\sinh(c+d*(-a)^{(1/2)}/b^{(1/2)})/b^{(5/2)}$

Rubi [A] time = 0.73, antiderivative size = 273, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {5293, 2637, 3296, 5281, 3303, 3298, 3301}

$$\frac{(-a)^{3/2} \cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}} + c\right) \text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}} - dx\right)}{2b^{5/2}} - \frac{(-a)^{3/2} \cosh\left(c - \frac{\sqrt{-a}d}{\sqrt{b}}\right) \text{Chi}\left(xd + \frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b^{5/2}} - \frac{(-a)^{3/2} \sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}} + c\right) \text{Shi}\left(xd + \frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[(x^4*Cosh[c + d*x])/(a + b*x^2), x]

[Out] $(-2*x*\text{Cosh}[c + d*x])/(b*d^2) + ((-a)^{(3/2)}*\text{Cosh}[c + (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{CoshIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] - d*x])/(2*b^{(5/2)}) - ((-a)^{(3/2)}*\text{Cosh}[c - (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{CoshIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] + d*x])/(2*b^{(5/2)}) + (2*\text{Sinh}[c + d*x])/(b*d^3) - (a*\text{Sinh}[c + d*x])/(b^2*d) + (x^2*\text{Sinh}[c + d*x])/(b*d) - ((-a)^{(3/2)}*\text{Sinh}[c + (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{SinhIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] - d*x])/(2*b^{(5/2)}) - ((-a)^{(3/2)}*\text{Sinh}[c - (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{SinhIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] + d*x])/(2*b^{(5/2)})$

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3296

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m-1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f,
fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz},
x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)
]/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

Rule 5281

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> In
t[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 5293

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4 \cosh(c + dx)}{a + bx^2} dx &= \int \left(-\frac{a \cosh(c + dx)}{b^2} + \frac{x^2 \cosh(c + dx)}{b} + \frac{a^2 \cosh(c + dx)}{b^2 (a + bx^2)} \right) dx \\
&= -\frac{a \int \cosh(c + dx) dx}{b^2} + \frac{a^2 \int \frac{\cosh(c+dx)}{a+bx^2} dx}{b^2} + \frac{\int x^2 \cosh(c + dx) dx}{b} \\
&= -\frac{a \sinh(c + dx)}{b^2 d} + \frac{x^2 \sinh(c + dx)}{bd} + \frac{a^2 \int \left(\frac{\sqrt{-a} \cosh(c+dx)}{2a(\sqrt{-a}-\sqrt{b}x)} + \frac{\sqrt{-a} \cosh(c+dx)}{2a(\sqrt{-a}+\sqrt{b}x)} \right) dx}{b^2} - \frac{2 \int x \sinh(c + dx) dx}{b} \\
&= -\frac{2x \cosh(c + dx)}{bd^2} - \frac{a \sinh(c + dx)}{b^2 d} + \frac{x^2 \sinh(c + dx)}{bd} - \frac{(-a)^{3/2} \int \frac{\cosh(c+dx)}{\sqrt{-a}-\sqrt{b}x} dx}{2b^2} - \frac{(-a)^{3/2} \int \frac{\cosh(c+dx)}{\sqrt{-a}+\sqrt{b}x} dx}{2b^2} \\
&= -\frac{2x \cosh(c + dx)}{bd^2} + \frac{2 \sinh(c + dx)}{bd^3} - \frac{a \sinh(c + dx)}{b^2 d} + \frac{x^2 \sinh(c + dx)}{bd} - \frac{((-a)^{3/2} \cosh(c + dx) \sqrt{-a} - (-a)^{3/2} \cosh(c + dx) \sqrt{-a})}{2b^2} \\
&= -\frac{2x \cosh(c + dx)}{bd^2} + \frac{(-a)^{3/2} \cosh\left(c + \frac{\sqrt{-a}d}{\sqrt{b}}\right) \text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}} - dx\right) - (-a)^{3/2} \cosh\left(c - \frac{\sqrt{-a}d}{\sqrt{b}}\right) \text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}} + dx\right)}{2b^{5/2}}
\end{aligned}$$

Mathematica [C] time = 0.45, size = 274, normalized size = 1.00

$$ia^{3/2}d^3 \cosh\left(c - \frac{i\sqrt{a}d}{\sqrt{b}}\right) \text{Ci}\left(idx - \frac{\sqrt{a}d}{\sqrt{b}}\right) - ia^{3/2}d^3 \cosh\left(c + \frac{i\sqrt{a}d}{\sqrt{b}}\right) \text{Ci}\left(ixd + \frac{\sqrt{a}d}{\sqrt{b}}\right) - a^{3/2}d^3 \sinh\left(c - \frac{i\sqrt{a}d}{\sqrt{b}}\right) \text{Si}\left(\frac{\sqrt{a}d}{\sqrt{b}} - dx\right) + a^{3/2}d^3 \sinh\left(c + \frac{i\sqrt{a}d}{\sqrt{b}}\right) \text{Si}\left(\frac{\sqrt{a}d}{\sqrt{b}} + dx\right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(x^4*Cosh[c + d*x])/(a + b*x^2), x]

[Out] $(-4*b^{(3/2)*d*x}*Cosh[c + d*x] + I*a^{(3/2)*d^3}*Cosh[c - (I*Sqrt[a]*d)/Sqrt[b]])*CosIntegral[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x] - I*a^{(3/2)*d^3}*Cosh[c + (I*Sqrt[a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x] + 4*b^{(3/2)*d*x}*Sinh[c + d*x] - 2*a*Sqrt[b]*d^2*Sinh[c + d*x] + 2*b^{(3/2)*d^2*x^2}*Sinh[c + d*x] - a^{(3/2)*d^3}*Sinh[c - (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d*x] - a^{(3/2)*d^3}*Sinh[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x])/(2*b^{(5/2)*d^3})$

fricas [B] time = 0.80, size = 605, normalized size = 2.22

$$8 b d x \cosh(dx + c) + \left((ad^2 \cosh(dx + c)^2 - ad^2 \sinh(dx + c)^2) \sqrt{-\frac{ad^2}{b}} \text{Ei}\left(dx - \sqrt{-\frac{ad^2}{b}}\right) + (ad^2 \cosh(dx + c))^2 - (ad^2 \sinh(dx + c))^2 \right) \sqrt{-\frac{ad^2}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*cosh(d*x+c)/(b*x^2+a),x, algorithm="fricas")

[Out]
$$\begin{aligned} & -1/4*(8*b*d*x*cosh(d*x + c) + ((a*d^2*cosh(d*x + c)^2 - a*d^2*sinh(d*x + c)^2)*sqrt(-a*d^2/b)*Ei(d*x - sqrt(-a*d^2/b)) + (a*d^2*cosh(d*x + c)^2 - a*d^2*sinh(d*x + c)^2)*sqrt(-a*d^2/b)*Ei(-d*x + sqrt(-a*d^2/b))) * cosh(c + sqrt(-a*d^2/b)) - ((a*d^2*cosh(d*x + c)^2 - a*d^2*sinh(d*x + c)^2)*sqrt(-a*d^2/b)*Ei(d*x + sqrt(-a*d^2/b)) + (a*d^2*cosh(d*x + c)^2 - a*d^2*sinh(d*x + c)^2)*sqrt(-a*d^2/b)*Ei(-d*x - sqrt(-a*d^2/b))) * cosh(-c + sqrt(-a*d^2/b)) - 4*(b*d^2*x^2 - a*d^2 + 2*b)*sinh(d*x + c) + ((a*d^2*cosh(d*x + c)^2 - a*d^2*sinh(d*x + c)^2)*sqrt(-a*d^2/b)*Ei(d*x - sqrt(-a*d^2/b)) - (a*d^2*cosh(d*x + c)^2 - a*d^2*sinh(d*x + c)^2)*sqrt(-a*d^2/b)*Ei(-d*x + sqrt(-a*d^2/b))) * sinh(c + sqrt(-a*d^2/b)) + ((a*d^2*cosh(d*x + c)^2 - a*d^2*sinh(d*x + c)^2)*sqrt(-a*d^2/b)*Ei(d*x + sqrt(-a*d^2/b)) - (a*d^2*cosh(d*x + c)^2 - a*d^2*sinh(d*x + c)^2)*sqrt(-a*d^2/b)*Ei(-d*x - sqrt(-a*d^2/b))) * sinh(-c + sqrt(-a*d^2/b)))/(b^2*d^3*cosh(d*x + c)^2 - b^2*d^3*sinh(d*x + c)^2) \end{aligned}$$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 \cosh(dx + c)}{bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*cosh(d*x+c)/(b*x^2+a),x, algorithm="giac")

[Out] integrate(x^4*cosh(d*x + c)/(b*x^2 + a), x)

maple [A] time = 0.29, size = 369, normalized size = 1.35

$$\frac{e^{-\frac{d\sqrt{-ab}+cb}{b}} Ei\left(1, \frac{d\sqrt{-ab}+(dx+c)b-cb}{b}\right) a^2 - e^{-\frac{d\sqrt{-ab}+cb}{b}} Ei\left(1, -\frac{d\sqrt{-ab}-(dx+c)b+cb}{b}\right) a^2}{4b^2\sqrt{-ab}} - \frac{e^{-dx-c}x^2}{2db} + \frac{e^{-dx-c}a}{2db^2} - \frac{e^{-dx-c}x}{d^2b} - \frac{e^{-dx-c}}{d^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4*cosh(d*x+c)/(b*x^2+a),x)

[Out]
$$\begin{aligned} & 1/4/b^2/(-a*b)^{(1/2)}*\exp(-(-d*(-a*b)^{(1/2)}+c*b)/b)*Ei(1, (d*(-a*b)^{(1/2)}+(d*x+c)*b-c*b)/b)*a^2-1/4/b^2/(-a*b)^{(1/2)}*\exp(-(-d*(-a*b)^{(1/2)}+c*b)/b)*Ei(1, -(d*(-a*b)^{(1/2)}-(d*x+c)*b+c*b)/b)*a^2-1/2/d*\exp(-d*x-c)/b*x^2+1/2/d*\exp(-d*x-c)/b^2*a-1/d^2*\exp(-d*x-c)/b*x-1/d^3*\exp(-d*x-c)/b-1/4/b^2/(-a*b)^{(1/2)}*\exp((d*(-a*b)^{(1/2)}+c*b)/b)*Ei(1, (d*(-a*b)^{(1/2)}-(d*x+c)*b+c*b)/b)*a^2+1/4/b^2/(-a*b)^{(1/2)}*\exp((-d*(-a*b)^{(1/2)}+c*b)/b)*Ei(1, -(d*(-a*b)^{(1/2)}+(d*x+c)*b-c*b)/b)*a^2+1/2/d*\exp(d*x+c)/b*x^2-1/2/d/b^2*a*\exp(d*x+c)-1/d^2/b*\exp(d*x+c)*x+1/d^3/b*\exp(d*x+c) \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{(bd^2x^4e^{(2c)} - 2bdx^3e^{(2c)} - 2adx^{(2c)} + 2bx^2e^{(2c)})e^{(dx)} - (bd^2x^4 + 2bdx^3 + 2adx + 2bx^2)e^{(-dx)}}{2(b^2d^3x^2e^c + abd^3e^c)} + \frac{1}{2} \int \frac{2(abdx^2e^c}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*cosh(d*x+c)/(b*x^2+a),x, algorithm="maxima")

[Out] 1/2*((b*d^2*x^4*e^(2*c) - 2*b*d*x^3*e^(2*c) - 2*a*d*x*e^(2*c) + 2*b*x^2*e^(2*c))*e^(d*x) - (b*d^2*x^4 + 2*b*d*x^3 + 2*a*d*x + 2*b*x^2)*e^(-d*x))/(b^2*d^3*x^2*e^c + a*b*d^3*e^c) + 1/2*integrate(2*(a*b*d*x^2*e^c + a^2*d*e^c + (a^2*d^2*e^c - 2*a*b*e^c)*x)*e^(d*x)/(b^3*d^3*x^4 + 2*a*b^2*d^3*x^2 + a^2*b*d^3), x) + 1/2*integrate(2*(a*b*d*x^2 + a^2*d - (a^2*d^2 - 2*a*b)*x)*e^(-d*x)/(b^3*d^3*x^4*e^c + 2*a*b^2*d^3*x^2*e^c + a^2*b*d^3*e^c), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4 \cosh(c + dx)}{bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4*cosh(c + d*x))/(a + b*x^2),x)

[Out] int((x^4*cosh(c + d*x))/(a + b*x^2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 \cosh(c + dx)}{a + bx^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4*cosh(d*x+c)/(b*x**2+a),x)

[Out] Integral(x**4*cosh(c + d*x)/(a + b*x**2), x)

$$3.58 \quad \int \frac{x^3 \cosh(c+dx)}{a+bx^2} dx$$

Optimal. Leaf size=209

$$\frac{a \cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}} + c\right) \text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}} - dx\right)}{2b^2} - \frac{a \cosh\left(c - \frac{\sqrt{-a}d}{\sqrt{b}}\right) \text{Chi}\left(xd + \frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b^2} + \frac{a \sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}} + c\right) \text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}} - dx\right)}{2b^2}$$

[Out] $-\cosh(d*x+c)/b/d^2-1/2*a*\text{Chi}(d*x+d*(-a)^{(1/2)}/b^{(1/2)})*\cosh(c-d*(-a)^{(1/2)}/b^{(1/2)})/b^2-1/2*a*\text{Chi}(-d*x+d*(-a)^{(1/2)}/b^{(1/2)})*\cosh(c+d*(-a)^{(1/2)}/b^{(1/2)})/b^2+x*\sinh(d*x+c)/b/d-1/2*a*\text{Shi}(d*x+d*(-a)^{(1/2)}/b^{(1/2)})*\sinh(c-d*(-a)^{(1/2)}/b^{(1/2)})/b^2-1/2*a*\text{Shi}(d*x-d*(-a)^{(1/2)}/b^{(1/2)})*\sinh(c+d*(-a)^{(1/2)}/b^{(1/2)})/b^2$

Rubi [A] time = 0.36, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {5293, 3296, 2638, 3303, 3298, 3301}

$$\frac{a \cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}} + c\right) \text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}} - dx\right)}{2b^2} - \frac{a \cosh\left(c - \frac{\sqrt{-a}d}{\sqrt{b}}\right) \text{Chi}\left(xd + \frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b^2} + \frac{a \sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}} + c\right) \text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}} - dx\right)}{2b^2}$$

Antiderivative was successfully verified.

[In] Int[(x^3*Cosh[c + d*x])/(a + b*x^2), x]

[Out] $-(\text{Cosh}[c + d*x]/(b*d^2)) - (a*\text{Cosh}[c + (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{CoshIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] - d*x])/(2*b^2) - (a*\text{Cosh}[c - (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{CoshIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] + d*x])/(2*b^2) + (x*\text{Sinh}[c + d*x])/(b*d) + (a*\text{Sinh}[c + (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{SinhIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] - d*x])/(2*b^2) - (a*\text{Sinh}[c - (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{SinhIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] + d*x])/(2*b^2)$

Rule 2638

Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3296

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x]
&& EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x]
&& EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*e - c*f)/d],
Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d],
Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

Rule 5293

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d}, x]
&& ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3 \cosh(c + dx)}{a + bx^2} dx &= \int \left(\frac{x \cosh(c + dx)}{b} - \frac{ax \cosh(c + dx)}{b(a + bx^2)} \right) dx \\
&= \frac{\int x \cosh(c + dx) dx}{b} - \frac{a \int \frac{x \cosh(c+dx)}{a+bx^2} dx}{b} \\
&= \frac{x \sinh(c + dx)}{bd} - \frac{a \int \left(-\frac{\cosh(c+dx)}{2\sqrt{b}(\sqrt{-a}-\sqrt{bx})} + \frac{\cosh(c+dx)}{2\sqrt{b}(\sqrt{-a}+\sqrt{bx})} \right) dx}{b} - \frac{\int \sinh(c + dx) dx}{bd} \\
&= -\frac{\cosh(c + dx)}{bd^2} + \frac{x \sinh(c + dx)}{bd} + \frac{a \int \frac{\cosh(c+dx)}{\sqrt{-a}-\sqrt{bx}} dx}{2b^{3/2}} - \frac{a \int \frac{\cosh(c+dx)}{\sqrt{-a}+\sqrt{bx}} dx}{2b^{3/2}} \\
&= -\frac{\cosh(c + dx)}{bd^2} + \frac{x \sinh(c + dx)}{bd} - \frac{\left(a \cosh \left(c - \frac{\sqrt{-ad}}{\sqrt{b}} \right) \right) \int \frac{\cosh \left(\frac{\sqrt{-ad}}{\sqrt{b}} + dx \right)}{\sqrt{-a} + \sqrt{bx}} dx}{2b^{3/2}} + \frac{\left(a \cosh \left(c + \frac{\sqrt{-ad}}{\sqrt{b}} \right) \right) \int \frac{\cosh \left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx \right)}{\sqrt{-a} - \sqrt{bx}} dx}{2b^{3/2}} \\
&= -\frac{\cosh(c + dx)}{bd^2} - \frac{a \cosh \left(c + \frac{\sqrt{-ad}}{\sqrt{b}} \right) \text{Chi} \left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx \right)}{2b^2} - \frac{a \cosh \left(c - \frac{\sqrt{-ad}}{\sqrt{b}} \right) \text{Chi} \left(\frac{\sqrt{-ad}}{\sqrt{b}} \right)}{2b^2}
\end{aligned}$$

Mathematica [C] time = 0.39, size = 210, normalized size = 1.00

$$\frac{ad^2 \cosh \left(c - \frac{i\sqrt{ad}}{\sqrt{b}} \right) \text{Ci} \left(idx - \frac{\sqrt{ad}}{\sqrt{b}} \right) + ad^2 \cosh \left(c + \frac{i\sqrt{ad}}{\sqrt{b}} \right) \text{Ci} \left(idx + \frac{\sqrt{ad}}{\sqrt{b}} \right) + iad^2 \sinh \left(c - \frac{i\sqrt{ad}}{\sqrt{b}} \right) \text{Si} \left(\frac{\sqrt{ad}}{\sqrt{b}} - idx \right) + iad^2 \sinh \left(c + \frac{i\sqrt{ad}}{\sqrt{b}} \right) \text{Si} \left(\frac{\sqrt{ad}}{\sqrt{b}} + idx \right)}{2b^2d^2}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(x^3*Cosh[c + d*x])/(a + b*x^2), x]

[Out] $-1/2*(2*b*Cosh[c + d*x] + a*d^2*Cosh[c - (I*sqrt[a]*d)/sqrt[b]]*CosIntegral[-((sqrt[a]*d)/sqrt[b]) + I*d*x] + a*d^2*Cosh[c + (I*sqrt[a]*d)/sqrt[b]]*CosIntegral[(sqrt[a]*d)/sqrt[b] + I*d*x] - 2*b*d*x*Sinh[c + d*x] + I*a*d^2*SinIntegral[(sqrt[a]*d)/sqrt[b] - I*d*x] - I*a*d^2*Sinh[c + (I*sqrt[a]*d)/sqrt[b]]*SinIntegral[(sqrt[a]*d)/sqrt[b] + I*d*x])/(b^2*d^2)$

fricas [B] time = 1.11, size = 502, normalized size = 2.40

$$\frac{4 b d x \sinh (d x + c) - 4 b \cosh (d x + c) - \left(\left(a d^2 \cosh (d x + c)^2 - a d^2 \sinh (d x + c)^2 \right) \text{Ei} \left(d x - \sqrt{-\frac{a d^2}{b}} \right) + \left(a d^2 \cosh (d x + c)^2 - a d^2 \sinh (d x + c)^2 \right) \text{Ei} \left(d x + \sqrt{-\frac{a d^2}{b}} \right)}{2 b^2 d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*cosh(d*x+c)/(b*x^2+a),x, algorithm="fricas")

[Out] $\frac{1}{4} \cdot (4 \cdot b \cdot d \cdot x \cdot \sinh(d \cdot x + c) - 4 \cdot b \cdot \cosh(d \cdot x + c) - ((a \cdot d^2 \cdot \cosh(d \cdot x + c))^2 - a \cdot d^2 \cdot \sinh(d \cdot x + c)^2) \cdot \text{Ei}(d \cdot x - \sqrt{-a \cdot d^2/b})) + (a \cdot d^2 \cdot \cosh(d \cdot x + c)^2 - a \cdot d^2 \cdot \sinh(d \cdot x + c)^2) \cdot \text{Ei}(-d \cdot x + \sqrt{-a \cdot d^2/b})) \cdot \cosh(c + \sqrt{-a \cdot d^2/b}) - ((a \cdot d^2 \cdot \cosh(d \cdot x + c))^2 - a \cdot d^2 \cdot \sinh(d \cdot x + c)^2) \cdot \text{Ei}(d \cdot x + \sqrt{-a \cdot d^2/b})) + (a \cdot d^2 \cdot \cosh(d \cdot x + c)^2 - a \cdot d^2 \cdot \sinh(d \cdot x + c)^2) \cdot \text{Ei}(-d \cdot x - \sqrt{-a \cdot d^2/b})) \cdot \cosh(-c + \sqrt{-a \cdot d^2/b}) - ((a \cdot d^2 \cdot \cosh(d \cdot x + c))^2 - a \cdot d^2 \cdot \sinh(d \cdot x + c)^2) \cdot \text{Ei}(d \cdot x - \sqrt{-a \cdot d^2/b})) - (a \cdot d^2 \cdot \cosh(d \cdot x + c)^2 - a \cdot d^2 \cdot \sinh(d \cdot x + c)^2) \cdot \text{Ei}(-d \cdot x + \sqrt{-a \cdot d^2/b})) \cdot \sinh(c + \sqrt{-a \cdot d^2/b}) + ((a \cdot d^2 \cdot \cosh(d \cdot x + c))^2 - a \cdot d^2 \cdot \sinh(d \cdot x + c)^2) \cdot \text{Ei}(d \cdot x + \sqrt{-a \cdot d^2/b})) - (a \cdot d^2 \cdot \cosh(d \cdot x + c)^2 - a \cdot d^2 \cdot \sinh(d \cdot x + c)^2) \cdot \text{Ei}(-d \cdot x - \sqrt{-a \cdot d^2/b})) \cdot \sinh(-c + \sqrt{-a \cdot d^2/b})) / (b^2 \cdot d^2 \cdot \cosh(d \cdot x + c)^2 - b^2 \cdot d^2 \cdot \sinh(d \cdot x + c)^2)$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \cosh(dx + c)}{bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*cosh(d*x+c)/(b*x^2+a),x, algorithm="giac")

[Out] integrate(x^3*cosh(d*x + c)/(b*x^2 + a), x)

maple [A] time = 0.21, size = 268, normalized size = 1.28

$$-\frac{e^{-dx-c}x}{2db} - \frac{e^{-dx-c}}{2d^2b} + \frac{e^{-\frac{d\sqrt{-ab}+cb}{b}} \text{Ei}\left(1, \frac{d\sqrt{-ab}+(dx+c)b-cb}{b}\right) a}{4b^2} + \frac{e^{-\frac{d\sqrt{-ab}+cb}{b}} \text{Ei}\left(1, -\frac{d\sqrt{-ab}-(dx+c)b+cb}{b}\right) a}{4b^2} + \frac{e^{dx+c}x}{2db} - \frac{e^{dx+c}}{2d^2b} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*cosh(d*x+c)/(b*x^2+a),x)

[Out] $-\frac{1}{2} \cdot \frac{1}{d} \cdot \exp(-d \cdot x - c) / b \cdot x - \frac{1}{2} \cdot \frac{1}{d^2} \cdot \exp(-d \cdot x - c) / b + \frac{1}{4} \cdot \frac{1}{b^2} \cdot \exp(-(-d \cdot (-a \cdot b)^{(1/2)} + c \cdot b) / b) \cdot \text{Ei}\left(1, \frac{(d \cdot (-a \cdot b)^{(1/2)} + (d \cdot x + c) \cdot b - c \cdot b)}{b}\right) \cdot a + \frac{1}{4} \cdot \frac{1}{b^2} \cdot \exp(-(-d \cdot (-a \cdot b)^{(1/2)} + c \cdot b) / b) \cdot \text{Ei}\left(1, -\frac{(d \cdot (-a \cdot b)^{(1/2)} - (d \cdot x + c) \cdot b + c \cdot b)}{b}\right) \cdot a + \frac{1}{2} \cdot \frac{1}{d} \cdot \frac{1}{b} \cdot \exp(d \cdot x + c) \cdot x - \frac{1}{2} \cdot \frac{1}{d^2} \cdot \frac{1}{b} \cdot \exp(d \cdot x + c) + \frac{1}{4} \cdot \frac{1}{b^2} \cdot \exp((d \cdot (-a \cdot b)^{(1/2)} + c \cdot b) / b) \cdot \text{Ei}\left(1, \frac{(d \cdot (-a \cdot b)^{(1/2)} - (d \cdot x + c) \cdot b + c \cdot b)}{b}\right) \cdot a + \frac{1}{4} \cdot \frac{1}{b^2} \cdot \exp((d \cdot (-a \cdot b)^{(1/2)} + c \cdot b) / b) \cdot \text{Ei}\left(1, -\frac{(d \cdot (-a \cdot b)^{(1/2)} + (d \cdot x + c) \cdot b - c \cdot b)}{b}\right) \cdot a$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{(dx^3 e^{(2c)} - x^2 e^{(2c)}) e^{(dx)} - (dx^3 + x^2) e^{(-dx)}}{2(bd^2 x^2 e^c + ad^2 e^c)} - \frac{1}{2} \int \frac{2(adx^2 e^c - axe^c) e^{(dx)}}{b^2 d^2 x^4 + 2abd^2 x^2 + a^2 d^2} dx + \frac{1}{2} \int \frac{2(adx^2 + ax) e^{(-dx)}}{b^2 d^2 x^4 e^c + 2abd^2 x^2 e^c + a^2 d^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*cosh(d*x+c)/(b*x^2+a),x, algorithm="maxima")

[Out] 1/2*((d*x^3*e^(2*c) - x^2*e^(2*c))*e^(d*x) - (d*x^3 + x^2)*e^(-d*x))/(b*d^2*x^2*e^c + a*d^2*e^c) - 1/2*integrate(2*(a*d*x^2*e^c - a*x*e^c)*e^(d*x)/(b^2*d^2*x^4 + 2*a*b*d^2*x^2 + a^2*d^2), x) + 1/2*integrate(2*(a*d*x^2 + a*x)*e^(-d*x)/(b^2*d^2*x^4*e^c + 2*a*b*d^2*x^2*e^c + a^2*d^2*e^c), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3 \cosh(c + dx)}{bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3*cosh(c + d*x))/(a + b*x^2),x)

[Out] int((x^3*cosh(c + d*x))/(a + b*x^2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \cosh(c + dx)}{a + bx^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*cosh(d*x+c)/(b*x**2+a),x)

[Out] Integral(x**3*cosh(c + d*x)/(a + b*x**2), x)

$$3.59 \quad \int \frac{x^2 \cosh(c+dx)}{a+bx^2} dx$$

Optimal. Leaf size=226

$$\frac{\sqrt{-a} \cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}} + c\right) \text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}} - dx\right)}{2b^{3/2}} - \frac{\sqrt{-a} \cosh\left(c - \frac{\sqrt{-a}d}{\sqrt{b}}\right) \text{Chi}\left(xd + \frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b^{3/2}} - \frac{\sqrt{-a} \sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}} + c\right) \text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b^{3/2}}$$

[Out] $\sinh(d*x+c)/b/d-1/2*\text{Chi}(d*x+d*(-a)^{(1/2)}/b^{(1/2)})*\cosh(c-d*(-a)^{(1/2)}/b^{(1/2)})*(-a)^{(1/2)}/b^{(3/2)}+1/2*\text{Chi}(-d*x+d*(-a)^{(1/2)}/b^{(1/2)})*\cosh(c+d*(-a)^{(1/2)}/b^{(1/2)})*(-a)^{(1/2)}/b^{(3/2)}-1/2*\text{Shi}(d*x+d*(-a)^{(1/2)}/b^{(1/2)})*\sinh(c-d*(-a)^{(1/2)}/b^{(1/2)})*(-a)^{(1/2)}/b^{(3/2)}+1/2*\text{Shi}(d*x-d*(-a)^{(1/2)}/b^{(1/2)})*\sinh(c+d*(-a)^{(1/2)}/b^{(1/2)})*(-a)^{(1/2)}/b^{(3/2)}$

Rubi [A] time = 0.38, antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {5293, 2637, 5281, 3303, 3298, 3301}

$$\frac{\sqrt{-a} \cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}} + c\right) \text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}} - dx\right)}{2b^{3/2}} - \frac{\sqrt{-a} \cosh\left(c - \frac{\sqrt{-a}d}{\sqrt{b}}\right) \text{Chi}\left(xd + \frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b^{3/2}} - \frac{\sqrt{-a} \sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}} + c\right) \text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b^{3/2}}$$

Antiderivative was successfully verified.

[In] `Int[(x^2*Cosh[c + d*x])/(a + b*x^2),x]`

[Out] $(\text{Sqrt}[-a]*\text{Cosh}[c + (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{CoshIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] - d*x])/(2*b^{(3/2)}) - (\text{Sqrt}[-a]*\text{Cosh}[c - (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{CoshIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] + d*x])/(2*b^{(3/2)}) + \text{Sinh}[c + d*x]/(b*d) - (\text{Sqrt}[-a]*\text{Sinh}[c + (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{SinhIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] - d*x])/(2*b^{(3/2)}) - (\text{Sqrt}[-a]*\text{Sinh}[c - (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{SinhIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] + d*x])/(2*b^{(3/2)})$

Rule 2637

`Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;`
`FreeQ[{c, d}, x]`

Rule 3298

`Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /;`
`FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]`

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x]
&& EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

Rule 5281

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> In
t[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d}, x]
&& ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 5293

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2 \cosh(c + dx)}{a + bx^2} dx &= \int \left(\frac{\cosh(c + dx)}{b} - \frac{a \cosh(c + dx)}{b(a + bx^2)} \right) dx \\
&= \frac{\int \cosh(c + dx) dx}{b} - \frac{a \int \frac{\cosh(c+dx)}{a+bx^2} dx}{b} \\
&= \frac{\sinh(c + dx)}{bd} - \frac{a \int \left(\frac{\sqrt{-a} \cosh(c+dx)}{2a(\sqrt{-a}-\sqrt{b}x)} + \frac{\sqrt{-a} \cosh(c+dx)}{2a(\sqrt{-a}+\sqrt{b}x)} \right) dx}{b} \\
&= \frac{\sinh(c + dx)}{bd} - \frac{\sqrt{-a} \int \frac{\cosh(c+dx)}{\sqrt{-a}-\sqrt{b}x} dx}{2b} - \frac{\sqrt{-a} \int \frac{\cosh(c+dx)}{\sqrt{-a}+\sqrt{b}x} dx}{2b} \\
&= \frac{\sinh(c + dx)}{bd} - \frac{\left(\sqrt{-a} \cosh\left(c - \frac{\sqrt{-a}d}{\sqrt{b}}\right) \right) \int \frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}} + dx\right)}{\sqrt{-a}+\sqrt{b}x} dx}{2b} - \frac{\left(\sqrt{-a} \cosh\left(c + \frac{\sqrt{-a}d}{\sqrt{b}}\right) \right) \int \frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}} + dx\right)}{\sqrt{-a}-\sqrt{b}x} dx}{2b} \\
&= \frac{\sqrt{-a} \cosh\left(c + \frac{\sqrt{-a}d}{\sqrt{b}}\right) \text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}} - dx\right)}{2b^{3/2}} - \frac{\sqrt{-a} \cosh\left(c - \frac{\sqrt{-a}d}{\sqrt{b}}\right) \text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}} + dx\right)}{2b^{3/2}} + \frac{\sinh(c + dx)}{bd}
\end{aligned}$$

Mathematica [C] time = 0.27, size = 213, normalized size = 0.94

$$\frac{-i\sqrt{a}d \cosh\left(c - \frac{i\sqrt{a}d}{\sqrt{b}}\right) \text{Ci}\left(idx - \frac{\sqrt{a}d}{\sqrt{b}}\right) + i\sqrt{a}d \cosh\left(c + \frac{i\sqrt{a}d}{\sqrt{b}}\right) \text{Ci}\left(ixd + \frac{\sqrt{a}d}{\sqrt{b}}\right) + \sqrt{a}d \sinh\left(c - \frac{i\sqrt{a}d}{\sqrt{b}}\right) \text{Si}\left(\frac{\sqrt{a}d}{\sqrt{b}} - dx\right) - \sqrt{a}d \sinh\left(c + \frac{i\sqrt{a}d}{\sqrt{b}}\right) \text{Si}\left(\frac{\sqrt{a}d}{\sqrt{b}} + dx\right)}{2b^{3/2}d}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(x^2*Cosh[c + d*x])/(a + b*x^2), x]

[Out] ((-1)*Sqrt[a]*d*Cosh[c - (I*Sqrt[a]*d)/Sqrt[b]]*CosIntegral[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x] + I*Sqrt[a]*d*Cosh[c + (I*Sqrt[a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x] + 2*Sqrt[b]*Sinh[c + d*x] + Sqrt[a]*d*Sinh[c - (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d*x] + Sqrt[a]*d*Sinh[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x])/(2*b^(3/2)*d)

fricas [B] time = 0.74, size = 496, normalized size = 2.19

$$\frac{\left(\sqrt{-\frac{ad^2}{b}} \left(\cosh(dx + c)^2 - \sinh(dx + c)^2\right) \text{Ei}\left(dx - \sqrt{-\frac{ad^2}{b}}\right) + \sqrt{-\frac{ad^2}{b}} \left(\cosh(dx + c)^2 - \sinh(dx + c)^2\right) \text{Ei}\left(-dx + \sqrt{-\frac{ad^2}{b}}\right)\right)}{2b^{3/2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*cosh(d*x+c)/(b*x^2+a),x, algorithm="fricas")

[Out] $\frac{1}{4} * ((\sqrt{-a*d^2/b}) * (\cosh(d*x + c)^2 - \sinh(d*x + c)^2) * \text{Ei}(d*x - \sqrt{-a*d^2/b}) + \sqrt{-a*d^2/b} * (\cosh(d*x + c)^2 - \sinh(d*x + c)^2) * \text{Ei}(-d*x + \sqrt{-a*d^2/b})) * \cosh(c + \sqrt{-a*d^2/b}) - (\sqrt{-a*d^2/b}) * (\cosh(d*x + c)^2 - \sinh(d*x + c)^2) * \text{Ei}(d*x + \sqrt{-a*d^2/b}) + \sqrt{-a*d^2/b} * (\cosh(d*x + c)^2 - \sinh(d*x + c)^2) * \text{Ei}(-d*x - \sqrt{-a*d^2/b})) * \cosh(-c + \sqrt{-a*d^2/b}) + (\sqrt{-a*d^2/b}) * (\cosh(d*x + c)^2 - \sinh(d*x + c)^2) * \text{Ei}(d*x - \sqrt{-a*d^2/b}) - \sqrt{-a*d^2/b} * (\cosh(d*x + c)^2 - \sinh(d*x + c)^2) * \text{Ei}(-d*x + \sqrt{-a*d^2/b})) * \sinh(c + \sqrt{-a*d^2/b}) + (\sqrt{-a*d^2/b}) * (\cosh(d*x + c)^2 - \sinh(d*x + c)^2) * \text{Ei}(d*x + \sqrt{-a*d^2/b}) - \sqrt{-a*d^2/b} * (\cosh(d*x + c)^2 - \sinh(d*x + c)^2) * \text{Ei}(-d*x - \sqrt{-a*d^2/b})) * \sinh(-c + \sqrt{-a*d^2/b}) + 4 * \sinh(d*x + c) / (b*d*\cosh(d*x + c)^2 - b*d*\sinh(d*x + c)^2)$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \cosh(dx + c)}{bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*cosh(d*x+c)/(b*x^2+a),x, algorithm="giac")

[Out] integrate(x^2*cosh(d*x + c)/(b*x^2 + a), x)

maple [A] time = 0.15, size = 259, normalized size = 1.15

$$\frac{e^{-dx-c}}{2db} \frac{e^{-\frac{d\sqrt{-ab}+cb}{b}} \text{Ei}\left(1, \frac{d\sqrt{-ab}+(dx+c)b-cb}{b}\right) a}{4b\sqrt{-ab}} + \frac{e^{-\frac{d\sqrt{-ab}+cb}{b}} \text{Ei}\left(1, -\frac{d\sqrt{-ab}-(dx+c)b+cb}{b}\right) a}{4b\sqrt{-ab}} + \frac{e^{dx+c}}{2db} + \frac{e^{\frac{d\sqrt{-ab}+cb}{b}} \text{Ei}\left(1, \frac{d\sqrt{-ab}+cb}{b}\right) a}{4b\sqrt{-ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*cosh(d*x+c)/(b*x^2+a),x)

[Out] $-1/2/d*\exp(-d*x-c)/b-1/4/b/(-a*b)^{(1/2)}*\exp(-(-d*(-a*b)^{(1/2)}+c*b)/b)*\text{Ei}(1, (d*(-a*b)^{(1/2)}+(d*x+c)*b-c*b)/b)*a+1/4/b/(-a*b)^{(1/2)}*\exp(-(-d*(-a*b)^{(1/2)}+c*b)/b)*\text{Ei}(1, -(d*(-a*b)^{(1/2)}-(d*x+c)*b+c*b)/b)*a+1/2/d/b*\exp(d*x+c)+1/4/b/(-a*b)^{(1/2)}*\exp((d*(-a*b)^{(1/2)}+c*b)/b)*\text{Ei}(1, (d*(-a*b)^{(1/2)}-(d*x+c)*b+c*b)/b)*a-1/4/b/(-a*b)^{(1/2)}*\exp((-d*(-a*b)^{(1/2)}+c*b)/b)*\text{Ei}(1, -(d*(-a*b)^{(1/2)}+(d*x+c)*b-c*b)/b)*a$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-a \int \frac{x e^{(dx+c)}}{b^2 dx^4 + 2 ab dx^2 + a^2 d} dx + a \int \frac{x e^{(-dx)}}{b^2 dx^4 e^c + 2 ab dx^2 e^c + a^2 d e^c} dx + \frac{x^2 e^{(dx+2c)} - x^2 e^{(-dx)}}{2(b dx^2 e^c + a d e^c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*cosh(d*x+c)/(b*x^2+a),x, algorithm="maxima")

[Out] -a*integrate(x*e^(d*x + c)/(b^2*d*x^4 + 2*a*b*d*x^2 + a^2*d), x) + a*integrate(x*e^(-d*x)/(b^2*d*x^4*e^c + 2*a*b*d*x^2*e^c + a^2*d*e^c), x) + 1/2*(x^2*e^(d*x + 2*c) - x^2*e^(-d*x))/(b*d*x^2*e^c + a*d*e^c)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2 \cosh(c + dx)}{bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2*cosh(c + d*x))/(a + b*x^2),x)

[Out] int((x^2*cosh(c + d*x))/(a + b*x^2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \cosh(c + dx)}{a + bx^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*cosh(d*x+c)/(b*x**2+a),x)

[Out] Integral(x**2*cosh(c + d*x)/(a + b*x**2), x)

$$3.60 \quad \int \frac{x \cosh(c+dx)}{a+bx^2} dx$$

Optimal. Leaf size=177

$$\frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}} + c\right) \text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}} - dx\right)}{2b} + \frac{\cosh\left(c - \frac{\sqrt{-a}d}{\sqrt{b}}\right) \text{Chi}\left(xd + \frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b} - \frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}} + c\right) \text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}} - dx\right)}{2b} + \frac{\sinh\left(c - \frac{\sqrt{-a}d}{\sqrt{b}}\right) \text{Shi}\left(xd + \frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b}$$

[Out] 1/2*Chi(d*x+d*(-a)^(1/2)/b^(1/2))*cosh(c-d*(-a)^(1/2)/b^(1/2))/b+1/2*Chi(-d*x+d*(-a)^(1/2)/b^(1/2))*cosh(c+d*(-a)^(1/2)/b^(1/2))/b+1/2*Shi(d*x+d*(-a)^(1/2)/b^(1/2))*sinh(c-d*(-a)^(1/2)/b^(1/2))/b+1/2*Shi(d*x-d*(-a)^(1/2)/b^(1/2))*sinh(c+d*(-a)^(1/2)/b^(1/2))/b

Rubi [A] time = 0.25, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {5293, 3303, 3298, 3301}

$$\frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}} + c\right) \text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}} - dx\right)}{2b} + \frac{\cosh\left(c - \frac{\sqrt{-a}d}{\sqrt{b}}\right) \text{Chi}\left(xd + \frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b} - \frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}} + c\right) \text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}} - dx\right)}{2b} + \frac{\sinh\left(c - \frac{\sqrt{-a}d}{\sqrt{b}}\right) \text{Shi}\left(xd + \frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b}$$

Antiderivative was successfully verified.

[In] Int[(x*Cosh[c + d*x])/(a + b*x^2), x]

[Out] (Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*b) + (Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*b) - (Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*b) + (Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*b)

Rule 3298

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]

Rule 3301

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]

Rule 3303

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x]

) / d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]

Rule 5293

Int[Cosh[(c_.) + (d_.)*(x_.)]*(x_.)^(m_.)*((a_.) + (b_.)*(x_.)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])

Rubi steps

$$\begin{aligned} \int \frac{x \cosh(c + dx)}{a + bx^2} dx &= \int \left(-\frac{\cosh(c + dx)}{2\sqrt{b}(\sqrt{-a} - \sqrt{b}x)} + \frac{\cosh(c + dx)}{2\sqrt{b}(\sqrt{-a} + \sqrt{b}x)} \right) dx \\ &= -\frac{\int \frac{\cosh(c+dx)}{\sqrt{-a}-\sqrt{b}x} dx}{2\sqrt{b}} + \frac{\int \frac{\cosh(c+dx)}{\sqrt{-a}+\sqrt{b}x} dx}{2\sqrt{b}} \\ &= \frac{\cosh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \int \frac{\cosh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + dx\right)}{\sqrt{-a} + \sqrt{b}x} dx}{2\sqrt{b}} - \frac{\cosh\left(c + \frac{\sqrt{-ad}}{\sqrt{b}}\right) \int \frac{\cosh\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{\sqrt{-a} - \sqrt{b}x} dx}{2\sqrt{b}} + \frac{\sinh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right)}{2b} \\ &= \frac{\cosh\left(c + \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{2b} + \frac{\cosh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} + dx\right)}{2b} - \frac{\sinh\left(c + \frac{\sqrt{-ad}}{\sqrt{b}}\right)}{2b} \end{aligned}$$

Mathematica [C] time = 0.19, size = 171, normalized size = 0.97

$$\frac{\cosh\left(c - \frac{i\sqrt{ad}}{\sqrt{b}}\right) \text{Ci}\left(idx - \frac{\sqrt{ad}}{\sqrt{b}}\right) + \cosh\left(c + \frac{i\sqrt{ad}}{\sqrt{b}}\right) \text{Ci}\left(ixd + \frac{\sqrt{ad}}{\sqrt{b}}\right) + i\left(\sinh\left(c - \frac{i\sqrt{ad}}{\sqrt{b}}\right) \text{Si}\left(\frac{\sqrt{ad}}{\sqrt{b}} - idx\right) - \sinh\left(c + \frac{i\sqrt{ad}}{\sqrt{b}}\right) \text{Si}\left(\frac{\sqrt{ad}}{\sqrt{b}} + idx\right)\right)}{2b}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(x*Cosh[c + d*x])/(a + b*x^2), x]

[Out] (Cosh[c - (I*Sqrt[a]*d)/Sqrt[b]]*CosIntegral[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x] + Cosh[c + (I*Sqrt[a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x] + I*(Sinh[c - (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d*x] - Sinh[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x]))/(2*b)

fricas [A] time = 0.66, size = 219, normalized size = 1.24

$$\left(\operatorname{Ei}\left(dx - \sqrt{-\frac{ad^2}{b}}\right) + \operatorname{Ei}\left(-dx + \sqrt{-\frac{ad^2}{b}}\right) \right) \cosh\left(c + \sqrt{-\frac{ad^2}{b}}\right) + \left(\operatorname{Ei}\left(dx + \sqrt{-\frac{ad^2}{b}}\right) + \operatorname{Ei}\left(-dx - \sqrt{-\frac{ad^2}{b}}\right) \right) \cosh\left(-c + \sqrt{-\frac{ad^2}{b}}\right) + \left(\operatorname{Ei}\left(dx - \sqrt{-\frac{ad^2}{b}}\right) - \operatorname{Ei}\left(-dx + \sqrt{-\frac{ad^2}{b}}\right) \right) \sinh\left(c + \sqrt{-\frac{ad^2}{b}}\right) - \left(\operatorname{Ei}\left(dx + \sqrt{-\frac{ad^2}{b}}\right) - \operatorname{Ei}\left(-dx - \sqrt{-\frac{ad^2}{b}}\right) \right) \sinh\left(-c + \sqrt{-\frac{ad^2}{b}}\right) \Big/ b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cosh(d*x+c)/(b*x^2+a),x, algorithm="fricas")

[Out] 1/4*((Ei(d*x - sqrt(-a*d^2/b)) + Ei(-d*x + sqrt(-a*d^2/b)))*cosh(c + sqrt(-a*d^2/b)) + (Ei(d*x + sqrt(-a*d^2/b)) + Ei(-d*x - sqrt(-a*d^2/b)))*cosh(-c + sqrt(-a*d^2/b)) + (Ei(d*x - sqrt(-a*d^2/b)) - Ei(-d*x + sqrt(-a*d^2/b)))*sinh(c + sqrt(-a*d^2/b)) - (Ei(d*x + sqrt(-a*d^2/b)) - Ei(-d*x - sqrt(-a*d^2/b)))*sinh(-c + sqrt(-a*d^2/b)))/b

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \cosh(dx + c)}{bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cosh(d*x+c)/(b*x^2+a),x, algorithm="giac")

[Out] integrate(x*cosh(d*x + c)/(b*x^2 + a), x)

maple [A] time = 0.11, size = 200, normalized size = 1.13

$$\frac{e^{-\frac{d\sqrt{-ab}+cb}{b}} \operatorname{Ei}\left(1, \frac{d\sqrt{-ab}+(dx+c)b-cb}{b}\right)}{4b} - \frac{e^{-\frac{d\sqrt{-ab}+cb}{b}} \operatorname{Ei}\left(1, -\frac{d\sqrt{-ab}-(dx+c)b+cb}{b}\right)}{4b} - \frac{e^{\frac{d\sqrt{-ab}+cb}{b}} \operatorname{Ei}\left(1, \frac{d\sqrt{-ab}-(dx+c)b+cb}{b}\right)}{4b} - \frac{e^{-\frac{d\sqrt{-ab}+cb}{b}} \operatorname{Ei}\left(1, \frac{d\sqrt{-ab}+(dx+c)b-cb}{b}\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*cosh(d*x+c)/(b*x^2+a),x)

[Out] -1/4/b*exp(-(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1, (d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b) - 1/4/b*exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1, -(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b) - 1/4/b*exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1, (d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b) - 1/4/b*exp((-d*(-a*b)^(1/2)+c*b)/b)*Ei(1, -(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{xe^{(dx+2c)} - xe^{-dx}}{2(bdx^2e^c + ade^c)} + \frac{1}{2} \int \frac{(bx^2e^c - ae^c)e^{(dx)}}{b^2dx^4 + 2abdx^2 + a^2d} dx - \frac{1}{2} \int \frac{(bx^2 - a)e^{-dx}}{b^2dx^4e^c + 2abdx^2e^c + a^2de^c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cosh(d*x+c)/(b*x^2+a),x, algorithm="maxima")

[Out] 1/2*(x*e^(d*x + 2*c) - x*e^(-d*x))/(b*d*x^2*e^c + a*d*e^c) + 1/2*integrate((b*x^2*e^c - a*e^c)*e^(d*x)/(b^2*d*x^4 + 2*a*b*d*x^2 + a^2*d), x) - 1/2*integrate((b*x^2 - a)*e^(-d*x)/(b^2*d*x^4*e^c + 2*a*b*d*x^2*e^c + a^2*d*e^c), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x \cosh(c + dx)}{bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x*cosh(c + d*x))/(a + b*x^2),x)

[Out] int((x*cosh(c + d*x))/(a + b*x^2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \cosh(c + dx)}{a + bx^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cosh(d*x+c)/(b*x**2+a),x)

[Out] Integral(x*cosh(c + d*x)/(a + b*x**2), x)

$$3.61 \quad \int \frac{\cosh(c+dx)}{a+bx^2} dx$$

Optimal. Leaf size=213

$$\frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}} + c\right) \text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}} - dx\right)}{2\sqrt{-a}\sqrt{b}} - \frac{\cosh\left(c - \frac{\sqrt{-a}d}{\sqrt{b}}\right) \text{Chi}\left(xd + \frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2\sqrt{-a}\sqrt{b}} - \frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}} + c\right) \text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}} - dx\right)}{2\sqrt{-a}\sqrt{b}} - \frac{\sinh\left(c - \frac{\sqrt{-a}d}{\sqrt{b}}\right) \text{Shi}\left(xd + \frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2\sqrt{-a}\sqrt{b}}$$

[Out] $-1/2*\text{Chi}(d*x+d*(-a)^{(1/2)}/b^{(1/2)})*\cosh(c-d*(-a)^{(1/2)}/b^{(1/2)})/(-a)^{(1/2)}/b^{(1/2)}+1/2*\text{Chi}(-d*x+d*(-a)^{(1/2)}/b^{(1/2)})*\cosh(c+d*(-a)^{(1/2)}/b^{(1/2)})/(-a)^{(1/2)}/b^{(1/2)}-1/2*\text{Shi}(d*x+d*(-a)^{(1/2)}/b^{(1/2)})*\sinh(c-d*(-a)^{(1/2)}/b^{(1/2)})/(-a)^{(1/2)}/b^{(1/2)}+1/2*\text{Shi}(d*x-d*(-a)^{(1/2)}/b^{(1/2)})*\sinh(c+d*(-a)^{(1/2)}/b^{(1/2)})/(-a)^{(1/2)}/b^{(1/2)}$

Rubi [A] time = 0.26, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5281, 3303, 3298, 3301}

$$\frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}} + c\right) \text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}} - dx\right)}{2\sqrt{-a}\sqrt{b}} - \frac{\cosh\left(c - \frac{\sqrt{-a}d}{\sqrt{b}}\right) \text{Chi}\left(xd + \frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2\sqrt{-a}\sqrt{b}} - \frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}} + c\right) \text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}} - dx\right)}{2\sqrt{-a}\sqrt{b}} - \frac{\sinh\left(c - \frac{\sqrt{-a}d}{\sqrt{b}}\right) \text{Shi}\left(xd + \frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2\sqrt{-a}\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]/(a + b*x^2), x]

[Out] $(\text{Cosh}[c + (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{CoshIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] - d*x])/((2*\text{Sqrt}[-a]*\text{Sqrt}[b]) - (\text{Cosh}[c - (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{CoshIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] + d*x]))/(2*\text{Sqrt}[-a]*\text{Sqrt}[b]) - (\text{Sinh}[c + (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{SinhIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] - d*x])/((2*\text{Sqrt}[-a]*\text{Sqrt}[b]) - (\text{Sinh}[c - (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{SinhIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] + d*x]))/(2*\text{Sqrt}[-a]*\text{Sqrt}[b])$

Rule 3298

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]

Rule 3301

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

Rule 5281

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := In
t[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rubi steps

$$\begin{aligned} \int \frac{\cosh(c + dx)}{a + bx^2} dx &= \int \left(\frac{\sqrt{-a} \cosh(c + dx)}{2a(\sqrt{-a} - \sqrt{b}x)} + \frac{\sqrt{-a} \cosh(c + dx)}{2a(\sqrt{-a} + \sqrt{b}x)} \right) dx \\ &= -\frac{\int \frac{\cosh(c+dx)}{\sqrt{-a}-\sqrt{b}x} dx}{2\sqrt{-a}} - \frac{\int \frac{\cosh(c+dx)}{\sqrt{-a}+\sqrt{b}x} dx}{2\sqrt{-a}} \\ &= -\frac{\cosh\left(c - \frac{\sqrt{-a}d}{\sqrt{b}}\right) \int \frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x} dx}{2\sqrt{-a}} - \frac{\cosh\left(c + \frac{\sqrt{-a}d}{\sqrt{b}}\right) \int \frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x} dx}{2\sqrt{-a}} - \frac{\sinh\left(c - \frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2\sqrt{-a}} \\ &= \frac{\cosh\left(c + \frac{\sqrt{-a}d}{\sqrt{b}}\right) \operatorname{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}} - dx\right)}{2\sqrt{-a}\sqrt{b}} - \frac{\cosh\left(c - \frac{\sqrt{-a}d}{\sqrt{b}}\right) \operatorname{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}} + dx\right)}{2\sqrt{-a}\sqrt{b}} - \frac{\sinh\left(c + \frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2\sqrt{-a}} \end{aligned}$$

Mathematica [C] time = 0.21, size = 180, normalized size = 0.85

$$\frac{i \left(\cosh\left(c - \frac{i\sqrt{ad}}{\sqrt{b}}\right) \operatorname{Ci}\left(idx - \frac{\sqrt{ad}}{\sqrt{b}}\right) - \cosh\left(c + \frac{i\sqrt{ad}}{\sqrt{b}}\right) \operatorname{Ci}\left(ixd + \frac{\sqrt{ad}}{\sqrt{b}}\right) + i \left(\sinh\left(c - \frac{i\sqrt{ad}}{\sqrt{b}}\right) \operatorname{Si}\left(\frac{\sqrt{ad}}{\sqrt{b}} - idx\right) + \sinh\left(c + \frac{i\sqrt{ad}}{\sqrt{b}}\right) \operatorname{Si}\left(\frac{\sqrt{ad}}{\sqrt{b}} + idx\right) \right)}{2\sqrt{a}\sqrt{b}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cosh[c + d*x]/(a + b*x^2), x]
```

```
[Out] ((I/2)*(Cosh[c - (I*Sqrt[a]*d)/Sqrt[b]]*CosIntegral[-((Sqrt[a]*d)/Sqrt[b])
+ I*d*x] - Cosh[c + (I*Sqrt[a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[a]*d)/Sqrt[b]
+ I*d*x] + I*(Sinh[c - (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[b]
```

b] - I*d*x] + Sinh[c + (I*sqrt[a]*d)/sqrt[b]]*SinIntegral[(sqrt[a]*d)/sqrt[b] + I*d*x]))/(sqrt[a]*sqrt[b])

fricas [B] time = 0.73, size = 316, normalized size = 1.48

$$\frac{\left(\sqrt{\frac{-ad^2}{b}} \operatorname{Ei}\left(dx - \sqrt{\frac{-ad^2}{b}}\right) + \sqrt{\frac{-ad^2}{b}} \operatorname{Ei}\left(-dx + \sqrt{\frac{-ad^2}{b}}\right)\right) \cosh\left(c + \sqrt{\frac{-ad^2}{b}}\right) - \left(\sqrt{\frac{-ad^2}{b}} \operatorname{Ei}\left(dx + \sqrt{\frac{-ad^2}{b}}\right) + \sqrt{\frac{-ad^2}{b}} \operatorname{Ei}\left(-dx - \sqrt{\frac{-ad^2}{b}}\right)\right) \cosh\left(-c + \sqrt{\frac{-ad^2}{b}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/(b*x^2+a),x, algorithm="fricas")

[Out] -1/4*((sqrt(-a*d^2/b)*Ei(d*x - sqrt(-a*d^2/b)) + sqrt(-a*d^2/b)*Ei(-d*x + sqrt(-a*d^2/b)))*cosh(c + sqrt(-a*d^2/b)) - (sqrt(-a*d^2/b)*Ei(d*x + sqrt(-a*d^2/b)) + sqrt(-a*d^2/b)*Ei(-d*x - sqrt(-a*d^2/b)))*cosh(-c + sqrt(-a*d^2/b)) + (sqrt(-a*d^2/b)*Ei(d*x - sqrt(-a*d^2/b)) - sqrt(-a*d^2/b)*Ei(-d*x + sqrt(-a*d^2/b)))*sinh(c + sqrt(-a*d^2/b)) + (sqrt(-a*d^2/b)*Ei(d*x + sqrt(-a*d^2/b)) - sqrt(-a*d^2/b)*Ei(-d*x - sqrt(-a*d^2/b)))*sinh(-c + sqrt(-a*d^2/b)))/(a*d)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(dx+c)}{bx^2+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/(b*x^2+a),x, algorithm="giac")

[Out] integrate(cosh(d*x + c)/(b*x^2 + a), x)

maple [A] time = 0.10, size = 212, normalized size = 1.00

$$\frac{e^{-\frac{d\sqrt{-ab}+cb}{b}} \operatorname{Ei}\left(1, -\frac{d\sqrt{-ab}-(dx+c)b+cb}{b}\right)}{4\sqrt{-ab}} + \frac{e^{-\frac{-d\sqrt{-ab}+cb}{b}} \operatorname{Ei}\left(1, \frac{d\sqrt{-ab}+(dx+c)b-cb}{b}\right)}{4\sqrt{-ab}} - \frac{e^{\frac{d\sqrt{-ab}+cb}{b}} \operatorname{Ei}\left(1, \frac{d\sqrt{-ab}-(dx+c)b+cb}{b}\right)}{4\sqrt{-ab}} + \frac{e^{\frac{-d\sqrt{-ab}+cb}{b}} \operatorname{Ei}\left(1, \frac{d\sqrt{-ab}+(dx+c)b-cb}{b}\right)}{4\sqrt{-ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d*x+c)/(b*x^2+a),x)

[Out] -1/4/(-a*b)^(1/2)*exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1, -(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)+1/4/(-a*b)^(1/2)*exp(-(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1, (d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)-1/4/(-a*b)^(1/2)*exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1, (d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)+1/4/(-a*b)^(1/2)*exp((-d*(-a*b)^(1/2)+c*b)/b)*Ei(1, -(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(dx + c)}{bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/(b*x^2+a),x, algorithm="maxima")

[Out] integrate(cosh(d*x + c)/(b*x^2 + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(c + dx)}{bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(c + d*x)/(a + b*x^2),x)

[Out] int(cosh(c + d*x)/(a + b*x^2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(c + dx)}{a + bx^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/(b*x**2+a),x)

[Out] Integral(cosh(c + d*x)/(a + b*x**2), x)

3.62 $\int \frac{\cosh(c+dx)}{x(a+bx^2)} dx$

Optimal. Leaf size=197

$$\frac{\cosh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right) \operatorname{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{2a} - \frac{\cosh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \operatorname{Chi}\left(xd + \frac{\sqrt{-ad}}{\sqrt{b}}\right)}{2a} + \frac{\sinh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right) \operatorname{Shi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{2a} - \frac{\sinh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \operatorname{Shi}\left(xd + \frac{\sqrt{-ad}}{\sqrt{b}}\right)}{2a}$$

[Out] Chi(d*x)*cosh(c)/a-1/2*Chi(d*x+d*(-a)^(1/2)/b^(1/2))*cosh(c-d*(-a)^(1/2)/b^(1/2))/a-1/2*Chi(-d*x+d*(-a)^(1/2)/b^(1/2))*cosh(c+d*(-a)^(1/2)/b^(1/2))/a+Shi(d*x)*sinh(c)/a-1/2*Shi(d*x+d*(-a)^(1/2)/b^(1/2))*sinh(c-d*(-a)^(1/2)/b^(1/2))/a-1/2*Shi(d*x-d*(-a)^(1/2)/b^(1/2))*sinh(c+d*(-a)^(1/2)/b^(1/2))/a

Rubi [A] time = 0.37, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {5293, 3303, 3298, 3301}

$$\frac{\cosh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right) \operatorname{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{2a} - \frac{\cosh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \operatorname{Chi}\left(xd + \frac{\sqrt{-ad}}{\sqrt{b}}\right)}{2a} + \frac{\sinh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right) \operatorname{Shi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{2a} - \frac{\sinh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \operatorname{Shi}\left(xd + \frac{\sqrt{-ad}}{\sqrt{b}}\right)}{2a}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]/(x*(a + b*x^2)), x]

[Out] (Cosh[c]*CoshIntegral[d*x])/a - (Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*a) - (Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*a) + (Sinh[c]*SinhIntegral[d*x])/a + (Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*a) - (Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*a)

Rule 3298

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]

Rule 3301

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

Rule 5293

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] := Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh(c+dx)}{x(a+bx^2)} dx &= \int \left(\frac{\cosh(c+dx)}{ax} - \frac{bx \cosh(c+dx)}{a(a+bx^2)} \right) dx \\
&= \frac{\int \frac{\cosh(c+dx)}{x} dx}{a} - \frac{b \int \frac{x \cosh(c+dx)}{a+bx^2} dx}{a} \\
&= -\frac{b \int \left(-\frac{\cosh(c+dx)}{2\sqrt{b}(\sqrt{-a}-\sqrt{bx})} + \frac{\cosh(c+dx)}{2\sqrt{b}(\sqrt{-a}+\sqrt{bx})} \right) dx}{a} + \frac{\cosh(c) \int \frac{\cosh(dx)}{x} dx}{a} + \frac{\sinh(c) \int \frac{\sinh(dx)}{x} dx}{a} \\
&= \frac{\cosh(c)\text{Chi}(dx)}{a} + \frac{\sinh(c)\text{Shi}(dx)}{a} + \frac{\sqrt{b} \int \frac{\cosh(c+dx)}{\sqrt{-a}-\sqrt{bx}} dx}{2a} - \frac{\sqrt{b} \int \frac{\cosh(c+dx)}{\sqrt{-a}+\sqrt{bx}} dx}{2a} \\
&= \frac{\cosh(c)\text{Chi}(dx)}{a} + \frac{\sinh(c)\text{Shi}(dx)}{a} - \frac{\left(\sqrt{b} \cosh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right)\right) \int \frac{\cosh\left(\frac{\sqrt{-ad}}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{bx}} dx}{2a} + \frac{\left(\sqrt{b} \cosh\left(c + \frac{\sqrt{-ad}}{\sqrt{b}}\right)\right) \int \frac{\cosh\left(\frac{\sqrt{-ad}}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{bx}} dx}{2a} \\
&= \frac{\cosh(c)\text{Chi}(dx)}{a} - \frac{\cosh\left(c + \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{2a} - \frac{\cosh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} + dx\right)}{2a}
\end{aligned}$$

Mathematica [C] time = 0.33, size = 187, normalized size = 0.95

$$\frac{\cosh\left(c - \frac{i\sqrt{ad}}{\sqrt{b}}\right) \text{Ci}\left(idx - \frac{\sqrt{ad}}{\sqrt{b}}\right) + \cosh\left(c + \frac{i\sqrt{ad}}{\sqrt{b}}\right) \text{Ci}\left(ixd + \frac{\sqrt{ad}}{\sqrt{b}}\right) + i \sinh\left(c - \frac{i\sqrt{ad}}{\sqrt{b}}\right) \text{Si}\left(\frac{\sqrt{ad}}{\sqrt{b}} - idx\right) - i \sinh\left(c + \frac{i\sqrt{ad}}{\sqrt{b}}\right) \text{Si}\left(\frac{\sqrt{ad}}{\sqrt{b}} + idx\right)}{2a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cosh[c + d*x]/(x*(a + b*x^2)), x]
```


$c*b)/b)*\text{Ei}(1, (d*(-a*b)^{(1/2)} - (d*x+c)*b+c*b)/b) + 1/4/a*\exp((-d*(-a*b)^{(1/2)} + c*b)/b)*\text{Ei}(1, -(d*(-a*b)^{(1/2)} + (d*x+c)*b-c*b)/b)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(dx + c)}{(bx^2 + a)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/x/(b*x^2+a),x, algorithm="maxima")

[Out] integrate(cosh(d*x + c)/((b*x^2 + a)*x), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c + dx)}{x(bx^2 + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(c + d*x)/(x*(a + b*x^2)),x)

[Out] int(cosh(c + d*x)/(x*(a + b*x^2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(c + dx)}{x(a + bx^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/x/(b*x**2+a),x)

[Out] Integral(cosh(c + d*x)/(x*(a + b*x**2)), x)

$$3.63 \quad \int \frac{\cosh(c+dx)}{x^2(a+bx^2)} dx$$

Optimal. Leaf size=249

$$\frac{\sqrt{b} \cosh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{2(-a)^{3/2}} - \frac{\sqrt{b} \cosh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(xd + \frac{\sqrt{-ad}}{\sqrt{b}}\right)}{2(-a)^{3/2}} - \frac{\sqrt{b} \sinh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right) \text{Shi}\left(\frac{\sqrt{-ad}}{\sqrt{b}}\right)}{2(-a)^{3/2}}$$

[Out] $-\cosh(d*x+c)/a/x+d*\cosh(c)*\text{Shi}(d*x)/a+d*\text{Chi}(d*x)*\sinh(c)/a-1/2*\text{Chi}(d*x+d*(-a)^{(1/2)}/b^{(1/2)})*\cosh(c-d*(-a)^{(1/2)}/b^{(1/2)})*b^{(1/2)}/(-a)^{(3/2)}+1/2*\text{Chi}(d*x+d*(-a)^{(1/2)}/b^{(1/2)})*\cosh(c+d*(-a)^{(1/2)}/b^{(1/2)})*b^{(1/2)}/(-a)^{(3/2)}-1/2*\text{Shi}(d*x+d*(-a)^{(1/2)}/b^{(1/2)})*\sinh(c-d*(-a)^{(1/2)}/b^{(1/2)})*b^{(1/2)}/(-a)^{(3/2)}+1/2*\text{Shi}(d*x-d*(-a)^{(1/2)}/b^{(1/2)})*\sinh(c+d*(-a)^{(1/2)}/b^{(1/2)})*b^{(1/2)}/(-a)^{(3/2)}$

Rubi [A] time = 0.50, antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {5293, 3297, 3303, 3298, 3301, 5281}

$$\frac{\sqrt{b} \cosh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{2(-a)^{3/2}} - \frac{\sqrt{b} \cosh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(xd + \frac{\sqrt{-ad}}{\sqrt{b}}\right)}{2(-a)^{3/2}} - \frac{\sqrt{b} \sinh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right) \text{Shi}\left(\frac{\sqrt{-ad}}{\sqrt{b}}\right)}{2(-a)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]/(x^2*(a + b*x^2)), x]

[Out] $-(\text{Cosh}[c + d*x]/(a*x)) + (\text{Sqrt}[b]*\text{Cosh}[c + (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{CoshIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] - d*x]/(2*(-a)^{(3/2)}) - (\text{Sqrt}[b]*\text{Cosh}[c - (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{CoshIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] + d*x]/(2*(-a)^{(3/2)}) + (d*\text{CoshIntegral}[d*x]*\text{Sinh}[c])/a + (d*\text{Cosh}[c]*\text{SinhIntegral}[d*x])/a - (\text{Sqrt}[b]*\text{Sinh}[c + (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{SinhIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] - d*x]/(2*(-a)^{(3/2)}) - (\text{Sqrt}[b]*\text{Sinh}[c - (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{SinhIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] + d*x]/(2*(-a)^{(3/2)})$

Rule 3297

Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]

Rule 3298

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)])/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f}

, fz}, x] && EqQ[d*e - c*f*fz*I, 0]

Rule 3301

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]

Rule 3303

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]

Rule 5281

Int[Cosh[(c_.) + (d_.)*(x_)]*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Int[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])

Rule 5293

Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])

Rubi steps

$$\begin{aligned}
\int \frac{\cosh(c+dx)}{x^2(a+bx^2)} dx &= \int \left(\frac{\cosh(c+dx)}{ax^2} - \frac{b \cosh(c+dx)}{a(a+bx^2)} \right) dx \\
&= \frac{\int \frac{\cosh(c+dx)}{x^2} dx}{a} - \frac{b \int \frac{\cosh(c+dx)}{a+bx^2} dx}{a} \\
&= -\frac{\cosh(c+dx)}{ax} - \frac{b \int \left(\frac{\sqrt{-a} \cosh(c+dx)}{2a(\sqrt{-a}-\sqrt{b}x)} + \frac{\sqrt{-a} \cosh(c+dx)}{2a(\sqrt{-a}+\sqrt{b}x)} \right) dx}{a} + \frac{d \int \frac{\sinh(c+dx)}{x} dx}{a} \\
&= -\frac{\cosh(c+dx)}{ax} - \frac{b \int \frac{\cosh(c+dx)}{\sqrt{-a}-\sqrt{b}x} dx}{2(-a)^{3/2}} - \frac{b \int \frac{\cosh(c+dx)}{\sqrt{-a}+\sqrt{b}x} dx}{2(-a)^{3/2}} + \frac{(d \cosh(c)) \int \frac{\sinh(dx)}{x} dx}{a} + \frac{(d \sinh(c)) \int \frac{\cosh(dx)}{x} dx}{a} \\
&= -\frac{\cosh(c+dx)}{ax} + \frac{d \operatorname{Chi}(dx) \sinh(c)}{a} + \frac{d \cosh(c) \operatorname{Shi}(dx)}{a} - \frac{\left(b \cosh\left(c - \frac{\sqrt{-a}d}{\sqrt{b}}\right) \right) \int \frac{\cosh\left(\frac{\sqrt{-a}}{\sqrt{b}}\right)}{\sqrt{-a}+}}{2(-a)^{3/2}} \\
&= -\frac{\cosh(c+dx)}{ax} + \frac{\sqrt{b} \cosh\left(c + \frac{\sqrt{-a}d}{\sqrt{b}}\right) \operatorname{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}} - dx\right)}{2(-a)^{3/2}} - \frac{\sqrt{b} \cosh\left(c - \frac{\sqrt{-a}d}{\sqrt{b}}\right) \operatorname{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2(-a)^{3/2}}
\end{aligned}$$

Mathematica [C] time = 0.38, size = 243, normalized size = 0.98

$$-i\sqrt{b}x \cosh\left(c - \frac{i\sqrt{a}d}{\sqrt{b}}\right) \operatorname{Ci}\left(idx - \frac{\sqrt{a}d}{\sqrt{b}}\right) + i\sqrt{b}x \cosh\left(c + \frac{i\sqrt{a}d}{\sqrt{b}}\right) \operatorname{Ci}\left(ixd + \frac{\sqrt{a}d}{\sqrt{b}}\right) + \sqrt{b}x \sinh\left(c - \frac{i\sqrt{a}d}{\sqrt{b}}\right) \operatorname{Si}\left(\frac{\sqrt{a}d}{\sqrt{b}}\right) - \sqrt{b}x \sinh\left(c + \frac{i\sqrt{a}d}{\sqrt{b}}\right) \operatorname{Si}\left(\frac{\sqrt{a}d}{\sqrt{b}}\right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[Cosh[c + d*x]/(x^2*(a + b*x^2)), x]

[Out] $(-2\sqrt{a}\operatorname{Cosh}[c + d*x] - I\sqrt{b}*x*\operatorname{Cosh}[c - (I\sqrt{a}*d)/\sqrt{b}])*\operatorname{CosIntegral}[-((\sqrt{a}*d)/\sqrt{b}) + I*d*x] + I\sqrt{b}*x*\operatorname{Cosh}[c + (I\sqrt{a}*d)/\sqrt{b}]*\operatorname{CosIntegral}[(\sqrt{a}*d)/\sqrt{b} + I*d*x] + 2\sqrt{a}*d*x*\operatorname{CoshIntegral}[d*x]*\operatorname{Sinh}[c] + 2\sqrt{a}*d*x*\operatorname{Cosh}[c]*\operatorname{SinhIntegral}[d*x] + \sqrt{b}*x*\operatorname{Sinh}[c - (I\sqrt{a}*d)/\sqrt{b}]*\operatorname{SinIntegral}[(\sqrt{a}*d)/\sqrt{b} - I*d*x] + \sqrt{b}*x*\operatorname{Sinh}[c + (I\sqrt{a}*d)/\sqrt{b}]*\operatorname{SinIntegral}[(\sqrt{a}*d)/\sqrt{b} + I*d*x])/(2*a^{(3/2)}*x)$

fricas [B] time = 0.51, size = 599, normalized size = 2.41

$$4ad \cosh(dx + c) - \left((bx \cosh(dx + c)^2 - bx \sinh(dx + c)^2) \sqrt{-\frac{ad^2}{b}} \operatorname{Ei}\left(dx - \sqrt{-\frac{ad^2}{b}}\right) + (bx \cosh(dx + c)^2 - bx \sinh(dx + c)^2) \sqrt{-\frac{ad^2}{b}} \operatorname{Ei}\left(dx + \sqrt{-\frac{ad^2}{b}}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/x^2/(b*x^2+a),x, algorithm="fricas")

[Out]
$$\begin{aligned} & -1/4*(4*a*d*cosh(d*x + c) - ((b*x*cosh(d*x + c))^2 - b*x*sinh(d*x + c)^2)*sqrt(-a*d^2/b)*Ei(d*x - sqrt(-a*d^2/b)) + (b*x*cosh(d*x + c)^2 - b*x*sinh(d*x + c)^2)*sqrt(-a*d^2/b)*Ei(-d*x + sqrt(-a*d^2/b))) * cosh(c + sqrt(-a*d^2/b)) \\ & - 2*(a*d^2*x*Ei(d*x) - a*d^2*x*Ei(-d*x))*cosh(c) + ((b*x*cosh(d*x + c))^2 - b*x*sinh(d*x + c)^2)*sqrt(-a*d^2/b)*Ei(d*x + sqrt(-a*d^2/b)) + (b*x*cosh(d*x + c)^2 - b*x*sinh(d*x + c)^2)*sqrt(-a*d^2/b)*Ei(-d*x - sqrt(-a*d^2/b))) * cosh(-c + sqrt(-a*d^2/b)) - ((b*x*cosh(d*x + c))^2 - b*x*sinh(d*x + c)^2)*sqrt(-a*d^2/b)*Ei(d*x - sqrt(-a*d^2/b)) - (b*x*cosh(d*x + c)^2 - b*x*sinh(d*x + c)^2)*sqrt(-a*d^2/b)*Ei(-d*x + sqrt(-a*d^2/b))) * sinh(c + sqrt(-a*d^2/b)) \\ & - 2*(a*d^2*x*Ei(d*x) + a*d^2*x*Ei(-d*x))*sinh(c) - ((b*x*cosh(d*x + c))^2 - b*x*sinh(d*x + c)^2)*sqrt(-a*d^2/b)*Ei(d*x + sqrt(-a*d^2/b)) - (b*x*cosh(d*x + c)^2 - b*x*sinh(d*x + c)^2)*sqrt(-a*d^2/b)*Ei(-d*x - sqrt(-a*d^2/b))) * sinh(-c + sqrt(-a*d^2/b)))/(a^2*d*x*cosh(d*x + c)^2 - a^2*d*x*sinh(d*x + c)^2) \end{aligned}$$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(dx + c)}{(bx^2 + a)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/x^2/(b*x^2+a),x, algorithm="giac")

[Out] integrate(cosh(d*x + c)/((b*x^2 + a)*x^2), x)

maple [A] time = 0.12, size = 288, normalized size = 1.16

$$-\frac{e^{-dx-c}}{2ax} + \frac{d e^{-c} Ei(1, dx)}{2a} + \frac{b e^{-\frac{d\sqrt{-ab}+cb}{b}} Ei\left(1, -\frac{d\sqrt{-ab}-(dx+c)b+cb}{b}\right)}{4a\sqrt{-ab}} - \frac{b e^{-\frac{-d\sqrt{-ab}+cb}{b}} Ei\left(1, \frac{d\sqrt{-ab}+(dx+c)b-cb}{b}\right)}{4a\sqrt{-ab}} - \frac{e^{dx+c}}{2ax} - \frac{d e^c}{2ax}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d*x+c)/x^2/(b*x^2+a),x)

[Out]
$$\begin{aligned} & -1/2*exp(-d*x-c)/a/x+1/2*d/a*exp(-c)*Ei(1,d*x)+1/4*b/a/(-a*b)^(1/2)*exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)-1/4*b/a/(-a*b)^(1/2)*exp(-(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)-1/2*exp(d*x+c)/a/x-1/2*d/a*exp(c)*Ei(1,-d*x)-1/4*b/a/(-a*b)^(1/2)*exp((-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)+1/4*b/a/(-a*b)^(1/2)*exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b) \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(dx + c)}{(bx^2 + a)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/x^2/(b*x^2+a),x, algorithm="maxima")

[Out] integrate(cosh(d*x + c)/((b*x^2 + a)*x^2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(c + dx)}{x^2 (bx^2 + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(c + d*x)/(x^2*(a + b*x^2)),x)

[Out] int(cosh(c + d*x)/(x^2*(a + b*x^2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(c + dx)}{x^2 (a + bx^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/x**2/(b*x**2+a),x)

[Out] Integral(cosh(c + d*x)/(x**2*(a + b*x**2)), x)

$$3.64 \quad \int \frac{\cosh(c+dx)}{x^3(a+bx^2)} dx$$

Optimal. Leaf size=270

$$-\frac{b \cosh(c) \operatorname{Chi}(dx)}{a^2} + \frac{b \cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}} + c\right) \operatorname{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}} - dx\right)}{2a^2} + \frac{b \cosh\left(c - \frac{\sqrt{-a}d}{\sqrt{b}}\right) \operatorname{Chi}\left(xd + \frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2a^2} - \frac{b \sinh(c) \operatorname{Shi}(dx)}{a^2}$$

[Out] $-b \operatorname{Chi}(d*x) \operatorname{cosh}(c)/a^2 + 1/2 d^2 \operatorname{Chi}(d*x) \operatorname{cosh}(c)/a - 1/2 \operatorname{cosh}(d*x+c)/a/x^2 + 1/2 b \operatorname{Chi}(d*x+d*(-a)^{1/2}/b^{1/2}) \operatorname{cosh}(c-d*(-a)^{1/2}/b^{1/2})/a^2 + 1/2 b \operatorname{Chi}(-d*x+d*(-a)^{1/2}/b^{1/2}) \operatorname{cosh}(c+d*(-a)^{1/2}/b^{1/2})/a^2 - b \operatorname{Shi}(d*x) \operatorname{sinh}(c)/a^2 + 1/2 d^2 \operatorname{Shi}(d*x) \operatorname{sinh}(c)/a - 1/2 d \operatorname{sinh}(d*x+c)/a/x + 1/2 b \operatorname{Shi}(d*x+d*(-a)^{1/2}/b^{1/2}) \operatorname{sinh}(c-d*(-a)^{1/2}/b^{1/2})/a^2 + 1/2 b \operatorname{Shi}(d*x-d*(-a)^{1/2}/b^{1/2}) \operatorname{sinh}(c+d*(-a)^{1/2}/b^{1/2})/a^2$

Rubi [A] time = 0.50, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {5293, 3297, 3303, 3298, 3301}

$$-\frac{b \cosh(c) \operatorname{Chi}(dx)}{a^2} + \frac{b \cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}} + c\right) \operatorname{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}} - dx\right)}{2a^2} + \frac{b \cosh\left(c - \frac{\sqrt{-a}d}{\sqrt{b}}\right) \operatorname{Chi}\left(xd + \frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2a^2} - \frac{b \sinh(c) \operatorname{Shi}(dx)}{a^2}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Cosh}[c + d*x]/(x^3*(a + b*x^2)), x]$

[Out] $-\operatorname{Cosh}[c + d*x]/(2*a*x^2) - (b*\operatorname{Cosh}[c]*\operatorname{CoshIntegral}[d*x])/a^2 + (d^2*\operatorname{Cosh}[c]*\operatorname{CoshIntegral}[d*x])/(2*a) + (b*\operatorname{Cosh}[c + (\operatorname{Sqrt}[-a]*d)/\operatorname{Sqrt}[b]]*\operatorname{CoshIntegral}[(\operatorname{Sqrt}[-a]*d)/\operatorname{Sqrt}[b] - d*x])/(2*a^2) + (b*\operatorname{Cosh}[c - (\operatorname{Sqrt}[-a]*d)/\operatorname{Sqrt}[b]]*\operatorname{CoshIntegral}[(\operatorname{Sqrt}[-a]*d)/\operatorname{Sqrt}[b] + d*x])/(2*a^2) - (d*\operatorname{Sinh}[c + d*x])/(2*a*x) - (b*\operatorname{Sinh}[c]*\operatorname{SinhIntegral}[d*x])/a^2 + (d^2*\operatorname{Sinh}[c]*\operatorname{SinhIntegral}[d*x])/(2*a) - (b*\operatorname{Sinh}[c + (\operatorname{Sqrt}[-a]*d)/\operatorname{Sqrt}[b]]*\operatorname{SinhIntegral}[(\operatorname{Sqrt}[-a]*d)/\operatorname{Sqrt}[b] - d*x])/(2*a^2) + (b*\operatorname{Sinh}[c - (\operatorname{Sqrt}[-a]*d)/\operatorname{Sqrt}[b]]*\operatorname{SinhIntegral}[(\operatorname{Sqrt}[-a]*d)/\operatorname{Sqrt}[b] + d*x])/(2*a^2)$

Rule 3297

$\operatorname{Int}[(c + d*x)^m \sin[e + f*x], x] := \operatorname{Simp}[(c + d*x)^{m+1} \operatorname{Sin}[e + f*x]/(d*(m+1)), x] - \operatorname{Dist}[f/(d*(m+1)), \operatorname{Int}[(c + d*x)^m \operatorname{Cos}[e + f*x], x], x] /;$ FreeQ[{c, d, e, f}, x] && LtQ[m, -1]

Rule 3298


```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x]
&& EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x]
&& EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*e - c*f)/d],
Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d],
Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

Rule 5293

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d}, x]
&& ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh(c+dx)}{x^3(a+bx^2)} dx &= \int \left(\frac{\cosh(c+dx)}{ax^3} - \frac{b \cosh(c+dx)}{a^2x} + \frac{b^2x \cosh(c+dx)}{a^2(a+bx^2)} \right) dx \\
&= \frac{\int \frac{\cosh(c+dx)}{x^3} dx}{a} - \frac{b \int \frac{\cosh(c+dx)}{x} dx}{a^2} + \frac{b^2 \int \frac{x \cosh(c+dx)}{a+bx^2} dx}{a^2} \\
&= -\frac{\cosh(c+dx)}{2ax^2} + \frac{b^2 \int \left(-\frac{\cosh(c+dx)}{2\sqrt{b}(\sqrt{-a}-\sqrt{bx})} + \frac{\cosh(c+dx)}{2\sqrt{b}(\sqrt{-a}+\sqrt{bx})} \right) dx}{a^2} + \frac{d \int \frac{\sinh(c+dx)}{x^2} dx}{2a} - \frac{(b \cosh(c+dx))}{2a^2} \\
&= -\frac{\cosh(c+dx)}{2ax^2} - \frac{b \cosh(c) \text{Chi}(dx)}{a^2} - \frac{d \sinh(c+dx)}{2ax} - \frac{b \sinh(c) \text{Shi}(dx)}{a^2} - \frac{b^{3/2} \int \frac{\cosh(c+dx)}{\sqrt{-a}-\sqrt{bx}} dx}{2a^2} \\
&= -\frac{\cosh(c+dx)}{2ax^2} - \frac{b \cosh(c) \text{Chi}(dx)}{a^2} - \frac{d \sinh(c+dx)}{2ax} - \frac{b \sinh(c) \text{Shi}(dx)}{a^2} + \frac{(d^2 \cosh(c)) \int \frac{1}{\sqrt{-a}-\sqrt{bx}} dx}{2a} \\
&= -\frac{\cosh(c+dx)}{2ax^2} - \frac{b \cosh(c) \text{Chi}(dx)}{a^2} + \frac{d^2 \cosh(c) \text{Chi}(dx)}{2a} + \frac{b \cosh\left(c + \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}}\right)}{2a^2}
\end{aligned}$$

Mathematica [C] time = 0.57, size = 257, normalized size = 0.95

$$-\left(x^2 \cosh(c) (2b - ad^2) \text{Chi}(dx)\right) + bx^2 \cosh\left(c - \frac{i\sqrt{ad}}{\sqrt{b}}\right) \text{Ci}\left(idx - \frac{\sqrt{ad}}{\sqrt{b}}\right) + bx^2 \cosh\left(c + \frac{i\sqrt{ad}}{\sqrt{b}}\right) \text{Ci}\left(ixd + \frac{\sqrt{ad}}{\sqrt{b}}\right) + \dots$$

Warning: Unable to verify antiderivative.

[In] Integrate[Cosh[c + d*x]/(x^3*(a + b*x^2)),x]

[Out] $(-a \text{Cosh}[c + d*x]) - (2*b - a*d^2)*x^2*\text{Cosh}[c]*\text{CoshIntegral}[d*x] + b*x^2*\text{Cosh}[c - (I*\text{Sqrt}[a]*d)/\text{Sqrt}[b]]*\text{CosIntegral}[-((\text{Sqrt}[a]*d)/\text{Sqrt}[b]) + I*d*x] + b*x^2*\text{Cosh}[c + (I*\text{Sqrt}[a]*d)/\text{Sqrt}[b]]*\text{CosIntegral}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b] + I*d*x] - a*d*x*\text{Sinh}[c + d*x] - 2*b*x^2*\text{Sinh}[c]*\text{SinhIntegral}[d*x] + a*d^2*x^2*\text{Sinh}[c]*\text{SinhIntegral}[d*x] + I*b*x^2*\text{Sinh}[c - (I*\text{Sqrt}[a]*d)/\text{Sqrt}[b]]*\text{SinIntegral}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b] - I*d*x] - I*b*x^2*\text{Sinh}[c + (I*\text{Sqrt}[a]*d)/\text{Sqrt}[b]]*\text{SinIntegral}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b] + I*d*x])/(2*a^2*x^2)$

fricas [B] time = 0.49, size = 583, normalized size = 2.16

$$2 adx \sinh(dx + c) + 2 a \cosh(dx + c) - \left((bx^2 \cosh(dx + c))^2 - bx^2 \sinh(dx + c)^2 \right) \text{Ei}\left(dx - \sqrt{-\frac{ad^2}{b}}\right) + (bx^2 \cosh(dx + c))^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/x^3/(b*x^2+a),x, algorithm="fricas")

[Out]
$$\begin{aligned} & -1/4*(2*a*d*x*\sinh(d*x + c) + 2*a*\cosh(d*x + c) - ((b*x^2*\cosh(d*x + c))^2 - \\ & b*x^2*\sinh(d*x + c)^2)*\text{Ei}(d*x - \sqrt{-a*d^2/b}) + (b*x^2*\cosh(d*x + c)^2 - \\ & b*x^2*\sinh(d*x + c)^2)*\text{Ei}(-d*x + \sqrt{-a*d^2/b}))*\cosh(c + \sqrt{-a*d^2/b}) \\ & - ((a*d^2 - 2*b)*x^2*\text{Ei}(d*x) + (a*d^2 - 2*b)*x^2*\text{Ei}(-d*x))*\cosh(c) - ((b*x \\ & ^2*\cosh(d*x + c)^2 - b*x^2*\sinh(d*x + c)^2)*\text{Ei}(d*x + \sqrt{-a*d^2/b}) + (b*x \\ & ^2*\cosh(d*x + c)^2 - b*x^2*\sinh(d*x + c)^2)*\text{Ei}(-d*x - \sqrt{-a*d^2/b}))*\cosh \\ & (-c + \sqrt{-a*d^2/b}) - ((b*x^2*\cosh(d*x + c)^2 - b*x^2*\sinh(d*x + c)^2)*\text{Ei} \\ & (d*x - \sqrt{-a*d^2/b}) - (b*x^2*\cosh(d*x + c)^2 - b*x^2*\sinh(d*x + c)^2)*\text{Ei} \\ & (-d*x + \sqrt{-a*d^2/b}))*\sinh(c + \sqrt{-a*d^2/b}) - ((a*d^2 - 2*b)*x^2*\text{Ei}(d \\ & *x) - (a*d^2 - 2*b)*x^2*\text{Ei}(-d*x))*\sinh(c) + ((b*x^2*\cosh(d*x + c)^2 - b*x^2 \\ & *\sinh(d*x + c)^2)*\text{Ei}(d*x + \sqrt{-a*d^2/b}) - (b*x^2*\cosh(d*x + c)^2 - b*x^2 \\ & *\sinh(d*x + c)^2)*\text{Ei}(-d*x - \sqrt{-a*d^2/b}))*\sinh(-c + \sqrt{-a*d^2/b}))/ (a^ \\ & 2*x^2*\cosh(d*x + c)^2 - a^2*x^2*\sinh(d*x + c)^2) \end{aligned}$$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(dx + c)}{(bx^2 + a)x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/x^3/(b*x^2+a),x, algorithm="giac")

[Out] integrate(cosh(d*x + c)/((b*x^2 + a)*x^3), x)

maple [A] time = 0.12, size = 330, normalized size = 1.22

$$\frac{d e^{-dx-c}}{4ax} - \frac{e^{-dx-c}}{4a x^2} - \frac{d^2 e^{-c} \text{Ei}(1, dx)}{4a} + \frac{e^{-c} \text{Ei}(1, dx) b}{2a^2} - \frac{b e^{-\frac{d\sqrt{-ab}+cb}}{b}} \text{Ei}\left(1, -\frac{d\sqrt{-ab}-(dx+c)b+cb}{b}\right)}{4a^2} - \frac{b e^{-\frac{-d\sqrt{-ab}+cb}{b}} \text{Ei}\left(1, \frac{d\sqrt{-ab}-(dx+c)b+cb}{b}\right)}{4a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d*x+c)/x^3/(b*x^2+a),x)

[Out]
$$\begin{aligned} & 1/4*d*\exp(-d*x-c)/a/x-1/4*\exp(-d*x-c)/a/x^2-1/4*d^2/a*\exp(-c)*\text{Ei}(1,d*x)+1/2 \\ & /a^2*\exp(-c)*\text{Ei}(1,d*x)*b-1/4*b/a^2*\exp(-(d*(-a*b)^(1/2)+c*b)/b)*\text{Ei}(1,-(d*(- \\ & a*b)^(1/2)-(d*x+c)*b+c*b)/b)-1/4*b/a^2*\exp(-(-d*(-a*b)^(1/2)+c*b)/b)*\text{Ei}(1,(\\ & d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)-1/4*d*\exp(d*x+c)/a/x-1/4*\exp(d*x+c)/a/x^2- \\ & 1/4*d^2/a*\exp(c)*\text{Ei}(1,-d*x)+1/2/a^2*\exp(c)*\text{Ei}(1,-d*x)*b-1/4*b/a^2*\exp((d*(- \\ & a*b)^(1/2)+c*b)/b)*\text{Ei}(1,(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)-1/4*b/a^2*\exp((-d \\ & *(-a*b)^(1/2)+c*b)/b)*\text{Ei}(1,-(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b) \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(dx + c)}{(bx^2 + a)x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/x^3/(b*x^2+a),x, algorithm="maxima")

[Out] integrate(cosh(d*x + c)/((b*x^2 + a)*x^3), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(c + dx)}{x^3 (bx^2 + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(c + d*x)/(x^3*(a + b*x^2)),x)

[Out] int(cosh(c + d*x)/(x^3*(a + b*x^2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(c + dx)}{x^3 (a + bx^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/x**3/(b*x**2+a),x)

[Out] Integral(cosh(c + d*x)/(x**3*(a + b*x**2)), x)

$$3.65 \quad \int \frac{x^4 \cosh(c+dx)}{(a+bx^2)^2} dx$$

Optimal. Leaf size=449

$$\frac{3\sqrt{-a} \cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}} + c\right) \text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}} - dx\right)}{4b^{5/2}} - \frac{3\sqrt{-a} \cosh\left(c - \frac{\sqrt{-a}d}{\sqrt{b}}\right) \text{Chi}\left(xd + \frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4b^{5/2}} - \frac{3\sqrt{-a} \sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}} + c\right) \text{Shi}\left(xd + \frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4b^{5/2}}$$

[Out] $1/2*x*\cosh(d*x+c)/b^2-1/2*x^3*\cosh(d*x+c)/b/(b*x^2+a)-1/4*a*d*\cosh(c+d*(-a)^{(1/2)}/b^{(1/2)})*Shi(d*x-d*(-a)^{(1/2)}/b^{(1/2)})/b^3-1/4*a*d*\cosh(c-d*(-a)^{(1/2)}/b^{(1/2)})*Shi(d*x+d*(-a)^{(1/2)}/b^{(1/2)})/b^3+\sinh(d*x+c)/b^2/d-1/4*a*d*Chi(d*x+d*(-a)^{(1/2)}/b^{(1/2)})*\sinh(c-d*(-a)^{(1/2)}/b^{(1/2)})/b^3-1/4*a*d*Chi(-d*x+d*(-a)^{(1/2)}/b^{(1/2)})*\sinh(c+d*(-a)^{(1/2)}/b^{(1/2)})/b^3-3/4*Chi(d*x+d*(-a)^{(1/2)}/b^{(1/2)})*\cosh(c-d*(-a)^{(1/2)}/b^{(1/2)})*(-a)^{(1/2)}/b^{(5/2)}+3/4*Chi(-d*x+d*(-a)^{(1/2)}/b^{(1/2)})*\cosh(c+d*(-a)^{(1/2)}/b^{(1/2)})*(-a)^{(1/2)}/b^{(5/2)}-3/4*Shi(d*x+d*(-a)^{(1/2)}/b^{(1/2)})*\sinh(c-d*(-a)^{(1/2)}/b^{(1/2)})*(-a)^{(1/2)}/b^{(5/2)}+3/4*Shi(d*x-d*(-a)^{(1/2)}/b^{(1/2)})*\sinh(c+d*(-a)^{(1/2)}/b^{(1/2)})*(-a)^{(1/2)}/b^{(5/2)}$

Rubi [A] time = 0.86, antiderivative size = 449, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {5291, 5293, 2637, 5281, 3303, 3298, 3301, 5292, 3296}

$$\frac{ad \sinh\left(c - \frac{\sqrt{-a}d}{\sqrt{b}}\right) \text{Chi}\left(xd + \frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4b^3} - \frac{ad \sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}} + c\right) \text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}} - dx\right)}{4b^3} + \frac{3\sqrt{-a} \cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}} + c\right) \text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}} + c\right)}{4b^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[(x^4*Cosh[c + d*x])/(a + b*x^2)^2,x]

[Out] $(x*\text{Cosh}[c + d*x])/(2*b^2) - (x^3*\text{Cosh}[c + d*x])/(2*b*(a + b*x^2)) + (3*\text{Sqrt}[-a]*\text{Cosh}[c + (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{CoshIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] - d*x])/(4*b^{(5/2)}) - (3*\text{Sqrt}[-a]*\text{Cosh}[c - (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{CoshIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] + d*x])/(4*b^{(5/2)}) - (a*d*\text{CoshIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] + d*x]*\text{Sinh}[c - (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]])/(4*b^3) - (a*d*\text{CoshIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] - d*x]*\text{Sinh}[c + (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]])/(4*b^3) + \text{Sinh}[c + d*x]/(b^2*d) + (a*d*\text{Cosh}[c + (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{SinhIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] - d*x])/(4*b^3) - (3*\text{Sqrt}[-a]*\text{Sinh}[c + (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{SinhIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] - d*x])/(4*b^{(5/2)}) - (a*d*\text{Cosh}[c - (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{SinhIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] + d*x])/(4*b^3) - (3*\text{Sqrt}[-a]*\text{Sinh}[c - (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{SinhIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] + d*x])/(4*b^{(5/2)})$

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;
FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]
```

Rule 5281

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Int[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 5291

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x^(m - n + 1)*(a + b*x^n)^(p + 1)*Cosh[c + d*x]/(b*n*(p + 1)), x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1)*Cosh[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^n)^(p + 1)*Sinh[c + d*x], x], x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1] && IGtQ[n, 0] && RationalQ[m] && (GtQ[m - n + 1, 0] || GtQ[n, 2])
```

Rule 5292

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*Sinh[(c_) + (d_)*(x_)], x_Symbol] :> Int[ExpandIntegrand[Sinh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])

Rule 5293

Int[Cosh[(c_) + (d_)*(x_)]*(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])

Rubi steps

$$\begin{aligned}
\int \frac{x^4 \cosh(c + dx)}{(a + bx^2)^2} dx &= -\frac{x^3 \cosh(c + dx)}{2b(a + bx^2)} + \frac{3 \int \frac{x^2 \cosh(c+dx)}{a+bx^2} dx}{2b} + \frac{d \int \frac{x^3 \sinh(c+dx)}{a+bx^2} dx}{2b} \\
&= -\frac{x^3 \cosh(c + dx)}{2b(a + bx^2)} + \frac{3 \int \left(\frac{\cosh(c+dx)}{b} - \frac{a \cosh(c+dx)}{b(a+bx^2)} \right) dx}{2b} + \frac{d \int \left(\frac{x \sinh(c+dx)}{b} - \frac{ax \sinh(c+dx)}{b(a+bx^2)} \right) dx}{2b} \\
&= -\frac{x^3 \cosh(c + dx)}{2b(a + bx^2)} + \frac{3 \int \cosh(c + dx) dx}{2b^2} - \frac{(3a) \int \frac{\cosh(c+dx)}{a+bx^2} dx}{2b^2} + \frac{d \int x \sinh(c + dx) dx}{2b^2} \\
&= \frac{x \cosh(c + dx)}{2b^2} - \frac{x^3 \cosh(c + dx)}{2b(a + bx^2)} + \frac{3 \sinh(c + dx)}{2b^2 d} - \frac{\int \cosh(c + dx) dx}{2b^2} - \frac{(3a) \int \left(\frac{\sqrt{-a}}{2a} \right) dx}{2b^2} \\
&= \frac{x \cosh(c + dx)}{2b^2} - \frac{x^3 \cosh(c + dx)}{2b(a + bx^2)} + \frac{\sinh(c + dx)}{b^2 d} - \frac{(3\sqrt{-a}) \int \frac{\cosh(c+dx)}{\sqrt{-a}-\sqrt{b}x} dx}{4b^2} - \frac{(3\sqrt{-a}) \int \frac{\cosh(c+dx)}{\sqrt{-a}+\sqrt{b}x} dx}{4b^2} \\
&= \frac{x \cosh(c + dx)}{2b^2} - \frac{x^3 \cosh(c + dx)}{2b(a + bx^2)} + \frac{\sinh(c + dx)}{b^2 d} - \frac{\left(3\sqrt{-a} \cosh\left(c - \frac{\sqrt{-a}d}{\sqrt{b}}\right) \right) \int \frac{\cosh\left(\frac{y}{\sqrt{-a}}\right) dy}{\sqrt{-a}}}{4b^2} \\
&= \frac{x \cosh(c + dx)}{2b^2} - \frac{x^3 \cosh(c + dx)}{2b(a + bx^2)} + \frac{3\sqrt{-a} \cosh\left(c + \frac{\sqrt{-a}d}{\sqrt{b}}\right) \operatorname{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}} - dx\right)}{4b^{5/2}} - \frac{3\sqrt{-a} \cosh\left(c - \frac{\sqrt{-a}d}{\sqrt{b}}\right) \operatorname{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}} + dx\right)}{4b^{5/2}}
\end{aligned}$$

Mathematica [C] time = 1.63, size = 621, normalized size = 1.38

$$\frac{3\sqrt{a} \sinh(c) \left(\sin\left(\frac{\sqrt{ad}}{\sqrt{b}}\right) \text{Ci}\left(ixd - \frac{\sqrt{ad}}{\sqrt{b}}\right) + \sin\left(\frac{\sqrt{ad}}{\sqrt{b}}\right) \text{Ci}\left(ixd + \frac{\sqrt{ad}}{\sqrt{b}}\right) - \cos\left(\frac{\sqrt{ad}}{\sqrt{b}}\right) \left(\text{Si}\left(\frac{\sqrt{ad}}{\sqrt{b}} - ixd\right) + \text{Si}\left(ixd + \frac{\sqrt{ad}}{\sqrt{b}}\right) \right) \right)}{\sqrt{b}} - \frac{ad \sinh(c) \left(\cos\left(\frac{\sqrt{ad}}{\sqrt{b}}\right) \text{Ci}\left(ixd - \frac{\sqrt{ad}}{\sqrt{b}}\right) + \right)}{\sqrt{b}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(x^4*Cosh[c + d*x])/(a + b*x^2)^2,x]

[Out] (2*Cosh[d*x]*((a*x*Cosh[c])/(a + b*x^2) + (2*Sinh[c])/d) + 2*((2*Cosh[c])/d + (a*x*Sinh[c])/(a + b*x^2))*Sinh[d*x] - ((3*I)*Sqrt[a]*Cosh[c]*(Cos[(Sqrt[a]*d)/Sqrt[b]]*CosIntegral[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x] - Cos[(Sqrt[a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x] + Sin[(Sqrt[a]*d)/Sqrt[b]]*(SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d*x] - SinIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x]))) / Sqrt[b] + (I*a*d*Cosh[c]*(CosIntegral[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x]*Sin[(Sqrt[a]*d)/Sqrt[b]] - CosIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x]*Sin[(Sqrt[a]*d)/Sqrt[b]] + Cos[(Sqrt[a]*d)/Sqrt[b]]*(-SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d*x] + SinIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x])))/b - (3*Sqrt[a]*Sinh[c]*(CosIntegral[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x]*Sin[(Sqrt[a]*d)/Sqrt[b]] + CosIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x]*Sin[(Sqrt[a]*d)/Sqrt[b]] - Cos[(Sqrt[a]*d)/Sqrt[b]]*(SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d*x] + SinIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x])))/Sqrt[b] - (a*d*Sinh[c]*(Cos[(Sqrt[a]*d)/Sqrt[b]]*CosIntegral[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x] + Cos[(Sqrt[a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x] + Sin[(Sqrt[a]*d)/Sqrt[b]]*(SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d*x] + SinIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x])))/b)/(4*b^2)

fricas [B] time = 0.64, size = 1179, normalized size = 2.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*cosh(d*x+c)/(b*x^2+a)^2,x, algorithm="fricas")

[Out] 1/8*(4*a*b*d*x*cosh(d*x + c) - (((a*b*d^2*x^2 + a^2*d^2)*cosh(d*x + c)^2 - (a*b*d^2*x^2 + a^2*d^2)*sinh(d*x + c)^2 - 3*((b^2*x^2 + a*b)*cosh(d*x + c)^2 - (b^2*x^2 + a*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x - sqrt(-a*d^2/b)) - ((a*b*d^2*x^2 + a^2*d^2)*cosh(d*x + c)^2 - (a*b*d^2*x^2 + a^2*d^2)*sinh(d*x + c)^2 + 3*((b^2*x^2 + a*b)*cosh(d*x + c)^2 - (b^2*x^2 + a*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x + sqrt(-a*d^2/b)))*cosh(c + sqrt(-a*d^2/b)) - (((a*b*d^2*x^2 + a^2*d^2)*cosh(d*x + c)^2 - (a*b*d^2*x^2 + a^2*d^2)*sinh(d*x + c)^2 + 3*((b^2*x^2 + a*b)*cosh(d*x + c)^2 - (b^2*x^2 + a*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x + sqrt(-a*d^2/b)) - ((a*b*d^2*x^2 + a^2*d^2)*cosh(d*x + c)^2 - (a*b*d^2*x^2 + a^2*d^2)*sinh(d*x + c)^2 - 3*((b^2*x^2 + a*b)*cosh(d*x + c)^2 - (b^2*x^2 + a*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x - sqrt(-a*d^2/b)))/b

$$\begin{aligned}
& + a*b)*\cosh(d*x + c)^2 - (b^2*x^2 + a*b)*\sinh(d*x + c)^2*\sqrt{-a*d^2/b))* \\
& \text{Ei}(-d*x - \sqrt{-a*d^2/b}))*\cosh(-c + \sqrt{-a*d^2/b}) + 8*(b^2*x^2 + a*b)*\text{si} \\
& \text{nh}(d*x + c) - (((a*b*d^2*x^2 + a^2*d^2)*\cosh(d*x + c)^2 - (a*b*d^2*x^2 + a^ \\
& 2*d^2)*\sinh(d*x + c)^2 - 3*((b^2*x^2 + a*b)*\cosh(d*x + c)^2 - (b^2*x^2 + a* \\
& b)*\sinh(d*x + c)^2)*\sqrt{-a*d^2/b}))*\text{Ei}(d*x - \sqrt{-a*d^2/b}) + ((a*b*d^2*x^ \\
& 2 + a^2*d^2)*\cosh(d*x + c)^2 - (a*b*d^2*x^2 + a^2*d^2)*\sinh(d*x + c)^2 + 3* \\
& ((b^2*x^2 + a*b)*\cosh(d*x + c)^2 - (b^2*x^2 + a*b)*\sinh(d*x + c)^2)*\sqrt{-a \\
& *d^2/b}))*\text{Ei}(-d*x + \sqrt{-a*d^2/b}))*\sinh(c + \sqrt{-a*d^2/b}) + (((a*b*d^2*x \\
& ^2 + a^2*d^2)*\cosh(d*x + c)^2 - (a*b*d^2*x^2 + a^2*d^2)*\sinh(d*x + c)^2 + 3 \\
& *((b^2*x^2 + a*b)*\cosh(d*x + c)^2 - (b^2*x^2 + a*b)*\sinh(d*x + c)^2)*\sqrt{- \\
& a*d^2/b}))*\text{Ei}(d*x + \sqrt{-a*d^2/b}) + ((a*b*d^2*x^2 + a^2*d^2)*\cosh(d*x + c) \\
& ^2 - (a*b*d^2*x^2 + a^2*d^2)*\sinh(d*x + c)^2 - 3*((b^2*x^2 + a*b)*\cosh(d*x \\
& + c)^2 - (b^2*x^2 + a*b)*\sinh(d*x + c)^2)*\sqrt{-a*d^2/b}))*\text{Ei}(-d*x - \sqrt{-a \\
& *d^2/b}))*\sinh(-c + \sqrt{-a*d^2/b}))/((b^4*d*x^2 + a*b^3*d)*\cosh(d*x + c)^2 \\
& - (b^4*d*x^2 + a*b^3*d)*\sinh(d*x + c)^2)
\end{aligned}$$

giac [F] time = 0.00, size = 0, normalized size = 0.00

sage0x

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*cosh(d*x+c)/(b*x^2+a)^2,x, algorithm="giac")

[Out] sage0*x

maple [A] time = 0.41, size = 532, normalized size = 1.18

$$\frac{d^2 e^{-dx-c} a x}{4b^2 (b d^2 x^2 + a d^2)} - \frac{e^{-dx-c}}{2d b^2} - \frac{d a e^{-\frac{d\sqrt{-ab}+cb}{b}} \text{Ei}\left(1, -\frac{d\sqrt{-ab}-(dx+c)b+cb}{b}\right)}{8b^3} - \frac{d a e^{-\frac{-d\sqrt{-ab}+cb}{b}} \text{Ei}\left(1, \frac{d\sqrt{-ab}+(dx+c)b-cb}{b}\right)}{8b^3} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4*cosh(d*x+c)/(b*x^2+a)^2,x)

[Out] $\frac{1}{4}d^2 \exp(-d*x-c)*a/b^2/(b*d^2*x^2+a*d^2)*x - \frac{1}{2}d \exp(-d*x-c)/b^2 - \frac{1}{8}d/b^3 * a \exp(-\frac{d*(-a*b)^{1/2}+c*b}{b}) * \text{Ei}\left(1, -\frac{d*(-a*b)^{1/2}-(d*x+c)*b+c*b}{b}\right) - \frac{1}{8}d/b^3 * a \exp(-\frac{-d*(-a*b)^{1/2}+c*b}{b}) * \text{Ei}\left(1, \frac{d*(-a*b)^{1/2}+(d*x+c)*b-c*b}{b}\right) + \frac{3}{8}d/b^2 * a / (-a*b)^{1/2} * \exp(-\frac{d*(-a*b)^{1/2}+c*b}{b}) * \text{Ei}\left(1, -\frac{d*(-a*b)^{1/2}-(d*x+c)*b+c*b}{b}\right) - \frac{3}{8}d/b^2 * a / (-a*b)^{1/2} * \exp(-\frac{-d*(-a*b)^{1/2}+c*b}{b}) * \text{Ei}\left(1, \frac{d*(-a*b)^{1/2}+(d*x+c)*b-c*b}{b}\right) + \frac{1}{2}d/b^2 \exp(d*x+c) + \frac{1}{4}d^2 \exp(d*x+c) * a/b^2 / (b*d^2*x^2+a*d^2) * x + \frac{1}{8}d/b^3 * a \exp(\frac{-d*(-a*b)^{1/2}+c*b}{b}) * \text{Ei}\left(1, -\frac{d*(-a*b)^{1/2}+(d*x+c)*b-c*b}{b}\right) + \frac{1}{8}d/b^3 * a \exp(\frac{d*(-a*b)^{1/2}+c*b}{b}) * \text{Ei}\left(1, \frac{d*(-a*b)^{1/2}-(d*x+c)*b+c*b}{b}\right) - \frac{3}{8}d/b^2 * a / (-a*b)^{1/2} * \exp(\frac{-d*(-a*b)^{1/2}+c*b}{b}) * \text{Ei}\left(1, -\frac{d*(-a*b)^{1/2}+(d*x+c)*b-c*b}{b}\right) + \frac{3}{8}d/b^2 * a / (-a*b)^{1/2} * \exp(\frac{d*(-a*b)^{1/2}+c*b}{b}) * \text{Ei}\left(1, \frac{d*(-a*b)^{1/2}-(d*x+c)*b+c*b}{b}\right)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{(bdx^4e^{(2c)} - 4axe^{(2c)})e^{(dx)} - (bdx^4 + 4ax)e^{(-dx)}}{2(b^3d^2x^4e^c + 2ab^2d^2x^2e^c + a^2bd^2e^c)} - \frac{1}{2} \int -\frac{4(a^2dxe^c - 3abx^2e^c + a^2e^c)e^{(dx)}}{b^4d^2x^6 + 3ab^3d^2x^4 + 3a^2b^2d^2x^2 + a^3bd^2} dx - \frac{1}{2} \int \frac{1}{b^4d^2x^6 + 3ab^3d^2x^4 + 3a^2b^2d^2x^2 + a^3bd^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*cosh(d*x+c)/(b*x^2+a)^2,x, algorithm="maxima")

[Out] 1/2*((b*d*x^4*e^(2*c) - 4*a*x*e^(2*c))*e^(d*x) - (b*d*x^4 + 4*a*x)*e^(-d*x))/ (b^3*d^2*x^4*e^c + 2*a*b^2*d^2*x^2*e^c + a^2*b*d^2*e^c) - 1/2*integrate(-4*(a^2*d*x*e^c - 3*a*b*x^2*e^c + a^2*e^c)*e^(d*x)/(b^4*d^2*x^6 + 3*a*b^3*d^2*x^4 + 3*a^2*b^2*d^2*x^2 + a^3*b*d^2), x) - 1/2*integrate(4*(a^2*d*x + 3*a*b*x^2 - a^2)*e^(-d*x)/(b^4*d^2*x^6*e^c + 3*a*b^3*d^2*x^4*e^c + 3*a^2*b^2*d^2*x^2*e^c + a^3*b*d^2*e^c), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4 \cosh(c + dx)}{(bx^2 + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4*cosh(c + d*x))/(a + b*x^2)^2,x)

[Out] int((x^4*cosh(c + d*x))/(a + b*x^2)^2, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 \cosh(c + dx)}{(a + bx^2)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4*cosh(d*x+c)/(b*x**2+a)**2,x)

[Out] Integral(x**4*cosh(c + d*x)/(a + b*x**2)**2, x)

$$3.66 \quad \int \frac{x^3 \cosh(c+dx)}{(a+bx^2)^2} dx$$

Optimal. Leaf size=431

$$\frac{\sqrt{-a} d \sinh\left(c - \frac{\sqrt{-a}d}{\sqrt{b}}\right) \text{Chi}\left(xd + \frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4b^{5/2}} + \frac{\sqrt{-a} d \sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}} + c\right) \text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}} - dx\right)}{4b^{5/2}} - \frac{\sqrt{-a} d \cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}} + c\right) \text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}} - dx\right)}{4b^{5/2}}$$

[Out] $\frac{1}{2} \frac{\cosh(dx+c)}{b^2 - 1/2 x^2 \cosh(dx+c)/b} + \frac{1}{2} \frac{\text{Chi}(dx+d(-a)^{1/2})}{b^{1/2}} \frac{\cosh(c-d(-a)^{1/2}/b^{1/2})}{b^2 + 1/2 \text{Chi}(-dx+d(-a)^{1/2}/b^{1/2})} \frac{\cosh(c+d(-a)^{1/2}/b^{1/2})}{b^2 + 1/2 \text{Shi}(dx+d(-a)^{1/2}/b^{1/2})} \frac{\sinh(c-d(-a)^{1/2}/b^{1/2})}{b^2 + 1/2 \text{Shi}(dx-d(-a)^{1/2}/b^{1/2})} \frac{\sinh(c+d(-a)^{1/2}/b^{1/2})}{b^2 + 1/4 d \cosh(c+d(-a)^{1/2}/b^{1/2})} \frac{\text{Shi}(dx-d(-a)^{1/2}/b^{1/2})}{b^{5/2}} \frac{\sinh(c-d(-a)^{1/2}/b^{1/2})}{b^2 + 1/4 d \cosh(c-d(-a)^{1/2}/b^{1/2})} \frac{\text{Shi}(dx+d(-a)^{1/2}/b^{1/2})}{b^{5/2}} - \frac{1}{4} d \frac{\text{Chi}(dx+d(-a)^{1/2}/b^{1/2})}{b^{5/2}} \frac{\sinh(c-d(-a)^{1/2}/b^{1/2})}{b^2 + 1/4 d \text{Chi}(-dx+d(-a)^{1/2}/b^{1/2})} \frac{\sinh(c+d(-a)^{1/2}/b^{1/2})}{b^2 + 1/4 d \text{Chi}(-dx+d(-a)^{1/2}/b^{1/2})} \frac{\sinh(c+d(-a)^{1/2}/b^{1/2})}{b^2 + 1/4 d \text{Chi}(-dx+d(-a)^{1/2}/b^{1/2})} \frac{\sinh(c+d(-a)^{1/2}/b^{1/2})}{b^2 + 1/4 d \text{Chi}(-dx+d(-a)^{1/2}/b^{1/2})}$

Rubi [A] time = 0.70, antiderivative size = 431, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {5291, 5293, 3303, 3298, 3301, 5292, 2638, 5280}

$$\frac{\sqrt{-a} d \sinh\left(c - \frac{\sqrt{-a}d}{\sqrt{b}}\right) \text{Chi}\left(xd + \frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4b^{5/2}} + \frac{\sqrt{-a} d \sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}} + c\right) \text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}} - dx\right)}{4b^{5/2}} + \frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}} + c\right) \text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}} - dx\right)}{2b^2}$$

Antiderivative was successfully verified.

[In] Int[(x^3*Cosh[c + d*x])/(a + b*x^2)^2,x]

[Out] $\frac{\text{Cosh}[c + d*x]}{(2*b^2) - (x^2*\text{Cosh}[c + d*x])/(2*b*(a + b*x^2))} + (\text{Cosh}[c + (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{CoshIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] - d*x])/(2*b^2) + (\text{Cosh}[c - (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{CoshIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] + d*x])/(2*b^2) - (\text{Sqrt}[-a]*d*\text{CoshIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] + d*x]*\text{Sinh}[c - (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]])/(4*b^{5/2}) + (\text{Sqrt}[-a]*d*\text{CoshIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] - d*x]*\text{Sinh}[c + (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]])/(4*b^{5/2}) - (\text{Sqrt}[-a]*d*\text{Cosh}[c + (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{SinhIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] - d*x])/(4*b^{5/2}) - (\text{Sinh}[c + (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{SinhIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] - d*x])/(2*b^2) - (\text{Sqrt}[-a]*d*\text{Cosh}[c - (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{SinhIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] + d*x])/(4*b^{5/2}) + (\text{Sinh}[c - (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{SinhIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] + d*x])/(2*b^2)$

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:= Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f,
fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:= Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz},
x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

Rule 5280

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Symbol] := In
t[ExpandIntegrand[Sinh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 5291

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] := Simp[(x^(m - n + 1)*(a + b*x^n)^(p + 1)*Cosh[c + d*x])/(b*n*(p +
1)), x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p +
1)*Cosh[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^
n)^(p + 1)*Sinh[c + d*x], x], x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1]
&& IGtQ[n, 0] && RationalQ[m] && (GtQ[m - n + 1, 0] || GtQ[n, 2])
```

Rule 5292

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Sy
mbol] := Int[ExpandIntegrand[Sinh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rule 5293

Int[Cosh[(c_.) + (d_.)*(x_.)]*(x_)^(m_.)*((a_.) + (b_.)*(x_)^(n_.))^p_, x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])

Rubi steps

$$\begin{aligned}
\int \frac{x^3 \cosh(c + dx)}{(a + bx^2)^2} dx &= -\frac{x^2 \cosh(c + dx)}{2b(a + bx^2)} + \frac{\int \frac{x \cosh(c+dx)}{a+bx^2} dx}{b} + \frac{d \int \frac{x^2 \sinh(c+dx)}{a+bx^2} dx}{2b} \\
&= -\frac{x^2 \cosh(c + dx)}{2b(a + bx^2)} + \frac{\int \left(-\frac{\cosh(c+dx)}{2\sqrt{b}(\sqrt{-a}-\sqrt{b}x)} + \frac{\cosh(c+dx)}{2\sqrt{b}(\sqrt{-a}+\sqrt{b}x)} \right) dx}{b} + \frac{d \int \left(\frac{\sinh(c+dx)}{b} - \frac{a \sinh(c+dx)}{b(a+bx^2)} \right) dx}{2b} \\
&= -\frac{x^2 \cosh(c + dx)}{2b(a + bx^2)} - \frac{\int \frac{\cosh(c+dx)}{\sqrt{-a}-\sqrt{b}x} dx}{2b^{3/2}} + \frac{\int \frac{\cosh(c+dx)}{\sqrt{-a}+\sqrt{b}x} dx}{2b^{3/2}} + \frac{d \int \sinh(c + dx) dx}{2b^2} - \frac{(ad) \int \frac{\sinh(c+dx)}{a+bx^2} dx}{2b} \\
&= \frac{\cosh(c + dx)}{2b^2} - \frac{x^2 \cosh(c + dx)}{2b(a + bx^2)} - \frac{(ad) \int \left(\frac{\sqrt{-a} \sinh(c+dx)}{2a(\sqrt{-a}-\sqrt{b}x)} + \frac{\sqrt{-a} \sinh(c+dx)}{2a(\sqrt{-a}+\sqrt{b}x)} \right) dx}{2b^2} + \frac{\cosh(c + dx)}{2b^2} \\
&= \frac{\cosh(c + dx)}{2b^2} - \frac{x^2 \cosh(c + dx)}{2b(a + bx^2)} + \frac{\cosh\left(c + \frac{\sqrt{-a}d}{\sqrt{b}}\right) \text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}} - dx\right)}{2b^2} + \frac{\cosh\left(c - \frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b^2} \\
&= \frac{\cosh(c + dx)}{2b^2} - \frac{x^2 \cosh(c + dx)}{2b(a + bx^2)} + \frac{\cosh\left(c + \frac{\sqrt{-a}d}{\sqrt{b}}\right) \text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}} - dx\right)}{2b^2} + \frac{\cosh\left(c - \frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b^2} \\
&= \frac{\cosh(c + dx)}{2b^2} - \frac{x^2 \cosh(c + dx)}{2b(a + bx^2)} + \frac{\cosh\left(c + \frac{\sqrt{-a}d}{\sqrt{b}}\right) \text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}} - dx\right)}{2b^2} + \frac{\cosh\left(c - \frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b^2}
\end{aligned}$$

Mathematica [C] time = 0.79, size = 582, normalized size = 1.35

$$a^{3/2}d \cosh\left(c - \frac{i\sqrt{a}d}{\sqrt{b}}\right) \text{Si}\left(\frac{\sqrt{a}d}{\sqrt{b}} - idx\right) + a^{3/2}d \cosh\left(c + \frac{i\sqrt{a}d}{\sqrt{b}}\right) \text{Si}\left(ixd + \frac{\sqrt{a}d}{\sqrt{b}}\right) + 2ib^{3/2}x^2 \sinh\left(c - \frac{i\sqrt{a}d}{\sqrt{b}}\right) \text{Si}\left(\frac{\sqrt{a}d}{\sqrt{b}} - idx\right) - 2ib^{3/2}x^2 \sinh\left(c + \frac{i\sqrt{a}d}{\sqrt{b}}\right) \text{Si}\left(ixd + \frac{\sqrt{a}d}{\sqrt{b}}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*Cosh[c + d*x])/(a + b*x^2)^2,x]
```

```
[Out] (2*a*Sqrt[b]*Cosh[c + d*x] + (a + b*x^2)*CosIntegral[-((Sqrt[a]*d)/Sqrt[b])
+ I*d*x]*(2*Sqrt[b]*Cosh[c - (I*Sqrt[a]*d)/Sqrt[b]] - I*Sqrt[a]*d*Sinh[c -
(I*Sqrt[a]*d)/Sqrt[b]]) + (a + b*x^2)*CosIntegral[(Sqrt[a]*d)/Sqrt[b] + I*
d*x]*(2*Sqrt[b]*Cosh[c + (I*Sqrt[a]*d)/Sqrt[b]] + I*Sqrt[a]*d*Sinh[c + (I*S
qrt[a]*d)/Sqrt[b]]) + a^(3/2)*d*Cosh[c - (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral
[(Sqrt[a]*d)/Sqrt[b] - I*d*x] + Sqrt[a]*b*d*x^2*Cosh[c - (I*Sqrt[a]*d)/Sqrt
[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d*x] + (2*I)*a*Sqrt[b]*Sinh[c - (I
*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d*x] + (2*I)*b^(3/
2)*x^2*Sinh[c - (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*
d*x] + a^(3/2)*d*Cosh[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sq
rt[b] + I*d*x] + Sqrt[a]*b*d*x^2*Cosh[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegra
l[(Sqrt[a]*d)/Sqrt[b] + I*d*x] - (2*I)*a*Sqrt[b]*Sinh[c + (I*Sqrt[a]*d)/Sqr
t[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x] - (2*I)*b^(3/2)*x^2*Sinh[c +
(I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x])/(4*b^(5/2)
)*(a + b*x^2))
```

fricas [B] time = 0.52, size = 931, normalized size = 2.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(d*x+c)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*a*cosh(d*x + c) + ((2*(b*x^2 + a)*cosh(d*x + c)^2 - 2*(b*x^2 + a)*si
nh(d*x + c)^2 + ((b*x^2 + a)*cosh(d*x + c)^2 - (b*x^2 + a)*sinh(d*x + c)^2)
*sqrt(-a*d^2/b))*Ei(d*x - sqrt(-a*d^2/b)) + (2*(b*x^2 + a)*cosh(d*x + c)^2
- 2*(b*x^2 + a)*sinh(d*x + c)^2 - ((b*x^2 + a)*cosh(d*x + c)^2 - (b*x^2 + a
)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x + sqrt(-a*d^2/b))*cosh(c + sqrt
(-a*d^2/b)) + ((2*(b*x^2 + a)*cosh(d*x + c)^2 - 2*(b*x^2 + a)*sinh(d*x + c)
^2 - ((b*x^2 + a)*cosh(d*x + c)^2 - (b*x^2 + a)*sinh(d*x + c)^2)*sqrt(-a*d^
2/b))*Ei(d*x + sqrt(-a*d^2/b)) + (2*(b*x^2 + a)*cosh(d*x + c)^2 - 2*(b*x^2
+ a)*sinh(d*x + c)^2 + ((b*x^2 + a)*cosh(d*x + c)^2 - (b*x^2 + a)*sinh(d*x
+ c)^2)*sqrt(-a*d^2/b))*Ei(-d*x - sqrt(-a*d^2/b))*cosh(-c + sqrt(-a*d^2/b)
) + ((2*(b*x^2 + a)*cosh(d*x + c)^2 - 2*(b*x^2 + a)*sinh(d*x + c)^2 + ((b*x
^2 + a)*cosh(d*x + c)^2 - (b*x^2 + a)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d
*x - sqrt(-a*d^2/b)) - (2*(b*x^2 + a)*cosh(d*x + c)^2 - 2*(b*x^2 + a)*sinh(
d*x + c)^2 - ((b*x^2 + a)*cosh(d*x + c)^2 - (b*x^2 + a)*sinh(d*x + c)^2)*sq
rt(-a*d^2/b))*Ei(-d*x + sqrt(-a*d^2/b))*sinh(c + sqrt(-a*d^2/b)) - ((2*(b*
x^2 + a)*cosh(d*x + c)^2 - 2*(b*x^2 + a)*sinh(d*x + c)^2 - ((b*x^2 + a)*cos
h(d*x + c)^2 - (b*x^2 + a)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x + sqrt(-
a*d^2/b)) - (2*(b*x^2 + a)*cosh(d*x + c)^2 - 2*(b*x^2 + a)*sinh(d*x + c)^2
+ ((b*x^2 + a)*cosh(d*x + c)^2 - (b*x^2 + a)*sinh(d*x + c)^2)*sqrt(-a*d^2/b
))*Ei(-d*x - sqrt(-a*d^2/b))*sinh(-c + sqrt(-a*d^2/b)))/((b^3*x^2 + a*b^2)
*cosh(d*x + c)^2 - (b^3*x^2 + a*b^2)*sinh(d*x + c)^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \cosh(dx + c)}{(bx^2 + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*cosh(d*x+c)/(b*x^2+a)^2,x, algorithm="giac")

[Out] integrate(x^3*cosh(d*x + c)/(b*x^2 + a)^2, x)

maple [A] time = 0.28, size = 495, normalized size = 1.15

$$\frac{d^2 e^{-dx-c} a}{4b^2 (b d^2 x^2 + a d^2)} - \frac{d a e^{-\frac{d\sqrt{-ab}+cb}{b}} \operatorname{Ei}\left(1, -\frac{d\sqrt{-ab}-(dx+c)b+cb}{b}\right)}{8b^2 \sqrt{-ab}} + \frac{d a e^{-\frac{-d\sqrt{-ab}+cb}{b}} \operatorname{Ei}\left(1, \frac{d\sqrt{-ab}+(dx+c)b-cb}{b}\right)}{8b^2 \sqrt{-ab}} - \frac{e^{-\frac{d\sqrt{-ab}+cb}{b}} \operatorname{Ei}\left(1, -\frac{d\sqrt{-ab}-(dx+c)b+cb}{b}\right)}{8b^2 \sqrt{-ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*cosh(d*x+c)/(b*x^2+a)^2,x)

[Out] $\frac{1}{4} d^2 \exp(-dx-c) a / b^2 / (b d^2 x^2 + a d^2) - \frac{1}{8} d / b^2 a / (-a b)^{1/2} \exp(-d(-a b)^{1/2} + c b) / b * \operatorname{Ei}\left(1, -\frac{d(-a b)^{1/2} - (d x + c) b + c b}{b}\right) + \frac{1}{8} d / b^2 a / (-a b)^{1/2} \exp(-d(-a b)^{1/2} + c b) / b * \operatorname{Ei}\left(1, \frac{d(-a b)^{1/2} + (d x + c) b - c b}{b}\right) - \frac{1}{4} / b^2 \exp(-d(-a b)^{1/2} + c b) / b * \operatorname{Ei}\left(1, -\frac{d(-a b)^{1/2} - (d x + c) b + c b}{b}\right) - \frac{1}{4} / b^2 \exp(-d(-a b)^{1/2} + c b) / b * \operatorname{Ei}\left(1, \frac{d(-a b)^{1/2} + (d x + c) b - c b}{b}\right) + \frac{1}{4} d^2 \exp(dx+c) a / b^2 / (b d^2 x^2 + a d^2) - \frac{1}{4} / b^2 \exp((d(-a b)^{1/2} + c b) / b) * \operatorname{Ei}\left(1, \frac{d(-a b)^{1/2} - (d x + c) b + c b}{b}\right) - \frac{1}{4} / b^2 \exp((d(-a b)^{1/2} + c b) / b) * \operatorname{Ei}\left(1, -\frac{d(-a b)^{1/2} + (d x + c) b - c b}{b}\right) + \frac{1}{8} d / b^2 a / (-a b)^{1/2} \exp((d(-a b)^{1/2} + c b) / b) * \operatorname{Ei}\left(1, \frac{d(-a b)^{1/2} - (d x + c) b + c b}{b}\right) - \frac{1}{8} d / b^2 a / (-a b)^{1/2} \exp((d(-a b)^{1/2} + c b) / b) * \operatorname{Ei}\left(1, -\frac{d(-a b)^{1/2} + (d x + c) b - c b}{b}\right)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{(d^2 x^3 e^{(2c)} + dx^2 e^{(2c)} + 2 x e^{(2c)}) e^{(dx)} - (d^2 x^3 - dx^2 + 2 x) e^{(-dx)}}{2 (b^2 d^3 x^4 e^c + 2 a b d^3 x^2 e^c + a^2 d^3 e^c)} - \frac{1}{2} \int \frac{2 (2 a d x e^c + (2 a d^2 e^c - 3 b e^c) x^2 + a e^c) e^{(dx)}}{b^3 d^3 x^6 + 3 a b^2 d^3 x^4 + 3 a^2 b d^3 x^2 + a^3 d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*cosh(d*x+c)/(b*x^2+a)^2,x, algorithm="maxima")

[Out] $\frac{1}{2} * ((d^2 x^3 e^{(2c)} + dx^2 e^{(2c)} + 2 x e^{(2c)}) * e^{(dx)} - (d^2 x^3 - dx^2 + 2 x) * e^{(-dx)}) / (b^2 d^3 x^4 e^c + 2 a b d^3 x^2 e^c + a^2 d^3 e^c) - \frac{1}{2} * \operatorname{integrate}(2 * (2 a d x e^c + (2 a d^2 e^c - 3 b e^c) x^2 + a e^c) * e^{(dx)}, x)$

)/(b^3*d^3*x^6 + 3*a*b^2*d^3*x^4 + 3*a^2*b*d^3*x^2 + a^3*d^3), x) + 1/2*integrate(-2*(2*a*d*x - (2*a*d^2 - 3*b)*x^2 - a)*e^(-d*x)/(b^3*d^3*x^6*e^c + 3*a*b^2*d^3*x^4*e^c + 3*a^2*b*d^3*x^2*e^c + a^3*d^3*e^c), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3 \cosh(c + dx)}{(bx^2 + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3*cosh(c + d*x))/(a + b*x^2)^2,x)

[Out] int((x^3*cosh(c + d*x))/(a + b*x^2)^2, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \cosh(c + dx)}{(a + bx^2)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*cosh(d*x+c)/(b*x**2+a)**2,x)

[Out] Integral(x**3*cosh(c + d*x)/(a + b*x**2)**2, x)

$$3.67 \quad \int \frac{x^2 \cosh(c+dx)}{(a+bx^2)^2} dx$$

Optimal. Leaf size=416

$$\frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}} + c\right) \operatorname{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}} - dx\right)}{4\sqrt{-a}b^{3/2}} - \frac{\cosh\left(c - \frac{\sqrt{-a}d}{\sqrt{b}}\right) \operatorname{Chi}\left(xd + \frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4\sqrt{-a}b^{3/2}} - \frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}} + c\right) \operatorname{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}} - dx\right)}{4\sqrt{-a}b^{3/2}} - \frac{\sinh\left(c - \frac{\sqrt{-a}d}{\sqrt{b}}\right) \operatorname{Shi}\left(xd + \frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4\sqrt{-a}b^{3/2}}$$

[Out] $-1/2*x*\cosh(d*x+c)/b/(b*x^2+a)+1/4*d*\cosh(c+d*(-a)^{(1/2)}/b^{(1/2)})*\operatorname{Shi}(d*x-d*(-a)^{(1/2)}/b^{(1/2)})/b^2+1/4*d*\cosh(c-d*(-a)^{(1/2)}/b^{(1/2)})*\operatorname{Shi}(d*x+d*(-a)^{(1/2)}/b^{(1/2)})/b^2+1/4*d*\operatorname{Chi}(d*x+d*(-a)^{(1/2)}/b^{(1/2)})*\sinh(c-d*(-a)^{(1/2)}/b^{(1/2)})/b^2+1/4*d*\operatorname{Chi}(-d*x+d*(-a)^{(1/2)}/b^{(1/2)})*\sinh(c+d*(-a)^{(1/2)}/b^{(1/2)})/b^2-1/4*\operatorname{Chi}(d*x+d*(-a)^{(1/2)}/b^{(1/2)})*\cosh(c-d*(-a)^{(1/2)}/b^{(1/2)})/b^{(3/2)}/(-a)^{(1/2)}+1/4*\operatorname{Chi}(-d*x+d*(-a)^{(1/2)}/b^{(1/2)})*\cosh(c+d*(-a)^{(1/2)}/b^{(1/2)})/b^{(3/2)}/(-a)^{(1/2)}-1/4*\operatorname{Shi}(d*x+d*(-a)^{(1/2)}/b^{(1/2)})*\sinh(c-d*(-a)^{(1/2)}/b^{(1/2)})/b^{(3/2)}/(-a)^{(1/2)}+1/4*\operatorname{Shi}(d*x-d*(-a)^{(1/2)}/b^{(1/2)})*\sinh(c+d*(-a)^{(1/2)}/b^{(1/2)})/b^{(3/2)}/(-a)^{(1/2)}$

Rubi [A] time = 0.61, antiderivative size = 416, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {5291, 5281, 3303, 3298, 3301, 5292}

$$\frac{d \sinh\left(c - \frac{\sqrt{-a}d}{\sqrt{b}}\right) \operatorname{Chi}\left(xd + \frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4b^2} + \frac{d \sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}} + c\right) \operatorname{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}} - dx\right)}{4b^2} + \frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}} + c\right) \operatorname{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}} - dx\right)}{4\sqrt{-a}b^{3/2}} - \frac{\cosh\left(c - \frac{\sqrt{-a}d}{\sqrt{b}}\right) \operatorname{Chi}\left(xd + \frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4\sqrt{-a}b^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(x^2*Cosh[c + d*x])/(a + b*x^2)^2,x]

[Out] $-(x*\operatorname{Cosh}[c + d*x])/(2*b*(a + b*x^2)) + (\operatorname{Cosh}[c + (\operatorname{Sqrt}[-a]*d)/\operatorname{Sqrt}[b]]*\operatorname{CoshIntegral}[(\operatorname{Sqrt}[-a]*d)/\operatorname{Sqrt}[b] - d*x])/(4*\operatorname{Sqrt}[-a]*b^{(3/2)}) - (\operatorname{Cosh}[c - (\operatorname{Sqrt}[-a]*d)/\operatorname{Sqrt}[b]]*\operatorname{CoshIntegral}[(\operatorname{Sqrt}[-a]*d)/\operatorname{Sqrt}[b] + d*x])/(4*\operatorname{Sqrt}[-a]*b^{(3/2)}) + (d*\operatorname{CoshIntegral}[(\operatorname{Sqrt}[-a]*d)/\operatorname{Sqrt}[b] + d*x]*\operatorname{Sinh}[c - (\operatorname{Sqrt}[-a]*d)/\operatorname{Sqrt}[b]])/(4*b^2) + (d*\operatorname{CoshIntegral}[(\operatorname{Sqrt}[-a]*d)/\operatorname{Sqrt}[b] - d*x]*\operatorname{Sinh}[c + (\operatorname{Sqrt}[-a]*d)/\operatorname{Sqrt}[b]])/(4*b^2) - (d*\operatorname{Cosh}[c + (\operatorname{Sqrt}[-a]*d)/\operatorname{Sqrt}[b]]*\operatorname{SinhIntegral}[(\operatorname{Sqrt}[-a]*d)/\operatorname{Sqrt}[b] - d*x])/(4*b^2) - (\operatorname{Sinh}[c + (\operatorname{Sqrt}[-a]*d)/\operatorname{Sqrt}[b]]*\operatorname{SinhIntegral}[(\operatorname{Sqrt}[-a]*d)/\operatorname{Sqrt}[b] - d*x])/(4*\operatorname{Sqrt}[-a]*b^{(3/2)}) + (d*\operatorname{Cosh}[c - (\operatorname{Sqrt}[-a]*d)/\operatorname{Sqrt}[b]]*\operatorname{SinhIntegral}[(\operatorname{Sqrt}[-a]*d)/\operatorname{Sqrt}[b] + d*x])/(4*b^2) - (\operatorname{Sinh}[c - (\operatorname{Sqrt}[-a]*d)/\operatorname{Sqrt}[b]]*\operatorname{SinhIntegral}[(\operatorname{Sqrt}[-a]*d)/\operatorname{Sqrt}[b] + d*x])/(4*\operatorname{Sqrt}[-a]*b^{(3/2)})$

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x]
&& EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x]
&& EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d],
Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]
```

Rule 5281

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Int[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d}, x]
&& ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 5291

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[(x^(m - n + 1)*(a + b*x^n)^(p + 1)*Cosh[c + d*x])/(b*n*(p + 1)), x] + (-Dist[(m - n + 1)/(b*n*(p + 1)),
Int[x^(m - n)*(a + b*x^n)^(p + 1)*Cosh[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^n)^(p + 1)*Sinh[c + d*x], x], x]) /; FreeQ[{a, b, c, d}, x]
&& ILtQ[p, -1] && IGtQ[n, 0] && RationalQ[m] && (GtQ[m - n + 1, 0] || GtQ[n, 2])
```

Rule 5292

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Symbol]
:> Int[ExpandIntegrand[Sinh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d}, x]
&& ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2 \cosh(c+dx)}{(a+bx^2)^2} dx &= -\frac{x \cosh(c+dx)}{2b(a+bx^2)} + \frac{\int \frac{\cosh(c+dx)}{a+bx^2} dx}{2b} + \frac{d \int \frac{x \sinh(c+dx)}{a+bx^2} dx}{2b} \\
&= -\frac{x \cosh(c+dx)}{2b(a+bx^2)} + \frac{\int \left(\frac{\sqrt{-a} \cosh(c+dx)}{2a(\sqrt{-a}-\sqrt{b}x)} + \frac{\sqrt{-a} \cosh(c+dx)}{2a(\sqrt{-a}+\sqrt{b}x)} \right) dx}{2b} + \frac{d \int \left(-\frac{\sinh(c+dx)}{2\sqrt{b}(\sqrt{-a}-\sqrt{b}x)} + \frac{\sinh(c+dx)}{2\sqrt{b}(\sqrt{-a}+\sqrt{b}x)} \right) dx}{2b} \\
&= -\frac{x \cosh(c+dx)}{2b(a+bx^2)} - \frac{\int \frac{\cosh(c+dx)}{\sqrt{-a}-\sqrt{b}x} dx}{4\sqrt{-a}b} - \frac{\int \frac{\cosh(c+dx)}{\sqrt{-a}+\sqrt{b}x} dx}{4\sqrt{-a}b} - \frac{d \int \frac{\sinh(c+dx)}{\sqrt{-a}-\sqrt{b}x} dx}{4b^{3/2}} + \frac{d \int \frac{\sinh(c+dx)}{\sqrt{-a}+\sqrt{b}x} dx}{4b^{3/2}} \\
&= -\frac{x \cosh(c+dx)}{2b(a+bx^2)} - \frac{\cosh\left(c - \frac{\sqrt{-a}d}{\sqrt{b}}\right) \int \frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x} dx}{4\sqrt{-a}b} + \frac{\left(d \cosh\left(c - \frac{\sqrt{-a}d}{\sqrt{b}}\right)\right) \int \frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}-\sqrt{b}x} dx}{4b^{3/2}} \\
&= -\frac{x \cosh(c+dx)}{2b(a+bx^2)} + \frac{\cosh\left(c + \frac{\sqrt{-a}d}{\sqrt{b}}\right) \text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4\sqrt{-a}b^{3/2}} - \frac{\cosh\left(c - \frac{\sqrt{-a}d}{\sqrt{b}}\right) \text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4\sqrt{-a}b^{3/2}}
\end{aligned}$$

Mathematica [C] time = 1.06, size = 364, normalized size = 0.88

$$(a+bx^2) \text{Ci}\left(ixd - \frac{\sqrt{ad}}{\sqrt{b}}\right) \left(\sqrt{a}d \sinh\left(c - \frac{i\sqrt{ad}}{\sqrt{b}}\right) + i\sqrt{b} \cosh\left(c - \frac{i\sqrt{ad}}{\sqrt{b}}\right)\right) + (a+bx^2) \text{Ci}\left(ixd + \frac{\sqrt{ad}}{\sqrt{b}}\right) \left(\sqrt{a}d \sinh\left(c + \frac{i\sqrt{ad}}{\sqrt{b}}\right) + i\sqrt{b} \cosh\left(c + \frac{i\sqrt{ad}}{\sqrt{b}}\right)\right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(x^2*Cosh[c + d*x])/(a + b*x^2)^2,x]

[Out] (-2*Sqrt[a]*b*x*Cosh[c + d*x] + (a + b*x^2)*CosIntegral[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x]*(I*Sqrt[b]*Cosh[c - (I*Sqrt[a]*d)/Sqrt[b]] + Sqrt[a]*d*Sinh[c - (I*Sqrt[a]*d)/Sqrt[b]]) + (a + b*x^2)*CosIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x]*((-I)*Sqrt[b]*Cosh[c + (I*Sqrt[a]*d)/Sqrt[b]] + Sqrt[a]*d*Sinh[c + (I*Sqrt[a]*d)/Sqrt[b]]) + (a + b*x^2)*(I*Sqrt[a]*d*Cosh[c - (I*Sqrt[a]*d)/Sqrt[b]] - Sqrt[b]*Sinh[c - (I*Sqrt[a]*d)/Sqrt[b]])*SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d*x] - (a + b*x^2)*(I*Sqrt[a]*d*Cosh[c + (I*Sqrt[a]*d)/Sqrt[b]] + Sqrt[b]*Sinh[c + (I*Sqrt[a]*d)/Sqrt[b]])*SinIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x)/(4*Sqrt[a]*b^2*(a + b*x^2))

fricas [B] time = 0.58, size = 1162, normalized size = 2.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*cosh(d*x+c)/(b*x^2+a)^2,x, algorithm="fricas")

[Out]
$$-1/8*(4*a*b*d*x*cosh(d*x + c) - ((a*b*d^2*x^2 + a^2*d^2)*cosh(d*x + c)^2 - (a*b*d^2*x^2 + a^2*d^2)*sinh(d*x + c)^2 - ((b^2*x^2 + a*b)*cosh(d*x + c)^2 - (b^2*x^2 + a*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x - sqrt(-a*d^2/b)) - ((a*b*d^2*x^2 + a^2*d^2)*cosh(d*x + c)^2 - (a*b*d^2*x^2 + a^2*d^2)*sinh(d*x + c)^2 + ((b^2*x^2 + a*b)*cosh(d*x + c)^2 - (b^2*x^2 + a*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x + sqrt(-a*d^2/b)))*cosh(c + sqrt(-a*d^2/b)) - (((a*b*d^2*x^2 + a^2*d^2)*cosh(d*x + c)^2 - (a*b*d^2*x^2 + a^2*d^2)*sinh(d*x + c)^2 + ((b^2*x^2 + a*b)*cosh(d*x + c)^2 - (b^2*x^2 + a*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x + sqrt(-a*d^2/b)) - ((a*b*d^2*x^2 + a^2*d^2)*cosh(d*x + c)^2 - (a*b*d^2*x^2 + a^2*d^2)*sinh(d*x + c)^2 - ((b^2*x^2 + a*b)*cosh(d*x + c)^2 - (b^2*x^2 + a*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x - sqrt(-a*d^2/b)))*cosh(-c + sqrt(-a*d^2/b)) - (((a*b*d^2*x^2 + a^2*d^2)*cosh(d*x + c)^2 - (a*b*d^2*x^2 + a^2*d^2)*sinh(d*x + c)^2 - ((b^2*x^2 + a*b)*cosh(d*x + c)^2 - (b^2*x^2 + a*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x - sqrt(-a*d^2/b)) + ((a*b*d^2*x^2 + a^2*d^2)*cosh(d*x + c)^2 - (a*b*d^2*x^2 + a^2*d^2)*sinh(d*x + c)^2 + ((b^2*x^2 + a*b)*cosh(d*x + c)^2 - (b^2*x^2 + a*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x + sqrt(-a*d^2/b)))*sinh(c + sqrt(-a*d^2/b)) + (((a*b*d^2*x^2 + a^2*d^2)*cosh(d*x + c)^2 - (a*b*d^2*x^2 + a^2*d^2)*sinh(d*x + c)^2 + ((b^2*x^2 + a*b)*cosh(d*x + c)^2 - (b^2*x^2 + a*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x + sqrt(-a*d^2/b)) + ((a*b*d^2*x^2 + a^2*d^2)*cosh(d*x + c)^2 - (a*b*d^2*x^2 + a^2*d^2)*sinh(d*x + c)^2 - ((b^2*x^2 + a*b)*cosh(d*x + c)^2 - (b^2*x^2 + a*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x - sqrt(-a*d^2/b)))*sinh(-c + sqrt(-a*d^2/b)))/((a*b^3*d*x^2 + a^2*b^2*d)*cosh(d*x + c)^2 - (a*b^3*d*x^2 + a^2*b^2*d)*sinh(d*x + c)^2)$$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \cosh(dx + c)}{(bx^2 + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*cosh(d*x+c)/(b*x^2+a)^2,x, algorithm="giac")

[Out] integrate(x^2*cosh(d*x + c)/(b*x^2 + a)^2, x)

maple [A] time = 0.23, size = 491, normalized size = 1.18

$$-\frac{d^2 e^{-dx-cx}}{4b(bd^2x^2 + ad^2)} + \frac{d e^{-\frac{d\sqrt{-ab}+cb}{b}} Ei\left(1, -\frac{d\sqrt{-ab}-(dx+c)b+cb}{b}\right)}{8b^2} + \frac{d e^{-\frac{-d\sqrt{-ab}+cb}{b}} Ei\left(1, \frac{d\sqrt{-ab}+(dx+c)b-cb}{b}\right)}{8b^2} - \frac{e^{-\frac{d\sqrt{-ab}+cb}{b}} Ei\left(1, \frac{d\sqrt{-ab}+cb}{b}\right)}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*cosh(d*x+c)/(b*x^2+a)^2,x)`

[Out]
$$\begin{aligned} & -1/4*d^2*exp(-d*x-c)/b/(b*d^2*x^2+a*d^2)*x+1/8*d/b^2*exp(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)+1/8*d/b^2*exp(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)-1/8/b/(-a*b)^(1/2)*exp(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)+1/8/b/(-a*b)^(1/2)*exp(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)-1/4*d^2*exp(d*x+c)/b/(b*d^2*x^2+a*d^2)*x-1/8*d/b^2*exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)-1/8*d/b^2*exp((-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)-1/8/b/(-a*b)^(1/2)*exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)+1/8/b/(-a*b)^(1/2)*exp((-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b) \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{(dx^2e^{2c} + 2xe^{2c})e^{dx} - (dx^2 - 2x)e^{-dx}}{2(b^2d^2x^4e^c + 2abd^2x^2e^c + a^2d^2e^c)} + \frac{1}{2} \int -\frac{2(2adxe^c - 3bx^2e^c + ae^c)e^{dx}}{b^3d^2x^6 + 3ab^2d^2x^4 + 3a^2bd^2x^2 + a^3d^2} dx + \frac{1}{2} \int \frac{1}{b^3d^2x^6e^c + 3ab^2d^2x^4e^c + 3a^2bd^2x^2e^c + a^3d^2e^c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*cosh(d*x+c)/(b*x^2+a)^2,x, algorithm="maxima")`

[Out]
$$\begin{aligned} & 1/2*((d*x^2*e^{2*c} + 2*x*e^{2*c})*e^{d*x} - (d*x^2 - 2*x)*e^{-d*x})/(b^2*d^2*x^4*e^c + 2*a*b*d^2*x^2*e^c + a^2*d^2*e^c) + 1/2*integrate(-2*(2*a*d*x*e^c - 3*b*x^2*e^c + a*e^c)*e^{d*x}/(b^3*d^2*x^6 + 3*a*b^2*d^2*x^4 + 3*a^2*b*d^2*x^2 + a^3*d^2), x) + 1/2*integrate(2*(2*a*d*x + 3*b*x^2 - a)*e^{-d*x}/(b^3*d^2*x^6*e^c + 3*a*b^2*d^2*x^4*e^c + 3*a^2*b*d^2*x^2*e^c + a^3*d^2*e^c), x) \end{aligned}$$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2 \cosh(c + dx)}{(bx^2 + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2*cosh(c + d*x))/(a + b*x^2)^2,x)`

[Out] `int((x^2*cosh(c + d*x))/(a + b*x^2)^2, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \cosh(c + dx)}{(a + bx^2)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cosh(d*x+c)/(b*x**2+a)**2,x)
```

```
[Out] Integral(x**2*cosh(c + d*x)/(a + b*x**2)**2, x)
```

$$3.68 \quad \int \frac{x \cosh(c+dx)}{(a+bx^2)^2} dx$$

Optimal. Leaf size=239

$$\frac{d \sinh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(xd + \frac{\sqrt{-ad}}{\sqrt{b}}\right)}{4\sqrt{-a} b^{3/2}} + \frac{d \sinh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{4\sqrt{-a} b^{3/2}} - \frac{d \cosh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right) \text{Shi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{4\sqrt{-a} b^{3/2}}$$

[Out] $-1/2*\cosh(d*x+c)/b/(b*x^2+a)+1/4*d*\cosh(c+d*(-a)^{(1/2)}/b^{(1/2)})*Shi(d*x-d*(-a)^{(1/2)}/b^{(1/2)})/b^{(3/2)}/(-a)^{(1/2)}-1/4*d*\cosh(c-d*(-a)^{(1/2)}/b^{(1/2)})*Shi(d*x+d*(-a)^{(1/2)}/b^{(1/2)})/b^{(3/2)}/(-a)^{(1/2)}-1/4*d*Chi(d*x+d*(-a)^{(1/2)}/b^{(1/2)})*sinh(c-d*(-a)^{(1/2)}/b^{(1/2)})/b^{(3/2)}/(-a)^{(1/2)}+1/4*d*Chi(-d*x+d*(-a)^{(1/2)}/b^{(1/2)})*sinh(c+d*(-a)^{(1/2)}/b^{(1/2)})/b^{(3/2)}/(-a)^{(1/2)}$

Rubi [A] time = 0.33, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {5289, 5280, 3303, 3298, 3301}

$$\frac{d \sinh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(xd + \frac{\sqrt{-ad}}{\sqrt{b}}\right)}{4\sqrt{-a} b^{3/2}} + \frac{d \sinh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{4\sqrt{-a} b^{3/2}} - \frac{d \cosh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right) \text{Shi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{4\sqrt{-a} b^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(x*Cosh[c + d*x])/(a + b*x^2)^2,x]

[Out] $-\text{Cosh}[c + d*x]/(2*b*(a + b*x^2)) - (d*\text{CoshIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] + d*x]*\text{Sinh}[c - (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]])/(4*\text{Sqrt}[-a]*b^{(3/2)}) + (d*\text{CoshIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] - d*x]*\text{Sinh}[c + (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]])/(4*\text{Sqrt}[-a]*b^{(3/2)}) - (d*\text{Cosh}[c + (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{SinhIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] - d*x])/(4*\text{Sqrt}[-a]*b^{(3/2)}) - (d*\text{Cosh}[c - (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{SinhIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] + d*x])/(4*\text{Sqrt}[-a]*b^{(3/2)})$

Rule 3298

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]

Rule 3301

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

Rule 5280

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Symbol] := In
t[ExpandIntegrand[Sinh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 5289

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_), x_Symbol] := Simp[(e^m*(a + b*x^n)^(p + 1)*Cosh[c + d*x]/(b*n*(p + 1))
, x] - Dist[(d*e^m)/(b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*Sinh[c + d*x], x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IntegerQ[p] && EqQ[m - n + 1, 0
] && LtQ[p, -1] && (IntegerQ[n] || GtQ[e, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x \cosh(c + dx)}{(a + bx^2)^2} dx &= -\frac{\cosh(c + dx)}{2b(a + bx^2)} + \frac{d \int \frac{\sinh(c+dx)}{a+bx^2} dx}{2b} \\
&= -\frac{\cosh(c + dx)}{2b(a + bx^2)} + \frac{d \int \left(\frac{\sqrt{-a} \sinh(c+dx)}{2a(\sqrt{-a} - \sqrt{b}x)} + \frac{\sqrt{-a} \sinh(c+dx)}{2a(\sqrt{-a} + \sqrt{b}x)} \right) dx}{2b} \\
&= -\frac{\cosh(c + dx)}{2b(a + bx^2)} - \frac{d \int \frac{\sinh(c+dx)}{\sqrt{-a} - \sqrt{b}x} dx}{4\sqrt{-a}b} - \frac{d \int \frac{\sinh(c+dx)}{\sqrt{-a} + \sqrt{b}x} dx}{4\sqrt{-a}b} \\
&= -\frac{\cosh(c + dx)}{2b(a + bx^2)} - \frac{\left(d \cosh\left(c - \frac{\sqrt{-a}d}{\sqrt{b}}\right) \right) \int \frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}} + dx\right)}{\sqrt{-a} + \sqrt{b}x} dx}{4\sqrt{-a}b} + \frac{\left(d \cosh\left(c + \frac{\sqrt{-a}d}{\sqrt{b}}\right) \right) \int \frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}} - dx\right)}{\sqrt{-a} - \sqrt{b}x} dx}{4\sqrt{-a}b} \\
&= -\frac{\cosh(c + dx)}{2b(a + bx^2)} - \frac{d \operatorname{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}} + dx\right) \sinh\left(c - \frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4\sqrt{-a}b^{3/2}} + \frac{d \operatorname{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}} - dx\right) \sinh\left(c + \frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4\sqrt{-a}b^{3/2}}
\end{aligned}$$

Mathematica [C] time = 0.53, size = 239, normalized size = 1.00

$$\frac{i \left(d (a + bx^2) \sinh \left(c - \frac{i\sqrt{ad}}{\sqrt{b}} \right) \text{Ci} \left(idx - \frac{\sqrt{ad}}{\sqrt{b}} \right) - d (a + bx^2) \sinh \left(c + \frac{i\sqrt{ad}}{\sqrt{b}} \right) \text{Ci} \left(idx + \frac{\sqrt{ad}}{\sqrt{b}} \right) + i \left(d (a + bx^2) \cosh \right. \right.}{4\sqrt{a} b^{3/2} (a + bx^2)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(x*Cosh[c + d*x])/(a + b*x^2)^2,x]

[Out] ((I/4)*(d*(a + b*x^2)*CosIntegral[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x]*Sinh[c - (I*Sqrt[a]*d)/Sqrt[b]] - d*(a + b*x^2)*CosIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x]*Sinh[c + (I*Sqrt[a]*d)/Sqrt[b]] + I*(2*Sqrt[a]*Sqrt[b]*Cosh[c + d*x] + d*(a + b*x^2)*Cosh[c - (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d*x] + d*(a + b*x^2)*Cosh[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x]))/(Sqrt[a]*b^(3/2)*(a + b*x^2))

fricas [B] time = 0.57, size = 641, normalized size = 2.68

$$4a \cosh(dx + c) + \left((bx^2 + a) \cosh(dx + c)^2 - (bx^2 + a) \sinh(dx + c)^2 \right) \sqrt{-\frac{ad^2}{b}} \text{Ei} \left(dx - \sqrt{-\frac{ad^2}{b}} \right) - \left((bx^2 + a) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cosh(d*x+c)/(b*x^2+a)^2,x, algorithm="fricas")

[Out] -1/8*(4*a*cosh(d*x + c) + (((b*x^2 + a)*cosh(d*x + c)^2 - (b*x^2 + a)*sinh(d*x + c)^2)*sqrt(-a*d^2/b)*Ei(d*x - sqrt(-a*d^2/b)) - ((b*x^2 + a)*cosh(d*x + c)^2 - (b*x^2 + a)*sinh(d*x + c)^2)*sqrt(-a*d^2/b)*Ei(-d*x + sqrt(-a*d^2/b))) *cosh(c + sqrt(-a*d^2/b)) - (((b*x^2 + a)*cosh(d*x + c)^2 - (b*x^2 + a)*sinh(d*x + c)^2)*sqrt(-a*d^2/b)*Ei(d*x + sqrt(-a*d^2/b)) - ((b*x^2 + a)*cosh(d*x + c)^2 - (b*x^2 + a)*sinh(d*x + c)^2)*sqrt(-a*d^2/b)*Ei(-d*x - sqrt(-a*d^2/b))) *cosh(-c + sqrt(-a*d^2/b)) + (((b*x^2 + a)*cosh(d*x + c)^2 - (b*x^2 + a)*sinh(d*x + c)^2)*sqrt(-a*d^2/b)*Ei(d*x - sqrt(-a*d^2/b)) + ((b*x^2 + a)*cosh(d*x + c)^2 - (b*x^2 + a)*sinh(d*x + c)^2)*sqrt(-a*d^2/b)*Ei(-d*x + sqrt(-a*d^2/b))) *sinh(c + sqrt(-a*d^2/b)) + (((b*x^2 + a)*cosh(d*x + c)^2 - (b*x^2 + a)*sinh(d*x + c)^2)*sqrt(-a*d^2/b)*Ei(d*x + sqrt(-a*d^2/b)) + ((b*x^2 + a)*cosh(d*x + c)^2 - (b*x^2 + a)*sinh(d*x + c)^2)*sqrt(-a*d^2/b)*Ei(-d*x - sqrt(-a*d^2/b))) *sinh(-c + sqrt(-a*d^2/b)))/((a*b^2*x^2 + a^2*b)*cosh(d*x + c)^2 - (a*b^2*x^2 + a^2*b)*sinh(d*x + c)^2)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \cosh(dx + c)}{(bx^2 + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cosh(d*x+c)/(b*x^2+a)^2,x, algorithm="giac")

[Out] integrate(x*cosh(d*x + c)/(b*x^2 + a)^2, x)

maple [A] time = 0.18, size = 291, normalized size = 1.22

$$\frac{d^2 e^{-dx-c}}{4b(bd^2x^2 + ad^2)} - \frac{d e^{-\frac{d\sqrt{-ab}+cb}{b}} \operatorname{Ei}\left(1, \frac{d\sqrt{-ab}+(dx+c)b-cb}{b}\right)}{8b\sqrt{-ab}} + \frac{d e^{-\frac{d\sqrt{-ab}-cb}{b}} \operatorname{Ei}\left(1, -\frac{d\sqrt{-ab}-(dx+c)b+cb}{b}\right)}{8b\sqrt{-ab}} - \frac{d^2 e^{dx+c}}{4b(bd^2x^2 + ad^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*cosh(d*x+c)/(b*x^2+a)^2,x)

[Out] $-1/4*d^2*\exp(-d*x-c)/b/(b*d^2*x^2+a*d^2)-1/8*d/b/(-a*b)^{(1/2)}*\exp(-(-d*(-a*b)^{(1/2)}+c*b)/b)*\operatorname{Ei}(1,(d*(-a*b)^{(1/2)}+(d*x+c)*b-c*b)/b)+1/8*d/b/(-a*b)^{(1/2)}*\exp(-(-d*(-a*b)^{(1/2)}+c*b)/b)*\operatorname{Ei}(1,-(d*(-a*b)^{(1/2)}-(d*x+c)*b+c*b)/b)-1/4*d^2*\exp(d*x+c)/b/(b*d^2*x^2+a*d^2)-1/8*d/b/(-a*b)^{(1/2)}*\exp((d*(-a*b)^{(1/2)}+c*b)/b)*\operatorname{Ei}(1,(d*(-a*b)^{(1/2)}-(d*x+c)*b+c*b)/b)+1/8*d/b/(-a*b)^{(1/2)}*\exp((-d*(-a*b)^{(1/2)}+c*b)/b)*\operatorname{Ei}(1,-(d*(-a*b)^{(1/2)}+(d*x+c)*b-c*b)/b)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{x e^{(dx+2c)} - x e^{-dx}}{2(b^2 dx^4 e^c + 2 ab dx^2 e^c + a^2 d e^c)} + \frac{1}{2} \int \frac{(3bx^2 e^c - ae^c) e^{(dx)}}{b^3 dx^6 + 3ab^2 dx^4 + 3a^2 b dx^2 + a^3 d} dx - \frac{1}{2} \int \frac{(3bx^2 - a) e^{(-dx)}}{b^3 dx^6 e^c + 3ab^2 dx^4 e^c + 3a^2 b dx^2 e^c + a^3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cosh(d*x+c)/(b*x^2+a)^2,x, algorithm="maxima")

[Out] $1/2*(x*e^{(d*x + 2*c)} - x*e^{(-d*x)})/(b^2*d*x^4*e^c + 2*a*b*d*x^2*e^c + a^2*d*e^c) + 1/2*\operatorname{integrate}((3*b*x^2*e^c - a*e^c)*e^{(d*x)}/(b^3*d*x^6 + 3*a*b^2*d*x^4 + 3*a^2*b*d*x^2 + a^3*d), x) - 1/2*\operatorname{integrate}((3*b*x^2 - a)*e^{(-d*x)}/(b^3*d*x^6*e^c + 3*a*b^2*d*x^4*e^c + 3*a^2*b*d*x^2*e^c + a^3*d*e^c), x)$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x \cosh(c + d x)}{(b x^2 + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x*cosh(c + d*x))/(a + b*x^2)^2,x)

```
[Out] int((x*cosh(c + d*x))/(a + b*x^2)^2, x)
```

```
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int \frac{x \cosh(c + dx)}{(a + bx^2)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(d*x+c)/(b*x**2+a)**2,x)
```

```
[Out] Integral(x*cosh(c + d*x)/(a + b*x**2)**2, x)
```

$$3.69 \quad \int \frac{\cosh(c+dx)}{(a+bx^2)^2} dx$$

Optimal. Leaf size=476

$$\frac{d \sinh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \operatorname{Chi}\left(xd + \frac{\sqrt{-ad}}{\sqrt{b}}\right)}{4ab} - \frac{d \sinh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right) \operatorname{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{4ab} - \frac{\cosh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right) \operatorname{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{4(-a)^{3/2}\sqrt{b}} + \dots$$

[Out] $-1/4*d*\cosh(c+d*(-a)^{(1/2)}/b^{(1/2)})*Shi(d*x-d*(-a)^{(1/2)}/b^{(1/2)})/a/b-1/4*d*\cosh(c-d*(-a)^{(1/2)}/b^{(1/2)})*Shi(d*x+d*(-a)^{(1/2)}/b^{(1/2)})/a/b-1/4*d*\operatorname{Chi}(d*x+d*(-a)^{(1/2)}/b^{(1/2)})*\sinh(c-d*(-a)^{(1/2)}/b^{(1/2)})/a/b-1/4*d*\operatorname{Chi}(-d*x+d*(-a)^{(1/2)}/b^{(1/2)})*\sinh(c+d*(-a)^{(1/2)}/b^{(1/2)})/a/b+1/4*\operatorname{Chi}(d*x+d*(-a)^{(1/2)}/b^{(1/2)})/b^{(1/2)}*\cosh(c-d*(-a)^{(1/2)}/b^{(1/2)})/(-a)^{(3/2)}/b^{(1/2)}-1/4*\operatorname{Chi}(-d*x+d*(-a)^{(1/2)}/b^{(1/2)})*\cosh(c+d*(-a)^{(1/2)}/b^{(1/2)})/(-a)^{(3/2)}/b^{(1/2)}+1/4*Shi(d*x+d*(-a)^{(1/2)}/b^{(1/2)})*\sinh(c-d*(-a)^{(1/2)}/b^{(1/2)})/(-a)^{(3/2)}/b^{(1/2)}-1/4*Shi(d*x-d*(-a)^{(1/2)}/b^{(1/2)})*\sinh(c+d*(-a)^{(1/2)}/b^{(1/2)})/(-a)^{(3/2)}/b^{(1/2)}-1/4*\cosh(d*x+c)/a/b^{(1/2)}/((-a)^{(1/2)}-x*b^{(1/2)})+1/4*\cosh(d*x+c)/a/b^{(1/2)}/((-a)^{(1/2)}+x*b^{(1/2)})$

Rubi [A] time = 0.82, antiderivative size = 476, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {5281, 3297, 3303, 3298, 3301}

$$\frac{d \sinh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \operatorname{Chi}\left(xd + \frac{\sqrt{-ad}}{\sqrt{b}}\right)}{4ab} - \frac{d \sinh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right) \operatorname{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{4ab} - \frac{\cosh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right) \operatorname{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{4(-a)^{3/2}\sqrt{b}} + \dots$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]/(a + b*x^2)^2,x]

[Out] $-\operatorname{Cosh}[c + d*x]/(4*a*\operatorname{Sqrt}[b]*(\operatorname{Sqrt}[-a] - \operatorname{Sqrt}[b]*x)) + \operatorname{Cosh}[c + d*x]/(4*a*\operatorname{Sqrt}[b]*(\operatorname{Sqrt}[-a] + \operatorname{Sqrt}[b]*x)) - (\operatorname{Cosh}[c + (\operatorname{Sqrt}[-a]*d)/\operatorname{Sqrt}[b]]*\operatorname{CoshIntegral}[(\operatorname{Sqrt}[-a]*d)/\operatorname{Sqrt}[b] - d*x])/(4*(-a)^{(3/2)}*\operatorname{Sqrt}[b]) + (\operatorname{Cosh}[c - (\operatorname{Sqrt}[-a]*d)/\operatorname{Sqrt}[b]]*\operatorname{CoshIntegral}[(\operatorname{Sqrt}[-a]*d)/\operatorname{Sqrt}[b] + d*x])/(4*(-a)^{(3/2)}*\operatorname{Sqrt}[b]) - (d*\operatorname{CoshIntegral}[(\operatorname{Sqrt}[-a]*d)/\operatorname{Sqrt}[b] + d*x]*\operatorname{Sinh}[c - (\operatorname{Sqrt}[-a]*d)/\operatorname{Sqrt}[b]])/(4*a*b) - (d*\operatorname{CoshIntegral}[(\operatorname{Sqrt}[-a]*d)/\operatorname{Sqrt}[b] - d*x]*\operatorname{Sinh}[c + (\operatorname{Sqrt}[-a]*d)/\operatorname{Sqrt}[b]])/(4*a*b) + (d*\operatorname{Cosh}[c + (\operatorname{Sqrt}[-a]*d)/\operatorname{Sqrt}[b]]*\operatorname{SinhIntegral}[(\operatorname{Sqrt}[-a]*d)/\operatorname{Sqrt}[b] - d*x])/(4*a*b) + (\operatorname{Sinh}[c + (\operatorname{Sqrt}[-a]*d)/\operatorname{Sqrt}[b]]*\operatorname{SinhIntegral}[(\operatorname{Sqrt}[-a]*d)/\operatorname{Sqrt}[b] - d*x])/(4*(-a)^{(3/2)}*\operatorname{Sqrt}[b]) - (d*\operatorname{Cosh}[c - (\operatorname{Sqrt}[-a]*d)/\operatorname{Sqrt}[b]]*\operatorname{SinhIntegral}[(\operatorname{Sqrt}[-a]*d)/\operatorname{Sqrt}[b] + d*x])/(4*a*b) + (\operatorname{Sinh}[c - (\operatorname{Sqrt}[-a]*d)/\operatorname{Sqrt}[b]]*\operatorname{SinhIntegral}[(\operatorname{Sqrt}[-a]*d)/\operatorname{Sqrt}[b] + d*x])/(4*(-a)^{(3/2)}*\operatorname{Sqrt}[b])$

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]
```

Rule 5281

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Int[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh(c+dx)}{(a+bx^2)^2} dx &= \int \left(-\frac{b \cosh(c+dx)}{4a(\sqrt{-a}\sqrt{b}-bx)^2} - \frac{b \cosh(c+dx)}{4a(\sqrt{-a}\sqrt{b}+bx)^2} - \frac{b \cosh(c+dx)}{2a(-ab-b^2x^2)} \right) dx \\
&= -\frac{b \int \frac{\cosh(c+dx)}{(\sqrt{-a}\sqrt{b}-bx)^2} dx}{4a} - \frac{b \int \frac{\cosh(c+dx)}{(\sqrt{-a}\sqrt{b}+bx)^2} dx}{4a} - \frac{b \int \frac{\cosh(c+dx)}{-ab-b^2x^2} dx}{2a} \\
&= -\frac{\cosh(c+dx)}{4a\sqrt{b}(\sqrt{-a}-\sqrt{b}x)} + \frac{\cosh(c+dx)}{4a\sqrt{b}(\sqrt{-a}+\sqrt{b}x)} - \frac{b \int \left(-\frac{\sqrt{-a}\cosh(c+dx)}{2ab(\sqrt{-a}-\sqrt{b}x)} - \frac{\sqrt{-a}\cosh(c+dx)}{2ab(\sqrt{-a}+\sqrt{b}x)} \right) dx}{2a} \\
&= -\frac{\cosh(c+dx)}{4a\sqrt{b}(\sqrt{-a}-\sqrt{b}x)} + \frac{\cosh(c+dx)}{4a\sqrt{b}(\sqrt{-a}+\sqrt{b}x)} + \frac{\int \frac{\cosh(c+dx)}{\sqrt{-a}-\sqrt{b}x} dx}{4(-a)^{3/2}} + \frac{\int \frac{\cosh(c+dx)}{\sqrt{-a}+\sqrt{b}x} dx}{4(-a)^{3/2}} - \frac{(d \cos)}{4a\sqrt{b}(\sqrt{-a}-\sqrt{b}x)} \\
&= -\frac{\cosh(c+dx)}{4a\sqrt{b}(\sqrt{-a}-\sqrt{b}x)} + \frac{\cosh(c+dx)}{4a\sqrt{b}(\sqrt{-a}+\sqrt{b}x)} - \frac{d \operatorname{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right) \sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4ab} - \frac{d \operatorname{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right) \sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4ab} \\
&= -\frac{\cosh(c+dx)}{4a\sqrt{b}(\sqrt{-a}-\sqrt{b}x)} + \frac{\cosh(c+dx)}{4a\sqrt{b}(\sqrt{-a}+\sqrt{b}x)} - \frac{\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right) \operatorname{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4(-a)^{3/2}\sqrt{b}} + \frac{\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right) \operatorname{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4(-a)^{3/2}\sqrt{b}}
\end{aligned}$$

Mathematica [C] time = 0.74, size = 590, normalized size = 1.24

$$\frac{-ia^{3/2}d \cosh\left(c-\frac{i\sqrt{a}d}{\sqrt{b}}\right) \operatorname{Si}\left(\frac{\sqrt{a}d}{\sqrt{b}}-idx\right) + ia^{3/2}d \cosh\left(c+\frac{i\sqrt{a}d}{\sqrt{b}}\right) \operatorname{Si}\left(ixd+\frac{\sqrt{a}d}{\sqrt{b}}\right) - b^{3/2}x^2 \sinh\left(c-\frac{i\sqrt{a}d}{\sqrt{b}}\right) \operatorname{Si}\left(\frac{\sqrt{a}d}{\sqrt{b}}-idx\right) + b^{3/2}x^2 \sinh\left(c+\frac{i\sqrt{a}d}{\sqrt{b}}\right) \operatorname{Si}\left(ixd+\frac{\sqrt{a}d}{\sqrt{b}}\right)}{4(-a)^{3/2}\sqrt{b}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Cosh[c + d*x]/(a + b*x^2)^2,x]

[Out] (2*Sqrt[a]*b*x*Cosh[c + d*x] - (a + b*x^2)*CosIntegral[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x]*((-I)*Sqrt[b]*Cosh[c - (I*Sqrt[a]*d)/Sqrt[b]] + Sqrt[a]*d*Sinh[c - (I*Sqrt[a]*d)/Sqrt[b]]) - (a + b*x^2)*CosIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x]*(I*Sqrt[b]*Cosh[c + (I*Sqrt[a]*d)/Sqrt[b]] + Sqrt[a]*d*Sinh[c + (I*Sqrt[a]*d)/Sqrt[b]]) - I*a^(3/2)*d*Cosh[c - (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d*x] - I*Sqrt[a]*b*d*x^2*Cosh[c - (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d*x] - a*Sqrt[b]*Sinh[c - (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d*x] - b^(3/2)*x^2*Sinh[c - (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d*x] +

$$I*a^{(3/2)*d}*Cosh[c + (I*sqrt[a]*d)/sqrt[b]]*SinIntegral[(sqrt[a]*d)/sqrt[b] + I*d*x] + I*sqrt[a]*b*d*x^2*Cosh[c + (I*sqrt[a]*d)/sqrt[b]]*SinIntegral[(sqrt[a]*d)/sqrt[b] + I*d*x] - a*sqrt[b]*sinh[c + (I*sqrt[a]*d)/sqrt[b]]*SinIntegral[(sqrt[a]*d)/sqrt[b] + I*d*x] - b^{(3/2)*x^2}*sinh[c + (I*sqrt[a]*d)/sqrt[b]]*SinIntegral[(sqrt[a]*d)/sqrt[b] + I*d*x]/(4*a^{(3/2)*b*(a + b*x^2)})$$

fricas [B] time = 0.51, size = 1162, normalized size = 2.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/(b*x^2+a)^2,x, algorithm="fricas")

[Out] $\frac{1}{8} * (4 * a * b * d * x * \cosh(d * x + c) - ((a * b * d^2 * x^2 + a^2 * d^2) * \cosh(d * x + c)^2 - (a * b * d^2 * x^2 + a^2 * d^2) * \sinh(d * x + c)^2 + ((b^2 * x^2 + a * b) * \cosh(d * x + c)^2 - (b^2 * x^2 + a * b) * \sinh(d * x + c)^2) * \sqrt{-a * d^2 / b}) * \text{Ei}(d * x - \sqrt{-a * d^2 / b}) - ((a * b * d^2 * x^2 + a^2 * d^2) * \cosh(d * x + c)^2 - (a * b * d^2 * x^2 + a^2 * d^2) * \sinh(d * x + c)^2 - ((b^2 * x^2 + a * b) * \cosh(d * x + c)^2 - (b^2 * x^2 + a * b) * \sinh(d * x + c)^2) * \sqrt{-a * d^2 / b}) * \text{Ei}(-d * x + \sqrt{-a * d^2 / b})) * \cosh(c + \sqrt{-a * d^2 / b}) - ((a * b * d^2 * x^2 + a^2 * d^2) * \cosh(d * x + c)^2 - (a * b * d^2 * x^2 + a^2 * d^2) * \sinh(d * x + c)^2 - ((b^2 * x^2 + a * b) * \cosh(d * x + c)^2 - (b^2 * x^2 + a * b) * \sinh(d * x + c)^2) * \sqrt{-a * d^2 / b}) * \text{Ei}(d * x + \sqrt{-a * d^2 / b}) - ((a * b * d^2 * x^2 + a^2 * d^2) * \cosh(d * x + c)^2 - (a * b * d^2 * x^2 + a^2 * d^2) * \sinh(d * x + c)^2 + ((b^2 * x^2 + a * b) * \cosh(d * x + c)^2 - (b^2 * x^2 + a * b) * \sinh(d * x + c)^2) * \sqrt{-a * d^2 / b}) * \text{Ei}(-d * x - \sqrt{-a * d^2 / b})) * \cosh(-c + \sqrt{-a * d^2 / b}) - (((a * b * d^2 * x^2 + a^2 * d^2) * \cosh(d * x + c)^2 - (a * b * d^2 * x^2 + a^2 * d^2) * \sinh(d * x + c)^2 + ((b^2 * x^2 + a * b) * \cosh(d * x + c)^2 - (b^2 * x^2 + a * b) * \sinh(d * x + c)^2) * \sqrt{-a * d^2 / b}) * \text{Ei}(d * x - \sqrt{-a * d^2 / b}) + ((a * b * d^2 * x^2 + a^2 * d^2) * \cosh(d * x + c)^2 - (a * b * d^2 * x^2 + a^2 * d^2) * \sinh(d * x + c)^2 - ((b^2 * x^2 + a * b) * \cosh(d * x + c)^2 - (b^2 * x^2 + a * b) * \sinh(d * x + c)^2) * \sqrt{-a * d^2 / b}) * \text{Ei}(-d * x + \sqrt{-a * d^2 / b})) * \sinh(c + \sqrt{-a * d^2 / b}) + (((a * b * d^2 * x^2 + a^2 * d^2) * \cosh(d * x + c)^2 - (a * b * d^2 * x^2 + a^2 * d^2) * \sinh(d * x + c)^2 - ((b^2 * x^2 + a * b) * \cosh(d * x + c)^2 - (b^2 * x^2 + a * b) * \sinh(d * x + c)^2) * \sqrt{-a * d^2 / b}) * \text{Ei}(d * x + \sqrt{-a * d^2 / b})) + ((a * b * d^2 * x^2 + a^2 * d^2) * \cosh(d * x + c)^2 - (a * b * d^2 * x^2 + a^2 * d^2) * \sinh(d * x + c)^2 + ((b^2 * x^2 + a * b) * \cosh(d * x + c)^2 - (b^2 * x^2 + a * b) * \sinh(d * x + c)^2) * \sqrt{-a * d^2 / b}) * \text{Ei}(-d * x - \sqrt{-a * d^2 / b})) * \sinh(-c + \sqrt{-a * d^2 / b})) / ((a^2 * b^2 * d * x^2 + a^3 * b * d) * \cosh(d * x + c)^2 - (a^2 * b^2 * d * x^2 + a^3 * b * d) * \sinh(d * x + c)^2)$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(dx + c)}{(bx^2 + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/(b*x^2+a)^2,x, algorithm="giac")

[Out] integrate(cosh(d*x + c)/(b*x^2 + a)^2, x)

maple [A] time = 0.11, size = 503, normalized size = 1.06

$$\frac{d^2 e^{-dx-c} x}{4a(b d^2 x^2 + a d^2)} - \frac{d e^{-\frac{d\sqrt{-ab}+cb}{b}} \operatorname{Ei}\left(1, -\frac{d\sqrt{-ab}-(dx+c)b+cb}{b}\right)}{8ba} - \frac{d e^{-\frac{-d\sqrt{-ab}+cb}{b}} \operatorname{Ei}\left(1, \frac{d\sqrt{-ab}+(dx+c)b-cb}{b}\right)}{8ba} - \frac{e^{-\frac{d\sqrt{-ab}+cb}{b}} \operatorname{Ei}\left(1, -\frac{d\sqrt{-ab}+cb}{b}\right)}{8a\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d*x+c)/(b*x^2+a)^2,x)

[Out] $\frac{1}{4}d^2 \exp(-dx-c) x/a / (bd^2x^2+ad^2) - \frac{1}{8}d/b/a \exp(-(d(-ab)^{1/2}+c)b/b) \operatorname{Ei}(1, -(d(-ab)^{1/2}-(dx+c)b+cb)/b) - \frac{1}{8}d/b/a \exp(-(-d(-ab)^{1/2}+cb)/b) \operatorname{Ei}(1, (d(-ab)^{1/2}+(dx+c)b-cb)/b) - \frac{1}{8}a/(-ab)^{1/2} \exp(-(d(-ab)^{1/2}+c)b/b) \operatorname{Ei}(1, -(d(-ab)^{1/2}-(dx+c)b+cb)/b) + \frac{1}{8}a/(-ab)^{1/2} \exp(-(-d(-ab)^{1/2}+cb)/b) \operatorname{Ei}(1, (d(-ab)^{1/2}+(dx+c)b-cb)/b) + \frac{1}{4}d^2 \exp(dx+c) x/a / (bd^2x^2+ad^2) + \frac{1}{8}d/b/a \exp((d(-ab)^{1/2}+c)b/b) \operatorname{Ei}(1, (d(-ab)^{1/2}-(dx+c)b+cb)/b) + \frac{1}{8}d/b/a \exp((-d(-ab)^{1/2}+cb)/b) \operatorname{Ei}(1, -(d(-ab)^{1/2}+(dx+c)b-cb)/b) - \frac{1}{8}a/(-ab)^{1/2} \exp((d(-ab)^{1/2}+c)b/b) \operatorname{Ei}(1, (d(-ab)^{1/2}-(dx+c)b+cb)/b) + \frac{1}{8}a/(-ab)^{1/2} \exp(-(-d(-ab)^{1/2}+cb)/b) \operatorname{Ei}(1, -(d(-ab)^{1/2}+(dx+c)b-cb)/b)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(dx+c)}{(bx^2+a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/(b*x^2+a)^2,x, algorithm="maxima")

[Out] integrate(cosh(d*x + c)/(b*x^2 + a)^2, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(c+dx)}{(bx^2+a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(c + d*x)/(a + b*x^2)^2,x)

[Out] int(cosh(c + d*x)/(a + b*x^2)^2, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(c + dx)}{(a + bx^2)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/(b*x**2+a)**2,x)

[Out] Integral(cosh(c + d*x)/(a + b*x**2)**2, x)

$$3.70 \quad \int \frac{\cosh(c+dx)}{x(a+bx^2)^2} dx$$

Optimal. Leaf size=435

$$\frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}} + c\right) \operatorname{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}} - dx\right)}{2a^2} - \frac{\cosh\left(c - \frac{\sqrt{-a}d}{\sqrt{b}}\right) \operatorname{Chi}\left(xd + \frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2a^2} + \frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}} + c\right) \operatorname{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}} - dx\right)}{2a^2} - \frac{\sinh\left(c - \frac{\sqrt{-a}d}{\sqrt{b}}\right) \operatorname{Shi}\left(xd + \frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2a^2}$$

[Out] Chi(d*x)*cosh(c)/a^2+1/2*cosh(d*x+c)/a/(b*x^2+a)-1/2*Chi(d*x+d*(-a)^(1/2)/b^(1/2))*cosh(c-d*(-a)^(1/2)/b^(1/2))/a^2-1/2*Chi(-d*x+d*(-a)^(1/2)/b^(1/2))*cosh(c+d*(-a)^(1/2)/b^(1/2))/a^2+Shi(d*x)*sinh(c)/a^2-1/2*Shi(d*x+d*(-a)^(1/2)/b^(1/2))*sinh(c-d*(-a)^(1/2)/b^(1/2))/a^2-1/2*Shi(d*x-d*(-a)^(1/2)/b^(1/2))*sinh(c+d*(-a)^(1/2)/b^(1/2))/a^2+1/4*d*cosh(c+d*(-a)^(1/2)/b^(1/2))*Shi(d*x-d*(-a)^(1/2)/b^(1/2))/(-a)^(3/2)/b^(1/2)-1/4*d*cosh(c-d*(-a)^(1/2)/b^(1/2))*Shi(d*x+d*(-a)^(1/2)/b^(1/2))/(-a)^(3/2)/b^(1/2)-1/4*d*Chi(d*x+d*(-a)^(1/2)/b^(1/2))*sinh(c-d*(-a)^(1/2)/b^(1/2))/(-a)^(3/2)/b^(1/2)+1/4*d*Chi(-d*x+d*(-a)^(1/2)/b^(1/2))*sinh(c+d*(-a)^(1/2)/b^(1/2))/(-a)^(3/2)/b^(1/2)

Rubi [A] time = 0.84, antiderivative size = 435, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {5293, 3303, 3298, 3301, 5289, 5280}

$$\frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}} + c\right) \operatorname{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}} - dx\right)}{2a^2} - \frac{\cosh\left(c - \frac{\sqrt{-a}d}{\sqrt{b}}\right) \operatorname{Chi}\left(xd + \frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2a^2} + \frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}} + c\right) \operatorname{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}} - dx\right)}{2a^2} - \frac{\sinh\left(c - \frac{\sqrt{-a}d}{\sqrt{b}}\right) \operatorname{Shi}\left(xd + \frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2a^2}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]/(x*(a + b*x^2)^2), x]

[Out] Cosh[c + d*x]/(2*a*(a + b*x^2)) + (Cosh[c]*CoshIntegral[d*x])/a^2 - (Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*a^2) - (Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*a^2) - (d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]])/(4*(-a)^(3/2)*Sqrt[b]) + (d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]])/(4*(-a)^(3/2)*Sqrt[b]) + (Sinh[c]*SinhIntegral[d*x])/a^2 - (d*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(4*(-a)^(3/2)*Sqrt[b]) + (Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*a^2) - (d*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(4*(-a)^(3/2)*Sqrt[b]) - (Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*a^2)

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
  := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x]
  && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
  := Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x]
  && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*e - c*f)/d],
  Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d],
  Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
  NeQ[d*e - c*f, 0]
```

Rule 5280

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Symbol] := Int[ExpandIntegrand[
  Sinh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IGtQ[n, 0]
  && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 5289

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
  := Simp[(e^m*(a + b*x^n)^(p + 1)*Cosh[c + d*x]/(b*n*(p + 1)), x] - Dist[(d*e^m)/(b*n*(p + 1)),
  Int[(a + b*x^n)^(p + 1)*Sinh[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IntegerQ[p]
  && EqQ[m - n + 1, 0] && LtQ[p, -1] && (IntegerQ[n] || GtQ[e, 0])
```

Rule 5293

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
  := Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d}, x]
  && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh(c+dx)}{x(a+bx^2)^2} dx &= \int \left(\frac{\cosh(c+dx)}{a^2 x} - \frac{bx \cosh(c+dx)}{a(a+bx^2)^2} - \frac{bx \cosh(c+dx)}{a^2(a+bx^2)} \right) dx \\
&= \frac{\int \frac{\cosh(c+dx)}{x} dx}{a^2} - \frac{b \int \frac{x \cosh(c+dx)}{a+bx^2} dx}{a^2} - \frac{b \int \frac{x \cosh(c+dx)}{(a+bx^2)^2} dx}{a} \\
&= \frac{\cosh(c+dx)}{2a(a+bx^2)} - \frac{b \int \left(-\frac{\cosh(c+dx)}{2\sqrt{b}(\sqrt{-a}-\sqrt{b}x)} + \frac{\cosh(c+dx)}{2\sqrt{b}(\sqrt{-a}+\sqrt{b}x)} \right) dx}{a^2} - \frac{d \int \frac{\sinh(c+dx)}{a+bx^2} dx}{2a} + \frac{\cosh(c) \int \frac{1}{a+bx^2} dx}{a^2} \\
&= \frac{\cosh(c+dx)}{2a(a+bx^2)} + \frac{\cosh(c)\text{Chi}(dx)}{a^2} + \frac{\sinh(c)\text{Shi}(dx)}{a^2} + \frac{\sqrt{b} \int \frac{\cosh(c+dx)}{\sqrt{-a}-\sqrt{b}x} dx}{2a^2} - \frac{\sqrt{b} \int \frac{\cosh(c+dx)}{\sqrt{-a}+\sqrt{b}x} dx}{2a^2} \\
&= \frac{\cosh(c+dx)}{2a(a+bx^2)} + \frac{\cosh(c)\text{Chi}(dx)}{a^2} + \frac{\sinh(c)\text{Shi}(dx)}{a^2} - \frac{d \int \frac{\sinh(c+dx)}{\sqrt{-a}-\sqrt{b}x} dx}{4(-a)^{3/2}} - \frac{d \int \frac{\sinh(c+dx)}{\sqrt{-a}+\sqrt{b}x} dx}{4(-a)^{3/2}} \\
&= \frac{\cosh(c+dx)}{2a(a+bx^2)} + \frac{\cosh(c)\text{Chi}(dx)}{a^2} - \frac{\cosh\left(c + \frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}} - dx\right)}{2a^2} - \frac{\cosh\left(c - \frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}} + dx\right)}{2a^2} \\
&= \frac{\cosh(c+dx)}{2a(a+bx^2)} + \frac{\cosh(c)\text{Chi}(dx)}{a^2} - \frac{\cosh\left(c + \frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}} - dx\right)}{2a^2} - \frac{\cosh\left(c - \frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}} + dx\right)}{2a^2}
\end{aligned}$$

Mathematica [C] time = 1.33, size = 501, normalized size = 1.15

$$\frac{\sqrt{a}d \left(-i \sinh\left(c - \frac{i\sqrt{a}d}{\sqrt{b}}\right) \text{Ci}\left(idx - \frac{\sqrt{a}d}{\sqrt{b}}\right) + i \sinh\left(c + \frac{i\sqrt{a}d}{\sqrt{b}}\right) \text{Ci}\left(ixd + \frac{\sqrt{a}d}{\sqrt{b}}\right) + \cosh\left(c - \frac{i\sqrt{a}d}{\sqrt{b}}\right) \text{Si}\left(\frac{\sqrt{a}d}{\sqrt{b}} - idx\right) + \cosh\left(c + \frac{i\sqrt{a}d}{\sqrt{b}}\right) \text{Si}\left(ixd + \frac{\sqrt{a}d}{\sqrt{b}}\right) \right)}{2\sqrt{b}} + i \sinh(c) \left(\int \frac{1}{a+bx^2} dx \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[Cosh[c + d*x]/(x*(a + b*x^2)^2), x]

[Out] ((a*Cosh[c]*Cosh[d*x])/(a + b*x^2) + (a*Sinh[c]*Sinh[d*x])/(a + b*x^2) + 2*(Cosh[c]*CoshIntegral[d*x] + Sinh[c]*SinhIntegral[d*x]) + (Sqrt[a]*d*((-I)*CosIntegral[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x]*Sinh[c - (I*Sqrt[a]*d)/Sqrt[b]] + I*CosIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x]*Sinh[c + (I*Sqrt[a]*d)/Sqrt[b]]) + Cosh[c - (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d*

$$\begin{aligned} & x] + \text{Cosh}[c + (I*\text{Sqrt}[a]*d)/\text{Sqrt}[b]]*\text{SinIntegral}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b] + I*d*x]) \\ &)/(2*\text{Sqrt}[b]) + I*\text{Sinh}[c]*(\text{CosIntegral}[-((\text{Sqrt}[a]*d)/\text{Sqrt}[b]) + I*d*x]*\text{Sin}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b]] \\ & - \text{CosIntegral}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b] + I*d*x]*\text{Sin}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b]] + \text{Cos}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b]]*(-\text{SinIntegral}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b] \\ & - I*d*x] + \text{SinIntegral}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b] + I*d*x])) - \text{Cosh}[c]*(\text{Cos}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b]]*\text{CosIntegral}[-((\text{Sqrt}[a]*d)/\text{Sqrt}[b]) + I*d*x] + \text{Cos}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b]]*\text{CosIntegral}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b] + I*d*x] + \text{Sin}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b]]*(\text{SinIntegral}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b] - I*d*x] + \text{SinIntegral}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b] + I*d*x])))/(2*a^2) \end{aligned}$$

fricas [B] time = 0.67, size = 992, normalized size = 2.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/x/(b*x^2+a)^2,x, algorithm="fricas")

[Out] $\frac{1}{8}*(4*a*\cosh(d*x + c) - ((2*(b*x^2 + a)*\cosh(d*x + c)^2 - 2*(b*x^2 + a)*\sinh(d*x + c)^2 - ((b*x^2 + a)*\cosh(d*x + c)^2 - (b*x^2 + a)*\sinh(d*x + c)^2)*\sqrt{-a*d^2/b})*\text{Ei}(d*x - \sqrt{-a*d^2/b})) + (2*(b*x^2 + a)*\cosh(d*x + c)^2 - 2*(b*x^2 + a)*\sinh(d*x + c)^2 + ((b*x^2 + a)*\cosh(d*x + c)^2 - (b*x^2 + a)*\sinh(d*x + c)^2)*\sqrt{-a*d^2/b})*\text{Ei}(-d*x + \sqrt{-a*d^2/b}))*\cosh(c + \sqrt{-a*d^2/b}) + 4*((b*x^2 + a)*\text{Ei}(d*x) + (b*x^2 + a)*\text{Ei}(-d*x))*\cosh(c) - ((2*(b*x^2 + a)*\cosh(d*x + c)^2 - 2*(b*x^2 + a)*\sinh(d*x + c)^2 + ((b*x^2 + a)*\cosh(d*x + c)^2 - (b*x^2 + a)*\sinh(d*x + c)^2)*\sqrt{-a*d^2/b})*\text{Ei}(d*x + \sqrt{-a*d^2/b}) + (2*(b*x^2 + a)*\cosh(d*x + c)^2 - 2*(b*x^2 + a)*\sinh(d*x + c)^2 - ((b*x^2 + a)*\cosh(d*x + c)^2 - (b*x^2 + a)*\sinh(d*x + c)^2)*\sqrt{-a*d^2/b})*\text{Ei}(-d*x - \sqrt{-a*d^2/b}))*\cosh(-c + \sqrt{-a*d^2/b}) - ((2*(b*x^2 + a)*\cosh(d*x + c)^2 - 2*(b*x^2 + a)*\sinh(d*x + c)^2 - ((b*x^2 + a)*\cosh(d*x + c)^2 - (b*x^2 + a)*\sinh(d*x + c)^2)*\sqrt{-a*d^2/b})*\text{Ei}(d*x - \sqrt{-a*d^2/b})) - (2*(b*x^2 + a)*\cosh(d*x + c)^2 - 2*(b*x^2 + a)*\sinh(d*x + c)^2 + ((b*x^2 + a)*\cosh(d*x + c)^2 - (b*x^2 + a)*\sinh(d*x + c)^2)*\sqrt{-a*d^2/b})*\text{Ei}(-d*x + \sqrt{-a*d^2/b}))*\sinh(c + \sqrt{-a*d^2/b}) + 4*((b*x^2 + a)*\text{Ei}(d*x) - (b*x^2 + a)*\text{Ei}(-d*x))*\sinh(c) + ((2*(b*x^2 + a)*\cosh(d*x + c)^2 - 2*(b*x^2 + a)*\sinh(d*x + c)^2 + ((b*x^2 + a)*\cosh(d*x + c)^2 - (b*x^2 + a)*\sinh(d*x + c)^2)*\sqrt{-a*d^2/b})*\text{Ei}(d*x + \sqrt{-a*d^2/b}) - (2*(b*x^2 + a)*\cosh(d*x + c)^2 - 2*(b*x^2 + a)*\sinh(d*x + c)^2 - ((b*x^2 + a)*\cosh(d*x + c)^2 - (b*x^2 + a)*\sinh(d*x + c)^2)*\sqrt{-a*d^2/b})*\text{Ei}(-d*x - \sqrt{-a*d^2/b}))*\sinh(-c + \sqrt{-a*d^2/b}))/((a^2*b*x^2 + a^3)*\cosh(d*x + c)^2 - (a^2*b*x^2 + a^3)*\sinh(d*x + c)^2)$

giac [F] time = 0.00, size = 0, normalized size = 0.00

sage0x

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/x/(b*x^2+a)^2,x, algorithm="giac")

[Out] sage0*x

maple [A] time = 0.16, size = 546, normalized size = 1.26

$$\frac{e^{-dx-c}d^2}{4a((dx+c)^2b-2(dx+c)bc+ad^2+bc^2)} - \frac{e^{-c}\operatorname{Ei}(1,dx)}{2a^2} - \frac{e^{-\frac{d\sqrt{-ab}+cb}}{b}}\operatorname{Ei}\left(1,-\frac{d\sqrt{-ab}-(dx+c)b+cb}{b}\right)d}{8a\sqrt{-ab}} + \frac{e^{-\frac{d\sqrt{-ab}+cb}}{b}}\operatorname{Ei}\left(1,-\frac{d\sqrt{-ab}-(dx+c)b+cb}{b}\right)}{8a\sqrt{-ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d*x+c)/x/(b*x^2+a)^2,x)

[Out] $\frac{1}{4}\exp(-dx-c)d^2/a/((dx+c)^2b-2(dx+c)bc+ad^2+bc^2)-1/2/a^2\exp(-c)\operatorname{Ei}(1,dx)-1/8/a/(-ab)^{1/2}\exp(-(d(-ab)^{1/2}+cb)/b)\operatorname{Ei}(1,-(d(-ab)^{1/2}-(dx+c)b+cb)/b)*d+1/8/a/(-ab)^{1/2}\exp(-(d(-ab)^{1/2}+cb)/b)\operatorname{Ei}(1,(d(-ab)^{1/2}+(dx+c)b-cb)/b)*d+1/4/a^2\exp(-(d(-ab)^{1/2}+cb)/b)\operatorname{Ei}(1,-(d(-ab)^{1/2}-(dx+c)b+cb)/b)+1/4/a^2\exp(-(d(-ab)^{1/2}+cb)/b)\operatorname{Ei}(1,(d(-ab)^{1/2}+(dx+c)b-cb)/b)+1/4\exp(dx+c)d^2/a/((dx+c)^2b-2(dx+c)bc+ad^2+bc^2)-1/2/a^2\exp(c)\operatorname{Ei}(1,-dx)+1/8/a/(-ab)^{1/2}\exp((d(-ab)^{1/2}+cb)/b)\operatorname{Ei}(1,(d(-ab)^{1/2}-(dx+c)b+cb)/b)*d-1/8/a/(-ab)^{1/2}\exp(-(d(-ab)^{1/2}+cb)/b)\operatorname{Ei}(1,-(d(-ab)^{1/2}+(dx+c)b-cb)/b)*d+1/4/a^2\exp((d(-ab)^{1/2}+cb)/b)\operatorname{Ei}(1,(d(-ab)^{1/2}-(dx+c)b+cb)/b)+1/4/a^2\exp(-(d(-ab)^{1/2}+cb)/b)\operatorname{Ei}(1,-(d(-ab)^{1/2}+(dx+c)b-cb)/b)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(dx+c)}{(bx^2+a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/x/(b*x^2+a)^2,x, algorithm="maxima")

[Out] integrate(cosh(d*x + c)/((b*x^2 + a)^2*x), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(c+dx)}{x(bx^2+a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(c + d*x)/(x*(a + b*x^2)^2),x)

```
[Out] int(cosh(c + d*x)/(x*(a + b*x^2)^2), x)
```

```
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int \frac{\cosh(c + dx)}{x(a + bx^2)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x/(b*x**2+a)**2,x)
```

```
[Out] Integral(cosh(c + d*x)/(x*(a + b*x**2)**2), x)
```

$$3.71 \quad \int \frac{\cosh(c+dx)}{x^2(a+bx^2)^2} dx$$

Optimal. Leaf size=500

$$\frac{d \sinh\left(c - \frac{\sqrt{-a}d}{\sqrt{b}}\right) \text{Chi}\left(xd + \frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4a^2} + \frac{d \sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}} + c\right) \text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}} - dx\right)}{4a^2} - \frac{d \cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}} + c\right) \text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}} - dx\right)}{4a^2} + \dots$$

[Out] $-\cosh(d*x+c)/a^2/x+d*\cosh(c)*\text{Shi}(d*x)/a^2+1/4*d*\cosh(c+d*(-a)^{(1/2)}/b^{(1/2)})*\text{Shi}(d*x-d*(-a)^{(1/2)}/b^{(1/2)})/a^2+1/4*d*\cosh(c-d*(-a)^{(1/2)}/b^{(1/2)})*\text{Shi}(d*x+d*(-a)^{(1/2)}/b^{(1/2)})/a^2+d*\text{Chi}(d*x)*\sinh(c)/a^2+1/4*d*\text{Chi}(d*x+d*(-a)^{(1/2)}/b^{(1/2)})*\sinh(c-d*(-a)^{(1/2)}/b^{(1/2)})/a^2+1/4*d*\text{Chi}(-d*x+d*(-a)^{(1/2)}/b^{(1/2)})*\sinh(c+d*(-a)^{(1/2)}/b^{(1/2)})/a^2+3/4*\text{Chi}(d*x+d*(-a)^{(1/2)}/b^{(1/2)})*\cosh(c-d*(-a)^{(1/2)}/b^{(1/2)})*b^{(1/2)}/(-a)^{(5/2)}-3/4*\text{Chi}(-d*x+d*(-a)^{(1/2)}/b^{(1/2)})*\cosh(c+d*(-a)^{(1/2)}/b^{(1/2)})*b^{(1/2)}/(-a)^{(5/2)}+3/4*\text{Shi}(d*x+d*(-a)^{(1/2)}/b^{(1/2)})*\sinh(c-d*(-a)^{(1/2)}/b^{(1/2)})*b^{(1/2)}/(-a)^{(5/2)}-3/4*\text{Shi}(d*x-d*(-a)^{(1/2)}/b^{(1/2)})*\sinh(c+d*(-a)^{(1/2)}/b^{(1/2)})*b^{(1/2)}/(-a)^{(5/2)}+1/4*\cosh(d*x+c)*b^{(1/2)}/a^2/((-a)^{(1/2)}-x*b^{(1/2)})-1/4*\cosh(d*x+c)*b^{(1/2)}/a^2/((-a)^{(1/2)}+x*b^{(1/2)})$

Rubi [A] time = 1.25, antiderivative size = 500, normalized size of antiderivative = 1.00, number of steps used = 32, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {5293, 3297, 3303, 3298, 3301, 5281}

$$\frac{d \sinh\left(c - \frac{\sqrt{-a}d}{\sqrt{b}}\right) \text{Chi}\left(xd + \frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4a^2} + \frac{d \sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}} + c\right) \text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}} - dx\right)}{4a^2} - \frac{d \cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}} + c\right) \text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}} - dx\right)}{4a^2} + \dots$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]/(x^2*(a + b*x^2)^2), x]

[Out] $-(\text{Cosh}[c + d*x]/(a^2*x)) + (\text{Sqrt}[b]*\text{Cosh}[c + d*x])/(4*a^2*(\text{Sqrt}[-a] - \text{Sqrt}[b]*x)) - (\text{Sqrt}[b]*\text{Cosh}[c + d*x])/(4*a^2*(\text{Sqrt}[-a] + \text{Sqrt}[b]*x)) - (3*\text{Sqrt}[b]*\text{Cosh}[c + (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{CoshIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] - d*x])/(4*(-a)^{(5/2)}) + (3*\text{Sqrt}[b]*\text{Cosh}[c - (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{CoshIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] + d*x])/(4*(-a)^{(5/2)}) + (d*\text{CoshIntegral}[d*x]*\text{Sinh}[c])/a^2 + (d*\text{CoshIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] + d*x]*\text{Sinh}[c - (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]])/(4*a^2) + (d*\text{CoshIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] - d*x]*\text{Sinh}[c + (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]])/(4*a^2) + (d*\text{Cosh}[c]*\text{SinhIntegral}[d*x])/a^2 - (d*\text{Cosh}[c + (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{SinhIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] - d*x])/(4*a^2) + (3*\text{Sqrt}[b]*\text{Sinh}[c + (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{SinhIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] - d*x])/(4*(-a)^{(5/2)}) + (d*\text{Cosh}[c - (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{SinhIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] + d*x])/(4*(-a)^{(5/2)})$

$t[-a*d)/\text{Sqrt}[b + d*x])/(4*a^2) + (3*\text{Sqrt}[b]*\text{Sinh}[c - (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{SinhIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b + d*x])/(4*(-a)^{(5/2)})$

Rule 3297

$\text{Int}[(c_.) + (d_.)*(x_.)]^{(m_.)}*\sin[(e_.) + (f_.)*(x_.)], x_Symbol] \rightarrow \text{Simp}[(c + d*x)^{(m + 1)}*\sin[e + f*x]/(d*(m + 1)), x] - \text{Dist}[f/(d*(m + 1)), \text{Int}[(c + d*x)^{(m + 1)}*\cos[e + f*x], x], x] /; \text{FreeQ}\{c, d, e, f\}, x \ \&\& \ \text{LtQ}[m, -1]$

Rule 3298

$\text{Int}[\sin[(e_.) + (\text{Complex}[0, fz_])*(f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x_Symbol] \rightarrow \text{Simp}[(I*\text{SinhIntegral}[(c*f*fz)/d + f*fz*x])/d, x] /; \text{FreeQ}\{c, d, e, f, fz\}, x \ \&\& \ \text{EqQ}[d*e - c*f*fz*I, 0]$

Rule 3301

$\text{Int}[\sin[(e_.) + (\text{Complex}[0, fz_])*(f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x_Symbol] \rightarrow \text{Simp}[\text{CoshIntegral}[(c*f*fz)/d + f*fz*x]/d, x] /; \text{FreeQ}\{c, d, e, f, fz\}, x \ \&\& \ \text{EqQ}[d*(e - \text{Pi}/2) - c*f*fz*I, 0]$

Rule 3303

$\text{Int}[\sin[(e_.) + (f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x_Symbol] \rightarrow \text{Dist}[\cos[(d*e - c*f)/d], \text{Int}[\sin[(c*f)/d + f*x]/(c + d*x), x], x] + \text{Dist}[\sin[(d*e - c*f)/d], \text{Int}[\cos[(c*f)/d + f*x]/(c + d*x), x], x] /; \text{FreeQ}\{c, d, e, f\}, x \ \&\& \ \text{NeQ}[d*e - c*f, 0]$

Rule 5281

$\text{Int}[\text{Cosh}[(c_.) + (d_.)*(x_.)]*((a_.) + (b_.)*(x_.)^{(n_.)})^{(p_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[\text{Cosh}[c + d*x], (a + b*x^n)^p, x], x] /; \text{FreeQ}\{a, b, c, d\}, x \ \&\& \ \text{ILtQ}[p, 0] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ (\text{EqQ}[n, 2] \ || \ \text{EqQ}[p, -1])$

Rule 5293

$\text{Int}[\text{Cosh}[(c_.) + (d_.)*(x_.)]*(x_.)^{(m_.)}*((a_.) + (b_.)*(x_.)^{(n_.)})^{(p_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[\text{Cosh}[c + d*x], x^m*(a + b*x^n)^p, x], x] /; \text{FreeQ}\{a, b, c, d\}, x \ \&\& \ \text{ILtQ}[p, 0] \ \&\& \ \text{IntegerQ}[m] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ (\text{EqQ}[n, 2] \ || \ \text{EqQ}[p, -1])$

Rubi steps

$$\begin{aligned}
\int \frac{\cosh(c+dx)}{x^2(a+bx^2)^2} dx &= \int \left(\frac{\cosh(c+dx)}{a^2x^2} - \frac{b \cosh(c+dx)}{a(a+bx^2)^2} - \frac{b \cosh(c+dx)}{a^2(a+bx^2)} \right) dx \\
&= \frac{\int \frac{\cosh(c+dx)}{x^2} dx}{a^2} - \frac{b \int \frac{\cosh(c+dx)}{a+bx^2} dx}{a^2} - \frac{b \int \frac{\cosh(c+dx)}{(a+bx^2)^2} dx}{a} \\
&= -\frac{\cosh(c+dx)}{a^2x} - \frac{b \int \left(\frac{\sqrt{-a} \cosh(c+dx)}{2a(\sqrt{-a}-\sqrt{b}x)} + \frac{\sqrt{-a} \cosh(c+dx)}{2a(\sqrt{-a}+\sqrt{b}x)} \right) dx}{a^2} - \frac{b \int \left(-\frac{b \cosh(c+dx)}{4a(\sqrt{-a}\sqrt{b}-bx)^2} - \frac{b \cosh(c+dx)}{4a(\sqrt{-a}\sqrt{b}+bx)^2} \right) dx}{a} \\
&= -\frac{\cosh(c+dx)}{a^2x} + \frac{b \int \frac{\cosh(c+dx)}{\sqrt{-a}-\sqrt{b}x} dx}{2(-a)^{5/2}} + \frac{b \int \frac{\cosh(c+dx)}{\sqrt{-a}+\sqrt{b}x} dx}{2(-a)^{5/2}} + \frac{b^2 \int \frac{\cosh(c+dx)}{(\sqrt{-a}\sqrt{b}-bx)^2} dx}{4a^2} + \frac{b^2 \int \frac{\cosh(c+dx)}{(\sqrt{-a}\sqrt{b}+bx)^2} dx}{4a^2} \\
&= -\frac{\cosh(c+dx)}{a^2x} + \frac{\sqrt{b} \cosh(c+dx)}{4a^2(\sqrt{-a}-\sqrt{b}x)} - \frac{\sqrt{b} \cosh(c+dx)}{4a^2(\sqrt{-a}+\sqrt{b}x)} + \frac{d \operatorname{Chi}(dx) \sinh(c)}{a^2} + \frac{d \cosh(c)}{a^2} \\
&= -\frac{\cosh(c+dx)}{a^2x} + \frac{\sqrt{b} \cosh(c+dx)}{4a^2(\sqrt{-a}-\sqrt{b}x)} - \frac{\sqrt{b} \cosh(c+dx)}{4a^2(\sqrt{-a}+\sqrt{b}x)} - \frac{\sqrt{b} \cosh\left(c + \frac{\sqrt{-a}d}{\sqrt{b}}\right) \operatorname{Chi}\left(\frac{\sqrt{-a}}{\sqrt{b}}\right)}{2(-a)^{5/2}} \\
&= -\frac{\cosh(c+dx)}{a^2x} + \frac{\sqrt{b} \cosh(c+dx)}{4a^2(\sqrt{-a}-\sqrt{b}x)} - \frac{\sqrt{b} \cosh(c+dx)}{4a^2(\sqrt{-a}+\sqrt{b}x)} - \frac{\sqrt{b} \cosh\left(c + \frac{\sqrt{-a}d}{\sqrt{b}}\right) \operatorname{Chi}\left(\frac{\sqrt{-a}}{\sqrt{b}}\right)}{2(-a)^{5/2}} \\
&= -\frac{\cosh(c+dx)}{a^2x} + \frac{\sqrt{b} \cosh(c+dx)}{4a^2(\sqrt{-a}-\sqrt{b}x)} - \frac{\sqrt{b} \cosh(c+dx)}{4a^2(\sqrt{-a}+\sqrt{b}x)} - \frac{3\sqrt{b} \cosh\left(c + \frac{\sqrt{-a}d}{\sqrt{b}}\right) \operatorname{Chi}\left(\frac{\sqrt{-a}}{\sqrt{b}}\right)}{4(-a)^{5/2}}
\end{aligned}$$

Mathematica [C] time = 1.09, size = 675, normalized size = 1.35

$$ia^{3/2} dx \cosh\left(c - \frac{i\sqrt{a}d}{\sqrt{b}}\right) \operatorname{Si}\left(\frac{\sqrt{a}d}{\sqrt{b}} - idx\right) - ia^{3/2} dx \cosh\left(c + \frac{i\sqrt{a}d}{\sqrt{b}}\right) \operatorname{Si}\left(ixd + \frac{\sqrt{a}d}{\sqrt{b}}\right) + 4a^{3/2} dx \sinh(c) \operatorname{Chi}(dx) + 4a^{3/2} dx$$

Warning: Unable to verify antiderivative.

[In] Integrate[Cosh[c + d*x]/(x^2*(a + b*x^2)^2), x]

[Out] (-4*a^(3/2)*Cosh[c + d*x] - 6*Sqrt[a]*b*x^2*Cosh[c + d*x] + 4*a^(3/2)*d*x*CoshIntegral[d*x]*Sinh[c] + 4*Sqrt[a]*b*d*x^3*CoshIntegral[d*x]*Sinh[c] + x*

$$\begin{aligned}
& (a + b*x^2)*\text{CosIntegral}[-((\text{Sqrt}[a]*d)/\text{Sqrt}[b]) + I*d*x]*((-3*I)*\text{Sqrt}[b]*\text{Cosh}[c - (I*\text{Sqrt}[a]*d)/\text{Sqrt}[b]] + \text{Sqrt}[a]*d*\text{Sinh}[c - (I*\text{Sqrt}[a]*d)/\text{Sqrt}[b]]) + \\
& x*(a + b*x^2)*\text{CosIntegral}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b] + I*d*x]*((3*I)*\text{Sqrt}[b]*\text{Cosh}[c + (I*\text{Sqrt}[a]*d)/\text{Sqrt}[b]] + \text{Sqrt}[a]*d*\text{Sinh}[c + (I*\text{Sqrt}[a]*d)/\text{Sqrt}[b]]) + \\
& 4*a^{(3/2)}*d*x*\text{Cosh}[c]*\text{SinhIntegral}[d*x] + 4*\text{Sqrt}[a]*b*d*x^3*\text{Cosh}[c]*\text{SinhIntegral}[d*x] + I*a^{(3/2)}*d*x*\text{Cosh}[c - (I*\text{Sqrt}[a]*d)/\text{Sqrt}[b]]*\text{SinIntegral}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b] - I*d*x] + I*\text{Sqrt}[a]*b*d*x^3*\text{Cosh}[c - (I*\text{Sqrt}[a]*d)/\text{Sqrt}[b]]*\text{SinIntegral}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b] - I*d*x] + 3*a*\text{Sqrt}[b]*x*\text{Sinh}[c - (I*\text{Sqrt}[a]*d)/\text{Sqrt}[b]]*\text{SinIntegral}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b] - I*d*x] + 3*b^{(3/2)}*x^3*\text{Sinh}[c - (I*\text{Sqrt}[a]*d)/\text{Sqrt}[b]]*\text{SinIntegral}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b] - I*d*x] - I*a^{(3/2)}*d*x*\text{Cosh}[c + (I*\text{Sqrt}[a]*d)/\text{Sqrt}[b]]*\text{SinIntegral}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b] + I*d*x] - I*\text{Sqrt}[a]*b*d*x^3*\text{Cosh}[c + (I*\text{Sqrt}[a]*d)/\text{Sqrt}[b]]*\text{SinIntegral}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b] + I*d*x] + 3*a*\text{Sqrt}[b]*x*\text{Sinh}[c + (I*\text{Sqrt}[a]*d)/\text{Sqrt}[b]]*\text{SinIntegral}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b] + I*d*x] + 3*b^{(3/2)}*x^3*\text{Sinh}[c + (I*\text{Sqrt}[a]*d)/\text{Sqrt}[b]]*\text{SinIntegral}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b] + I*d*x])/(4*a^{(5/2)}*x*(a + b*x^2))
\end{aligned}$$

fricas [B] time = 0.64, size = 1310, normalized size = 2.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/x^2/(b*x^2+a)^2,x, algorithm="fricas")

[Out]
$$\begin{aligned}
& -1/8*(4*(3*a*b*d*x^2 + 2*a^2*d)*\text{cosh}(d*x + c) - (((a*b*d^2*x^3 + a^2*d^2*x) * \text{cosh}(d*x + c)^2 - (a*b*d^2*x^3 + a^2*d^2*x)*\text{sinh}(d*x + c)^2 + 3*((b^2*x^3 + a*b*x)*\text{cosh}(d*x + c)^2 - (b^2*x^3 + a*b*x)*\text{sinh}(d*x + c)^2)*\text{sqrt}(-a*d^2/b)) * \text{Ei}(d*x - \text{sqrt}(-a*d^2/b)) - ((a*b*d^2*x^3 + a^2*d^2*x)*\text{cosh}(d*x + c)^2 - (a*b*d^2*x^3 + a^2*d^2*x)*\text{sinh}(d*x + c)^2 - 3*((b^2*x^3 + a*b*x)*\text{cosh}(d*x + c)^2 - (b^2*x^3 + a*b*x)*\text{sinh}(d*x + c)^2)*\text{sqrt}(-a*d^2/b)) * \text{Ei}(-d*x + \text{sqrt}(-a*d^2/b))) * \text{cosh}(c + \text{sqrt}(-a*d^2/b)) - 4*((a*b*d^2*x^3 + a^2*d^2*x)*\text{Ei}(d*x) - (a*b*d^2*x^3 + a^2*d^2*x)*\text{Ei}(-d*x)) * \text{cosh}(c) - (((a*b*d^2*x^3 + a^2*d^2*x) * \text{cosh}(d*x + c)^2 - (a*b*d^2*x^3 + a^2*d^2*x)*\text{sinh}(d*x + c)^2 - 3*((b^2*x^3 + a*b*x)*\text{cosh}(d*x + c)^2 - (b^2*x^3 + a*b*x)*\text{sinh}(d*x + c)^2)*\text{sqrt}(-a*d^2/b)) * \text{Ei}(d*x + \text{sqrt}(-a*d^2/b)) - ((a*b*d^2*x^3 + a^2*d^2*x)*\text{cosh}(d*x + c)^2 - (a*b*d^2*x^3 + a^2*d^2*x)*\text{sinh}(d*x + c)^2 + 3*((b^2*x^3 + a*b*x)*\text{cosh}(d*x + c)^2 - (b^2*x^3 + a*b*x)*\text{sinh}(d*x + c)^2)*\text{sqrt}(-a*d^2/b)) * \text{Ei}(-d*x - \text{sqrt}(-a*d^2/b))) * \text{cosh}(-c + \text{sqrt}(-a*d^2/b)) - (((a*b*d^2*x^3 + a^2*d^2*x)*\text{cosh}(d*x + c)^2 - (a*b*d^2*x^3 + a^2*d^2*x)*\text{sinh}(d*x + c)^2 + 3*((b^2*x^3 + a*b*x)*\text{cosh}(d*x + c)^2 - (b^2*x^3 + a*b*x)*\text{sinh}(d*x + c)^2)*\text{sqrt}(-a*d^2/b)) * \text{Ei}(d*x - \text{sqrt}(-a*d^2/b)) + ((a*b*d^2*x^3 + a^2*d^2*x)*\text{cosh}(d*x + c)^2 - (a*b*d^2*x^3 + a^2*d^2*x)*\text{sinh}(d*x + c)^2 - 3*((b^2*x^3 + a*b*x)*\text{cosh}(d*x + c)^2 - (b^2*x^3 + a*b*x)*\text{sinh}(d*x + c)^2)*\text{sqrt}(-a*d^2/b)) * \text{Ei}(-d*x + \text{sqrt}(-a*d^2/b))) * \text{sinh}(c + \text{sqrt}(-a*d^2/b)) - 4*((a*b*d^2*x^3 + a^2*d^2*x)*\text{Ei}(d*x) + (a*b*d^2*x^3 + a^2*d^2*x)*\text{Ei}(-d*x)) * \text{sinh}(c) + (((a*b*d^2*x^3 + a^2*d^2*x)*\text{cosh}(d*x
\end{aligned}$$

+ c)^2 - (a*b*d^2*x^3 + a^2*d^2*x)*sinh(d*x + c)^2 - 3*((b^2*x^3 + a*b*x)*cosh(d*x + c)^2 - (b^2*x^3 + a*b*x)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x + sqrt(-a*d^2/b)) + ((a*b*d^2*x^3 + a^2*d^2*x)*cosh(d*x + c)^2 - (a*b*d^2*x^3 + a^2*d^2*x)*sinh(d*x + c)^2 + 3*((b^2*x^3 + a*b*x)*cosh(d*x + c)^2 - (b^2*x^3 + a*b*x)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x - sqrt(-a*d^2/b)))*sinh(-c + sqrt(-a*d^2/b)))/((a^3*b*d*x^3 + a^4*d*x)*cosh(d*x + c)^2 - (a^3*b*d*x^3 + a^4*d*x)*sinh(d*x + c)^2)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(dx + c)}{(bx^2 + a)^2 x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/x^2/(b*x^2+a)^2,x, algorithm="giac")

[Out] integrate(cosh(d*x + c)/((b*x^2 + a)^2*x^2), x)

maple [A] time = 0.17, size = 595, normalized size = 1.19

$$-\frac{3e^{-dx-c}x d^2 b}{4a^2(b d^2 x^2 + a d^2)} - \frac{e^{-dx-c}d^2}{2ax(b d^2 x^2 + a d^2)} + \frac{d e^{-c} \operatorname{Ei}(1, dx)}{2a^2} + \frac{d e^{-\frac{d\sqrt{-ab}+cb}{b}} \operatorname{Ei}\left(1, -\frac{d\sqrt{-ab}-(dx+c)b+cb}{b}\right)}{8a^2} + \frac{d e^{-\frac{-d\sqrt{-ab}+cb}{b}} \operatorname{Ei}\left(1, -\frac{d\sqrt{-ab}-(dx+c)b+cb}{b}\right)}{8a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d*x+c)/x^2/(b*x^2+a)^2,x)

[Out]
$$-3/4*\exp(-d*x-c)/a^2*x*d^2/(b*d^2*x^2+a*d^2)*b-1/2*\exp(-d*x-c)/a/x*d^2/(b*d^2*x^2+a*d^2)+1/2*d/a^2*\exp(-c)*\operatorname{Ei}(1,d*x)+1/8*d/a^2*\exp(-(d*(-a*b)^(1/2)+c*b)/b)*\operatorname{Ei}(1,-(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)+1/8*d/a^2*\exp(-(d*(-a*b)^(1/2)+c*b)/b)*\operatorname{Ei}(1,(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)+3/8/a^2/(-a*b)^(1/2)*\exp(-(d*(-a*b)^(1/2)+c*b)/b)*\operatorname{Ei}(1,-(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)*b-3/8/a^2/(-a*b)^(1/2)*\exp(-(d*(-a*b)^(1/2)+c*b)/b)*\operatorname{Ei}(1,(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)*b-3/4*\exp(d*x+c)/a^2*x*d^2/(b*d^2*x^2+a*d^2)*b-1/2*\exp(d*x+c)/a/x*d^2/(b*d^2*x^2+a*d^2)-1/2*d/a^2*\exp(c)*\operatorname{Ei}(1,-d*x)-1/8*d/a^2*\exp((d*(-a*b)^(1/2)+c*b)/b)*\operatorname{Ei}(1,(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)-1/8*d/a^2*\exp((-d*(-a*b)^(1/2)+c*b)/b)*\operatorname{Ei}(1,-(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)+3/8/a^2/(-a*b)^(1/2)*\exp((d*(-a*b)^(1/2)+c*b)/b)*\operatorname{Ei}(1,(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)*b-3/8/a^2/(-a*b)^(1/2)*\exp((-d*(-a*b)^(1/2)+c*b)/b)*\operatorname{Ei}(1,-(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)*b$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(dx + c)}{(bx^2 + a)^2 x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x^2/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(cosh(d*x + c)/((b*x^2 + a)^2*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(c + dx)}{x^2 (bx^2 + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)/(x^2*(a + b*x^2)^2), x)
```

```
[Out] int(cosh(c + d*x)/(x^2*(a + b*x^2)^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x**2/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

$$3.72 \quad \int \frac{x^3 \cosh(c+dx)}{(a+bx^2)^3} dx$$

Optimal. Leaf size=476

$$\frac{3d \sinh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(xd + \frac{\sqrt{-ad}}{\sqrt{b}}\right)}{16\sqrt{-a} b^{5/2}} + \frac{3d \sinh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{16\sqrt{-a} b^{5/2}} - \frac{3d \cosh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right) \text{Shi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{16\sqrt{-a} b^{5/2}}$$

[Out] $-1/4*x^2*\cosh(d*x+c)/b/(b*x^2+a)^2-1/4*\cosh(d*x+c)/b^2/(b*x^2+a)+1/16*d^2*\text{Chi}(d*x+d*(-a)^{(1/2)}/b^{(1/2)})*\cosh(c-d*(-a)^{(1/2)}/b^{(1/2)})/b^3+1/16*d^2*\text{Chi}(-d*x+d*(-a)^{(1/2)}/b^{(1/2)})*\cosh(c+d*(-a)^{(1/2)}/b^{(1/2)})/b^3-1/8*d*x*\sinh(d*x+c)/b^2/(b*x^2+a)+1/16*d^2*\text{Shi}(d*x+d*(-a)^{(1/2)}/b^{(1/2)})*\sinh(c-d*(-a)^{(1/2)}/b^{(1/2)})/b^3+1/16*d^2*\text{Shi}(d*x-d*(-a)^{(1/2)}/b^{(1/2)})*\sinh(c+d*(-a)^{(1/2)}/b^{(1/2)})/b^3+3/16*d*\cosh(c+d*(-a)^{(1/2)}/b^{(1/2)})*\text{Shi}(d*x-d*(-a)^{(1/2)}/b^{(1/2)})/b^{(5/2)}/(-a)^{(1/2)}-3/16*d*\cosh(c-d*(-a)^{(1/2)}/b^{(1/2)})*\text{Shi}(d*x+d*(-a)^{(1/2)}/b^{(1/2)})/b^{(5/2)}/(-a)^{(1/2)}-3/16*d*\text{Chi}(d*x+d*(-a)^{(1/2)}/b^{(1/2)})*\sinh(c-d*(-a)^{(1/2)}/b^{(1/2)})/b^{(5/2)}/(-a)^{(1/2)}+3/16*d*\text{Chi}(-d*x+d*(-a)^{(1/2)}/b^{(1/2)})*\sinh(c+d*(-a)^{(1/2)}/b^{(1/2)})/b^{(5/2)}/(-a)^{(1/2)}$

Rubi [A] time = 1.06, antiderivative size = 476, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {5291, 5289, 5280, 3303, 3298, 3301, 5290, 5293}

$$\frac{d^2 \cosh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{16b^3} + \frac{d^2 \cosh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(xd + \frac{\sqrt{-ad}}{\sqrt{b}}\right)}{16b^3} - \frac{3d \sinh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(xd + \frac{\sqrt{-ad}}{\sqrt{b}}\right)}{16\sqrt{-a} b^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[(x^3*Cosh[c + d*x])/(a + b*x^2)^3,x]

[Out] $-(x^2*\text{Cosh}[c + d*x])/(4*b*(a + b*x^2)^2) - \text{Cosh}[c + d*x]/(4*b^2*(a + b*x^2)) + (d^2*\text{Cosh}[c + (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{CoshIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] - d*x])/(16*b^3) + (d^2*\text{Cosh}[c - (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{CoshIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] + d*x])/(16*b^3) - (3*d*\text{CoshIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] + d*x]*\text{Sinh}[c - (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]])/(16*\text{Sqrt}[-a]*b^{(5/2)}) + (3*d*\text{CoshIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] - d*x]*\text{Sinh}[c + (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]])/(16*\text{Sqrt}[-a]*b^{(5/2)}) - (d*x*\text{Sinh}[c + d*x])/(8*b^2*(a + b*x^2)) - (3*d*\text{Cosh}[c + (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{SinhIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] - d*x])/(16*\text{Sqrt}[-a]*b^{(5/2)}) - (d^2*\text{Sinh}[c + (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{SinhIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] - d*x])/(16*b^3) - (3*d*\text{Cosh}[c - (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{SinhIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] + d*x])/(16*\text{Sqrt}[-a]*b^{(5/2)}) + (d^2*\text{Sinh}[c - (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{SinhIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] + d*x])/(16*b^3)$

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x]
&& EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x]
&& EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d],
Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

Rule 5280

```
Int[((a_.) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Symbol]
:> Int[ExpandIntegrand[Sinh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d}, x]
&& ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 5289

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_)^(m_.))*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[(e^m*(a + b*x^n)^(p + 1)*Cosh[c + d*x])/(b*n*(p + 1)), x] - Dist[(d*e^m)/(b*n*(p + 1)),
Int[(a + b*x^n)^(p + 1)*Sinh[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
&& IntegerQ[p] && EqQ[m - n + 1, 0] && LtQ[p, -1] && (IntegerQ[n] || GtQ[e, 0])
```

Rule 5290

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Symbol]
:> Simp[(x^(m - n + 1)*(a + b*x^n)^(p + 1)*Sinh[c + d*x])/(b*n*(p + 1)), x] + (-Dist[(m - n + 1)/(b*n*(p + 1)),
Int[x^(m - n)*(a + b*x^n)^(p + 1)*Sinh[c + d*x], x], x] - Dist[d/(b*n*(p + 1)),
Int[x^(m - n + 1)*(a + b*x^n)^(p + 1)*Cosh[c + d*x], x], x]) /; FreeQ[{a, b, c, d}, x]
&& ILtQ[p, -1] && IGtQ[n, 0] && RationalQ[m] && (GtQ[m - n + 1, 0] || GtQ[n, 2])
```

Rule 5291

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
```

```

mbol] := Simp[(x^(m - n + 1)*(a + b*x^n)^(p + 1)*Cosh[c + d*x])/(b*n*(p + 1)), x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1)*Cosh[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^n)^(p + 1)*Sinh[c + d*x], x], x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1] && IGtQ[n, 0] && RationalQ[m] && (GtQ[m - n + 1, 0] || GtQ[n, 2])

```

Rule 5293

```

Int[Cosh[(c_.) + (d_.)*(x_.)]*(x_)^(m_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] := Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])

```

Rubi steps

$$\begin{aligned}
\int \frac{x^3 \cosh(c + dx)}{(a + bx^2)^3} dx &= -\frac{x^2 \cosh(c + dx)}{4b(a + bx^2)^2} + \frac{\int \frac{x \cosh(c + dx)}{(a + bx^2)^2} dx}{2b} + \frac{d \int \frac{x^2 \sinh(c + dx)}{(a + bx^2)^2} dx}{4b} \\
&= -\frac{x^2 \cosh(c + dx)}{4b(a + bx^2)^2} - \frac{\cosh(c + dx)}{4b^2(a + bx^2)} - \frac{dx \sinh(c + dx)}{8b^2(a + bx^2)} + \frac{d \int \frac{\sinh(c + dx)}{a + bx^2} dx}{8b^2} + \frac{d \int \frac{\sinh(c + dx)}{a + bx^2} dx}{4b^2} \\
&= -\frac{x^2 \cosh(c + dx)}{4b(a + bx^2)^2} - \frac{\cosh(c + dx)}{4b^2(a + bx^2)} - \frac{dx \sinh(c + dx)}{8b^2(a + bx^2)} + \frac{d \int \left(\frac{\sqrt{-a} \sinh(c + dx)}{2a(\sqrt{-a} - \sqrt{b}x)} + \frac{\sqrt{-a} \sinh(c + dx)}{2a(\sqrt{-a} + \sqrt{b}x)} \right) dx}{8b^2} \\
&= -\frac{x^2 \cosh(c + dx)}{4b(a + bx^2)^2} - \frac{\cosh(c + dx)}{4b^2(a + bx^2)} - \frac{dx \sinh(c + dx)}{8b^2(a + bx^2)} - \frac{d \int \frac{\sinh(c + dx)}{\sqrt{-a} - \sqrt{b}x} dx}{16\sqrt{-a}b^2} - \frac{d \int \frac{\sinh(c + dx)}{\sqrt{-a} + \sqrt{b}x} dx}{16\sqrt{-a}b^2} \\
&= -\frac{x^2 \cosh(c + dx)}{4b(a + bx^2)^2} - \frac{\cosh(c + dx)}{4b^2(a + bx^2)} - \frac{dx \sinh(c + dx)}{8b^2(a + bx^2)} - \frac{\left(d \cosh\left(c - \frac{\sqrt{-a}d}{\sqrt{b}}\right) \right) \int \frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}} + \dots\right)}{\sqrt{-a} + \sqrt{b}x} dx}{16\sqrt{-a}b^2} \\
&= -\frac{x^2 \cosh(c + dx)}{4b(a + bx^2)^2} - \frac{\cosh(c + dx)}{4b^2(a + bx^2)} + \frac{d^2 \cosh\left(c + \frac{\sqrt{-a}d}{\sqrt{b}}\right) \operatorname{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}} - dx\right)}{16b^3} + \frac{d^2 \cosh\left(c - \frac{\sqrt{-a}d}{\sqrt{b}}\right) \operatorname{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}} + dx\right)}{16b^3}
\end{aligned}$$

$$2 - (a^2 b^2 d^2 x^4 + 2 a^2 b d^2 x^2 + a^3 d^2) \sinh(dx + c)^2 + 3((b^3 x^4 + 2 a b^2 x^2 + a^2 b) \cosh(dx + c)^2 - (b^3 x^4 + 2 a b^2 x^2 + a^2 b) \sinh(dx + c)^2) \sqrt{-a d^2 / b} \operatorname{Ei}(dx + \sqrt{-a d^2 / b}) + ((a^2 b^2 d^2 x^4 + 2 a^2 b d^2 x^2 + a^3 d^2) \cosh(dx + c)^2 - (a^2 b^2 d^2 x^4 + 2 a^2 b d^2 x^2 + a^3 d^2) \sinh(dx + c)^2) \sqrt{-a d^2 / b} \operatorname{Ei}(-dx - \sqrt{-a d^2 / b}) \cosh(-c + \sqrt{-a d^2 / b}) + 4(a^2 b^2 d^2 x^3 + a^2 b d^2 x) \sinh(dx + c) - (((a^2 b^2 d^2 x^4 + 2 a^2 b d^2 x^2 + a^3 d^2) \cosh(dx + c)^2 - (a^2 b^2 d^2 x^4 + 2 a^2 b d^2 x^2 + a^3 d^2) \sinh(dx + c)^2) \sqrt{-a d^2 / b} \operatorname{Ei}(dx - \sqrt{-a d^2 / b}) - ((a^2 b^2 d^2 x^4 + 2 a^2 b d^2 x^2 + a^3 d^2) \cosh(dx + c)^2 - (a^2 b^2 d^2 x^4 + 2 a^2 b d^2 x^2 + a^3 d^2) \sinh(dx + c)^2) \sqrt{-a d^2 / b} \operatorname{Ei}(-dx + \sqrt{-a d^2 / b})) \sinh(c + \sqrt{-a d^2 / b}) + (((a^2 b^2 d^2 x^4 + 2 a^2 b d^2 x^2 + a^3 d^2) \cosh(dx + c)^2 - (a^2 b^2 d^2 x^4 + 2 a^2 b d^2 x^2 + a^3 d^2) \sinh(dx + c)^2) \sqrt{-a d^2 / b} \operatorname{Ei}(dx + \sqrt{-a d^2 / b}) - ((a^2 b^2 d^2 x^4 + 2 a^2 b d^2 x^2 + a^3 d^2) \cosh(dx + c)^2 - (a^2 b^2 d^2 x^4 + 2 a^2 b d^2 x^2 + a^3 d^2) \sinh(dx + c)^2) \sqrt{-a d^2 / b} \operatorname{Ei}(-dx - \sqrt{-a d^2 / b})) \sinh(-c + \sqrt{-a d^2 / b})) / ((a^2 b^5 x^4 + 2 a^2 b^4 x^2 + a^3 b^3) \cosh(dx + c)^2 - (a^2 b^5 x^4 + 2 a^2 b^4 x^2 + a^3 b^3) \sinh(dx + c)^2)$$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \cosh(dx + c)}{(bx^2 + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*cosh(d*x+c)/(b*x^2+a)^3,x, algorithm="giac")

[Out] integrate(x^3*cosh(d*x + c)/(b*x^2 + a)^3, x)

maple [B] time = 0.48, size = 820, normalized size = 1.72

$$\frac{d^2 e^{-\frac{d\sqrt{-ab}+cb}{b}} \operatorname{Ei}\left(1, \frac{d\sqrt{-ab}+(dx+c)b-cb}{b}\right)}{32b^3} - \frac{d^2 e^{-\frac{d\sqrt{-ab}+cb}{b}} \operatorname{Ei}\left(1, -\frac{d\sqrt{-ab}-(dx+c)b+cb}{b}\right)}{32b^3} - \frac{3d e^{-\frac{d\sqrt{-ab}+cb}{b}} \operatorname{Ei}\left(1, \frac{d\sqrt{-ab}+(dx+c)b-cb}{b}\right)}{32b^2\sqrt{-ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*cosh(d*x+c)/(b*x^2+a)^3,x)

```
[Out] -1/32*d^2/b^3*exp(-(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1, (d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)-1/32*d^2/b^3*exp(-(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1, -(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)-3/32*d/b^2/(-a*b)^(1/2)*exp(-(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1, (d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)+3/32*d/b^2/(-a*b)^(1/2)*exp(-(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1, -(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)-1/8*d^4*exp(-d*x-c)*a/b^2/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)+1/16*d^5*exp(-d*x-c)/b/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*x^3+1/16*d^5*exp(-d*x-c)*a/b^2/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*x-1/4*d^4*exp(-d*x-c)/b/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*x^2-1/32*d^2/b^3*exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1, (d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)-1/32*d^2/b^3*exp((-d*(-a*b)^(1/2)+c*b)/b)*Ei(1, -(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)-3/32*d/b^2/(-a*b)^(1/2)*exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1, (d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)+3/32*d/b^2/(-a*b)^(1/2)*exp((-d*(-a*b)^(1/2)+c*b)/b)*Ei(1, -(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)-1/8*d^4*exp(d*x+c)*a/b^2/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)-1/4*d^4*exp(d*x+c)/b/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*x^2-1/16*d^5*exp(d*x+c)/b/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*x^3-1/16*d^5*exp(d*x+c)*a/b^2/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{(d^2x^3e^{(2c)} + 3dx^2e^{(2c)} + 12xe^{(2c)})e^{(dx)} - (d^2x^3 - 3dx^2 + 12x)e^{(-dx)}}{2(b^3d^3x^6e^c + 3ab^2d^3x^4e^c + 3a^2bd^3x^2e^c + a^3d^3e^c)} - \frac{1}{2} \int \frac{6(3adx e^c + (ad^2e^c - 10be^c)x^2 + 4ad^3e^c - 10bd^3e^c)x^3}{b^4d^3x^8 + 4ab^3d^3x^6 + 6a^2b^2d^3x^4 + 4a^3d^3e^c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(d*x+c)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*((d^2*x^3*e^(2*c) + 3*d*x^2*e^(2*c) + 12*x*e^(2*c))*e^(d*x) - (d^2*x^3 - 3*d*x^2 + 12*x)*e^(-d*x))/(b^3*d^3*x^6*e^c + 3*a*b^2*d^3*x^4*e^c + 3*a^2*b*d^3*x^2*e^c + a^3*d^3*e^c) - 1/2*integrate(6*(3*a*d*x*e^c + (a*d^2*e^c - 10*b*e^c)*x^2 + 2*a*e^c)*e^(d*x)/(b^4*d^3*x^8 + 4*a*b^3*d^3*x^6 + 6*a^2*b^2*d^3*x^4 + 4*a^3*b*d^3*x^2 + a^4*d^3), x) + 1/2*integrate(-6*(3*a*d*x - (a*d^2 - 10*b)*x^2 - 2*a)*e^(-d*x)/(b^4*d^3*x^8*e^c + 4*a*b^3*d^3*x^6*e^c + 6*a^2*b^2*d^3*x^4*e^c + 4*a^3*b*d^3*x^2*e^c + a^4*d^3*e^c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3 \cosh(c + dx)}{(bx^2 + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*cosh(c + d*x))/(a + b*x^2)^3,x)
```

```
[Out] int((x^3*cosh(c + d*x))/(a + b*x^2)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*cosh(d*x+c)/(b*x**2+a)**3,x)

[Out] Timed out

$$3.73 \quad \int \frac{x^2 \cosh(c+dx)}{(a+bx^2)^3} dx$$

Optimal. Leaf size=746

$$\frac{d^2 \cosh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{16\sqrt{-a} b^{5/2}} - \frac{d^2 \cosh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(xd + \frac{\sqrt{-ad}}{\sqrt{b}}\right)}{16\sqrt{-a} b^{5/2}} - \frac{\cosh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{16(-a)^{3/2} b^{3/2}}$$

[Out] $-1/4*x*\cosh(d*x+c)/b/(b*x^2+a)^2+1/16*\text{Chi}(d*x+d*(-a)^{(1/2)}/b^{(1/2)})*\cosh(c-d*(-a)^{(1/2)}/b^{(1/2)})/(-a)^{(3/2)}/b^{(3/2)}-1/16*\text{Chi}(-d*x+d*(-a)^{(1/2)}/b^{(1/2)})*\cosh(c+d*(-a)^{(1/2)}/b^{(1/2)})/(-a)^{(3/2)}/b^{(3/2)}-1/16*d*\cosh(c+d*(-a)^{(1/2)}/b^{(1/2)})*\text{Shi}(d*x-d*(-a)^{(1/2)}/b^{(1/2)})/a/b^2-1/16*d*\cosh(c-d*(-a)^{(1/2)}/b^{(1/2)})*\text{Shi}(d*x+d*(-a)^{(1/2)}/b^{(1/2)})/a/b^2-1/8*d*\sinh(d*x+c)/b^2/(b*x^2+a)-1/16*d*\text{Chi}(d*x+d*(-a)^{(1/2)}/b^{(1/2)})*\sinh(c-d*(-a)^{(1/2)}/b^{(1/2)})/a/b^2+1/16*\text{Shi}(d*x+d*(-a)^{(1/2)}/b^{(1/2)})*\sinh(c-d*(-a)^{(1/2)}/b^{(1/2)})/(-a)^{(3/2)}/b^{(3/2)}-1/16*d*\text{Chi}(-d*x+d*(-a)^{(1/2)}/b^{(1/2)})*\sinh(c+d*(-a)^{(1/2)}/b^{(1/2)})/a/b^2-1/16*\text{Shi}(d*x-d*(-a)^{(1/2)}/b^{(1/2)})*\sinh(c+d*(-a)^{(1/2)}/b^{(1/2)})/(-a)^{(3/2)}/b^{(3/2)}-1/16*d^2*\text{Chi}(d*x+d*(-a)^{(1/2)}/b^{(1/2)})*\cosh(c-d*(-a)^{(1/2)}/b^{(1/2)})/b^{(5/2)}/(-a)^{(1/2)}+1/16*d^2*\text{Chi}(-d*x+d*(-a)^{(1/2)}/b^{(1/2)})*\cosh(c+d*(-a)^{(1/2)}/b^{(1/2)})/b^{(5/2)}/(-a)^{(1/2)}-1/16*d^2*\text{Shi}(d*x+d*(-a)^{(1/2)}/b^{(1/2)})*\sinh(c-d*(-a)^{(1/2)}/b^{(1/2)})/b^{(5/2)}/(-a)^{(1/2)}+1/16*d^2*\text{Shi}(d*x-d*(-a)^{(1/2)}/b^{(1/2)})*\sinh(c+d*(-a)^{(1/2)}/b^{(1/2)})/b^{(5/2)}/(-a)^{(1/2)}-1/16*\cosh(d*x+c)/a/b^{(3/2)}/((-a)^{(1/2)}-x*b^{(1/2)})+1/16*\cosh(d*x+c)/a/b^{(3/2)}/((-a)^{(1/2)}+x*b^{(1/2)})$

Rubi [A] time = 1.10, antiderivative size = 746, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {5291, 5281, 3297, 3303, 3298, 3301, 5288}

$$\frac{d^2 \cosh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{16\sqrt{-a} b^{5/2}} - \frac{d^2 \cosh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(xd + \frac{\sqrt{-ad}}{\sqrt{b}}\right)}{16\sqrt{-a} b^{5/2}} - \frac{d \sinh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(xd + \frac{\sqrt{-ad}}{\sqrt{b}}\right)}{16ab^2}$$

Antiderivative was successfully verified.

[In] Int[(x^2*Cosh[c + d*x])/(a + b*x^2)^3,x]

[Out] $-\text{Cosh}[c + d*x]/(16*a*b^{(3/2)}*(\text{Sqrt}[-a] - \text{Sqrt}[b]*x)) + \text{Cosh}[c + d*x]/(16*a*b^{(3/2)}*(\text{Sqrt}[-a] + \text{Sqrt}[b]*x)) - (x*\text{Cosh}[c + d*x])/(4*b*(a + b*x^2)^2) - (\text{Cosh}[c + (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{CoshIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] - d*x])/(16*(-a)^{(3/2)}*b^{(3/2)}) + (d^2*\text{Cosh}[c + (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{CoshIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] - d*x])/(16*\text{Sqrt}[-a]*b^{(5/2)}) + (\text{Cosh}[c - (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{CoshIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] + d*x])/(16*(-a)^{(3/2)}*b^{(3/2)}) -$

$$\begin{aligned} & (d^2 \cosh[c - (\sqrt{-a}d)/\sqrt{b}] \operatorname{CoshIntegral}[(\sqrt{-a}d)/\sqrt{b} + dx]) / (16\sqrt{-a}b^{5/2}) - (d \operatorname{CoshIntegral}[(\sqrt{-a}d)/\sqrt{b} + dx] \operatorname{Sinh}[c - (\sqrt{-a}d)/\sqrt{b}]) / (16ab^2) - (d \operatorname{CoshIntegral}[(\sqrt{-a}d)/\sqrt{b} - dx] \operatorname{Sinh}[c + (\sqrt{-a}d)/\sqrt{b}]) / (16ab^2) - (d \operatorname{Sinh}[c + dx]) / (8b^2(a + bx^2)) + (d \cosh[c + (\sqrt{-a}d)/\sqrt{b}] \operatorname{SinhIntegral}[(\sqrt{-a}d)/\sqrt{b} - dx]) / (16ab^2) + (\operatorname{Sinh}[c + (\sqrt{-a}d)/\sqrt{b}] \operatorname{SinhIntegral}[(\sqrt{-a}d)/\sqrt{b} - dx]) / (16(-a)^{3/2}b^{3/2}) - (d^2 \operatorname{Sinh}[c + (\sqrt{-a}d)/\sqrt{b}] \operatorname{SinhIntegral}[(\sqrt{-a}d)/\sqrt{b} - dx]) / (16\sqrt{-a}b^{5/2}) - (d \cosh[c - (\sqrt{-a}d)/\sqrt{b}] \operatorname{SinhIntegral}[(\sqrt{-a}d)/\sqrt{b} + dx]) / (16ab^2) + (\operatorname{Sinh}[c - (\sqrt{-a}d)/\sqrt{b}] \operatorname{SinhIntegral}[(\sqrt{-a}d)/\sqrt{b} + dx]) / (16(-a)^{3/2}b^{3/2}) - (d^2 \operatorname{Sinh}[c - (\sqrt{-a}d)/\sqrt{b}] \operatorname{SinhIntegral}[(\sqrt{-a}d)/\sqrt{b} + dx]) / (16\sqrt{-a}b^{5/2}) \end{aligned}$$

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((c + dx)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + dx)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)])/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)])/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]
```

Rule 5281

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Int[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 5288

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_
)], x_Symbol] := Simp[(e^m*(a + b*x^n)^(p + 1)*Sinh[c + d*x])/(b*n*(p + 1))
, x] - Dist[(d*e^m)/(b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*Cosh[c + d*x], x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IntegerQ[p] && EqQ[m - n + 1, 0
] && LtQ[p, -1] && (IntegerQ[n] || GtQ[e, 0])
```

Rule 5291

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] := Simp[(x^(m - n + 1)*(a + b*x^n)^(p + 1)*Cosh[c + d*x])/(b*n*(p + 1
)), x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1
)*Cosh[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^
n)^(p + 1)*Sinh[c + d*x], x], x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1]
&& IGtQ[n, 0] && RationalQ[m] && (GtQ[m - n + 1, 0] || GtQ[n, 2])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2 \cosh(c+dx)}{(a+bx^2)^3} dx &= -\frac{x \cosh(c+dx)}{4b(a+bx^2)^2} + \frac{\int \frac{\cosh(c+dx)}{(a+bx^2)^2} dx}{4b} + \frac{d \int \frac{x \sinh(c+dx)}{(a+bx^2)^2} dx}{4b} \\
&= -\frac{x \cosh(c+dx)}{4b(a+bx^2)^2} - \frac{d \sinh(c+dx)}{8b^2(a+bx^2)} + \frac{\int \left(-\frac{b \cosh(c+dx)}{4a(\sqrt{-a}\sqrt{b}-bx)^2} - \frac{b \cosh(c+dx)}{4a(\sqrt{-a}\sqrt{b}+bx)^2} - \frac{b \cosh(c+dx)}{2a(-ab-b^2x^2)} \right) dx}{4b} \\
&= -\frac{x \cosh(c+dx)}{4b(a+bx^2)^2} - \frac{d \sinh(c+dx)}{8b^2(a+bx^2)} - \frac{\int \frac{\cosh(c+dx)}{(\sqrt{-a}\sqrt{b}-bx)^2} dx}{16a} - \frac{\int \frac{\cosh(c+dx)}{(\sqrt{-a}\sqrt{b}+bx)^2} dx}{16a} - \frac{\int \frac{\cosh(c+dx)}{-ab-b^2x^2} dx}{8a} \\
&= -\frac{\cosh(c+dx)}{16ab^{3/2}(\sqrt{-a}-\sqrt{b}x)} + \frac{\cosh(c+dx)}{16ab^{3/2}(\sqrt{-a}+\sqrt{b}x)} - \frac{x \cosh(c+dx)}{4b(a+bx^2)^2} - \frac{d \sinh(c+dx)}{8b^2(a+bx^2)} - \frac{d^2 \cosh\left(c + \frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16\sqrt{-a}} \\
&= -\frac{\cosh(c+dx)}{16ab^{3/2}(\sqrt{-a}-\sqrt{b}x)} + \frac{\cosh(c+dx)}{16ab^{3/2}(\sqrt{-a}+\sqrt{b}x)} - \frac{x \cosh(c+dx)}{4b(a+bx^2)^2} - \frac{d \sinh(c+dx)}{8b^2(a+bx^2)} + \frac{d^2 \cosh\left(c + \frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16\sqrt{-a}} \\
&= -\frac{\cosh(c+dx)}{16ab^{3/2}(\sqrt{-a}-\sqrt{b}x)} + \frac{\cosh(c+dx)}{16ab^{3/2}(\sqrt{-a}+\sqrt{b}x)} - \frac{x \cosh(c+dx)}{4b(a+bx^2)^2} - \frac{\cosh\left(c + \frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16(-a)^{3/2}}
\end{aligned}$$

Mathematica [C] time = 2.67, size = 932, normalized size = 1.25

$$\frac{2\sqrt{a}b^2 \cosh(c) \cosh(dx)x^3}{(bx^2+a)^2} + \frac{2\sqrt{a}b^2 \sinh(c) \sinh(dx)x^3}{(bx^2+a)^2} - \frac{2a^{3/2}bd \cosh(dx) \sinh(c)x^2}{(bx^2+a)^2} - \frac{2a^{3/2}bd \cosh(c) \sinh(dx)x^2}{(bx^2+a)^2} - \frac{2a^{3/2}b \cosh(c) \cosh(dx)x}{(bx^2+a)^2}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(x^2*Cosh[c + d*x])/(a + b*x^2)^3, x]

[Out] ((-2*a^(3/2)*b*x*Cosh[c]*Cosh[d*x])/(a + b*x^2)^2 + (2*sqrt[a]*b^2*x^3*Cosh[c]*Cosh[d*x])/(a + b*x^2)^2 - (2*a^(5/2)*d*Cosh[d*x]*Sinh[c])/(a + b*x^2)^2)

$$\begin{aligned}
& 2 - (2a^{3/2} b d x^2 \cosh[dx] \sinh[c]) / (a + b x^2)^2 + (I \cosIntegral[-(\\
& (\sqrt{a} d) / \sqrt{b}] + I d x) * ((b + a d^2) \cosh[c - (I \sqrt{a} d) / \sqrt{b}] \\
& + I \sqrt{a} \sqrt{b} d \sinh[c - (I \sqrt{a} d) / \sqrt{b}])) / \sqrt{b} - (I \cosInt \\
& egral[(\sqrt{a} d) / \sqrt{b} + I d x] * ((b + a d^2) \cosh[c + (I \sqrt{a} d) / \sqrt{b}] \\
& - I \sqrt{a} \sqrt{b} d \sinh[c + (I \sqrt{a} d) / \sqrt{b}]))) / \sqrt{b} - (2 a \\
& ^{(5/2)} d \cosh[c] \sinh[dx]) / (a + b x^2)^2 - (2 a^{3/2} b d x^2 \cosh[c] \sinh \\
& [dx]) / (a + b x^2)^2 - (2 a^{3/2} b x \sinh[c] \sinh[dx]) / (a + b x^2)^2 + (2 \\
& * \sqrt{a} b^2 x^3 \sinh[c] \sinh[dx]) / (a + b x^2)^2 - I \sqrt{a} d \cos[(\sqrt{a} \\
&] d) / \sqrt{b}] * \cosh[c] * \sinIntegral[(\sqrt{a} d) / \sqrt{b} - I d x] + I \sqrt{b} * \\
& \cosh[c] * \sin[(\sqrt{a} d) / \sqrt{b}] * \sinIntegral[(\sqrt{a} d) / \sqrt{b} - I d x] + \\
& (I a d^2 \cosh[c] * \sin[(\sqrt{a} d) / \sqrt{b}] * \sinIntegral[(\sqrt{a} d) / \sqrt{b} \\
& - I d x]) / \sqrt{b} - \sqrt{b} * \cos[(\sqrt{a} d) / \sqrt{b}] * \sinh[c] * \sinIntegral[(S \\
& qrt[a] d) / \sqrt{b} - I d x] - (a d^2 \cos[(\sqrt{a} d) / \sqrt{b}] * \sinh[c] * \sinInt \\
& egral[(\sqrt{a} d) / \sqrt{b} - I d x]) / \sqrt{b} - \sqrt{a} d * \sin[(\sqrt{a} d) / \sqrt{b}] * \sinh[c] * \sinIntegral[(\sqrt{a} d) / \sqrt{b} - I d x] + I \sqrt{a} d * \cos[(S \\
& qrt[a] d) / \sqrt{b}] * \cosh[c] * \sinIntegral[(\sqrt{a} d) / \sqrt{b} + I d x] - I \sqrt{b} * \cosh[c] * \sin[(\sqrt{a} d) / \sqrt{b}] * \sinIntegral[(\sqrt{a} d) / \sqrt{b} + I d \\
& x] - (I a d^2 \cosh[c] * \sin[(\sqrt{a} d) / \sqrt{b}] * \sinIntegral[(\sqrt{a} d) / \sqrt{b} \\
& + I d x]) / \sqrt{b} - \sqrt{b} * \cos[(\sqrt{a} d) / \sqrt{b}] * \sinh[c] * \sinIntegr \\
& al[(\sqrt{a} d) / \sqrt{b} + I d x] - (a d^2 \cos[(\sqrt{a} d) / \sqrt{b}] * \sinh[c] * \sin \\
& inIntegral[(\sqrt{a} d) / \sqrt{b} + I d x]) / \sqrt{b} - \sqrt{a} d * \sin[(\sqrt{a} d) \\
&) / \sqrt{b}] * \sinh[c] * \sinIntegral[(\sqrt{a} d) / \sqrt{b} + I d x]) / (16 a^{3/2} b^2)
\end{aligned}$$

fricas [B] time = 0.48, size = 2047, normalized size = 2.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*cosh(dx+c)/(b*x^2+a)^3,x, algorithm="fricas")

[Out] 1/32*(4*(a*b^2*d*x^3 - a^2*b*d*x)*cosh(dx + c) - (((a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*cosh(dx + c)^2 - (a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*sinh(dx + c)^2 + ((a^3*d^2 + (a*b^2*d^2 + b^3)*x^4 + a^2*b + 2*(a^2*b*d^2 + a*b^2)*x^2)*cosh(dx + c)^2 - (a^3*d^2 + (a*b^2*d^2 + b^3)*x^4 + a^2*b + 2*(a^2*b*d^2 + a*b^2)*x^2)*sinh(dx + c)^2)*sqrt(-a*d^2/b))*Ei(dx - sqrt(-a*d^2/b)) - ((a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*cosh(dx + c)^2 - (a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*sinh(dx + c)^2 - ((a^3*d^2 + (a*b^2*d^2 + b^3)*x^4 + a^2*b + 2*(a^2*b*d^2 + a*b^2)*x^2)*cosh(dx + c)^2 - (a^3*d^2 + (a*b^2*d^2 + b^3)*x^4 + a^2*b + 2*(a^2*b*d^2 + a*b^2)*x^2)*sinh(dx + c)^2)*sqrt(-a*d^2/b))*Ei(-dx + sqrt(-a*d^2/b))*cosh(c + sqrt(-a*d^2/b)) - (((a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*cosh(dx + c)^2 - (a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*sinh(dx + c)^2 - ((a^3*d^2 + (a*b^2*d^2 + b^3)*x^4 + a^2*b + 2*(a^2*b*d^2 + a*b^2)*x^2)*cosh(dx + c)^2 - (a^3*d^2 + (a*b^2*d^2 + b^3)*x^4 + a^2*b + 2*(a^2*b*d^2 + a*b^2)*x^2)*sinh(dx + c)^2)

```

*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x + sqrt(-a*d^2/b)) - ((a*b^2*d^2*x^
4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*cosh(d*x + c)^2 - (a*b^2*d^2*x^4 + 2*a^2*b*d
^2*x^2 + a^3*d^2)*sinh(d*x + c)^2 + ((a^3*d^2 + (a*b^2*d^2 + b^3)*x^4 + a^2
*b + 2*(a^2*b*d^2 + a*b^2)*x^2)*cosh(d*x + c)^2 - (a^3*d^2 + (a*b^2*d^2 + b
^3)*x^4 + a^2*b + 2*(a^2*b*d^2 + a*b^2)*x^2)*sinh(d*x + c)^2)*sqrt(-a*d^2/b
))*Ei(-d*x - sqrt(-a*d^2/b))*cosh(-c + sqrt(-a*d^2/b)) - 4*(a^2*b*d^2*x^2
+ a^3*d^2)*sinh(d*x + c) - (((a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*co
sh(d*x + c)^2 - (a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*sinh(d*x + c)^2
+ ((a^3*d^2 + (a*b^2*d^2 + b^3)*x^4 + a^2*b + 2*(a^2*b*d^2 + a*b^2)*x^2)*c
osh(d*x + c)^2 - (a^3*d^2 + (a*b^2*d^2 + b^3)*x^4 + a^2*b + 2*(a^2*b*d^2 +
a*b^2)*x^2)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x - sqrt(-a*d^2/b)) + ((a
*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*cosh(d*x + c)^2 - (a*b^2*d^2*x^4
+ 2*a^2*b*d^2*x^2 + a^3*d^2)*sinh(d*x + c)^2 - ((a^3*d^2 + (a*b^2*d^2 + b^3
)*x^4 + a^2*b + 2*(a^2*b*d^2 + a*b^2)*x^2)*cosh(d*x + c)^2 - (a^3*d^2 + (a
*b^2*d^2 + b^3)*x^4 + a^2*b + 2*(a^2*b*d^2 + a*b^2)*x^2)*sinh(d*x + c)^2)*sq
rt(-a*d^2/b))*Ei(-d*x + sqrt(-a*d^2/b))*sinh(c + sqrt(-a*d^2/b)) + (((a*b^
2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*cosh(d*x + c)^2 - (a*b^2*d^2*x^4 + 2
*a^2*b*d^2*x^2 + a^3*d^2)*sinh(d*x + c)^2 - ((a^3*d^2 + (a*b^2*d^2 + b^3)*x
^4 + a^2*b + 2*(a^2*b*d^2 + a*b^2)*x^2)*cosh(d*x + c)^2 - (a^3*d^2 + (a*b^
2*d^2 + b^3)*x^4 + a^2*b + 2*(a^2*b*d^2 + a*b^2)*x^2)*sinh(d*x + c)^2)*sqrt(
-a*d^2/b))*Ei(d*x + sqrt(-a*d^2/b)) + ((a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a
^3*d^2)*cosh(d*x + c)^2 - (a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*sinh(
d*x + c)^2 + ((a^3*d^2 + (a*b^2*d^2 + b^3)*x^4 + a^2*b + 2*(a^2*b*d^2 + a*b
^2)*x^2)*cosh(d*x + c)^2 - (a^3*d^2 + (a*b^2*d^2 + b^3)*x^4 + a^2*b + 2*(a^
2*b*d^2 + a*b^2)*x^2)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x - sqrt(-a*d^
2/b))*sinh(-c + sqrt(-a*d^2/b)))/((a^2*b^4*d*x^4 + 2*a^3*b^3*d*x^2 + a^4*b
^2*d)*cosh(d*x + c)^2 - (a^2*b^4*d*x^4 + 2*a^3*b^3*d*x^2 + a^4*b^2*d)*sinh(
d*x + c)^2)

```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \cosh(dx + c)}{(bx^2 + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*cosh(d*x+c)/(b*x^2+a)^3,x, algorithm="giac")

[Out] integrate(x^2*cosh(d*x + c)/(b*x^2 + a)^3, x)

maple [A] time = 0.33, size = 1064, normalized size = 1.43

$$\frac{d^5 e^{-dx-c} x^2}{16b(b^2 d^4 x^4 + 2ab d^4 x^2 + a^2 d^4)} + \frac{d^4 e^{-dx-c} x^3}{16a(b^2 d^4 x^4 + 2ab d^4 x^2 + a^2 d^4)} + \frac{d^5 e^{-dx-c} a}{16b^2(b^2 d^4 x^4 + 2ab d^4 x^2 + a^2 d^4)} - \frac{d^4 e^{-dx-c} x^3}{16b(b^2 d^4 x^4 + 2ab d^4 x^2 + a^2 d^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*cosh(c + d*x))/(a + b*x^2)^3,x)
```

```
[Out] int((x^2*cosh(c + d*x))/(a + b*x^2)^3, x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cosh(d*x+c)/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

$$3.74 \quad \int \frac{x \cosh(c+dx)}{(a+bx^2)^3} dx$$

Optimal. Leaf size=512

$$\frac{d \sinh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(xd + \frac{\sqrt{-ad}}{\sqrt{b}}\right)}{16(-a)^{3/2}b^{3/2}} - \frac{d \sinh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{16(-a)^{3/2}b^{3/2}} + \frac{d \cosh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right) \text{Shi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{16(-a)^{3/2}b^{3/2}} +$$

[Out] $-1/4*\cosh(d*x+c)/b/(b*x^2+a)^2-1/16*d^2*\text{Chi}(d*x+d*(-a)^{(1/2)}/b^{(1/2)})*\cosh(c-d*(-a)^{(1/2)}/b^{(1/2)})/a/b^2-1/16*d^2*\text{Chi}(-d*x+d*(-a)^{(1/2)}/b^{(1/2)})*\cosh(c+d*(-a)^{(1/2)}/b^{(1/2)})/a/b^2-1/16*d*\cosh(c+d*(-a)^{(1/2)}/b^{(1/2)})*\text{Shi}(d*x-d*(-a)^{(1/2)}/b^{(1/2)})/(-a)^{(3/2)}/b^{(3/2)}+1/16*d*\cosh(c-d*(-a)^{(1/2)}/b^{(1/2)})*\text{Shi}(d*x+d*(-a)^{(1/2)}/b^{(1/2)})/(-a)^{(3/2)}/b^{(3/2)}+1/16*d*\text{Chi}(d*x+d*(-a)^{(1/2)}/b^{(1/2)})*\sinh(c-d*(-a)^{(1/2)}/b^{(1/2)})/(-a)^{(3/2)}/b^{(3/2)}-1/16*d^2*\text{Shi}(d*x+d*(-a)^{(1/2)}/b^{(1/2)})*\sinh(c-d*(-a)^{(1/2)}/b^{(1/2)})/a/b^2-1/16*d*\text{Chi}(-d*x+d*(-a)^{(1/2)}/b^{(1/2)})*\sinh(c+d*(-a)^{(1/2)}/b^{(1/2)})/(-a)^{(3/2)}/b^{(3/2)}-1/16*d^2*\text{Shi}(d*x-d*(-a)^{(1/2)}/b^{(1/2)})*\sinh(c+d*(-a)^{(1/2)}/b^{(1/2)})/a/b^2-1/16*d*\sinh(d*x+c)/a/b^{(3/2)}/((-a)^{(1/2)}-x*b^{(1/2)})+1/16*d*\sinh(d*x+c)/a/b^{(3/2)}/((-a)^{(1/2)}+x*b^{(1/2)})$

Rubi [A] time = 0.83, antiderivative size = 512, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {5289, 5280, 3297, 3303, 3298, 3301}

$$\frac{d^2 \cosh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{16ab^2} - \frac{d^2 \cosh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(xd + \frac{\sqrt{-ad}}{\sqrt{b}}\right)}{16ab^2} + \frac{d \sinh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(xd + \frac{\sqrt{-ad}}{\sqrt{b}}\right)}{16(-a)^{3/2}b^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(x*Cosh[c + d*x])/(a + b*x^2)^3, x]

[Out] $-\text{Cosh}[c + d*x]/(4*b*(a + b*x^2)^2) - (d^2*\text{Cosh}[c + (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{CoshIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] - d*x])/(16*a*b^2) - (d^2*\text{Cosh}[c - (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{CoshIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] + d*x])/(16*a*b^2) + (d*\text{CoshIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] + d*x]*\text{Sinh}[c - (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]])/(16*(-a)^{(3/2)}*b^{(3/2)}) - (d*\text{CoshIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] - d*x]*\text{Sinh}[c + (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]])/(16*(-a)^{(3/2)}*b^{(3/2)}) - (d*\text{Sinh}[c + d*x])/(16*a*b^{(3/2)}*(\text{Sqrt}[-a] - \text{Sqrt}[b]*x)) + (d*\text{Sinh}[c + d*x])/(16*a*b^{(3/2)}*(\text{Sqrt}[-a] + \text{Sqrt}[b]*x)) + (d*\text{Cosh}[c + (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{SinhIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] - d*x])/(16*(-a)^{(3/2)}*b^{(3/2)}) + (d^2*\text{Sinh}[c + (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{SinhIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] - d*x])/(16*a*b^2) + (d*\text{Cosh}[c - (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{SinhIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] + d*x])/(16*(-a)^{(3/2)}*b^{(3/2)})$

$(3/2)) - (d^2 \sinh[c - (\sqrt{-a}d)/\sqrt{b}] \operatorname{SinhIntegral}[(\sqrt{-a}d)/\sqrt{b} + dx]) / (16ab^2)$

Rule 3297

$\operatorname{Int}[(c_.) + (d_.)x]^{(m_.)} \sin[(e_.) + (f_.)x], x_{\text{Symbol}}] \rightarrow \operatorname{Simp}[(c + dx)^{(m+1)} \sin[e + fx] / (d(m+1)), x] - \operatorname{Dist}[f / (d(m+1)), \operatorname{Int}[(c + dx)^{(m+1)} \cos[e + fx], x], x] /;$ $\operatorname{FreeQ}\{c, d, e, f\}, x] \ \&\& \operatorname{LtQ}[m, -1]$

Rule 3298

$\operatorname{Int}[\sin[(e_.) + (\operatorname{Complex}[0, fz_])x] / ((c_.) + (d_.)x), x_{\text{Symbol}}] \rightarrow \operatorname{Simp}[(I \operatorname{SinhIntegral}[(c*fxz)/d + f*fxz*x]) / d, x] /;$ $\operatorname{FreeQ}\{c, d, e, f, fz\}, x] \ \&\& \operatorname{EqQ}[d*e - c*f*fz*I, 0]$

Rule 3301

$\operatorname{Int}[\sin[(e_.) + (\operatorname{Complex}[0, fz_])x] / ((c_.) + (d_.)x), x_{\text{Symbol}}] \rightarrow \operatorname{Simp}[\operatorname{CoshIntegral}[(c*fxz)/d + f*fxz*x] / d, x] /;$ $\operatorname{FreeQ}\{c, d, e, f, fz\}, x] \ \&\& \operatorname{EqQ}[d*(e - \pi/2) - c*f*fz*I, 0]$

Rule 3303

$\operatorname{Int}[\sin[(e_.) + (f_.)x] / ((c_.) + (d_.)x), x_{\text{Symbol}}] \rightarrow \operatorname{Dist}[\cos[(d*e - c*f)/d], \operatorname{Int}[\sin[(c*f)/d + fx] / (c + dx), x], x] + \operatorname{Dist}[\sin[(d*e - c*f)/d], \operatorname{Int}[\cos[(c*f)/d + fx] / (c + dx), x], x] /;$ $\operatorname{FreeQ}\{c, d, e, f\}, x] \ \&\& \operatorname{NeQ}[d*e - c*f, 0]$

Rule 5280

$\operatorname{Int}[(a_.) + (b_.)x]^{(n_.)} \operatorname{Sinh}[(c_.) + (d_.)x], x_{\text{Symbol}}] \rightarrow \operatorname{Int}[\operatorname{ExpandIntegrand}[\operatorname{Sinh}[c + dx], (a + b*x^n)^p, x], x] /;$ $\operatorname{FreeQ}\{a, b, c, d\}, x] \ \&\& \operatorname{ILtQ}[p, 0] \ \&\& \operatorname{IGtQ}[n, 0] \ \&\& (\operatorname{EqQ}[n, 2] \ || \ \operatorname{EqQ}[p, -1])$

Rule 5289

$\operatorname{Int}[\operatorname{Cosh}[(c_.) + (d_.)x] * ((e_.)x)^{(m_.)} * ((a_.) + (b_.)x]^{(n_.)} \operatorname{Sinh}[(c_.) + (d_.)x], x_{\text{Symbol}}] \rightarrow \operatorname{Simp}[(e^m * (a + b*x^n)^{(p+1)} \operatorname{Cosh}[c + dx]) / (b*n*(p+1)), x] - \operatorname{Dist}[(d*e^m) / (b*n*(p+1)), \operatorname{Int}[(a + b*x^n)^{(p+1)} \operatorname{Sinh}[c + dx], x], x] /;$ $\operatorname{FreeQ}\{a, b, c, d, e, m, n\}, x] \ \&\& \operatorname{IntegerQ}[p] \ \&\& \operatorname{EqQ}[m - n + 1, 0] \ \&\& \operatorname{LtQ}[p, -1] \ \&\& (\operatorname{IntegerQ}[n] \ || \ \operatorname{GtQ}[e, 0])$

Rubi steps

$$\begin{aligned}
\int \frac{x \cosh(c + dx)}{(a + bx^2)^3} dx &= -\frac{\cosh(c + dx)}{4b(a + bx^2)^2} + \frac{d \int \frac{\sinh(c+dx)}{(a+bx^2)^2} dx}{4b} \\
&= -\frac{\cosh(c + dx)}{4b(a + bx^2)^2} + \frac{d \int \left(-\frac{b \sinh(c+dx)}{4a(\sqrt{-a} \sqrt{b} - bx)^2} - \frac{b \sinh(c+dx)}{4a(\sqrt{-a} \sqrt{b} + bx)^2} - \frac{b \sinh(c+dx)}{2a(-ab - b^2x^2)} \right) dx}{4b} \\
&= -\frac{\cosh(c + dx)}{4b(a + bx^2)^2} - \frac{d \int \frac{\sinh(c+dx)}{(\sqrt{-a} \sqrt{b} - bx)^2} dx}{16a} - \frac{d \int \frac{\sinh(c+dx)}{(\sqrt{-a} \sqrt{b} + bx)^2} dx}{16a} - \frac{d \int \frac{\sinh(c+dx)}{-ab - b^2x^2} dx}{8a} \\
&= -\frac{\cosh(c + dx)}{4b(a + bx^2)^2} - \frac{d \sinh(c + dx)}{16ab^{3/2}(\sqrt{-a} - \sqrt{b}x)} + \frac{d \sinh(c + dx)}{16ab^{3/2}(\sqrt{-a} + \sqrt{b}x)} - \frac{d \int \left(-\frac{\sqrt{-a} \sinh(c+dx)}{2ab(\sqrt{-a} - \sqrt{b}x)} \right) dx}{8a} \\
&= -\frac{\cosh(c + dx)}{4b(a + bx^2)^2} - \frac{d \sinh(c + dx)}{16ab^{3/2}(\sqrt{-a} - \sqrt{b}x)} + \frac{d \sinh(c + dx)}{16ab^{3/2}(\sqrt{-a} + \sqrt{b}x)} + \frac{d \int \frac{\sinh(c+dx)}{\sqrt{-a} - \sqrt{b}x} dx}{16(-a)^{3/2}b} + \\
&= -\frac{\cosh(c + dx)}{4b(a + bx^2)^2} - \frac{d^2 \cosh\left(c + \frac{\sqrt{-a}d}{\sqrt{b}}\right) \text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}} - dx\right)}{16ab^2} - \frac{d^2 \cosh\left(c - \frac{\sqrt{-a}d}{\sqrt{b}}\right) \text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16ab^2} \\
&= -\frac{\cosh(c + dx)}{4b(a + bx^2)^2} - \frac{d^2 \cosh\left(c + \frac{\sqrt{-a}d}{\sqrt{b}}\right) \text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}} - dx\right)}{16ab^2} - \frac{d^2 \cosh\left(c - \frac{\sqrt{-a}d}{\sqrt{b}}\right) \text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16ab^2}
\end{aligned}$$

Mathematica [C] time = 1.85, size = 637, normalized size = 1.24

$$\frac{id^2 \sinh(c) \left(\sin\left(\frac{\sqrt{a}d}{\sqrt{b}}\right) \text{Ci}\left(idx - \frac{\sqrt{a}d}{\sqrt{b}}\right) - \sin\left(\frac{\sqrt{a}d}{\sqrt{b}}\right) \text{Ci}\left(ixd + \frac{\sqrt{a}d}{\sqrt{b}}\right) + \cos\left(\frac{\sqrt{a}d}{\sqrt{b}}\right) \left(\text{Si}\left(ixd + \frac{\sqrt{a}d}{\sqrt{b}}\right) - \text{Si}\left(\frac{\sqrt{a}d}{\sqrt{b}} - idx\right) \right) \right)}{b} - \frac{d^2 \cosh(c) \left(\cos\left(\frac{\sqrt{a}d}{\sqrt{b}}\right) \text{Ci}\left(idx - \frac{\sqrt{a}d}{\sqrt{b}}\right) + \cos\left(\frac{\sqrt{a}d}{\sqrt{b}}\right) \text{Ci}\left(ixd + \frac{\sqrt{a}d}{\sqrt{b}}\right) \right)}{16ab^2}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(x*Cosh[c + d*x])/(a + b*x^2)^3, x]

[Out] ((2*Cosh[d*x]*(-2*a*Cosh[c] + d*x*(a + b*x^2)*Sinh[c]))/(a + b*x^2)^2 + (2*(d*x*(a + b*x^2)*Cosh[c] - 2*a*Sinh[c])*Sinh[d*x])/(a + b*x^2)^2 + (I*d*Sin

$$\begin{aligned}
& h[c] * (\text{Cos}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b]] * \text{CosIntegral}[-((\text{Sqrt}[a]*d)/\text{Sqrt}[b]) + I*d*x] \\
& - \text{Cos}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b]] * \text{CosIntegral}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b] + I*d*x] + \text{Sin}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b]] * (\text{SinIntegral}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b] - I*d*x] - \text{SinIntegral}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b] + I*d*x])) / (\text{Sqrt}[a]*\text{Sqrt}[b]) + (I*d^2*\text{Sinh}[c] * (\text{CosIntegral}[-((\text{Sqrt}[a]*d)/\text{Sqrt}[b]) + I*d*x] * \text{Sin}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b]] - \text{CosIntegral}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b] + I*d*x] * \text{Sin}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b]] + \text{Cos}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b]] * (-\text{SinIntegral}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b] - I*d*x] + \text{SinIntegral}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b] + I*d*x])) / b + (d*\text{Cosh}[c] * (\text{CosIntegral}[-((\text{Sqrt}[a]*d)/\text{Sqrt}[b]) + I*d*x] * \text{Sin}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b]] + \text{CosIntegral}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b] + I*d*x] * \text{Sin}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b]] - \text{Cos}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b]] * (\text{SinIntegral}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b] - I*d*x] + \text{SinIntegral}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b] + I*d*x])))) / (\text{Sqrt}[a]*\text{Sqrt}[b]) - (d^2*\text{Cosh}[c] * (\text{Cos}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b]] * \text{CosIntegral}[-((\text{Sqrt}[a]*d)/\text{Sqrt}[b]) + I*d*x] + \text{Cos}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b]] * \text{CosIntegral}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b] + I*d*x] + \text{Sin}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b]] * (\text{SinIntegral}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b] - I*d*x] + \text{SinIntegral}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b] + I*d*x])))) / b) / (16*a*b)
\end{aligned}$$

fricas [B] time = 0.59, size = 1607, normalized size = 3.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cosh(d*x+c)/(b*x^2+a)^3,x, algorithm="fricas")

[Out]
$$\begin{aligned}
& -1/32*(8*a^2*b*cosh(d*x + c) + ((a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2) * cosh(d*x + c)^2 - (a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2) * sinh(d*x + c)^2 + ((b^3*x^4 + 2*a*b^2*x^2 + a^2*b) * cosh(d*x + c)^2 - (b^3*x^4 + 2*a*b^2*x^2 + a^2*b) * sinh(d*x + c)^2) * sqrt(-a*d^2/b)) * Ei(d*x - sqrt(-a*d^2/b)) + ((a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2) * cosh(d*x + c)^2 - (a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2) * sinh(d*x + c)^2 - ((b^3*x^4 + 2*a*b^2*x^2 + a^2*b) * cosh(d*x + c)^2 - (b^3*x^4 + 2*a*b^2*x^2 + a^2*b) * sinh(d*x + c)^2) * sqrt(-a*d^2/b)) * Ei(-d*x + sqrt(-a*d^2/b)) * cosh(c + sqrt(-a*d^2/b)) + ((a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2) * cosh(d*x + c)^2 - (a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2) * sinh(d*x + c)^2 + ((b^3*x^4 + 2*a*b^2*x^2 + a^2*b) * cosh(d*x + c)^2 - (b^3*x^4 + 2*a*b^2*x^2 + a^2*b) * sinh(d*x + c)^2) * sqrt(-a*d^2/b)) * Ei(d*x + sqrt(-a*d^2/b)) + ((a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2) * cosh(d*x + c)^2 - (a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2) * sinh(d*x + c)^2 + ((b^3*x^4 + 2*a*b^2*x^2 + a^2*b) * cosh(d*x + c)^2 - (b^3*x^4 + 2*a*b^2*x^2 + a^2*b) * sinh(d*x + c)^2) * sqrt(-a*d^2/b)) * Ei(-d*x - sqrt(-a*d^2/b))) * cosh(-c + sqrt(-a*d^2/b)) - 4*(a*b^2*d*x^3 + a^2*b*d*x) * sinh(d*x + c) + (((a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2) * cosh(d*x + c)^2 - (a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2) * sinh(d*x + c)^2 + ((b^3*x^4 + 2*a*b^2*x^2 + a^2*b) * cosh(d*x + c)^2 - (b^3*x^4 + 2*a*b^2*x^2 + a^2*b) * sinh(d*x + c)^2) * sqrt(-a*d^2/b)) * Ei(d*x - sqrt(-a*d^2/b)) - ((a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2) * cosh(d*x + c)^2 - (a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2) * sinh(d*x + c)^2 - ((b^3*x^4 + 2*a*b^2*x^2 + a^2*b) * cosh(d*x + c)^2 - (b^3*x^4 + 2*a*b^2*x^2 + a^2*b) * sinh(d*x + c)^2) * sqrt(-a*d^2/b)) * Ei(d*x + sqrt(-a*d^2/b)) * cosh(c + sqrt(-a*d^2/b)) + ((a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2) * cosh(d*x + c)^2 - (a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2) * sinh(d*x + c)^2 - ((b^3*x^4 + 2*a*b^2*x^2 + a^2*b) * cosh(d*x + c)^2 - (b^3*x^4 + 2*a*b^2*x^2 + a^2*b) * sinh(d*x + c)^2) * sqrt(-a*d^2/b)) * Ei(-d*x - sqrt(-a*d^2/b)) * cosh(c + sqrt(-a*d^2/b))
\end{aligned}$$

$$b^3x^4 + 2ab^2x^2 + a^2b) \sinh(dx + c)^2 \sqrt{-ad^2/b}) Ei(-dx + \sqrt{-ad^2/b}) \sinh(c + \sqrt{-ad^2/b}) - ((ab^2d^2x^4 + 2a^2bd^2x^2 + a^3d^2) \cosh(dx + c)^2 - (ab^2d^2x^4 + 2a^2bd^2x^2 + a^3d^2) \sinh(dx + c)^2 - ((b^3x^4 + 2ab^2x^2 + a^2b) \cosh(dx + c)^2 - (b^3x^4 + 2ab^2x^2 + a^2b) \sinh(dx + c)^2) \sqrt{-ad^2/b}) Ei(dx + \sqrt{-ad^2/b}) - ((ab^2d^2x^4 + 2a^2bd^2x^2 + a^3d^2) \cosh(dx + c)^2 - (ab^2d^2x^4 + 2a^2bd^2x^2 + a^3d^2) \sinh(dx + c)^2 + ((b^3x^4 + 2ab^2x^2 + a^2b) \cosh(dx + c)^2 - (b^3x^4 + 2ab^2x^2 + a^2b) \sinh(dx + c)^2) \sqrt{-ad^2/b}) Ei(-dx - \sqrt{-ad^2/b}) \sinh(-c + \sqrt{-ad^2/b})) / ((a^2b^4x^4 + 2a^3b^3x^2 + a^4b^2) \cosh(dx + c)^2 - (a^2b^4x^4 + 2a^3b^3x^2 + a^4b^2) \sinh(dx + c)^2)$$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \cosh(dx + c)}{(bx^2 + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cosh(dx+c)/(b*x^2+a)^3,x, algorithm="giac")

[Out] integrate(x*cosh(dx + c)/(b*x^2 + a)^3, x)

maple [A] time = 0.23, size = 743, normalized size = 1.45

$$\frac{d^5 e^{-dx-c} x^3}{16a(b^2 d^4 x^4 + 2ab d^4 x^2 + a^2 d^4)} - \frac{d^5 e^{-dx-c} x}{16b(b^2 d^4 x^4 + 2ab d^4 x^2 + a^2 d^4)} - \frac{d^4 e^{-dx-c}}{8b(b^2 d^4 x^4 + 2ab d^4 x^2 + a^2 d^4)} + \frac{d^2 e^{-\frac{d\sqrt{-ab}+c}{b}}}{16b(b^2 d^4 x^4 + 2ab d^4 x^2 + a^2 d^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*cosh(dx+c)/(b*x^2+a)^3,x)

[Out]
$$-1/16*d^5*\exp(-d*x-c)/a/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*x^3-1/16*d^5*\exp(-d*x-c)/b/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*x-1/8*d^4*\exp(-d*x-c)/b/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)+1/32*d^2/b^2/a*\exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1, -(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)+1/32*d^2/b^2/a*\exp(-(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1, (d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)+1/32*d/b/a/(-a*b)^(1/2)*\exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1, -(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)-1/32*d/b/a/(-a*b)^(1/2)*\exp(-(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1, (d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)+1/16*d^5*\exp(d*x+c)/a/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*x^3+1/16*d^5*\exp(d*x+c)/b/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*x-1/8*d^4*\exp(d*x+c)/b/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)+1/32*d^2/b^2/a*\exp((-d*(-a*b)^(1/2)+c*b)/b)*Ei(1, -(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)+1/32*d^2/b^2/a*\exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1, (d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)+1/32*d/b/a/(-a*b)^(1/2)$$

$(1/2)*\exp((-d*(-a*b)^{(1/2)+c*b}/b)*\text{Ei}(1, -(d*(-a*b)^{(1/2)+(d*x+c)*b-c*b)/b)-1/32*d/b/a/(-a*b)^{(1/2)*\exp((d*(-a*b)^{(1/2)+c*b}/b)*\text{Ei}(1, (d*(-a*b)^{(1/2)-(d*x+c)*b+c*b)/b)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{x e^{(dx+2c)} - x e^{(-dx)}}{2(b^3 dx^6 e^c + 3 ab^2 dx^4 e^c + 3 a^2 b dx^2 e^c + a^3 d e^c)} + \frac{1}{2} \int \frac{(5 b x^2 e^c - a e^c) e^{(dx)}}{b^4 dx^8 + 4 a b^3 dx^6 + 6 a^2 b^2 dx^4 + 4 a^3 b dx^2 + a^4 d} dx - \frac{1}{2} \int \frac{1}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cosh(d*x+c)/(b*x^2+a)^3,x, algorithm="maxima")

[Out] $1/2*(x*e^{(d*x + 2*c)} - x*e^{(-d*x)})/(b^3*d*x^6*e^c + 3*a*b^2*d*x^4*e^c + 3*a^2*b*d*x^2*e^c + a^3*d*e^c) + 1/2*\text{integrate}((5*b*x^2*e^c - a*e^c)*e^{(d*x)})/(b^4*d*x^8 + 4*a*b^3*d*x^6 + 6*a^2*b^2*d*x^4 + 4*a^3*b*d*x^2 + a^4*d), x) - 1/2*\text{integrate}((5*b*x^2 - a)*e^{(-d*x)})/(b^4*d*x^8*e^c + 4*a*b^3*d*x^6*e^c + 6*a^2*b^2*d*x^4*e^c + 4*a^3*b*d*x^2*e^c + a^4*d*e^c), x)$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x \cosh(c + d x)}{(b x^2 + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x*cosh(c + d*x))/(a + b*x^2)^3,x)

[Out] int((x*cosh(c + d*x))/(a + b*x^2)^3, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cosh(d*x+c)/(b*x**2+a)**3,x)

[Out] Timed out

$$3.75 \quad \int \frac{\cosh(c+dx)}{(a+bx^2)^3} dx$$

Optimal. Leaf size=856

$$\frac{\cosh\left(c + \frac{\sqrt{-ad}}{\sqrt{b}}\right) \operatorname{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right) d^2}{16(-a)^{3/2} b^{3/2}} - \frac{\cosh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \operatorname{Chi}\left(xd + \frac{\sqrt{-ad}}{\sqrt{b}}\right) d^2}{16(-a)^{3/2} b^{3/2}} - \frac{\sinh\left(c + \frac{\sqrt{-ad}}{\sqrt{b}}\right) \operatorname{Shi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right) d^2}{16(-a)^{3/2} b^{3/2}}$$

[Out] $-1/16*d^2*\operatorname{Chi}(d*x+d*(-a)^{(1/2)}/b^{(1/2)})*\cosh(c-d*(-a)^{(1/2)}/b^{(1/2)})/(-a)^{(3/2)}/b^{(3/2)}+1/16*d^2*\operatorname{Chi}(-d*x+d*(-a)^{(1/2)}/b^{(1/2)})*\cosh(c+d*(-a)^{(1/2)}/b^{(1/2)})/(-a)^{(3/2)}/b^{(3/2)}-3/16*d*\cosh(c+d*(-a)^{(1/2)}/b^{(1/2)})*\operatorname{Shi}(d*x-d*(-a)^{(1/2)}/b^{(1/2)})/a^2/b-3/16*d*\cosh(c-d*(-a)^{(1/2)}/b^{(1/2)})*\operatorname{Shi}(d*x+d*(-a)^{(1/2)}/b^{(1/2)})/a^2/b-3/16*d*\operatorname{Chi}(d*x+d*(-a)^{(1/2)}/b^{(1/2)})*\sinh(c-d*(-a)^{(1/2)}/b^{(1/2)})/a^2/b-1/16*d^2*\operatorname{Shi}(d*x+d*(-a)^{(1/2)}/b^{(1/2)})*\sinh(c-d*(-a)^{(1/2)}/b^{(1/2)})/(-a)^{(3/2)}/b^{(3/2)}-3/16*d*\operatorname{Chi}(-d*x+d*(-a)^{(1/2)}/b^{(1/2)})*\sinh(c+d*(-a)^{(1/2)}/b^{(1/2)})/a^2/b+1/16*d^2*\operatorname{Shi}(d*x-d*(-a)^{(1/2)}/b^{(1/2)})*\sinh(c+d*(-a)^{(1/2)}/b^{(1/2)})/(-a)^{(3/2)}/b^{(3/2)}-3/16*\operatorname{Chi}(d*x+d*(-a)^{(1/2)}/b^{(1/2)})*\cosh(c-d*(-a)^{(1/2)}/b^{(1/2)})/(-a)^{(5/2)}/b^{(1/2)}+3/16*\operatorname{Chi}(-d*x+d*(-a)^{(1/2)}/b^{(1/2)})*\cosh(c+d*(-a)^{(1/2)}/b^{(1/2)})/(-a)^{(5/2)}/b^{(1/2)}-3/16*\operatorname{Shi}(d*x+d*(-a)^{(1/2)}/b^{(1/2)})*\sinh(c-d*(-a)^{(1/2)}/b^{(1/2)})/(-a)^{(5/2)}/b^{(1/2)}+3/16*\operatorname{Shi}(d*x-d*(-a)^{(1/2)}/b^{(1/2)})*\sinh(c+d*(-a)^{(1/2)}/b^{(1/2)})/(-a)^{(5/2)}/b^{(1/2)}-1/16*\cosh(d*x+c)/(-a)^{(3/2)}/b^{(1/2)}/((-a)^{(1/2)}-x*b^{(1/2)})^2+1/16*d*\sinh(d*x+c)/(-a)^{(3/2)}/b/((-a)^{(1/2)}-x*b^{(1/2)})-3/16*\cosh(d*x+c)/a^2/b^{(1/2)}/((-a)^{(1/2)}-x*b^{(1/2)})+1/16*\cosh(d*x+c)/(-a)^{(3/2)}/b^{(1/2)}/((-a)^{(1/2)}+x*b^{(1/2)})^2+1/16*d*\sinh(d*x+c)/(-a)^{(3/2)}/b/((-a)^{(1/2)}+x*b^{(1/2)})+3/16*\cosh(d*x+c)/a^2/b^{(1/2)}/((-a)^{(1/2)}+x*b^{(1/2)})$

Rubi [A] time = 1.26, antiderivative size = 856, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {5281, 3297, 3303, 3298, 3301}

$$\frac{\cosh\left(c + \frac{\sqrt{-ad}}{\sqrt{b}}\right) \operatorname{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right) d^2}{16(-a)^{3/2} b^{3/2}} - \frac{\cosh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \operatorname{Chi}\left(xd + \frac{\sqrt{-ad}}{\sqrt{b}}\right) d^2}{16(-a)^{3/2} b^{3/2}} - \frac{\sinh\left(c + \frac{\sqrt{-ad}}{\sqrt{b}}\right) \operatorname{Shi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right) d^2}{16(-a)^{3/2} b^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]/(a + b*x^2)^3,x]

[Out] $-\operatorname{Cosh}[c + d*x]/(16*(-a)^{(3/2)}*\operatorname{Sqrt}[b]*(\operatorname{Sqrt}[-a] - \operatorname{Sqrt}[b]*x)^2) - (3*\operatorname{Cosh}[c + d*x])/((16*a^2*\operatorname{Sqrt}[b]*(\operatorname{Sqrt}[-a] - \operatorname{Sqrt}[b]*x)) + \operatorname{Cosh}[c + d*x]/(16*(-a)^{(3/2)}*\operatorname{Sqrt}[b]*(\operatorname{Sqrt}[-a] + \operatorname{Sqrt}[b]*x)^2) + (3*\operatorname{Cosh}[c + d*x])/((16*a^2*\operatorname{Sqrt}[b]*(\operatorname{Sqrt}[-a] + \operatorname{Sqrt}[b]*x)) + (3*\operatorname{Cosh}[c + (\operatorname{Sqrt}[-a]*d)/\operatorname{Sqrt}[b]]*\operatorname{CoshIntegral}[(S$

$$\begin{aligned} & \text{qrt}[-a]*d)/\text{Sqrt}[b] - d*x]/(16*(-a)^{(5/2)}*\text{Sqrt}[b]) + (d^2*\text{Cosh}[c + (\text{Sqrt}[-a] \\ & *d)/\text{Sqrt}[b]]*\text{CoshIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] - d*x]/(16*(-a)^{(3/2)}*b^{(3/2)}) \\ & - (3*\text{Cosh}[c - (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{CoshIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] \\ & + d*x]/(16*(-a)^{(5/2)}*\text{Sqrt}[b]) - (d^2*\text{Cosh}[c - (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{CoshI} \\ & ntegral[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] + d*x]/(16*(-a)^{(3/2)}*b^{(3/2)}) - (3*d*\text{CoshInt} \\ & egral[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] + d*x]*\text{Sinh}[c - (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]/(16*a^2*b \\ &) - (3*d*\text{CoshIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] - d*x]*\text{Sinh}[c + (\text{Sqrt}[-a]*d)/\text{Sqr} \\ & t[b]]/(16*a^2*b) + (d*\text{Sinh}[c + d*x])/(16*(-a)^{(3/2)}*b*(\text{Sqrt}[-a] - \text{Sqrt}[b]* \\ & x)) + (d*\text{Sinh}[c + d*x])/(16*(-a)^{(3/2)}*b*(\text{Sqrt}[-a] + \text{Sqrt}[b]*x)) + (3*d*\text{Cos} \\ & h[c + (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{SinhIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] - d*x]/(16*a \\ & ^2*b) - (3*\text{Sinh}[c + (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{SinhIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] \\ & - d*x]/(16*(-a)^{(5/2)}*\text{Sqrt}[b]) - (d^2*\text{Sinh}[c + (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{Sinh} \\ & Integral[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] - d*x]/(16*(-a)^{(3/2)}*b^{(3/2)}) - (3*d*\text{Cosh}[c \\ & - (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{SinhIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] + d*x]/(16*a^2* \\ & b) - (3*\text{Sinh}[c - (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{SinhIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] + \\ & d*x]/(16*(-a)^{(5/2)}*\text{Sqrt}[b]) - (d^2*\text{Sinh}[c - (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{SinhInt} \\ & egral[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] + d*x]/(16*(-a)^{(3/2)}*b^{(3/2)}) \end{aligned}$$

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)])/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)])/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]
```

Rule 5281

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := In
t[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rubi steps

$$\begin{aligned} \int \frac{\cosh(c+dx)}{(a+bx^2)^3} dx &= \int \left(-\frac{b^{3/2} \cosh(c+dx)}{8(-a)^{3/2} (\sqrt{-a}\sqrt{b}-bx)^3} - \frac{3b \cosh(c+dx)}{16a^2 (\sqrt{-a}\sqrt{b}-bx)^2} - \frac{b^{3/2} \cosh(c+dx)}{8(-a)^{3/2} (\sqrt{-a}\sqrt{b}+bx)^3} - \frac{3b \cosh(c+dx)}{16a^2 (\sqrt{-a}\sqrt{b}+bx)^2} \right) dx \\ &= -\frac{(3b) \int \frac{\cosh(c+dx)}{(\sqrt{-a}\sqrt{b}-bx)^2} dx}{16a^2} - \frac{(3b) \int \frac{\cosh(c+dx)}{(\sqrt{-a}\sqrt{b}+bx)^2} dx}{16a^2} - \frac{(3b) \int \frac{\cosh(c+dx)}{-ab-b^2x^2} dx}{8a^2} - \frac{b^{3/2} \int \frac{\cosh(c+dx)}{(\sqrt{-a}\sqrt{b}-bx)^3} dx}{8(-a)^{3/2}} \\ &= -\frac{\cosh(c+dx)}{16(-a)^{3/2}\sqrt{b}(\sqrt{-a}-\sqrt{b}x)^2} - \frac{3 \cosh(c+dx)}{16a^2\sqrt{b}(\sqrt{-a}-\sqrt{b}x)} + \frac{\cosh(c+dx)}{16(-a)^{3/2}\sqrt{b}(\sqrt{-a}+\sqrt{b}x)^2} \\ &= -\frac{\cosh(c+dx)}{16(-a)^{3/2}\sqrt{b}(\sqrt{-a}-\sqrt{b}x)^2} - \frac{3 \cosh(c+dx)}{16a^2\sqrt{b}(\sqrt{-a}-\sqrt{b}x)} + \frac{\cosh(c+dx)}{16(-a)^{3/2}\sqrt{b}(\sqrt{-a}+\sqrt{b}x)^2} \\ &= -\frac{\cosh(c+dx)}{16(-a)^{3/2}\sqrt{b}(\sqrt{-a}-\sqrt{b}x)^2} - \frac{3 \cosh(c+dx)}{16a^2\sqrt{b}(\sqrt{-a}-\sqrt{b}x)} + \frac{\cosh(c+dx)}{16(-a)^{3/2}\sqrt{b}(\sqrt{-a}+\sqrt{b}x)^2} \\ &= -\frac{\cosh(c+dx)}{16(-a)^{3/2}\sqrt{b}(\sqrt{-a}-\sqrt{b}x)^2} - \frac{3 \cosh(c+dx)}{16a^2\sqrt{b}(\sqrt{-a}-\sqrt{b}x)} + \frac{\cosh(c+dx)}{16(-a)^{3/2}\sqrt{b}(\sqrt{-a}+\sqrt{b}x)^2} \end{aligned}$$

Mathematica [C] time = 2.60, size = 933, normalized size = 1.09

$$\frac{6b^{5/2} \cosh(c) \cosh(dx)x^3}{(bx^2+a)^2} + \frac{6b^{5/2} \sinh(c) \sinh(dx)x^3}{(bx^2+a)^2} + \frac{2ab^{3/2}d \cosh(dx) \sinh(c)x^2}{(bx^2+a)^2} + \frac{2ab^{3/2}d \cosh(c) \sinh(dx)x^2}{(bx^2+a)^2} + \frac{10ab^{3/2} \cosh(c) \cosh(dx)x}{(bx^2+a)^2} + \dots$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cosh[c + d*x]/(a + b*x^2)^3, x]
```

```
[Out] ((10*a*b^(3/2)*x*Cosh[c]*Cosh[d*x])/(a + b*x^2)^2 + (6*b^(5/2)*x^3*Cosh[c]*
Cosh[d*x])/(a + b*x^2)^2 + (2*a^2*Sqrt[b]*d*Cosh[d*x]*Sinh[c])/(a + b*x^2)^
```

$$2 + (2*a*b^{(3/2)}*d*x^2*\text{Cosh}[d*x]*\text{Sinh}[c])/(a + b*x^2)^2 + (\text{CosIntegral}[-((\text{Sqrt}[a]*d)/\text{Sqrt}[b]) + I*d*x]*(I*(3*b - a*d^2)*\text{Cosh}[c - (I*\text{Sqrt}[a]*d)/\text{Sqrt}[b]] - 3*\text{Sqrt}[a]*\text{Sqrt}[b]*d*\text{Sinh}[c - (I*\text{Sqrt}[a]*d)/\text{Sqrt}[b]]))/\text{Sqrt}[a] + (I*\text{CosIntegral}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b] + I*d*x]*((-3*b + a*d^2)*\text{Cosh}[c + (I*\text{Sqrt}[a]*d)/\text{Sqrt}[b]] + (3*I)*\text{Sqrt}[a]*\text{Sqrt}[b]*d*\text{Sinh}[c + (I*\text{Sqrt}[a]*d)/\text{Sqrt}[b]]))/\text{Sqrt}[a] + (2*a^2*\text{Sqrt}[b]*d*\text{Cosh}[c]*\text{Sinh}[d*x])/(a + b*x^2)^2 + (2*a*b^{(3/2)}*d*x^2*\text{Cosh}[c]*\text{Sinh}[d*x])/(a + b*x^2)^2 + (10*a*b^{(3/2)}*x*\text{Sinh}[c]*\text{Sinh}[d*x])/(a + b*x^2)^2 + (6*b^{(5/2)}*x^3*\text{Sinh}[c]*\text{Sinh}[d*x])/(a + b*x^2)^2 - (3*I)*\text{Sqrt}[b]*d*\text{Cos}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b]]*\text{Cosh}[c]*\text{SinIntegral}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b] - I*d*x] + ((3*I)*b*\text{Cosh}[c]*\text{Sin}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b]]*\text{SinIntegral}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b] - I*d*x])/ \text{Sqrt}[a] - I*\text{Sqrt}[a]*d^2*\text{Cosh}[c]*\text{Sin}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b]]*\text{SinIntegral}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b] - I*d*x] - (3*b*\text{Cos}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b]]*\text{Sinh}[c]*\text{SinIntegral}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b] - I*d*x])/ \text{Sqrt}[a] + \text{Sqrt}[a]*d^2*\text{Cos}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b]]*\text{Sinh}[c]*\text{SinIntegral}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b] - I*d*x] - 3*\text{Sqrt}[b]*d*\text{Sin}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b]]*\text{Sinh}[c]*\text{SinIntegral}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b] - I*d*x] + (3*I)*\text{Sqrt}[b]*d*\text{Cos}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b]]*\text{Cosh}[c]*\text{SinIntegral}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b] + I*d*x] - ((3*I)*b*\text{Cosh}[c]*\text{Sin}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b]]*\text{SinIntegral}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b] + I*d*x])/ \text{Sqrt}[a] + I*\text{Sqrt}[a]*d^2*\text{Cosh}[c]*\text{Sin}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b]]*\text{SinIntegral}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b] + I*d*x] - (3*b*\text{Cos}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b]]*\text{Sinh}[c]*\text{SinIntegral}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b] + I*d*x])/ \text{Sqrt}[a] + \text{Sqrt}[a]*d^2*\text{Cos}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b]]*\text{Sinh}[c]*\text{SinIntegral}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b] + I*d*x] - 3*\text{Sqrt}[b]*d*\text{Sin}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b]]*\text{Sinh}[c]*\text{SinIntegral}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b] + I*d*x])/ (16*a^2*b^{(3/2)})$$

fricas [B] time = 0.66, size = 2116, normalized size = 2.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/(b*x^2+a)^3,x, algorithm="fricas")

[Out] $\frac{1}{32}*(4*(3*a*b^2*d*x^3 + 5*a^2*b*d*x)*\text{cosh}(d*x + c) - ((3*(a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*\text{cosh}(d*x + c)^2 - 3*(a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*\text{sinh}(d*x + c)^2 - ((a^3*d^2 + (a*b^2*d^2 - 3*b^3)*x^4 - 3*a^2*b + 2*(a^2*b*d^2 - 3*a*b^2)*x^2)*\text{cosh}(d*x + c)^2 - (a^3*d^2 + (a*b^2*d^2 - 3*b^3)*x^4 - 3*a^2*b + 2*(a^2*b*d^2 - 3*a*b^2)*x^2)*\text{sinh}(d*x + c)^2)*\text{sqrt}(-a*d^2/b))*\text{Ei}(d*x - \text{sqrt}(-a*d^2/b)) - (3*(a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*\text{cosh}(d*x + c)^2 - 3*(a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*\text{sinh}(d*x + c)^2 + ((a^3*d^2 + (a*b^2*d^2 - 3*b^3)*x^4 - 3*a^2*b + 2*(a^2*b*d^2 - 3*a*b^2)*x^2)*\text{cosh}(d*x + c)^2 - (a^3*d^2 + (a*b^2*d^2 - 3*b^3)*x^4 - 3*a^2*b + 2*(a^2*b*d^2 - 3*a*b^2)*x^2)*\text{sinh}(d*x + c)^2)*\text{sqrt}(-a*d^2/b))*\text{Ei}(-d*x + \text{sqrt}(-a*d^2/b)))*\text{cosh}(c + \text{sqrt}(-a*d^2/b)) - ((3*(a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*\text{cosh}(d*x + c)^2 - 3*(a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*\text{sinh}(d*x + c)^2 + ((a^3*d^2 + (a*b^2*d^2 - 3*b^3)*x^4 - 3*a^2*b + 2*(a^2*b*d^2 - 3*a*b^2)*x^2)*\text{cosh}(d*x + c)^2 - (a^3*d^2 + (a*b^2*d^2 - 3*b^3)*x^4 - 3*a^2*b + 2*(a^2*b*d^2 - 3*a*b^2)*x^2)*\text{sinh}(d*x + c)^2)*\text{sqrt}(-a*d^2/b))$

$$\begin{aligned}
& 3*b^3)*x^4 - 3*a^2*b + 2*(a^2*b*d^2 - 3*a*b^2)*x^2)*\sinh(d*x + c)^2)*\sqrt{(-a*d^2/b)}*Ei(d*x + \sqrt{-a*d^2/b}) - (3*(a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*\cosh(d*x + c)^2 - 3*(a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*\sinh(d*x + c)^2 - ((a^3*d^2 + (a*b^2*d^2 - 3*b^3)*x^4 - 3*a^2*b + 2*(a^2*b*d^2 - 3*a*b^2)*x^2)*\cosh(d*x + c)^2 - (a^3*d^2 + (a*b^2*d^2 - 3*b^3)*x^4 - 3*a^2*b + 2*(a^2*b*d^2 - 3*a*b^2)*x^2)*\sinh(d*x + c)^2)*\sqrt{-a*d^2/b)}*Ei(-d*x - \sqrt{-a*d^2/b}))*\cosh(-c + \sqrt{-a*d^2/b}) + 4*(a^2*b*d^2*x^2 + a^3*d^2)*\sinh(d*x + c) - ((3*(a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*\cosh(d*x + c)^2 - 3*(a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*\sinh(d*x + c)^2 - ((a^3*d^2 + (a*b^2*d^2 - 3*b^3)*x^4 - 3*a^2*b + 2*(a^2*b*d^2 - 3*a*b^2)*x^2)*\cosh(d*x + c)^2 - (a^3*d^2 + (a*b^2*d^2 - 3*b^3)*x^4 - 3*a^2*b + 2*(a^2*b*d^2 - 3*a*b^2)*x^2)*\sinh(d*x + c)^2)*\sqrt{-a*d^2/b)}*Ei(d*x - \sqrt{-a*d^2/b})) + (3*(a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*\cosh(d*x + c)^2 - 3*(a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*\sinh(d*x + c)^2 + ((a^3*d^2 + (a*b^2*d^2 - 3*b^3)*x^4 - 3*a^2*b + 2*(a^2*b*d^2 - 3*a*b^2)*x^2)*\cosh(d*x + c)^2 - (a^3*d^2 + (a*b^2*d^2 - 3*b^3)*x^4 - 3*a^2*b + 2*(a^2*b*d^2 - 3*a*b^2)*x^2)*\sinh(d*x + c)^2)*\sqrt{-a*d^2/b)}*Ei(-d*x + \sqrt{-a*d^2/b}))*\sinh(c + \sqrt{-a*d^2/b}) + ((3*(a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*\cosh(d*x + c)^2 - 3*(a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*\sinh(d*x + c)^2 + ((a^3*d^2 + (a*b^2*d^2 - 3*b^3)*x^4 - 3*a^2*b + 2*(a^2*b*d^2 - 3*a*b^2)*x^2)*\cosh(d*x + c)^2 - (a^3*d^2 + (a*b^2*d^2 - 3*b^3)*x^4 - 3*a^2*b + 2*(a^2*b*d^2 - 3*a*b^2)*x^2)*\sinh(d*x + c)^2)*\sqrt{-a*d^2/b)}*Ei(d*x + \sqrt{-a*d^2/b})) + (3*(a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*\cosh(d*x + c)^2 - 3*(a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*\sinh(d*x + c)^2 - ((a^3*d^2 + (a*b^2*d^2 - 3*b^3)*x^4 - 3*a^2*b + 2*(a^2*b*d^2 - 3*a*b^2)*x^2)*\cosh(d*x + c)^2 - (a^3*d^2 + (a*b^2*d^2 - 3*b^3)*x^4 - 3*a^2*b + 2*(a^2*b*d^2 - 3*a*b^2)*x^2)*\sinh(d*x + c)^2)*\sqrt{-a*d^2/b)}*Ei(-d*x - \sqrt{-a*d^2/b}))*\sinh(-c + \sqrt{-a*d^2/b}))/((a^3*b^3*d*x^4 + 2*a^4*b^2*d*x^2 + a^5*b*d)*\cosh(d*x + c)^2 - (a^3*b^3*d*x^4 + 2*a^4*b^2*d*x^2 + a^5*b*d)*\sinh(d*x + c)^2)
\end{aligned}$$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(dx+c)}{(bx^2+a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/(b*x^2+a)^3,x, algorithm="giac")

[Out] integrate(cosh(d*x + c)/(b*x^2 + a)^3, x)

maple [A] time = 0.15, size = 1064, normalized size = 1.24

$$\frac{d^5 e^{-dx-c} x^2}{16a(b^2 d^4 x^4 + 2ab d^4 x^2 + a^2 d^4)} + \frac{3d^4 e^{-dx-c} b x^3}{16a^2(b^2 d^4 x^4 + 2ab d^4 x^2 + a^2 d^4)} - \frac{d^5 e^{-dx-c}}{16b(b^2 d^4 x^4 + 2ab d^4 x^2 + a^2 d^4)} + \frac{d^5 e^{-dx-c}}{16a(b^2 d^4 x^4 + 2ab d^4 x^2 + a^2 d^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cosh(d*x+c)/(b*x^2+a)^3,x)`

[Out]
$$\begin{aligned} & -1/16*d^5*exp(-d*x-c)/a/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*x^2+3/16*d^4*exp(-d*x-c)/a^2*b/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*x^3-1/16*d^5*exp(-d*x-c)/b/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)+5/16*d^4*exp(-d*x-c)/a/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*x+1/32*d^2/b/a/(-a*b)^(1/2)*exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1, -(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)-1/32*d^2/b/a/(-a*b)^(1/2)*exp(-(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1, (d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)-3/32*d/b/a^2*exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1, -(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)-3/32*d/b/a^2*exp(-(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1, (d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)-3/32/a^2/(-a*b)^(1/2)*exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1, -(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)+3/32/a^2/(-a*b)^(1/2)*exp(-(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1, (d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)+1/16*d^5*exp(d*x+c)/a/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*x^2+3/16*d^4*exp(d*x+c)/a^2*b/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*x^3+1/16*d^5*exp(d*x+c)/b/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)+5/16*d^4*exp(d*x+c)/a/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*x+1/32*d^2/b/a/(-a*b)^(1/2)*exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1, (d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)-1/32*d^2/b/a/(-a*b)^(1/2)*exp(-(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1, -(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)+3/32*d/b/a^2*exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1, (d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)+3/32*d/b/a^2*exp(-(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1, -(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)-3/32/a^2/(-a*b)^(1/2)*exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1, (d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)+3/32/a^2/(-a*b)^(1/2)*exp(-(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1, -(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b) \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(dx+c)}{(bx^2+a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(d*x+c)/(b*x^2+a)^3,x, algorithm="maxima")`

[Out] `integrate(cosh(d*x + c)/(b*x^2 + a)^3, x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(c+dx)}{(bx^2+a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cosh(c + d*x)/(a + b*x^2)^3,x)`


```
[Out] int(cosh(c + d*x)/(a + b*x^2)^3, x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

$$3.76 \quad \int \frac{\cosh(c+dx)}{x(a+bx^2)^3} dx$$

Optimal. Leaf size=730

$$\frac{\cosh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right) \operatorname{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{2a^3} - \frac{\cosh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \operatorname{Chi}\left(xd + \frac{\sqrt{-ad}}{\sqrt{b}}\right)}{2a^3} + \frac{\sinh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right) \operatorname{Shi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{2a^3} - \frac{\sinh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \operatorname{Shi}\left(xd + \frac{\sqrt{-ad}}{\sqrt{b}}\right)}{2a^3}$$

[Out] Chi(d*x)*cosh(c)/a^3+1/4*cosh(d*x+c)/a/(b*x^2+a)^2+1/2*cosh(d*x+c)/a^2/(b*x^2+a)-1/2*Chi(d*x+d*(-a)^(1/2)/b^(1/2))*cosh(c-d*(-a)^(1/2)/b^(1/2))/a^3+1/16*d^2*Chi(d*x+d*(-a)^(1/2)/b^(1/2))*cosh(c-d*(-a)^(1/2)/b^(1/2))/a^2/b-1/2*Chi(-d*x+d*(-a)^(1/2)/b^(1/2))*cosh(c+d*(-a)^(1/2)/b^(1/2))/a^3+1/16*d^2*Chi(-d*x+d*(-a)^(1/2)/b^(1/2))*cosh(c+d*(-a)^(1/2)/b^(1/2))/a^2/b+Shi(d*x)*sinh(c)/a^3-1/2*Shi(d*x+d*(-a)^(1/2)/b^(1/2))*sinh(c-d*(-a)^(1/2)/b^(1/2))/a^3+1/16*d^2*Shi(d*x+d*(-a)^(1/2)/b^(1/2))*sinh(c-d*(-a)^(1/2)/b^(1/2))/a^2/b-1/2*Shi(d*x-d*(-a)^(1/2)/b^(1/2))*sinh(c+d*(-a)^(1/2)/b^(1/2))/a^3+1/16*d^2*Shi(d*x-d*(-a)^(1/2)/b^(1/2))*sinh(c+d*(-a)^(1/2)/b^(1/2))/a^2/b-5/16*d*cosh(c+d*(-a)^(1/2)/b^(1/2))*Shi(d*x-d*(-a)^(1/2)/b^(1/2))/(-a)^(5/2)/b^(1/2)+5/16*d*cosh(c-d*(-a)^(1/2)/b^(1/2))*Shi(d*x+d*(-a)^(1/2)/b^(1/2))/(-a)^(5/2)/b^(1/2)+5/16*d*Chi(d*x+d*(-a)^(1/2)/b^(1/2))*sinh(c-d*(-a)^(1/2)/b^(1/2))/(-a)^(5/2)/b^(1/2)-5/16*d*Chi(-d*x+d*(-a)^(1/2)/b^(1/2))*sinh(c+d*(-a)^(1/2)/b^(1/2))/(-a)^(5/2)/b^(1/2)+1/16*d*sinh(d*x+c)/a^2/b^(1/2)/((-a)^(1/2)-x*b^(1/2))-1/16*d*sinh(d*x+c)/a^2/b^(1/2)/((-a)^(1/2)+x*b^(1/2))

Rubi [A] time = 1.68, antiderivative size = 730, normalized size of antiderivative = 1.00, number of steps used = 41, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {5293, 3303, 3298, 3301, 5289, 5280, 3297}

$$\frac{d^2 \cosh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right) \operatorname{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{16a^2b} + \frac{d^2 \cosh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \operatorname{Chi}\left(xd + \frac{\sqrt{-ad}}{\sqrt{b}}\right)}{16a^2b} - \frac{\cosh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right) \operatorname{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{2a^3} - \frac{\sinh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \operatorname{Chi}\left(xd + \frac{\sqrt{-ad}}{\sqrt{b}}\right)}{2a^3}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]/(x*(a + b*x^2)^3), x]

[Out] Cosh[c + d*x]/(4*a*(a + b*x^2)^2) + Cosh[c + d*x]/(2*a^2*(a + b*x^2)) + (Cosh[c]*CoshIntegral[d*x])/a^3 - (Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*a^3) + (d^2*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*a^2*b) - (Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*a^3) + (d^2*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*a^2*b) + (5*d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]])/(2*a^3)

$$\begin{aligned} & \text{qrt}[b]]/(16*(-a)^{(5/2)}*\text{Sqrt}[b]) - (5*d*\text{CoshIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] - \\ & d*x]*\text{Sinh}[c + (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]/(16*(-a)^{(5/2)}*\text{Sqrt}[b]) + (d*\text{Sinh}[c + \\ & d*x])/(16*a^2*\text{Sqrt}[b]*(\text{Sqrt}[-a] - \text{Sqrt}[b]*x)) - (d*\text{Sinh}[c + d*x])/(16*a^2* \\ & \text{Sqrt}[b]*(\text{Sqrt}[-a] + \text{Sqrt}[b]*x)) + (\text{Sinh}[c]*\text{SinhIntegral}[d*x])/a^3 + (5*d*\text{Co} \\ & \text{sh}[c + (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{SinhIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] - d*x])/(16* \\ & (-a)^{(5/2)}*\text{Sqrt}[b]) + (\text{Sinh}[c + (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{SinhIntegral}[(\text{Sqrt}[-a] \\ &]*d)/\text{Sqrt}[b] - d*x])/(2*a^3) - (d^2*\text{Sinh}[c + (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{SinhInte} \\ & \text{gral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] - d*x])/(16*a^2*b) + (5*d*\text{Cosh}[c - (\text{Sqrt}[-a]*d)/\text{S} \\ & \text{qrt}[b]]*\text{SinhIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] + d*x])/(16*(-a)^{(5/2)}*\text{Sqrt}[b]) - \\ & (\text{Sinh}[c - (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{SinhIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] + d*x])/(\\ & (2*a^3) + (d^2*\text{Sinh}[c - (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{SinhIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqr} \\ & \text{t}[b] + d*x])/(16*a^2*b) \end{aligned}$$
Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
+ d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

Rule 5280

```
Int[((a_.) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Symbol] := In
t[ExpandIntegrand[Sinh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 5289

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_), x_Symbol] := Simp[(e^m*(a + b*x^n)^(p + 1)*Cosh[c + d*x]/(b*n*(p + 1))
, x] - Dist[(d*e^m)/(b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*Sinh[c + d*x], x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IntegerQ[p] && EqQ[m - n + 1, 0
] && LtQ[p, -1] && (IntegerQ[n] || GtQ[e, 0])
```

Rule 5293

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] := Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh(c+dx)}{x(a+bx^2)^3} dx &= \int \left(\frac{\cosh(c+dx)}{a^3 x} - \frac{bx \cosh(c+dx)}{a(a+bx^2)^3} - \frac{bx \cosh(c+dx)}{a^2(a+bx^2)^2} - \frac{bx \cosh(c+dx)}{a^3(a+bx^2)} \right) dx \\
&= \frac{\int \frac{\cosh(c+dx)}{x} dx}{a^3} - \frac{b \int \frac{x \cosh(c+dx)}{a+bx^2} dx}{a^3} - \frac{b \int \frac{x \cosh(c+dx)}{(a+bx^2)^2} dx}{a^2} - \frac{b \int \frac{x \cosh(c+dx)}{(a+bx^2)^3} dx}{a} \\
&= \frac{\cosh(c+dx)}{4a(a+bx^2)^2} + \frac{\cosh(c+dx)}{2a^2(a+bx^2)} - \frac{b \int \left(-\frac{\cosh(c+dx)}{2\sqrt{b}(\sqrt{-a}-\sqrt{bx})} + \frac{\cosh(c+dx)}{2\sqrt{b}(\sqrt{-a}+\sqrt{bx})} \right) dx}{a^3} - \frac{d \int \frac{\sinh(c+dx)}{a+bx^2} dx}{2a^2} \\
&= \frac{\cosh(c+dx)}{4a(a+bx^2)^2} + \frac{\cosh(c+dx)}{2a^2(a+bx^2)} + \frac{\cosh(c)\text{Chi}(dx)}{a^3} + \frac{\sinh(c)\text{Shi}(dx)}{a^3} + \frac{\sqrt{b} \int \frac{\cosh(c+dx)}{\sqrt{-a}-\sqrt{bx}} dx}{2a^3} \\
&= \frac{\cosh(c+dx)}{4a(a+bx^2)^2} + \frac{\cosh(c+dx)}{2a^2(a+bx^2)} + \frac{\cosh(c)\text{Chi}(dx)}{a^3} + \frac{\sinh(c)\text{Shi}(dx)}{a^3} + \frac{d \int \frac{\sinh(c+dx)}{\sqrt{-a}-\sqrt{bx}} dx}{4(-a)^{5/2}} + \\
&= \frac{\cosh(c+dx)}{4a(a+bx^2)^2} + \frac{\cosh(c+dx)}{2a^2(a+bx^2)} + \frac{\cosh(c)\text{Chi}(dx)}{a^3} - \frac{\cosh\left(c + \frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}} - dx\right)}{2a^3} - \\
&= \frac{\cosh(c+dx)}{4a(a+bx^2)^2} + \frac{\cosh(c+dx)}{2a^2(a+bx^2)} + \frac{\cosh(c)\text{Chi}(dx)}{a^3} - \frac{\cosh\left(c + \frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}} - dx\right)}{2a^3} - \\
&= \frac{\cosh(c+dx)}{4a(a+bx^2)^2} + \frac{\cosh(c+dx)}{2a^2(a+bx^2)} + \frac{\cosh(c)\text{Chi}(dx)}{a^3} - \frac{\cosh\left(c + \frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}} - dx\right)}{2a^3} + \\
&= \frac{\cosh(c+dx)}{4a(a+bx^2)^2} + \frac{\cosh(c+dx)}{2a^2(a+bx^2)} + \frac{\cosh(c)\text{Chi}(dx)}{a^3} - \frac{\cosh\left(c + \frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}} - dx\right)}{2a^3} +
\end{aligned}$$

Mathematica [C] time = 2.61, size = 1046, normalized size = 1.43

result too large to display

Warning: Unable to verify antiderivative.

[In] Integrate[Cosh[c + d*x]/(x*(a + b*x^2)^3),x]

[Out]
$$\begin{aligned} & ((12*a^2*Cosh[c + d*x])/(a + b*x^2)^2 + (8*a*b*x^2*Cosh[c + d*x])/(a + b*x^2)^2 + 16*Cosh[c]*CoshIntegral[d*x] - 8*\cos[(\sqrt{a}*d)/\sqrt{b}]*Cosh[c]*\cosIntegral[(\sqrt{a}*d)/\sqrt{b} + I*d*x] + (a*d^2*\cos[(\sqrt{a}*d)/\sqrt{b}]*\cosh[c]*\cosIntegral[(\sqrt{a}*d)/\sqrt{b} + I*d*x])/b - (5*\sqrt{a}*d*Cosh[c]*\cosIntegral[(\sqrt{a}*d)/\sqrt{b} + I*d*x]*\sin[(\sqrt{a}*d)/\sqrt{b}])/ \sqrt{b} + ((5*I)*\sqrt{a}*d*\cos[(\sqrt{a}*d)/\sqrt{b}]*\cosIntegral[(\sqrt{a}*d)/\sqrt{b} + I*d*x]*\sinh[c])/ \sqrt{b} - (8*I)*\cosIntegral[(\sqrt{a}*d)/\sqrt{b} + I*d*x]*\sin[(\sqrt{a}*d)/\sqrt{b}]*\sinh[c] + (I*a*d^2*\cosIntegral[(\sqrt{a}*d)/\sqrt{b} + I*d*x]*\sin[(\sqrt{a}*d)/\sqrt{b}]*\sinh[c])/b - (\cosIntegral[-((\sqrt{a}*d)/\sqrt{b}) + I*d*x]*((8*b - a*d^2)*\cosh[c - (I*\sqrt{a}*d)/\sqrt{b}] + (5*I)*\sqrt{a}* \sqrt{b}*d*\sinh[c - (I*\sqrt{a}*d)/\sqrt{b}]))/b - (2*a^2*d*x*\sinh[c + d*x])/(a + b*x^2)^2 - (2*a*b*d*x^3*\sinh[c + d*x])/(a + b*x^2)^2 + (16*a^2*\sinh[c]*\sinhIntegral[d*x])/(a + b*x^2)^2 + (32*a*b*x^2*\sinh[c]*\sinhIntegral[d*x])/(a + b*x^2)^2 + (16*b^2*x^4*\sinh[c]*\sinhIntegral[d*x])/(a + b*x^2)^2 + (5*\sqrt{a}*d*\cos[(\sqrt{a}*d)/\sqrt{b}]*\cosh[c]*\sinIntegral[(\sqrt{a}*d)/\sqrt{b} - I*d*x])/ \sqrt{b} - 8*\cosh[c]*\sin[(\sqrt{a}*d)/\sqrt{b}]*\sinIntegral[(\sqrt{a}*d)/\sqrt{b} - I*d*x] + (a*d^2*\cosh[c]*\sin[(\sqrt{a}*d)/\sqrt{b}]*\sinIntegral[(\sqrt{a}*d)/\sqrt{b} - I*d*x])/b - (8*I)*\cos[(\sqrt{a}*d)/\sqrt{b}]*\sinh[c]*\sinIntegral[(\sqrt{a}*d)/\sqrt{b} - I*d*x] + (I*a*d^2*\cos[(\sqrt{a}*d)/\sqrt{b}]*\sinh[c]*\sinIntegral[(\sqrt{a}*d)/\sqrt{b} - I*d*x])/b - ((5*I)*\sqrt{a}*d*\sin[(\sqrt{a}*d)/\sqrt{b}]*\sinh[c]*\sinIntegral[(\sqrt{a}*d)/\sqrt{b} - I*d*x])/ \sqrt{b} + (5*\sqrt{a}*d*\cos[(\sqrt{a}*d)/\sqrt{b}]*\cosh[c]*\sinIntegral[(\sqrt{a}*d)/\sqrt{b} + I*d*x])/ \sqrt{b} - 8*\cosh[c]*\sin[(\sqrt{a}*d)/\sqrt{b}]*\sinIntegral[(\sqrt{a}*d)/\sqrt{b} + I*d*x] + (a*d^2*\cosh[c]*\sin[(\sqrt{a}*d)/\sqrt{b}]*\sinIntegral[(\sqrt{a}*d)/\sqrt{b} + I*d*x])/b + (8*I)*\cos[(\sqrt{a}*d)/\sqrt{b}]*\sinh[c]*\sinIntegral[(\sqrt{a}*d)/\sqrt{b} + I*d*x] - (I*a*d^2*\cos[(\sqrt{a}*d)/\sqrt{b}]*\sinh[c]*\sinIntegral[(\sqrt{a}*d)/\sqrt{b} + I*d*x])/b + ((5*I)*\sqrt{a}*d*\sin[(\sqrt{a}*d)/\sqrt{b}]*\sinh[c]*\sinIntegral[(\sqrt{a}*d)/\sqrt{b} + I*d*x])/ \sqrt{b})/(16*a^3) \end{aligned}$$

fricas [B] time = 0.55, size = 2076, normalized size = 2.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/x/(b*x^2+a)^3,x, algorithm="fricas")

[Out]
$$\begin{aligned} & 1/32*(8*(2*a*b^2*x^2 + 3*a^2*b)*\cosh(d*x + c) + (((a^3*d^2 + (a*b^2*d^2 - 8*b^3)*x^4 - 8*a^2*b + 2*(a^2*b*d^2 - 8*a*b^2)*x^2)*\cosh(d*x + c)^2 - (a^3*d^2 + (a*b^2*d^2 - 8*b^3)*x^4 - 8*a^2*b + 2*(a^2*b*d^2 - 8*a*b^2)*x^2)*\sinh(d*x + c)^2 + 5*((b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*\cosh(d*x + c)^2 - (b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*\sinh(d*x + c)^2)*\sqrt{-a*d^2/b})*\text{Ei}(d*x - \sqrt{-a*d^2/b}) + ((a^3*d^2 + (a*b^2*d^2 - 8*b^3)*x^4 - 8*a^2*b + 2*(a^2*b*d^2 - 8*a \end{aligned}$$

$b^2*x^2)*\cosh(d*x + c)^2 - (a^3*d^2 + (a*b^2*d^2 - 8*b^3)*x^4 - 8*a^2*b + 2*(a^2*b*d^2 - 8*a*b^2)*x^2)*\sinh(d*x + c)^2 - 5*((b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*\cosh(d*x + c)^2 - (b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*\sinh(d*x + c)^2)*\sqrt{-a*d^2/b})*\text{Ei}(-d*x + \sqrt{-a*d^2/b}))*\cosh(c + \sqrt{-a*d^2/b}) + 16*((b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*\text{Ei}(d*x) + (b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*\text{Ei}(-d*x))*\cosh(c) + (((a^3*d^2 + (a*b^2*d^2 - 8*b^3)*x^4 - 8*a^2*b + 2*(a^2*b*d^2 - 8*a*b^2)*x^2)*\cosh(d*x + c)^2 - (a^3*d^2 + (a*b^2*d^2 - 8*b^3)*x^4 - 8*a^2*b + 2*(a^2*b*d^2 - 8*a*b^2)*x^2)*\sinh(d*x + c)^2 - 5*((b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*\cosh(d*x + c)^2 - (b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*\sinh(d*x + c)^2)*\sqrt{-a*d^2/b})*\text{Ei}(d*x + \sqrt{-a*d^2/b}) + ((a^3*d^2 + (a*b^2*d^2 - 8*b^3)*x^4 - 8*a^2*b + 2*(a^2*b*d^2 - 8*a*b^2)*x^2)*\cosh(d*x + c)^2 - (a^3*d^2 + (a*b^2*d^2 - 8*b^3)*x^4 - 8*a^2*b + 2*(a^2*b*d^2 - 8*a*b^2)*x^2)*\sinh(d*x + c)^2 + 5*((b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*\cosh(d*x + c)^2 - (b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*\sinh(d*x + c)^2)*\sqrt{-a*d^2/b})*\text{Ei}(-d*x - \sqrt{-a*d^2/b}))*\cosh(-c + \sqrt{-a*d^2/b}) - 4*(a*b^2*d*x^3 + a^2*b*d*x)*\sinh(d*x + c) + (((a^3*d^2 + (a*b^2*d^2 - 8*b^3)*x^4 - 8*a^2*b + 2*(a^2*b*d^2 - 8*a*b^2)*x^2)*\cosh(d*x + c)^2 - (a^3*d^2 + (a*b^2*d^2 - 8*b^3)*x^4 - 8*a^2*b + 2*(a^2*b*d^2 - 8*a*b^2)*x^2)*\sinh(d*x + c)^2 + 5*((b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*\cosh(d*x + c)^2 - (b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*\sinh(d*x + c)^2)*\sqrt{-a*d^2/b})*\text{Ei}(d*x - \sqrt{-a*d^2/b}) - ((a^3*d^2 + (a*b^2*d^2 - 8*b^3)*x^4 - 8*a^2*b + 2*(a^2*b*d^2 - 8*a*b^2)*x^2)*\cosh(d*x + c)^2 - (a^3*d^2 + (a*b^2*d^2 - 8*b^3)*x^4 - 8*a^2*b + 2*(a^2*b*d^2 - 8*a*b^2)*x^2)*\sinh(d*x + c)^2 - 5*((b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*\cosh(d*x + c)^2 - (b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*\sinh(d*x + c)^2)*\sqrt{-a*d^2/b})*\text{Ei}(-d*x + \sqrt{-a*d^2/b}))*\sinh(c + \sqrt{-a*d^2/b}) + 16*((b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*\text{Ei}(d*x) - (b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*\text{Ei}(-d*x))*\sinh(c) - (((a^3*d^2 + (a*b^2*d^2 - 8*b^3)*x^4 - 8*a^2*b + 2*(a^2*b*d^2 - 8*a*b^2)*x^2)*\cosh(d*x + c)^2 - (a^3*d^2 + (a*b^2*d^2 - 8*b^3)*x^4 - 8*a^2*b + 2*(a^2*b*d^2 - 8*a*b^2)*x^2)*\sinh(d*x + c)^2 - 5*((b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*\cosh(d*x + c)^2 - (b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*\sinh(d*x + c)^2)*\sqrt{-a*d^2/b})*\text{Ei}(d*x + \sqrt{-a*d^2/b}) - ((a^3*d^2 + (a*b^2*d^2 - 8*b^3)*x^4 - 8*a^2*b + 2*(a^2*b*d^2 - 8*a*b^2)*x^2)*\cosh(d*x + c)^2 - (a^3*d^2 + (a*b^2*d^2 - 8*b^3)*x^4 - 8*a^2*b + 2*(a^2*b*d^2 - 8*a*b^2)*x^2)*\sinh(d*x + c)^2 + 5*((b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*\cosh(d*x + c)^2 - (b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*\sinh(d*x + c)^2)*\sqrt{-a*d^2/b})*\text{Ei}(-d*x - \sqrt{-a*d^2/b}))*\sinh(-c + \sqrt{-a*d^2/b}))/(((a^3*b^3*x^4 + 2*a^4*b^2*x^2 + a^5*b)*\cosh(d*x + c)^2 - (a^3*b^3*x^4 + 2*a^4*b^2*x^2 + a^5*b)*\sinh(d*x + c)^2)$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(dx + c)}{(bx^2 + a)^3 x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/x/(b*x^2+a)^3,x, algorithm="giac")

[Out] integrate(cosh(d*x + c)/((b*x^2 + a)^3*x), x)

maple [A] time = 0.18, size = 1090, normalized size = 1.49

$$\frac{e^{-dx-c} d^2 \left((dx+c)^3 b - 3(dx+c)^2 bc + (dx+c) a d^2 + 3(dx+c) b c^2 - a c d^2 - b c^3 + 4(dx+c)^2 b - 8(dx+c) b c \right)}{16a^2 \left((dx+c)^4 b^2 - 4(dx+c)^3 c b^2 + 2(dx+c)^2 a b d^2 + 6(dx+c)^2 c^2 b^2 - 4ab(dx+c) c d^2 - 4b^2(dx+c) c^3 + a^2 d^2 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d*x+c)/x/(b*x^2+a)^3,x)

[Out] $\frac{1}{16} \exp(-d*x-c) * d^2 * ((d*x+c)^3 * b - 3 * (d*x+c)^2 * b * c + (d*x+c) * a * d^2 + 3 * (d*x+c) * b * c^2 - a * c * d^2 - b * c^3 + 4 * (d*x+c)^2 * b - 8 * (d*x+c) * b * c) / a^2 / ((d*x+c)^4 * b^2 - 4 * (d*x+c)^3 * c * b^2 + 2 * (d*x+c)^2 * a * b * d^2 + 6 * (d*x+c)^2 * c^2 * b^2 - 4 * a * b * (d*x+c) * c * d^2 - 4 * b^2 * (d*x+c) * c^3 + a^2 * d^2) - 1/2/a^3 * \exp(-c) * Ei(1, d*x) - 1/32/b/a^2 * \exp(-(d*(-a*b)^(1/2)+c*b)/b) * Ei(1, -(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b) * d^2 - 1/32/b/a^2 * \exp(-(-d*(-a*b)^(1/2)+c*b)/b) * Ei(1, (d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b) * d^2 - 5/32/a^2/(-a*b)^(1/2) * \exp(-(d*(-a*b)^(1/2)+c*b)/b) * Ei(1, -(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b) * d + 5/32/a^2/(-a*b)^(1/2) * \exp(-(-d*(-a*b)^(1/2)+c*b)/b) * Ei(1, (d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b) * d + 1/4/a^3 * \exp(-(d*(-a*b)^(1/2)+c*b)/b) * Ei(1, -(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b) + 1/4/a^3 * \exp(-(-d*(-a*b)^(1/2)+c*b)/b) * Ei(1, (d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b) - 1/16 * \exp(d*x+c) * d^2 * ((d*x+c)^3 * b - 3 * (d*x+c)^2 * b * c + (d*x+c) * a * d^2 + 3 * (d*x+c) * b * c^2 - a * c * d^2 - b * c^3 + 4 * (d*x+c)^2 * b - 8 * (d*x+c) * b * c) / a^2 / ((d*x+c)^4 * b^2 - 4 * (d*x+c)^3 * c * b^2 + 2 * (d*x+c)^2 * a * b * d^2 + 6 * (d*x+c)^2 * c^2 * b^2 - 4 * a * b * (d*x+c) * c * d^2 - 4 * b^2 * (d*x+c) * c^3 + a^2 * d^2) - 1/2/a^3 * \exp(c) * Ei(1, -d*x) - 1/32/b/a^2 * \exp((-d*(-a*b)^(1/2)+c*b)/b) * Ei(1, -(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b) * d^2 - 1/32/b/a^2 * \exp((d*(-a*b)^(1/2)+c*b)/b) * Ei(1, (d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b) * d^2 - 5/32/a^2/(-a*b)^(1/2) * \exp((-d*(-a*b)^(1/2)+c*b)/b) * Ei(1, -(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b) * d + 5/32/a^2/(-a*b)^(1/2) * \exp((d*(-a*b)^(1/2)+c*b)/b) * Ei(1, (d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b) * d + 1/4/a^3 * \exp((-d*(-a*b)^(1/2)+c*b)/b) * Ei(1, -(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b) + 1/4/a^3 * \exp((d*(-a*b)^(1/2)+c*b)/b) * Ei(1, (d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(dx+c)}{(bx^2+a)^3 x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/x/(b*x^2+a)^3,x, algorithm="maxima")

[Out] integrate(cosh(d*x + c)/((b*x^2 + a)^3*x), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(c + dx)}{x(bx^2 + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(c + d*x)/(x*(a + b*x^2)^3), x)

[Out] int(cosh(c + d*x)/(x*(a + b*x^2)^3), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/x/(b*x**2+a)**3, x)

[Out] Timed out

$$3.77 \quad \int \frac{\cosh(c+dx)}{x^2(a+bx^2)^3} dx$$

Optimal. Leaf size=874

$$\frac{\cosh\left(c + \frac{\sqrt{-ad}}{\sqrt{b}}\right) \operatorname{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right) d^2}{16(-a)^{5/2}\sqrt{b}} - \frac{\cosh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \operatorname{Chi}\left(xd + \frac{\sqrt{-ad}}{\sqrt{b}}\right) d^2}{16(-a)^{5/2}\sqrt{b}} - \frac{\sinh\left(c + \frac{\sqrt{-ad}}{\sqrt{b}}\right) \operatorname{Shi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right) d^2}{16(-a)^{5/2}\sqrt{b}}$$

[Out] $-\cosh(d*x+c)/a^3/x+d*\cosh(c)*\operatorname{Shi}(d*x)/a^3+7/16*d*\cosh(c+d*(-a)^{(1/2)}/b^{(1/2)})*\operatorname{Shi}(d*x-d*(-a)^{(1/2)}/b^{(1/2)})/a^3+7/16*d*\cosh(c-d*(-a)^{(1/2)}/b^{(1/2)})*\operatorname{Shi}(d*x+d*(-a)^{(1/2)}/b^{(1/2)})/a^3+d*\operatorname{Chi}(d*x)*\sinh(c)/a^3+7/16*d*\operatorname{Chi}(d*x+d*(-a)^{(1/2)}/b^{(1/2)})*\sinh(c-d*(-a)^{(1/2)}/b^{(1/2)})/a^3+7/16*d*\operatorname{Chi}(-d*x+d*(-a)^{(1/2)}/b^{(1/2)})*\sinh(c+d*(-a)^{(1/2)}/b^{(1/2)})/a^3-1/16*d^2*\operatorname{Chi}(d*x+d*(-a)^{(1/2)}/b^{(1/2)})*\cosh(c-d*(-a)^{(1/2)}/b^{(1/2)})/(-a)^{(5/2)}/b^{(1/2)}+1/16*d^2*\operatorname{Chi}(-d*x+d*(-a)^{(1/2)}/b^{(1/2)})*\cosh(c+d*(-a)^{(1/2)}/b^{(1/2)})/(-a)^{(5/2)}/b^{(1/2)}-1/16*d^2*\operatorname{Shi}(d*x+d*(-a)^{(1/2)}/b^{(1/2)})*\sinh(c-d*(-a)^{(1/2)}/b^{(1/2)})/(-a)^{(5/2)}/b^{(1/2)}+1/16*d^2*\operatorname{Shi}(d*x-d*(-a)^{(1/2)}/b^{(1/2)})*\sinh(c+d*(-a)^{(1/2)}/b^{(1/2)})/(-a)^{(5/2)}/b^{(1/2)}-15/16*\operatorname{Chi}(d*x+d*(-a)^{(1/2)}/b^{(1/2)})*\cosh(c-d*(-a)^{(1/2)}/b^{(1/2)})*b^{(1/2)}/(-a)^{(7/2)}+15/16*\operatorname{Chi}(-d*x+d*(-a)^{(1/2)}/b^{(1/2)})*\cosh(c+d*(-a)^{(1/2)}/b^{(1/2)})*b^{(1/2)}/(-a)^{(7/2)}-15/16*\operatorname{Shi}(d*x+d*(-a)^{(1/2)}/b^{(1/2)})*\sinh(c-d*(-a)^{(1/2)}/b^{(1/2)})*b^{(1/2)}/(-a)^{(7/2)}+15/16*\operatorname{Shi}(d*x-d*(-a)^{(1/2)}/b^{(1/2)})*\sinh(c+d*(-a)^{(1/2)}/b^{(1/2)})*b^{(1/2)}/(-a)^{(7/2)}-1/16*\cosh(d*x+c)*b^{(1/2)}/(-a)^{(5/2)}/((-a)^{(1/2)}-x*b^{(1/2)})^2+1/16*d*\sinh(d*x+c)/(-a)^{(5/2)}/((-a)^{(1/2)}-x*b^{(1/2)})+7/16*\cosh(d*x+c)*b^{(1/2)}/a^3/((-a)^{(1/2)}-x*b^{(1/2)})+1/16*\cosh(d*x+c)*b^{(1/2)}/(-a)^{(5/2)}/((-a)^{(1/2)}+x*b^{(1/2)})^2+1/16*d*\sinh(d*x+c)/(-a)^{(5/2)}/((-a)^{(1/2)}+x*b^{(1/2)})-7/16*\cosh(d*x+c)*b^{(1/2)}/a^3/((-a)^{(1/2)}+x*b^{(1/2)})$

Rubi [A] time = 2.69, antiderivative size = 874, normalized size of antiderivative = 1.00, number of steps used = 60, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {5293, 3297, 3303, 3298, 3301, 5281}

$$\frac{\cosh\left(c + \frac{\sqrt{-ad}}{\sqrt{b}}\right) \operatorname{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right) d^2}{16(-a)^{5/2}\sqrt{b}} - \frac{\cosh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \operatorname{Chi}\left(xd + \frac{\sqrt{-ad}}{\sqrt{b}}\right) d^2}{16(-a)^{5/2}\sqrt{b}} - \frac{\sinh\left(c + \frac{\sqrt{-ad}}{\sqrt{b}}\right) \operatorname{Shi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right) d^2}{16(-a)^{5/2}\sqrt{b}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Cosh}[c + d*x]/(x^2*(a + b*x^2)^3), x]$

[Out] $-(\operatorname{Cosh}[c + d*x]/(a^3*x)) - (\operatorname{Sqrt}[b]*\operatorname{Cosh}[c + d*x])/((16*(-a)^{(5/2)}*(\operatorname{Sqrt}[-a] - \operatorname{Sqrt}[b]*x)^2) + (7*\operatorname{Sqrt}[b]*\operatorname{Cosh}[c + d*x])/((16*a^3*(\operatorname{Sqrt}[-a] - \operatorname{Sqrt}[b]*x)) + (\operatorname{Sqrt}[b]*\operatorname{Cosh}[c + d*x])/((16*(-a)^{(5/2)}*(\operatorname{Sqrt}[-a] + \operatorname{Sqrt}[b]*x)^2) - (7*S$

```

qrt[b]*Cosh[c + d*x))/(16*a^3*(Sqrt[-a] + Sqrt[b]*x)) + (15*Sqrt[b]*Cosh[c
+ (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x))/(16*(-a)^
(7/2)) + (d^2*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt
[b] - d*x))/(16*(-a)^(5/2)*Sqrt[b]) - (15*Sqrt[b]*Cosh[c - (Sqrt[-a]*d)/Sqr
t[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x))/(16*(-a)^(7/2)) - (d^2*Cosh
[c - (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x))/(16*(-
a)^(5/2)*Sqrt[b]) + (d*CoshIntegral[d*x]*Sinh[c])/a^3 + (7*d*CoshIntegral[(
Sqrt[-a]*d)/Sqrt[b] + d*x]*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]])/(16*a^3) + (7*d*
CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]])/(1
6*a^3) + (d*Sinh[c + d*x))/(16*(-a)^(5/2)*(Sqrt[-a] - Sqrt[b]*x)) + (d*Sinh
[c + d*x))/(16*(-a)^(5/2)*(Sqrt[-a] + Sqrt[b]*x)) + (d*Cosh[c]*SinhIntegral
[d*x])/a^3 - (7*d*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/
Sqrt[b] - d*x))/(16*a^3) - (15*Sqrt[b]*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhI
ntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x))/(16*(-a)^(7/2)) - (d^2*Sinh[c + (Sqrt[
-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x))/(16*(-a)^(5/2)*Sq
rt[b]) + (7*d*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt
[b] + d*x))/(16*a^3) - (15*Sqrt[b]*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhInteg
ral[(Sqrt[-a]*d)/Sqrt[b] + d*x))/(16*(-a)^(7/2)) - (d^2*Sinh[c - (Sqrt[-a]*
d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x))/(16*(-a)^(5/2)*Sqrt[b
])

```

Rule 3297

```

Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
+ d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]

```

Rule 3298

```

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)])/((c_.) + (d_.)*(x_)), x_Symbol]
:= Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]

```

Rule 3301

```

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)])/((c_.) + (d_.)*(x_)), x_Symbol]
:= Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]

```

Rule 3303

```

Int[sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&

```

NeQ[d*e - c*f, 0]

Rule 5281

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Int[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 5293

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh(c+dx)}{x^2(a+bx^2)^3} dx &= \int \left(\frac{\cosh(c+dx)}{a^3 x^2} - \frac{b \cosh(c+dx)}{a(a+bx^2)^3} - \frac{b \cosh(c+dx)}{a^2(a+bx^2)^2} - \frac{b \cosh(c+dx)}{a^3(a+bx^2)} \right) dx \\
&= \frac{\int \frac{\cosh(c+dx)}{x^2} dx}{a^3} - \frac{b \int \frac{\cosh(c+dx)}{a+bx^2} dx}{a^3} - \frac{b \int \frac{\cosh(c+dx)}{(a+bx^2)^2} dx}{a^2} - \frac{b \int \frac{\cosh(c+dx)}{(a+bx^2)^3} dx}{a} \\
&= -\frac{\cosh(c+dx)}{a^3 x} - \frac{b \int \left(\frac{\sqrt{-a} \cosh(c+dx)}{2a(\sqrt{-a}-\sqrt{b}x)} + \frac{\sqrt{-a} \cosh(c+dx)}{2a(\sqrt{-a}+\sqrt{b}x)} \right) dx}{a^3} - \frac{b \int \left(-\frac{b \cosh(c+dx)}{4a(\sqrt{-a}\sqrt{b}-bx)^2} - \frac{b \cosh(c+dx)}{4a(\sqrt{-a}+\sqrt{b}x)^2} \right) dx}{a^2} \\
&= -\frac{\cosh(c+dx)}{a^3 x} - \frac{b \int \frac{\cosh(c+dx)}{\sqrt{-a}-\sqrt{b}x} dx}{2(-a)^{7/2}} - \frac{b \int \frac{\cosh(c+dx)}{\sqrt{-a}+\sqrt{b}x} dx}{2(-a)^{7/2}} + \frac{(3b^2) \int \frac{\cosh(c+dx)}{(\sqrt{-a}\sqrt{b}-bx)^2} dx}{16a^3} + \frac{(3b^2) \int \frac{\cosh(c+dx)}{(\sqrt{-a}+\sqrt{b}x)^2} dx}{16a^3} \\
&= -\frac{\cosh(c+dx)}{a^3 x} - \frac{\sqrt{b} \cosh(c+dx)}{16(-a)^{5/2}(\sqrt{-a}-\sqrt{b}x)^2} + \frac{7\sqrt{b} \cosh(c+dx)}{16a^3(\sqrt{-a}-\sqrt{b}x)} + \frac{\sqrt{b} \cosh(c+dx)}{16(-a)^{5/2}(\sqrt{-a}+\sqrt{b}x)^2} - \frac{7\sqrt{b} \cosh(c+dx)}{16a^3(\sqrt{-a}+\sqrt{b}x)} \\
&= -\frac{\cosh(c+dx)}{a^3 x} - \frac{\sqrt{b} \cosh(c+dx)}{16(-a)^{5/2}(\sqrt{-a}-\sqrt{b}x)^2} + \frac{7\sqrt{b} \cosh(c+dx)}{16a^3(\sqrt{-a}-\sqrt{b}x)} + \frac{\sqrt{b} \cosh(c+dx)}{16(-a)^{5/2}(\sqrt{-a}+\sqrt{b}x)^2} - \frac{7\sqrt{b} \cosh(c+dx)}{16a^3(\sqrt{-a}+\sqrt{b}x)} \\
&= -\frac{\cosh(c+dx)}{a^3 x} - \frac{\sqrt{b} \cosh(c+dx)}{16(-a)^{5/2}(\sqrt{-a}-\sqrt{b}x)^2} + \frac{7\sqrt{b} \cosh(c+dx)}{16a^3(\sqrt{-a}-\sqrt{b}x)} + \frac{\sqrt{b} \cosh(c+dx)}{16(-a)^{5/2}(\sqrt{-a}+\sqrt{b}x)^2} - \frac{7\sqrt{b} \cosh(c+dx)}{16a^3(\sqrt{-a}+\sqrt{b}x)} \\
&= -\frac{\cosh(c+dx)}{a^3 x} - \frac{\sqrt{b} \cosh(c+dx)}{16(-a)^{5/2}(\sqrt{-a}-\sqrt{b}x)^2} + \frac{7\sqrt{b} \cosh(c+dx)}{16a^3(\sqrt{-a}-\sqrt{b}x)} + \frac{\sqrt{b} \cosh(c+dx)}{16(-a)^{5/2}(\sqrt{-a}+\sqrt{b}x)^2} - \frac{7\sqrt{b} \cosh(c+dx)}{16a^3(\sqrt{-a}+\sqrt{b}x)}
\end{aligned}$$

Mathematica [C] time = 2.94, size = 1009, normalized size = 1.15

$$\frac{30\sqrt{a}b^2 \cosh(c) \cosh(dx)x^3}{(bx^2+a)^2} - \frac{30\sqrt{a}b^2 \sinh(c) \sinh(dx)x^3}{(bx^2+a)^2} - \frac{2a^{3/2}bd \cosh(dx) \sinh(c)x^2}{(bx^2+a)^2} - \frac{2a^{3/2}bd \cosh(c) \sinh(dx)x^2}{(bx^2+a)^2} - \frac{50a^{3/2}b \cosh(c) \cosh(dx)}{(bx^2+a)^2}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Cosh[c + d*x]/(x^2*(a + b*x^2)^3), x]

```
[Out] ((-16*a^(5/2)*Cosh[c]*Cosh[d*x])/(x*(a + b*x^2)^2) - (50*a^(3/2)*b*x*Cosh[c]*Cosh[d*x])/(a + b*x^2)^2 - (30*Sqrt[a]*b^2*x^3*Cosh[c]*Cosh[d*x])/(a + b*x^2)^2 - (2*a^(5/2)*d*Cosh[d*x]*Sinh[c])/(a + b*x^2)^2 - (2*a^(3/2)*b*d*x^2*Cosh[d*x]*Sinh[c])/(a + b*x^2)^2 + 16*Sqrt[a]*d*CoshIntegral[d*x]*Sinh[c] + (CosIntegral[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x]*((-I)*(15*b - a*d^2)*Cosh[c - (I*Sqrt[a]*d)/Sqrt[b]) + 7*Sqrt[a]*Sqrt[b]*d*Sinh[c - (I*Sqrt[a]*d)/Sqrt[b]])/Sqrt[b] + (CosIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x]*(I*(15*b - a*d^2)*Cosh[c + (I*Sqrt[a]*d)/Sqrt[b]) + 7*Sqrt[a]*Sqrt[b]*d*Sinh[c + (I*Sqrt[a]*d)/Sqrt[b]])/Sqrt[b] - (2*a^(5/2)*d*Cosh[c]*Sinh[d*x])/(a + b*x^2)^2 - (2*a^(3/2)*b*d*x^2*Cosh[c]*Sinh[d*x])/(a + b*x^2)^2 - (16*a^(5/2)*Sinh[c]*Sinh[d*x])/(x*(a + b*x^2)^2) - (50*a^(3/2)*b*x*Sinh[c]*Sinh[d*x])/(a + b*x^2)^2 - (30*Sqrt[a]*b^2*x^3*Sinh[c]*Sinh[d*x])/(a + b*x^2)^2 + 16*Sqrt[a]*d*Cosh[c]*SinhIntegral[d*x] + (7*I)*Sqrt[a]*d*Cos[(Sqrt[a]*d)/Sqrt[b]]*Cosh[c]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d*x] - (15*I)*Sqrt[b]*Cosh[c]*Sin[(Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d*x] + (I*a*d^2*Cosh[c]*Sin[(Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d*x])/Sqrt[b] + 15*Sqrt[b]*Cos[(Sqrt[a]*d)/Sqrt[b]]*Sinh[c]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d*x] - (a*d^2*Cos[(Sqrt[a]*d)/Sqrt[b]]*Sinh[c]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d*x])/Sqrt[b] + 7*Sqrt[a]*d*Ssin[(Sqrt[a]*d)/Sqrt[b]]*Sinh[c]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d*x] - (7*I)*Sqrt[a]*d*Cos[(Sqrt[a]*d)/Sqrt[b]]*Cosh[c]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x] + (15*I)*Sqrt[b]*Cosh[c]*Sin[(Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x] - (I*a*d^2*Cosh[c]*Sin[(Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x])/Sqrt[b] + 15*Sqrt[b]*Cos[(Sqrt[a]*d)/Sqrt[b]]*Sinh[c]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x] - (a*d^2*Cos[(Sqrt[a]*d)/Sqrt[b]]*Sinh[c]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x])/Sqrt[b] + 7*Sqrt[a]*d*Ssin[(Sqrt[a]*d)/Sqrt[b]]*Sinh[c]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x])/(16*a^(7/2))
```

fricas [B] time = 0.61, size = 2346, normalized size = 2.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x^2/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/32*(4*(15*a*b^2*d*x^4 + 25*a^2*b*d*x^2 + 8*a^3*d)*cosh(d*x + c) - ((7*(a*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*cosh(d*x + c)^2 - 7*(a*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*sinh(d*x + c)^2 - (((a*b^2*d^2 - 15*b^3)*x^5 + 2*(a^2*b*d^2 - 15*a*b^2)*x^3 + (a^3*d^2 - 15*a^2*b)*x)*cosh(d*x + c)^2 - ((a*b^2*d^2 - 15*b^3)*x^5 + 2*(a^2*b*d^2 - 15*a*b^2)*x^3 + (a^3*d^2 - 15*a^2*b)*x)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x - sqrt(-a*d^2/b)) - (7*(a*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*cosh(d*x + c)^2 - 7*(a*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*sinh(d*x + c)^2 + (((a*b^2*d^2 - 15*b^3)*x^5 + 2*(a^2*b*d^2 - 15*a*b^2)*x^3 + (a^3*d^2 - 15*a^2*b)*x)*cosh(d*x + c)^2 - ((a*b^2*d^2 - 15*b^3)*x^5 + 2*(a^2*b*d^2 - 15*a*b^2)*x^3 + (a^3*d^2 - 15*a^2*b)*x)*sinh(d*x + c)^2)
```

```

- 15*a^2*b)*x)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x + sqrt(-a*d^2/b)))
*cosh(c + sqrt(-a*d^2/b)) - 16*((a*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)
*x)*Ei(d*x) - (a*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*Ei(-d*x))*cosh(c
) - ((7*(a*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*cosh(d*x + c)^2 - 7*(
a*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*sinh(d*x + c)^2 + (((a*b^2*d^2
- 15*b^3)*x^5 + 2*(a^2*b*d^2 - 15*a*b^2)*x^3 + (a^3*d^2 - 15*a^2*b)*x)*cos
h(d*x + c)^2 - ((a*b^2*d^2 - 15*b^3)*x^5 + 2*(a^2*b*d^2 - 15*a*b^2)*x^3 + (
a^3*d^2 - 15*a^2*b)*x)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x + sqrt(-a*d^
2/b)) - (7*(a*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*cosh(d*x + c)^2 -
7*(a*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*sinh(d*x + c)^2 - (((a*b^2*
d^2 - 15*b^3)*x^5 + 2*(a^2*b*d^2 - 15*a*b^2)*x^3 + (a^3*d^2 - 15*a^2*b)*x)*
cosh(d*x + c)^2 - ((a*b^2*d^2 - 15*b^3)*x^5 + 2*(a^2*b*d^2 - 15*a*b^2)*x^3
+ (a^3*d^2 - 15*a^2*b)*x)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x - sqrt(-
a*d^2/b))*cosh(-c + sqrt(-a*d^2/b)) + 4*(a^2*b*d^2*x^3 + a^3*d^2*x)*sinh(d
*x + c) - ((7*(a*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*cosh(d*x + c)^2
- 7*(a*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*sinh(d*x + c)^2 - (((a*b
^2*d^2 - 15*b^3)*x^5 + 2*(a^2*b*d^2 - 15*a*b^2)*x^3 + (a^3*d^2 - 15*a^2*b)*
x)*cosh(d*x + c)^2 - ((a*b^2*d^2 - 15*b^3)*x^5 + 2*(a^2*b*d^2 - 15*a*b^2)*x
^3 + (a^3*d^2 - 15*a^2*b)*x)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x - sqrt
(-a*d^2/b)) + (7*(a*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*cosh(d*x + c
)^2 - 7*(a*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*sinh(d*x + c)^2 + (((
a*b^2*d^2 - 15*b^3)*x^5 + 2*(a^2*b*d^2 - 15*a*b^2)*x^3 + (a^3*d^2 - 15*a^2*
b)*x)*cosh(d*x + c)^2 - ((a*b^2*d^2 - 15*b^3)*x^5 + 2*(a^2*b*d^2 - 15*a*b^2
)*x^3 + (a^3*d^2 - 15*a^2*b)*x)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x +
sqrt(-a*d^2/b))*sinh(c + sqrt(-a*d^2/b)) - 16*((a*b^2*d^2*x^5 + 2*a^2*b*d^
2*x^3 + a^3*d^2*x)*Ei(d*x) + (a*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*
Ei(-d*x))*sinh(c) + ((7*(a*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*cosh(
d*x + c)^2 - 7*(a*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*sinh(d*x + c)^
2 + (((a*b^2*d^2 - 15*b^3)*x^5 + 2*(a^2*b*d^2 - 15*a*b^2)*x^3 + (a^3*d^2 -
15*a^2*b)*x)*cosh(d*x + c)^2 - ((a*b^2*d^2 - 15*b^3)*x^5 + 2*(a^2*b*d^2 - 1
5*a*b^2)*x^3 + (a^3*d^2 - 15*a^2*b)*x)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(
d*x + sqrt(-a*d^2/b)) + (7*(a*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*co
sh(d*x + c)^2 - 7*(a*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*sinh(d*x +
c)^2 - (((a*b^2*d^2 - 15*b^3)*x^5 + 2*(a^2*b*d^2 - 15*a*b^2)*x^3 + (a^3*d^2
- 15*a^2*b)*x)*cosh(d*x + c)^2 - ((a*b^2*d^2 - 15*b^3)*x^5 + 2*(a^2*b*d^2
- 15*a*b^2)*x^3 + (a^3*d^2 - 15*a^2*b)*x)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*
Ei(-d*x - sqrt(-a*d^2/b))*sinh(-c + sqrt(-a*d^2/b)))/((a^4*b^2*d*x^5 + 2*a
^5*b*d*x^3 + a^6*d*x)*cosh(d*x + c)^2 - (a^4*b^2*d*x^5 + 2*a^5*b*d*x^3 + a^
6*d*x)*sinh(d*x + c)^2)

```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(dx + c)}{(bx^2 + a)^3 x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/x^2/(b*x^2+a)^3,x, algorithm="giac")

[Out] integrate(cosh(d*x + c)/((b*x^2 + a)^3*x^2), x)

maple [A] time = 0.20, size = 1178, normalized size = 1.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d*x+c)/x^2/(b*x^2+a)^3,x)

[Out] $\frac{1}{16} \exp(-d*x-c)/a^2*x^2*d^5/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*b-15/16*\exp(-d*x-c)/a^3*x^3*d^4/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*b^2+1/16*\exp(-d*x-c)/a*d^5/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)-25/16*\exp(-d*x-c)/a^2*x*d^4/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*b-1/2*\exp(-d*x-c)/a/x*d^4/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)+1/2*d/a^3*\exp(-c)*Ei(1,d*x)+1/32/a^2*d^2/(-a*b)^(1/2)*\exp(-(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)-1/32/a^2*d^2/(-a*b)^(1/2)*\exp(-(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)+7/32*d/a^3*\exp(-(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)+7/32*d/a^3*\exp(-(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)-15/32/a^3/(-a*b)^(1/2)*\exp(-(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)*b+15/32/a^3/(-a*b)^(1/2)*\exp(-(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)*b-1/16*\exp(d*x+c)/a^2*x^2*d^5/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*b-15/16*\exp(d*x+c)/a^3*x^3*d^4/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*b^2-1/16*\exp(d*x+c)/a*d^5/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)-25/16*\exp(d*x+c)/a^2*x*d^4/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*b-1/2*\exp(d*x+c)/a/x*d^4/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)-1/2*d/a^3*\exp(c)*Ei(1,-d*x)-1/32/a^2*d^2/(-a*b)^(1/2)*\exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)+1/32/a^2*d^2/(-a*b)^(1/2)*\exp((-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)-7/32*d/a^3*\exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)-7/32*d/a^3*\exp((-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)+15/32/a^3/(-a*b)^(1/2)*\exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)*b-15/32/a^3/(-a*b)^(1/2)*\exp((-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)*b$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(dx+c)}{(bx^2+a)^3 x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/x^2/(b*x^2+a)^3,x, algorithm="maxima")

[Out] integrate(cosh(d*x + c)/((b*x^2 + a)^3*x^2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(c + dx)}{x^2 (bx^2 + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(c + d*x)/(x^2*(a + b*x^2)^3), x)

[Out] int(cosh(c + d*x)/(x^2*(a + b*x^2)^3), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/x**2/(b*x**2+a)**3, x)

[Out] Timed out

$$3.78 \quad \int \frac{\cosh(c+dx)}{x^3(a+bx^2)^3} dx$$

Optimal. Leaf size=791

$$-\frac{3b \cosh(c) \operatorname{Chi}(dx)}{a^4} + \frac{3b \cosh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right) \operatorname{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{2a^4} + \frac{3b \cosh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \operatorname{Chi}\left(xd + \frac{\sqrt{-ad}}{\sqrt{b}}\right)}{2a^4} - \frac{3b \sinh(c) \operatorname{Shi}(dx)}{a^4}$$

[Out] $-3*b*\operatorname{Chi}(d*x)*\cosh(c)/a^4+1/2*d^2*\operatorname{Chi}(d*x)*\cosh(c)/a^3-1/2*\cosh(d*x+c)/a^3/x^2-1/4*b*\cosh(d*x+c)/a^2/(b*x^2+a)^2-b*\cosh(d*x+c)/a^3/(b*x^2+a)+3/2*b*\operatorname{Chi}(d*x+d*(-a)^{(1/2)}/b^{(1/2)})*\cosh(c-d*(-a)^{(1/2)}/b^{(1/2)})/a^4-1/16*d^2*\operatorname{Chi}(d*x+d*(-a)^{(1/2)}/b^{(1/2)})*\cosh(c-d*(-a)^{(1/2)}/b^{(1/2)})/a^3+3/2*b*\operatorname{Chi}(-d*x+d*(-a)^{(1/2)}/b^{(1/2)})*\cosh(c+d*(-a)^{(1/2)}/b^{(1/2)})/a^4-1/16*d^2*\operatorname{Chi}(-d*x+d*(-a)^{(1/2)}/b^{(1/2)})*\cosh(c+d*(-a)^{(1/2)}/b^{(1/2)})/a^3-3*b*\operatorname{Shi}(d*x)*\sinh(c)/a^4+1/2*d^2*\operatorname{Shi}(d*x)*\sinh(c)/a^3-1/2*d*\sinh(d*x+c)/a^3/x+3/2*b*\operatorname{Shi}(d*x+d*(-a)^{(1/2)}/b^{(1/2)})*\sinh(c-d*(-a)^{(1/2)}/b^{(1/2)})/a^4-1/16*d^2*\operatorname{Shi}(d*x+d*(-a)^{(1/2)}/b^{(1/2)})*\sinh(c-d*(-a)^{(1/2)}/b^{(1/2)})/a^3+3/2*b*\operatorname{Shi}(d*x-d*(-a)^{(1/2)}/b^{(1/2)})*\sinh(c+d*(-a)^{(1/2)}/b^{(1/2)})/a^4-1/16*d^2*\operatorname{Shi}(d*x-d*(-a)^{(1/2)}/b^{(1/2)})*\sinh(c+d*(-a)^{(1/2)}/b^{(1/2)})/a^3-9/16*d*\cosh(c+d*(-a)^{(1/2)}/b^{(1/2)})*\operatorname{Shi}(d*x-d*(-a)^{(1/2)}/b^{(1/2)})*b^{(1/2)}/(-a)^{(7/2)}+9/16*d*\cosh(c-d*(-a)^{(1/2)}/b^{(1/2)})*\operatorname{Shi}(d*x+d*(-a)^{(1/2)}/b^{(1/2)})*b^{(1/2)}/(-a)^{(7/2)}+9/16*d*\operatorname{Chi}(d*x+d*(-a)^{(1/2)}/b^{(1/2)})*\sinh(c-d*(-a)^{(1/2)}/b^{(1/2)})*b^{(1/2)}/(-a)^{(7/2)}-9/16*d*\operatorname{Chi}(-d*x+d*(-a)^{(1/2)}/b^{(1/2)})*\sinh(c+d*(-a)^{(1/2)}/b^{(1/2)})*b^{(1/2)}/(-a)^{(7/2)}-1/16*d*\sinh(d*x+c)*b^{(1/2)}/a^3/((-a)^{(1/2)}-x*b^{(1/2)})+1/16*d*\sinh(d*x+c)*b^{(1/2)}/a^3/((-a)^{(1/2)}+x*b^{(1/2)})$

Rubi [A] time = 1.87, antiderivative size = 791, normalized size of antiderivative = 1.00, number of steps used = 46, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {5293, 3297, 3303, 3298, 3301, 5289, 5280}

$$-\frac{d^2 \cosh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right) \operatorname{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{16a^3} - \frac{d^2 \cosh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \operatorname{Chi}\left(xd + \frac{\sqrt{-ad}}{\sqrt{b}}\right)}{16a^3} - \frac{3b \cosh(c) \operatorname{Chi}(dx)}{a^4} + \frac{3b \cosh\left(\frac{\sqrt{-ad}}{\sqrt{b}}\right) \operatorname{Shi}(dx)}{a^4}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Cosh}[c + d*x]/(x^3*(a + b*x^2)^3), x]$

[Out] $-\operatorname{Cosh}[c + d*x]/(2*a^3*x^2) - (b*\operatorname{Cosh}[c + d*x])/(4*a^2*(a + b*x^2)^2) - (b*\operatorname{Cosh}[c + d*x])/(a^3*(a + b*x^2)) - (3*b*\operatorname{Cosh}[c]*\operatorname{CoshIntegral}[d*x])/a^4 + (d^2*\operatorname{Cosh}[c]*\operatorname{CoshIntegral}[d*x])/(2*a^3) + (3*b*\operatorname{Cosh}[c + (\operatorname{Sqrt}[-a]*d)/\operatorname{Sqrt}[b]])*\operatorname{CoshIntegral}[(\operatorname{Sqrt}[-a]*d)/\operatorname{Sqrt}[b] - d*x]/(2*a^4) - (d^2*\operatorname{Cosh}[c + (\operatorname{Sqrt}[-a]*d)/\operatorname{Sqrt}[b]])*\operatorname{CoshIntegral}[(\operatorname{Sqrt}[-a]*d)/\operatorname{Sqrt}[b] - d*x]/(16*a^3) + (3*b*\operatorname{Cosh}[c + (\operatorname{Sqrt}[-a]*d)/\operatorname{Sqrt}[b]])*\operatorname{CoshIntegral}[(\operatorname{Sqrt}[-a]*d)/\operatorname{Sqrt}[b] + d*x]/(16*a^3)$

$$\begin{aligned}
& [c - (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{CoshIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] + d*x]/(2*a^4) \\
& - (d^2*\text{Cosh}[c - (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{CoshIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] + \\
& d*x]/(16*a^3) + (9*\text{Sqrt}[b]*d*\text{CoshIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] + d*x]*\text{Sin} \\
& \text{h}[c - (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]])/(16*(-a)^(7/2)) - (9*\text{Sqrt}[b]*d*\text{CoshIntegral}[(\text{S} \\
& \text{qrt}[-a]*d)/\text{Sqrt}[b] - d*x]*\text{Sinh}[c + (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]])/(16*(-a)^(7/2)) - \\
& (d*\text{Sinh}[c + d*x])/(2*a^3*x) - (\text{Sqrt}[b]*d*\text{Sinh}[c + d*x])/(16*a^3*(\text{Sqrt}[-a] \\
& - \text{Sqrt}[b]*x)) + (\text{Sqrt}[b]*d*\text{Sinh}[c + d*x])/(16*a^3*(\text{Sqrt}[-a] + \text{Sqrt}[b]*x)) - \\
& (3*b*\text{Sinh}[c]*\text{SinhIntegral}[d*x])/a^4 + (d^2*\text{Sinh}[c]*\text{SinhIntegral}[d*x])/(2*a \\
& ^3) + (9*\text{Sqrt}[b]*d*\text{Cosh}[c + (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{SinhIntegral}[(\text{Sqrt}[-a]*d) \\
& / \text{Sqrt}[b] - d*x]/(16*(-a)^(7/2)) - (3*b*\text{Sinh}[c + (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{Sinh} \\
& \text{Integral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] - d*x]/(2*a^4) + (d^2*\text{Sinh}[c + (\text{Sqrt}[-a]*d)/ \\
& \text{Sqrt}[b]]*\text{SinhIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] - d*x]/(16*a^3) + (9*\text{Sqrt}[b]*d* \\
& \text{Cosh}[c - (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{SinhIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] + d*x]/(1 \\
& 6*(-a)^(7/2)) + (3*b*\text{Sinh}[c - (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{SinhIntegral}[(\text{Sqrt}[-a]* \\
& d)/\text{Sqrt}[b] + d*x]/(2*a^4) - (d^2*\text{Sinh}[c - (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{SinhIntegr} \\
& \text{al}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] + d*x]/(16*a^3)
\end{aligned}$$
Rule 3297

```

Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[(((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
+ d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]

```

Rule 3298

```

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_) ]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]

```

Rule 3301

```

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_) ]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]

```

Rule 3303

```

Int[sin[(e_.) + (f_.)*(x_) ]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]

```

Rule 5280

```

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_) ], x_Symbol] := In

```

```
t[ExpandIntegrand[Sinh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 5289

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(e^m*(a + b*x^n)^(p + 1)*Cosh[c + d*x]/(b*n*(p + 1)), x] - Dist[(d*e^m)/(b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*Sinh[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IntegerQ[p] && EqQ[m - n + 1, 0] && LtQ[p, -1] && (IntegerQ[n] || GtQ[e, 0])
```

Rule 5293

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh(c+dx)}{x^3(a+bx^2)^3} dx &= \int \left(\frac{\cosh(c+dx)}{a^3 x^3} - \frac{3b \cosh(c+dx)}{a^4 x} + \frac{b^2 x \cosh(c+dx)}{a^2 (a+bx^2)^3} + \frac{2b^2 x \cosh(c+dx)}{a^3 (a+bx^2)^2} + \frac{3b^2 x \cosh(c+dx)}{a^4 (a+bx^2)} \right) dx \\
&= \frac{\int \frac{\cosh(c+dx)}{x^3} dx}{a^3} - \frac{(3b) \int \frac{\cosh(c+dx)}{x} dx}{a^4} + \frac{(3b^2) \int \frac{x \cosh(c+dx)}{a+bx^2} dx}{a^4} + \frac{(2b^2) \int \frac{x \cosh(c+dx)}{(a+bx^2)^2} dx}{a^3} + \frac{(3b^2) \int \frac{x \cosh(c+dx)}{(a+bx^2)^3} dx}{a^2} \\
&= -\frac{\cosh(c+dx)}{2a^3 x^2} - \frac{b \cosh(c+dx)}{4a^2 (a+bx^2)^2} - \frac{b \cosh(c+dx)}{a^3 (a+bx^2)} + \frac{(3b^2) \int \left(-\frac{\cosh(c+dx)}{2\sqrt{b}(\sqrt{-a}-\sqrt{b}x)} + \frac{\cosh(c+dx)}{2\sqrt{b}(\sqrt{-a}+\sqrt{b}x)} \right) dx}{a^4} \\
&= -\frac{\cosh(c+dx)}{2a^3 x^2} - \frac{b \cosh(c+dx)}{4a^2 (a+bx^2)^2} - \frac{b \cosh(c+dx)}{a^3 (a+bx^2)} - \frac{3b \cosh(c) \text{Chi}(dx)}{a^4} - \frac{d \sinh(c+dx)}{2a^3 x} \\
&= -\frac{\cosh(c+dx)}{2a^3 x^2} - \frac{b \cosh(c+dx)}{4a^2 (a+bx^2)^2} - \frac{b \cosh(c+dx)}{a^3 (a+bx^2)} - \frac{3b \cosh(c) \text{Chi}(dx)}{a^4} - \frac{d \sinh(c+dx)}{2a^3 x} \\
&= -\frac{\cosh(c+dx)}{2a^3 x^2} - \frac{b \cosh(c+dx)}{4a^2 (a+bx^2)^2} - \frac{b \cosh(c+dx)}{a^3 (a+bx^2)} - \frac{3b \cosh(c) \text{Chi}(dx)}{a^4} + \frac{d^2 \cosh(c) \text{Chi}(dx)}{2a^3} \\
&= -\frac{\cosh(c+dx)}{2a^3 x^2} - \frac{b \cosh(c+dx)}{4a^2 (a+bx^2)^2} - \frac{b \cosh(c+dx)}{a^3 (a+bx^2)} - \frac{3b \cosh(c) \text{Chi}(dx)}{a^4} + \frac{d^2 \cosh(c) \text{Chi}(dx)}{2a^3} \\
&= -\frac{\cosh(c+dx)}{2a^3 x^2} - \frac{b \cosh(c+dx)}{4a^2 (a+bx^2)^2} - \frac{b \cosh(c+dx)}{a^3 (a+bx^2)} - \frac{3b \cosh(c) \text{Chi}(dx)}{a^4} + \frac{d^2 \cosh(c) \text{Chi}(dx)}{2a^3} \\
&= -\frac{\cosh(c+dx)}{2a^3 x^2} - \frac{b \cosh(c+dx)}{4a^2 (a+bx^2)^2} - \frac{b \cosh(c+dx)}{a^3 (a+bx^2)} - \frac{3b \cosh(c) \text{Chi}(dx)}{a^4} + \frac{d^2 \cosh(c) \text{Chi}(dx)}{2a^3}
\end{aligned}$$

Mathematica [C] time = 4.02, size = 998, normalized size = 1.26

$$-ia \sinh(c) \left(\text{Ci} \left(idx - \frac{\sqrt{ad}}{\sqrt{b}} \right) \sin \left(\frac{\sqrt{ad}}{\sqrt{b}} \right) - \text{Ci} \left(idx + \frac{\sqrt{ad}}{\sqrt{b}} \right) \sin \left(\frac{\sqrt{ad}}{\sqrt{b}} \right) + \cos \left(\frac{\sqrt{ad}}{\sqrt{b}} \right) \left(\text{Si} \left(idx + \frac{\sqrt{ad}}{\sqrt{b}} \right) - \text{Si} \left(\frac{\sqrt{ad}}{\sqrt{b}} - idx \right) \right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[Cosh[c + d*x]/(x^3*(a + b*x^2)^3), x]

[Out]
$$-1/16 * ((2*a*Cosh[d*x] * (2*(2*a^2 + 9*a*b*x^2 + 6*b^2*x^4)*Cosh[c] + d*x*(4*a^2 + 7*a*b*x^2 + 3*b^2*x^4)*Sinh[c])) / (x^2*(a + b*x^2)^2) + (2*a*(d*x*(4*a^2 + 7*a*b*x^2 + 3*b^2*x^4)*Cosh[c] + 2*(2*a^2 + 9*a*b*x^2 + 6*b^2*x^4)*Sinh[c])*Sinh[d*x]) / (x^2*(a + b*x^2)^2) + 8*(6*b - a*d^2)*(Cosh[c]*CoshIntegral[d*x] + Sinh[c]*SinhIntegral[d*x]) - (9*I)*Sqrt[a]*Sqrt[b]*d*Sinh[c]*(Cos[(Sqrt[a]*d)/Sqrt[b]]*CosIntegral[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x] - Cos[(Sqrt[a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x] + Sin[(Sqrt[a]*d)/Sqrt[b]]*(SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d*x] - SinIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x])) + (24*I)*b*Sinh[c]*(CosIntegral[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x]*Sin[(Sqrt[a]*d)/Sqrt[b]] - CosIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x]*Sin[(Sqrt[a]*d)/Sqrt[b]] + Cos[(Sqrt[a]*d)/Sqrt[b]]*(-SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d*x] + SinIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x])) - I*a*d^2*Sinh[c]*(CosIntegral[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x]*Sin[(Sqrt[a]*d)/Sqrt[b]] - CosIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x]*Sin[(Sqrt[a]*d)/Sqrt[b]] + Cos[(Sqrt[a]*d)/Sqrt[b]]*(-SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d*x] + SinIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x])) - 9*Sqrt[a]*Sqrt[b]*d*Cosh[c]*(CosIntegral[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x]*Sin[(Sqrt[a]*d)/Sqrt[b]] + CosIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x]*Sin[(Sqrt[a]*d)/Sqrt[b]] - Cos[(Sqrt[a]*d)/Sqrt[b]]*(SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d*x] + SinIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x])) - 24*b*Cosh[c]*(Cos[(Sqrt[a]*d)/Sqrt[b]]*CosIntegral[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x] + Cos[(Sqrt[a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x] + Sin[(Sqrt[a]*d)/Sqrt[b]]*(SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d*x] + SinIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x])) + a*d^2*Cosh[c]*(Cos[(Sqrt[a]*d)/Sqrt[b]]*CosIntegral[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x] + Cos[(Sqrt[a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x] + Sin[(Sqrt[a]*d)/Sqrt[b]]*(SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d*x] + SinIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x])))/a^4$$

fricas [B] time = 0.53, size = 2363, normalized size = 2.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/x^3/(b*x^2+a)^3,x, algorithm="fricas")

[Out]
$$-1/32*(8*(6*a*b^2*x^4 + 9*a^2*b*x^2 + 2*a^3)*cosh(d*x + c) + (((a*b^2*d^2 - 24*b^3)*x^6 + 2*(a^2*b*d^2 - 24*a*b^2)*x^4 + (a^3*d^2 - 24*a^2*b)*x^2)*cosh(d*x + c)^2 - ((a*b^2*d^2 - 24*b^3)*x^6 + 2*(a^2*b*d^2 - 24*a*b^2)*x^4 + (a^3*d^2 - 24*a^2*b)*x^2)*sinh(d*x + c)^2 + 9*((b^3*x^6 + 2*a*b^2*x^4 + a^2*b*x^2)*cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^4 + a^2*b*x^2)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x - sqrt(-a*d^2/b)) + (((a*b^2*d^2 - 24*b^3)*x^6 +$$

$$\begin{aligned}
& 2*(a^2*b*d^2 - 24*a*b^2)*x^4 + (a^3*d^2 - 24*a^2*b)*x^2)*\cosh(d*x + c)^2 - \\
& ((a*b^2*d^2 - 24*b^3)*x^6 + 2*(a^2*b*d^2 - 24*a*b^2)*x^4 + (a^3*d^2 - 24*a \\
& ^2*b)*x^2)*\sinh(d*x + c)^2 - 9*((b^3*x^6 + 2*a*b^2*x^4 + a^2*b*x^2)*\cosh(d* \\
& x + c)^2 - (b^3*x^6 + 2*a*b^2*x^4 + a^2*b*x^2)*\sinh(d*x + c)^2)*\sqrt{-a*d^2 \\
& /b})*\text{Ei}(-d*x + \sqrt{-a*d^2/b}))*\cosh(c + \sqrt{-a*d^2/b}) - 8*((a*b^2*d^2 - \\
& 6*b^3)*x^6 + 2*(a^2*b*d^2 - 6*a*b^2)*x^4 + (a^3*d^2 - 6*a^2*b)*x^2)*\text{Ei}(d*x \\
&) + ((a*b^2*d^2 - 6*b^3)*x^6 + 2*(a^2*b*d^2 - 6*a*b^2)*x^4 + (a^3*d^2 - 6*a \\
& ^2*b)*x^2)*\text{Ei}(-d*x))*\cosh(c) + (((a*b^2*d^2 - 24*b^3)*x^6 + 2*(a^2*b*d^2 - \\
& 24*a*b^2)*x^4 + (a^3*d^2 - 24*a^2*b)*x^2)*\cosh(d*x + c)^2 - ((a*b^2*d^2 - \\
& 24*b^3)*x^6 + 2*(a^2*b*d^2 - 24*a*b^2)*x^4 + (a^3*d^2 - 24*a^2*b)*x^2)*\sinh \\
& (d*x + c)^2 - 9*((b^3*x^6 + 2*a*b^2*x^4 + a^2*b*x^2)*\cosh(d*x + c)^2 - (b^3 \\
& *x^6 + 2*a*b^2*x^4 + a^2*b*x^2)*\sinh(d*x + c)^2)*\sqrt{-a*d^2/b})*\text{Ei}(d*x + s \\
& \sqrt{-a*d^2/b}) + (((a*b^2*d^2 - 24*b^3)*x^6 + 2*(a^2*b*d^2 - 24*a*b^2)*x^4 \\
& + (a^3*d^2 - 24*a^2*b)*x^2)*\cosh(d*x + c)^2 - ((a*b^2*d^2 - 24*b^3)*x^6 + 2 \\
& *(a^2*b*d^2 - 24*a*b^2)*x^4 + (a^3*d^2 - 24*a^2*b)*x^2)*\sinh(d*x + c)^2 + 9 \\
& *((b^3*x^6 + 2*a*b^2*x^4 + a^2*b*x^2)*\cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2* \\
& x^4 + a^2*b*x^2)*\sinh(d*x + c)^2)*\sqrt{-a*d^2/b})*\text{Ei}(-d*x - \sqrt{-a*d^2/b}) \\
&)*\cosh(-c + \sqrt{-a*d^2/b}) + 4*(3*a*b^2*d*x^5 + 7*a^2*b*d*x^3 + 4*a^3*d*x) \\
& *\sinh(d*x + c) + (((a*b^2*d^2 - 24*b^3)*x^6 + 2*(a^2*b*d^2 - 24*a*b^2)*x^4 \\
& + (a^3*d^2 - 24*a^2*b)*x^2)*\cosh(d*x + c)^2 - ((a*b^2*d^2 - 24*b^3)*x^6 + \\
& 2*(a^2*b*d^2 - 24*a*b^2)*x^4 + (a^3*d^2 - 24*a^2*b)*x^2)*\sinh(d*x + c)^2 + \\
& 9*((b^3*x^6 + 2*a*b^2*x^4 + a^2*b*x^2)*\cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2 \\
& *x^4 + a^2*b*x^2)*\sinh(d*x + c)^2)*\sqrt{-a*d^2/b})*\text{Ei}(d*x - \sqrt{-a*d^2/b}) \\
& - (((a*b^2*d^2 - 24*b^3)*x^6 + 2*(a^2*b*d^2 - 24*a*b^2)*x^4 + (a^3*d^2 - 2 \\
& 4*a^2*b)*x^2)*\cosh(d*x + c)^2 - ((a*b^2*d^2 - 24*b^3)*x^6 + 2*(a^2*b*d^2 - \\
& 24*a*b^2)*x^4 + (a^3*d^2 - 24*a^2*b)*x^2)*\sinh(d*x + c)^2 - 9*((b^3*x^6 + 2 \\
& *a*b^2*x^4 + a^2*b*x^2)*\cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^4 + a^2*b*x^ \\
& ^2)*\sinh(d*x + c)^2)*\sqrt{-a*d^2/b})*\text{Ei}(-d*x + \sqrt{-a*d^2/b}))*\sinh(c + \sqrt{ \\
& t(-a*d^2/b)} - 8*((a*b^2*d^2 - 6*b^3)*x^6 + 2*(a^2*b*d^2 - 6*a*b^2)*x^4 + \\
& (a^3*d^2 - 6*a^2*b)*x^2)*\text{Ei}(d*x) - ((a*b^2*d^2 - 6*b^3)*x^6 + 2*(a^2*b*d^2 \\
& - 6*a*b^2)*x^4 + (a^3*d^2 - 6*a^2*b)*x^2)*\text{Ei}(-d*x))*\sinh(c) - (((a*b^2*d^2 \\
& - 24*b^3)*x^6 + 2*(a^2*b*d^2 - 24*a*b^2)*x^4 + (a^3*d^2 - 24*a^2*b)*x^2)*\c \\
& osh(d*x + c)^2 - ((a*b^2*d^2 - 24*b^3)*x^6 + 2*(a^2*b*d^2 - 24*a*b^2)*x^4 + \\
& (a^3*d^2 - 24*a^2*b)*x^2)*\sinh(d*x + c)^2 - 9*((b^3*x^6 + 2*a*b^2*x^4 + a^ \\
& 2*b*x^2)*\cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^4 + a^2*b*x^2)*\sinh(d*x + c \\
&)^2)*\sqrt{-a*d^2/b})*\text{Ei}(d*x + \sqrt{-a*d^2/b}) - (((a*b^2*d^2 - 24*b^3)*x^6 \\
& + 2*(a^2*b*d^2 - 24*a*b^2)*x^4 + (a^3*d^2 - 24*a^2*b)*x^2)*\cosh(d*x + c)^2 \\
& - ((a*b^2*d^2 - 24*b^3)*x^6 + 2*(a^2*b*d^2 - 24*a*b^2)*x^4 + (a^3*d^2 - 24* \\
& a^2*b)*x^2)*\sinh(d*x + c)^2 + 9*((b^3*x^6 + 2*a*b^2*x^4 + a^2*b*x^2)*\cosh(d \\
& *x + c)^2 - (b^3*x^6 + 2*a*b^2*x^4 + a^2*b*x^2)*\sinh(d*x + c)^2)*\sqrt{-a*d^ \\
& 2/b})*\text{Ei}(-d*x - \sqrt{-a*d^2/b}))*\sinh(-c + \sqrt{-a*d^2/b}))/((a^4*b^2*x^6 + \\
& 2*a^5*b*x^4 + a^6*x^2)*\cosh(d*x + c)^2 - (a^4*b^2*x^6 + 2*a^5*b*x^4 + a^6* \\
& x^2)*\sinh(d*x + c)^2)
\end{aligned}$$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(dx + c)}{(bx^2 + a)^3 x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/x^3/(b*x^2+a)^3,x, algorithm="giac")

[Out] integrate(cosh(d*x + c)/((b*x^2 + a)^3*x^3), x)

maple [B] time = 0.23, size = 1294, normalized size = 1.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d*x+c)/x^3/(b*x^2+a)^3,x)

[Out]
$$\begin{aligned} & 3/16*d^5*exp(-d*x-c)/a^3*x^3/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*b^2+7/16*d^5*exp(-d*x-c)/a^2*x/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*b-3/4*d^4*exp(-d*x-c)/a^3*x^2/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*b^2+9/32*d/a^3/(-a*b)^(1/2)*exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1, -(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)*b-9/32*d/a^3/(-a*b)^(1/2)*exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1, (d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)*b-1/4*d^5*exp(d*x+c)/a/x/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)-9/8*d^4*exp(d*x+c)/a^2/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*b-1/4*d^4*exp(d*x+c)/a/x^2/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)-3/16*d^5*exp(d*x+c)/a^3*x^3/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*b^2-1/4*d^2/a^3*exp(c)*Ei(1, -d*x)+3/2/a^4*exp(c)*Ei(1, -d*x)*b+1/32*d^2/a^3*exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1, (d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)+1/32*d^2/a^3*exp((-d*(-a*b)^(1/2)+c*b)/b)*Ei(1, -(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)-3/4/a^4*exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1, (d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)*b-3/4/a^4*exp((-d*(-a*b)^(1/2)+c*b)/b)*Ei(1, -(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)*b-1/4*d^2/a^3*exp(-c)*Ei(1, d*x)+3/2/a^4*exp(-c)*Ei(1, d*x)*b+1/32*d^2/a^3*exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1, -(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)+1/32*d^2/a^3*exp((-d*(-a*b)^(1/2)+c*b)/b)*Ei(1, (d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)-3/4/a^4*exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1, -(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)*b-3/4/a^4*exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1, (d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)*b-1/4*d^4*exp(-d*x-c)/a/x^2/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)+1/4*d^5*exp(-d*x-c)/a/x/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)-9/8*d^4*exp(-d*x-c)/a^2/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*b-7/16*d^5*exp(d*x+c)/a^2*x/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*b-3/4*d^4*exp(d*x+c)/a^3*x^2/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*b^2-9/32*d/a^3/(-a*b)^(1/2)*exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1, (d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)*b+9/32*d/a^3/(-a*b)^(1/2)*exp((-d*(-a*b)^(1/2)+c*b)/b)*Ei(1, -(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)*b \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(dx + c)}{(bx^2 + a)^3 x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/x^3/(b*x^2+a)^3,x, algorithm="maxima")

[Out] integrate(cosh(d*x + c)/((b*x^2 + a)^3*x^3), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(c + dx)}{x^3 (bx^2 + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(c + d*x)/(x^3*(a + b*x^2)^3), x)

[Out] int(cosh(c + d*x)/(x^3*(a + b*x^2)^3), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/x**3/(b*x**2+a)**3,x)

[Out] Timed out

3.79 $\int x^3 (a + bx^3) \cosh(c + dx) dx$

Optimal. Leaf size=154

$$-\frac{6a \cosh(c + dx)}{d^4} + \frac{6ax \sinh(c + dx)}{d^3} - \frac{3ax^2 \cosh(c + dx)}{d^2} + \frac{ax^3 \sinh(c + dx)}{d} + \frac{720b \sinh(c + dx)}{d^7} - \frac{720bx \cosh(c + dx)}{d^6}$$

[Out] $-6*a*\cosh(d*x+c)/d^4-720*b*x*\cosh(d*x+c)/d^6-3*a*x^2*\cosh(d*x+c)/d^2-120*b*x^3*\cosh(d*x+c)/d^4-6*b*x^5*\cosh(d*x+c)/d^2+720*b*\sinh(d*x+c)/d^7+6*a*x*\sinh(d*x+c)/d^3+360*b*x^2*\sinh(d*x+c)/d^5+a*x^3*\sinh(d*x+c)/d+30*b*x^4*\sinh(d*x+c)/d^3+b*x^6*\sinh(d*x+c)/d$

Rubi [A] time = 0.30, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {5287, 3296, 2638, 2637}

$$-\frac{3ax^2 \cosh(c + dx)}{d^2} + \frac{6ax \sinh(c + dx)}{d^3} - \frac{6a \cosh(c + dx)}{d^4} + \frac{ax^3 \sinh(c + dx)}{d} + \frac{30bx^4 \sinh(c + dx)}{d^3} + \frac{360bx^2 \sinh(c + dx)}{d^5}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^3*(a + b*x^3)*\text{Cosh}[c + d*x], x]$

[Out] $(-6*a*\text{Cosh}[c + d*x])/d^4 - (720*b*x*\text{Cosh}[c + d*x])/d^6 - (3*a*x^2*\text{Cosh}[c + d*x])/d^2 - (120*b*x^3*\text{Cosh}[c + d*x])/d^4 - (6*b*x^5*\text{Cosh}[c + d*x])/d^2 + (720*b*\text{Sinh}[c + d*x])/d^7 + (6*a*x*\text{Sinh}[c + d*x])/d^3 + (360*b*x^2*\text{Sinh}[c + d*x])/d^5 + (a*x^3*\text{Sinh}[c + d*x])/d + (30*b*x^4*\text{Sinh}[c + d*x])/d^3 + (b*x^6*\text{Sinh}[c + d*x])/d$

Rule 2637

$\text{Int}[\sin[\text{Pi}/2 + (c_.) + (d_.)*(x_.)], x_Symbol] \text{ :> } \text{Simp}[\sin[c + d*x]/d, x] /;$
FreeQ[{c, d}, x]

Rule 2638

$\text{Int}[\sin[(c_.) + (d_.)*(x_.)], x_Symbol] \text{ :> } -\text{Simp}[\cos[c + d*x]/d, x] /;$ FreeQ[{c, d}, x]

Rule 3296

$\text{Int}[((c_.) + (d_.)*(x_.))^{(m_.)}*\sin[(e_.) + (f_.)*(x_.)], x_Symbol] \text{ :> } -\text{Simp}[(c + d*x)^m*\cos[e + f*x]/f, x] + \text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{(m-1)}*\cos[e + f*x], x], x] /;$ FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 5287

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
 \int x^3 (a + bx^3) \cosh(c + dx) dx &= \int (ax^3 \cosh(c + dx) + bx^6 \cosh(c + dx)) dx \\
 &= a \int x^3 \cosh(c + dx) dx + b \int x^6 \cosh(c + dx) dx \\
 &= \frac{ax^3 \sinh(c + dx)}{d} + \frac{bx^6 \sinh(c + dx)}{d} - \frac{(3a) \int x^2 \sinh(c + dx) dx}{d} - \frac{(6b) \int x^5 \sinh(c + dx) dx}{d} \\
 &= -\frac{3ax^2 \cosh(c + dx)}{d^2} - \frac{6bx^5 \cosh(c + dx)}{d^2} + \frac{ax^3 \sinh(c + dx)}{d} + \frac{bx^6 \sinh(c + dx)}{d} \\
 &= -\frac{3ax^2 \cosh(c + dx)}{d^2} - \frac{6bx^5 \cosh(c + dx)}{d^2} + \frac{6ax \sinh(c + dx)}{d^3} + \frac{ax^3 \sinh(c + dx)}{d} \\
 &= -\frac{6a \cosh(c + dx)}{d^4} - \frac{3ax^2 \cosh(c + dx)}{d^2} - \frac{120bx^3 \cosh(c + dx)}{d^4} - \frac{6bx^5 \cosh(c + dx)}{d^2} \\
 &= -\frac{6a \cosh(c + dx)}{d^4} - \frac{3ax^2 \cosh(c + dx)}{d^2} - \frac{120bx^3 \cosh(c + dx)}{d^4} - \frac{6bx^5 \cosh(c + dx)}{d^2} \\
 &= -\frac{6a \cosh(c + dx)}{d^4} - \frac{720bx \cosh(c + dx)}{d^6} - \frac{3ax^2 \cosh(c + dx)}{d^2} - \frac{120bx^3 \cosh(c + dx)}{d^4} \\
 &= -\frac{6a \cosh(c + dx)}{d^4} - \frac{720bx \cosh(c + dx)}{d^6} - \frac{3ax^2 \cosh(c + dx)}{d^2} - \frac{120bx^3 \cosh(c + dx)}{d^4}
 \end{aligned}$$

Mathematica [A] time = 0.19, size = 100, normalized size = 0.65

$$\frac{(ad^4x(d^2x^2 + 6) + b(d^6x^6 + 30d^4x^4 + 360d^2x^2 + 720)) \sinh(c + dx) - 3d(ad^2(d^2x^2 + 2) + 2bx(d^4x^4 + 20d^2x^2 + 60)) \cosh(c + dx)}{d^7}$$

Antiderivative was successfully verified.

[In] Integrate[x^3*(a + b*x^3)*Cosh[c + d*x], x]

[Out] (-3*d*(a*d^2*(2 + d^2*x^2) + 2*b*x*(120 + 20*d^2*x^2 + d^4*x^4))*Cosh[c + d*x] + (a*d^4*x*(6 + d^2*x^2) + b*(720 + 360*d^2*x^2 + 30*d^4*x^4 + d^6*x^6))*Sinh[c + d*x])/d^7

fricas [A] time = 0.47, size = 105, normalized size = 0.68

$$\frac{3(2bd^5x^5 + ad^5x^2 + 40bd^3x^3 + 2ad^3 + 240bdx) \cosh(dx + c) - (bd^6x^6 + ad^6x^3 + 30bd^4x^4 + 6ad^4x + 360bd^2) \sinh(dx + c)}{d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(b*x^3+a)*cosh(d*x+c),x, algorithm="fricas")

[Out] $-(3*(2*b*d^5*x^5 + a*d^5*x^2 + 40*b*d^3*x^3 + 2*a*d^3 + 240*b*d*x)*\cosh(d*x + c) - (b*d^6*x^6 + a*d^6*x^3 + 30*b*d^4*x^4 + 6*a*d^4*x + 360*b*d^2*x^2 + 720*b)*\sinh(d*x + c))/d^7$

giac [A] time = 0.12, size = 192, normalized size = 1.25

$$\frac{(bd^6x^6 - 6bd^5x^5 + ad^6x^3 + 30bd^4x^4 - 3ad^5x^2 - 120bd^3x^3 + 6ad^4x + 360bd^2x^2 - 6ad^3 - 720bdx + 720b)e^{(dx+c)}}{2d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(b*x^3+a)*cosh(d*x+c),x, algorithm="giac")

[Out] $1/2*(b*d^6*x^6 - 6*b*d^5*x^5 + a*d^6*x^3 + 30*b*d^4*x^4 - 3*a*d^5*x^2 - 120*b*d^3*x^3 + 6*a*d^4*x + 360*b*d^2*x^2 - 6*a*d^3 - 720*b*d*x + 720*b)*e^{(d*x + c)}/d^7 - 1/2*(b*d^6*x^6 + 6*b*d^5*x^5 + a*d^6*x^3 + 30*b*d^4*x^4 + 3*a*d^5*x^2 + 120*b*d^3*x^3 + 6*a*d^4*x + 360*b*d^2*x^2 + 6*a*d^3 + 720*b*d*x + 720*b)*e^{(-d*x - c)}/d^7$

maple [B] time = 0.04, size = 551, normalized size = 3.58

$$\frac{b((dx+c)^6 \sinh(dx+c) - 6(dx+c)^5 \cosh(dx+c) + 30(dx+c)^4 \sinh(dx+c) - 120(dx+c)^3 \cosh(dx+c) + 360(dx+c)^2 \sinh(dx+c) - 720(dx+c) \cosh(dx+c) + 720 \sinh(dx+c))}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(b*x^3+a)*cosh(d*x+c),x)

[Out] $1/d^4*(1/d^3*b*((d*x+c)^6*\sinh(d*x+c)-6*(d*x+c)^5*\cosh(d*x+c)+30*(d*x+c)^4*\sinh(d*x+c)-120*(d*x+c)^3*\cosh(d*x+c)+360*(d*x+c)^2*\sinh(d*x+c)-720*(d*x+c)*\cosh(d*x+c)+720*\sinh(d*x+c))-6/d^3*b*c*((d*x+c)^5*\sinh(d*x+c)-5*(d*x+c)^4*\cosh(d*x+c)+20*(d*x+c)^3*\sinh(d*x+c)-60*(d*x+c)^2*\cosh(d*x+c)+120*(d*x+c)*\sinh(d*x+c)-120*\cosh(d*x+c))+15/d^3*b*c^2*((d*x+c)^4*\sinh(d*x+c)-4*(d*x+c)^3*\cosh(d*x+c)+12*(d*x+c)^2*\sinh(d*x+c)-24*(d*x+c)*\cosh(d*x+c)+24*\sinh(d*x+c))-20/d^3*b*c^3*((d*x+c)^3*\sinh(d*x+c)-3*(d*x+c)^2*\cosh(d*x+c)+6*(d*x+c)*\sinh(d*x+c)-6*\cosh(d*x+c))+15/d^3*b*c^4*((d*x+c)^2*\sinh(d*x+c)-2*(d*x+c)*\cosh(d*x+c)+2*\sinh(d*x+c))-6/d^3*b*c^5*((d*x+c)*\sinh(d*x+c)-\cosh(d*x+c))+1/d^3*b*c^6*\sinh(d*x+c)+a*((d*x+c)^3*\sinh(d*x+c)-3*(d*x+c)^2*\cosh(d*x+c)+6*(d*x+c)*\sinh(d*x+c)-6*\cosh(d*x+c))-3*a*c*((d*x+c)^2*\sinh(d*x+c)-2*(d*x+c)*\cosh(d*x+c)+2*\sinh(d*x+c))+3*a*c^2*((d*x+c)*\sinh(d*x+c)-\cosh(d*x+c))-a*c^3*\sinh(d*x+c))$

maxima [A] time = 1.09, size = 268, normalized size = 1.74

$$-\frac{1}{56}d \left(\frac{7(d^4x^4e^c - 4d^3x^3e^c + 12d^2x^2e^c - 24dxe^c + 24e^c)ae^{(dx)}}{d^5} + \frac{7(d^4x^4 + 4d^3x^3 + 12d^2x^2 + 24dx + 24)ae^{(-d}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(b*x^3+a)*cosh(d*x+c),x, algorithm="maxima")

[Out] $-1/56*d*(7*(d^4*x^4*e^c - 4*d^3*x^3*e^c + 12*d^2*x^2*e^c - 24*d*x*e^c + 24*e^c)*a*e^{(d*x)}/d^5 + 7*(d^4*x^4 + 4*d^3*x^3 + 12*d^2*x^2 + 24*d*x + 24)*a*e^{(-d*x - c)}/d^5 + 4*(d^7*x^7*e^c - 7*d^6*x^6*e^c + 42*d^5*x^5*e^c - 210*d^4*x^4*e^c + 840*d^3*x^3*e^c - 2520*d^2*x^2*e^c + 5040*d*x*e^c - 5040*e^c)*b*e^{(d*x)}/d^8 + 4*(d^7*x^7 + 7*d^6*x^6 + 42*d^5*x^5 + 210*d^4*x^4 + 840*d^3*x^3 + 2520*d^2*x^2 + 5040*d*x + 5040)*b*e^{(-d*x - c)}/d^8) + 1/28*(4*b*x^7 + 7*a*x^4)*cosh(d*x + c)$

mupad [B] time = 1.05, size = 152, normalized size = 0.99

$$\frac{30bx^4 \sinh(c+dx) + 6ax \sinh(c+dx)}{d^3} - \frac{3ax^2 \cosh(c+dx) + 6bx^5 \cosh(c+dx)}{d^2} + \frac{ax^3 \sinh(c+dx) + b}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*cosh(c+d*x)*(a+b*x^3),x)

[Out] $(30*b*x^4*\sinh(c+d*x) + 6*a*x*\sinh(c+d*x))/d^3 - (3*a*x^2*\cosh(c+d*x) + 6*b*x^5*\cosh(c+d*x))/d^2 + (a*x^3*\sinh(c+d*x) + b*x^6*\sinh(c+d*x))/d - (6*a*\cosh(c+d*x) + 120*b*x^3*\cosh(c+d*x))/d^4 + (720*b*\sinh(c+d*x))/d^7 - (720*b*x*\cosh(c+d*x))/d^6 + (360*b*x^2*\sinh(c+d*x))/d^5$

sympy [A] time = 5.71, size = 185, normalized size = 1.20

$$\left\{ \begin{array}{l} \frac{ax^3 \sinh(c+dx)}{d} - \frac{3ax^2 \cosh(c+dx)}{d^2} + \frac{6ax \sinh(c+dx)}{d^3} - \frac{6a \cosh(c+dx)}{d^4} + \frac{bx^6 \sinh(c+dx)}{d} - \frac{6bx^5 \cosh(c+dx)}{d^2} + \frac{30bx^4 \sinh(c+dx)}{d^3} - \frac{120bx^3 \cosh(c+dx)}{d^4} \\ \left(\frac{ax^4}{4} + \frac{bx^7}{7} \right) \cosh(c) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*(b*x**3+a)*cosh(d*x+c),x)

[Out] Piecewise((a*x**3*sinh(c+d*x)/d - 3*a*x**2*cosh(c+d*x)/d**2 + 6*a*x*sinh(c+d*x)/d**3 - 6*a*cosh(c+d*x)/d**4 + b*x**6*sinh(c+d*x)/d - 6*b*x**5*cosh(c+d*x)/d**2 + 30*b*x**4*sinh(c+d*x)/d**3 - 120*b*x**3*cosh(c+d*x)/d**4 + 360*b*x**2*sinh(c+d*x)/d**5 - 720*b*x*cosh(c+d*x)/d**6 + 720*b*sinh(c+d*x)/d**7, Ne(d, 0)), ((a*x**4/4 + b*x**7/7)*cosh(c), True))

3.80 $\int x^2 (a + bx^3) \cosh(c + dx) dx$

Optimal. Leaf size=124

$$\frac{2a \sinh(c + dx)}{d^3} - \frac{2ax \cosh(c + dx)}{d^2} + \frac{ax^2 \sinh(c + dx)}{d} - \frac{120b \cosh(c + dx)}{d^6} + \frac{120bx \sinh(c + dx)}{d^5} - \frac{60bx^2 \cosh(c + dx)}{d^4}$$

[Out] $-120*b*\cosh(d*x+c)/d^6-2*a*x*\cosh(d*x+c)/d^2-60*b*x^2*\cosh(d*x+c)/d^4-5*b*x^4*\cosh(d*x+c)/d^2+2*a*\sinh(d*x+c)/d^3+120*b*x*\sinh(d*x+c)/d^5+a*x^2*\sinh(d*x+c)/d+20*b*x^3*\sinh(d*x+c)/d^3+b*x^5*\sinh(d*x+c)/d$

Rubi [A] time = 0.23, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {5287, 3296, 2637, 2638}

$$\frac{2a \sinh(c + dx)}{d^3} - \frac{2ax \cosh(c + dx)}{d^2} + \frac{ax^2 \sinh(c + dx)}{d} + \frac{20bx^3 \sinh(c + dx)}{d^3} - \frac{5bx^4 \cosh(c + dx)}{d^2} - \frac{60bx^2 \cosh(c + dx)}{d^4}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^2*(a + b*x^3)*\text{Cosh}[c + d*x], x]$

[Out] $(-120*b*\text{Cosh}[c + d*x])/d^6 - (2*a*x*\text{Cosh}[c + d*x])/d^2 - (60*b*x^2*\text{Cosh}[c + d*x])/d^4 - (5*b*x^4*\text{Cosh}[c + d*x])/d^2 + (2*a*\text{Sinh}[c + d*x])/d^3 + (120*b*x*\text{Sinh}[c + d*x])/d^5 + (a*x^2*\text{Sinh}[c + d*x])/d + (20*b*x^3*\text{Sinh}[c + d*x])/d^3 + (b*x^5*\text{Sinh}[c + d*x])/d$

Rule 2637

$\text{Int}[\sin[\text{Pi}/2 + (c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow \text{Simp}[\sin[c + d*x]/d, x] /;$
FreeQ[{c, d}, x]

Rule 2638

$\text{Int}[\sin[(c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow -\text{Simp}[\cos[c + d*x]/d, x] /;$ FreeQ[{c, d}, x]

Rule 3296

$\text{Int}[((c_.) + (d_.)*(x_.))^{(m_.)}*\sin[(e_.) + (f_.)*(x_.)], x_Symbol] \rightarrow -\text{Simp}[(c + d*x)^m*\cos[e + f*x]/f, x] + \text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{(m-1)}*\cos[e + f*x], x], x] /;$ FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 5287

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
 \int x^2 (a + bx^3) \cosh(c + dx) dx &= \int (ax^2 \cosh(c + dx) + bx^5 \cosh(c + dx)) dx \\
 &= a \int x^2 \cosh(c + dx) dx + b \int x^5 \cosh(c + dx) dx \\
 &= \frac{ax^2 \sinh(c + dx)}{d} + \frac{bx^5 \sinh(c + dx)}{d} - \frac{(2a) \int x \sinh(c + dx) dx}{d} - \frac{(5b) \int x^4 \sinh(c + dx) dx}{d} \\
 &= -\frac{2ax \cosh(c + dx)}{d^2} - \frac{5bx^4 \cosh(c + dx)}{d^2} + \frac{ax^2 \sinh(c + dx)}{d} + \frac{bx^5 \sinh(c + dx)}{d} \\
 &= -\frac{2ax \cosh(c + dx)}{d^2} - \frac{5bx^4 \cosh(c + dx)}{d^2} + \frac{2a \sinh(c + dx)}{d^3} + \frac{ax^2 \sinh(c + dx)}{d} \\
 &= -\frac{2ax \cosh(c + dx)}{d^2} - \frac{60bx^2 \cosh(c + dx)}{d^4} - \frac{5bx^4 \cosh(c + dx)}{d^2} + \frac{2a \sinh(c + dx)}{d^3} \\
 &= -\frac{2ax \cosh(c + dx)}{d^2} - \frac{60bx^2 \cosh(c + dx)}{d^4} - \frac{5bx^4 \cosh(c + dx)}{d^2} + \frac{2a \sinh(c + dx)}{d^3} \\
 &= -\frac{120b \cosh(c + dx)}{d^6} - \frac{2ax \cosh(c + dx)}{d^2} - \frac{60bx^2 \cosh(c + dx)}{d^4} - \frac{5bx^4 \cosh(c + dx)}{d^2} + \frac{2a \sinh(c + dx)}{d^3}
 \end{aligned}$$

Mathematica [A] time = 0.15, size = 84, normalized size = 0.68

$$\frac{d(ad^2(d^2x^2 + 2) + bx(d^4x^4 + 20d^2x^2 + 120)) \sinh(c + dx) - (2ad^4x + 5b(d^4x^4 + 12d^2x^2 + 24)) \cosh(c + dx)}{d^6}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*(a + b*x^3)*Cosh[c + d*x], x]

[Out] (-(2*a*d^4*x + 5*b*(24 + 12*d^2*x^2 + d^4*x^4))*Cosh[c + d*x]) + d*(a*d^2*(2 + d^2*x^2) + b*x*(120 + 20*d^2*x^2 + d^4*x^4))*Sinh[c + d*x])/d^6

fricas [A] time = 0.51, size = 87, normalized size = 0.70

$$\frac{(5bd^4x^4 + 2ad^4x + 60bd^2x^2 + 120b) \cosh(dx + c) - (bd^5x^5 + ad^5x^2 + 20bd^3x^3 + 2ad^3 + 120bdx) \sinh(dx + c)}{d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(b*x^3+a)*cosh(d*x+c),x, algorithm="fricas")

[Out] $-\left(\left(5*b*d^4*x^4 + 2*a*d^4*x + 60*b*d^2*x^2 + 120*b\right)*\cosh(d*x + c) - \left(b*d^5*x^5 + a*d^5*x^2 + 20*b*d^3*x^3 + 2*a*d^3 + 120*b*d*x\right)*\sinh(d*x + c)\right)/d^6$

giac [A] time = 0.12, size = 156, normalized size = 1.26

$$\frac{(bd^5x^5 - 5bd^4x^4 + ad^5x^2 + 20bd^3x^3 - 2ad^4x - 60bd^2x^2 + 2ad^3 + 120bdx - 120b)e^{(dx+c)} - (bd^5x^5 + 5bd^4x^4 + a$$

$$2d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(b*x^3+a)*cosh(d*x+c),x, algorithm="giac")

[Out] $1/2*(b*d^5*x^5 - 5*b*d^4*x^4 + a*d^5*x^2 + 20*b*d^3*x^3 - 2*a*d^4*x - 60*b*d^2*x^2 + 2*a*d^3 + 120*b*d*x - 120*b)*e^{(d*x + c)}/d^6 - 1/2*(b*d^5*x^5 + 5*b*d^4*x^4 + a*d^5*x^2 + 20*b*d^3*x^3 + 2*a*d^4*x + 60*b*d^2*x^2 + 2*a*d^3 + 120*b*d*x + 120*b)*e^{(-d*x - c)}/d^6$

maple [B] time = 0.05, size = 389, normalized size = 3.14

$$\frac{b((dx+c)^5 \sinh(dx+c) - 5(dx+c)^4 \cosh(dx+c) + 20(dx+c)^3 \sinh(dx+c) - 60(dx+c)^2 \cosh(dx+c) + 120(dx+c) \sinh(dx+c) - 120 \cosh(dx+c)) - 5bc(dx+c)^4 s$$

$$d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(b*x^3+a)*cosh(d*x+c),x)

[Out] $1/d^3*(1/d^3*b*((d*x+c)^5*\sinh(d*x+c)-5*(d*x+c)^4*\cosh(d*x+c)+20*(d*x+c)^3*\sinh(d*x+c)-60*(d*x+c)^2*\cosh(d*x+c)+120*(d*x+c)*\sinh(d*x+c)-120*\cosh(d*x+c))-5/d^3*b*c*((d*x+c)^4*\sinh(d*x+c)-4*(d*x+c)^3*\cosh(d*x+c)+12*(d*x+c)^2*\sinh(d*x+c)-24*(d*x+c)*\cosh(d*x+c)+24*\sinh(d*x+c))+10/d^3*b*c^2*((d*x+c)^3*\sinh(d*x+c)-3*(d*x+c)^2*\cosh(d*x+c)+6*(d*x+c)*\sinh(d*x+c)-6*\cosh(d*x+c))-10/d^3*b*c^3*((d*x+c)^2*\sinh(d*x+c)-2*(d*x+c)*\cosh(d*x+c)+2*\sinh(d*x+c))+5/d^3*b*c^4*((d*x+c)*\sinh(d*x+c)-\cosh(d*x+c))-1/d^3*b*c^5*\sinh(d*x+c)+a*((d*x+c)^2*\sinh(d*x+c)-2*(d*x+c)*\cosh(d*x+c)+2*\sinh(d*x+c))-2*a*c*((d*x+c)*\sinh(d*x+c)-\cosh(d*x+c))+a*c^2*\sinh(d*x+c))$

maxima [B] time = 0.34, size = 267, normalized size = 2.15

$$\frac{(bx^3 + a)^2 \cosh(dx + c)}{6b} - \left(\frac{a^2 e^{(dx+c)}}{d} + \frac{a^2 e^{(-dx-c)}}{d} + \frac{2(d^3 x^3 e^c - 3d^2 x^2 e^c + 6dx e^c - 6e^c) a b e^{(dx)}}{d^4} + \frac{2(d^3 x^3 + 3d^2 x^2 + 6dx + 6) a b e^{(-dx-c)}}{d^4} + \frac{(d^6 x^5 + 5bd^4x^4 + a$$

$$2d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(b*x^3+a)*cosh(d*x+c),x, algorithm="maxima")

[Out] $\frac{1}{6}(bx^3 + a)^2 \cosh(dx + c)/b - \frac{1}{12}(a^2 e^{(dx + c)}/d + a^2 e^{-(dx - c)}/d + 2(d^3 x^3 e^c - 3d^2 x^2 e^c + 6d x e^c - 6e^c) a b e^{(dx)}/d^4 + 2(d^3 x^3 + 3d^2 x^2 + 6d x + 6) a b e^{-(dx - c)}/d^4 + (d^6 x^6 e^c - 6d^5 x^5 e^c + 30d^4 x^4 e^c - 120d^3 x^3 e^c + 360d^2 x^2 e^c - 720d x e^c + 720e^c) b^2 e^{(dx)}/d^7 + (d^6 x^6 + 6d^5 x^5 + 30d^4 x^4 + 120d^3 x^3 + 360d^2 x^2 + 720d x + 720) b^2 e^{-(dx - c)}/d^7) d/b$

mupad [B] time = 0.96, size = 122, normalized size = 0.98

$$\frac{ax^2 \sinh(c + dx) + bx^5 \sinh(c + dx)}{d} - \frac{2ax \cosh(c + dx) + 5bx^4 \cosh(c + dx)}{d^2} + \frac{2a \sinh(c + dx) + 20bx^3 \sinh(c + dx)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*cosh(c + d*x)*(a + b*x^3),x)`

[Out] $(ax^2 \sinh(c + dx) + bx^5 \sinh(c + dx))/d - (2ax \cosh(c + dx) + 5bx^4 \cosh(c + dx))/d^2 + (2a \sinh(c + dx) + 20bx^3 \sinh(c + dx))/d^3 - (120b \cosh(c + dx))/d^6 + (120bx \sinh(c + dx))/d^5 - (60bx^2 \cosh(c + dx))/d^4$

sympy [A] time = 3.25, size = 151, normalized size = 1.22

$$\left\{ \begin{array}{l} \frac{ax^2 \sinh(c+dx)}{d} - \frac{2ax \cosh(c+dx)}{d^2} + \frac{2a \sinh(c+dx)}{d^3} + \frac{bx^5 \sinh(c+dx)}{d} - \frac{5bx^4 \cosh(c+dx)}{d^2} + \frac{20bx^3 \sinh(c+dx)}{d^3} - \frac{60bx^2 \cosh(c+dx)}{d^4} + \frac{120bx \sinh(c+dx)}{d^5} - \frac{120b \cosh(c+dx)}{d^6} \\ \left(\frac{ax^3}{3} + \frac{bx^6}{6} \right) \cosh(c) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(b*x**3+a)*cosh(d*x+c),x)`

[Out] `Piecewise((a*x**2*sinh(c + d*x)/d - 2*a*x*cosh(c + d*x)/d**2 + 2*a*sinh(c + d*x)/d**3 + b*x**5*sinh(c + d*x)/d - 5*b*x**4*cosh(c + d*x)/d**2 + 20*b*x**3*sinh(c + d*x)/d**3 - 60*b*x**2*cosh(c + d*x)/d**4 + 120*b*x*sinh(c + d*x)/d**5 - 120*b*cosh(c + d*x)/d**6, Ne(d, 0)), ((a*x**3/3 + b*x**6/6)*cosh(c), True))`

3.81 $\int x (a + bx^3) \cosh(c + dx) dx$

Optimal. Leaf size=94

$$-\frac{a \cosh(c + dx)}{d^2} + \frac{ax \sinh(c + dx)}{d} + \frac{24b \sinh(c + dx)}{d^5} - \frac{24bx \cosh(c + dx)}{d^4} + \frac{12bx^2 \sinh(c + dx)}{d^3} - \frac{4bx^3 \cosh(c + dx)}{d^2}$$

[Out] $-a*\cosh(d*x+c)/d^2-24*b*x*\cosh(d*x+c)/d^4-4*b*x^3*\cosh(d*x+c)/d^2+24*b*\sinh(d*x+c)/d^5+a*x*\sinh(d*x+c)/d+12*b*x^2*\sinh(d*x+c)/d^3+b*x^4*\sinh(d*x+c)/d$

Rubi [A] time = 0.16, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {5287, 3296, 2638, 2637}

$$-\frac{a \cosh(c + dx)}{d^2} + \frac{ax \sinh(c + dx)}{d} + \frac{12bx^2 \sinh(c + dx)}{d^3} - \frac{4bx^3 \cosh(c + dx)}{d^2} + \frac{24b \sinh(c + dx)}{d^5} - \frac{24bx \cosh(c + dx)}{d^4}$$

Antiderivative was successfully verified.

[In] Int[x*(a + b*x^3)*Cosh[c + d*x], x]

[Out] $-((a*\cosh[c + d*x])/d^2) - (24*b*x*\cosh[c + d*x])/d^4 - (4*b*x^3*\cosh[c + d*x])/d^2 + (24*b*\sinh[c + d*x])/d^5 + (a*x*\sinh[c + d*x])/d + (12*b*x^2*\sinh[c + d*x])/d^3 + (b*x^4*\sinh[c + d*x])/d$

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 2638

Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3296

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[((c + d*x)^m*cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m-1)*cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 5287

Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] := Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,

`x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]`

Rubi steps

$$\begin{aligned}
 \int x(a + bx^3) \cosh(c + dx) dx &= \int (ax \cosh(c + dx) + bx^4 \cosh(c + dx)) dx \\
 &= a \int x \cosh(c + dx) dx + b \int x^4 \cosh(c + dx) dx \\
 &= \frac{ax \sinh(c + dx)}{d} + \frac{bx^4 \sinh(c + dx)}{d} - \frac{a \int \sinh(c + dx) dx}{d} - \frac{(4b) \int x^3 \sinh(c + dx) dx}{d} \\
 &= -\frac{a \cosh(c + dx)}{d^2} - \frac{4bx^3 \cosh(c + dx)}{d^2} + \frac{ax \sinh(c + dx)}{d} + \frac{bx^4 \sinh(c + dx)}{d} + \dots \\
 &= -\frac{a \cosh(c + dx)}{d^2} - \frac{4bx^3 \cosh(c + dx)}{d^2} + \frac{ax \sinh(c + dx)}{d} + \frac{12bx^2 \sinh(c + dx)}{d^3} \\
 &= -\frac{a \cosh(c + dx)}{d^2} - \frac{24bx \cosh(c + dx)}{d^4} - \frac{4bx^3 \cosh(c + dx)}{d^2} + \frac{ax \sinh(c + dx)}{d} \\
 &= -\frac{a \cosh(c + dx)}{d^2} - \frac{24bx \cosh(c + dx)}{d^4} - \frac{4bx^3 \cosh(c + dx)}{d^2} + \frac{24b \sinh(c + dx)}{d^5}
 \end{aligned}$$

Mathematica [A] time = 0.12, size = 66, normalized size = 0.70

$$\frac{(ad^4x + b(d^4x^4 + 12d^2x^2 + 24)) \sinh(c + dx) - d(ad^2 + 4bx(d^2x^2 + 6)) \cosh(c + dx)}{d^5}$$

Antiderivative was successfully verified.

[In] Integrate[x*(a + b*x^3)*Cosh[c + d*x], x]

[Out] $-(d*(a*d^2 + 4*b*x*(6 + d^2*x^2))*Cosh[c + d*x]) + (a*d^4*x + b*(24 + 12*d^2*x^2 + d^4*x^4))*Sinh[c + d*x])/d^5$

fricas [A] time = 0.60, size = 68, normalized size = 0.72

$$\frac{(4bd^3x^3 + ad^3 + 24bdx) \cosh(dx + c) - (bd^4x^4 + ad^4x + 12bd^2x^2 + 24b) \sinh(dx + c)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x^3+a)*cosh(d*x+c), x, algorithm="fricas")

[Out] $-((4*b*d^3*x^3 + a*d^3 + 24*b*d*x)*cosh(d*x + c) - (b*d^4*x^4 + a*d^4*x + 12*b*d^2*x^2 + 24*b)*sinh(d*x + c))/d^5$

giac [A] time = 0.13, size = 119, normalized size = 1.27

$$\frac{(bd^4x^4 - 4bd^3x^3 + ad^4x + 12bd^2x^2 - ad^3 - 24bdx + 24b)e^{(dx+c)}}{2d^5} - \frac{(bd^4x^4 + 4bd^3x^3 + ad^4x + 12bd^2x^2 + ad^3 + 24bdx + 24b)e^{-(dx+c)}}{2d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x^3+a)*cosh(d*x+c),x, algorithm="giac")

[Out] $\frac{1}{2} \frac{(bd^4x^4 - 4bd^3x^3 + ad^4x + 12bd^2x^2 - ad^3 - 24bdx + 24b)e^{(dx+c)}}{d^5} - \frac{1}{2} \frac{(bd^4x^4 + 4bd^3x^3 + ad^4x + 12bd^2x^2 + ad^3 + 24bdx + 24b)e^{-(dx+c)}}{d^5}$

maple [B] time = 0.04, size = 257, normalized size = 2.73

$$\frac{b((dx+c)^4 \sinh(dx+c) - 4(dx+c)^3 \cosh(dx+c) + 12(dx+c)^2 \sinh(dx+c) - 24(dx+c) \cosh(dx+c) + 24 \sinh(dx+c))}{d^3} - \frac{4bc((dx+c)^3 \sinh(dx+c) - 3(dx+c)^2 \cosh(dx+c) + 6(dx+c) \sinh(dx+c) - 6 \cosh(dx+c))}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(b*x^3+a)*cosh(d*x+c),x)

[Out] $\frac{1}{d^2} \left(\frac{1}{d^3} b \left((dx+c)^4 \sinh(dx+c) - 4(dx+c)^3 \cosh(dx+c) + 12(dx+c)^2 \sinh(dx+c) - 24(dx+c) \cosh(dx+c) + 24 \sinh(dx+c) \right) - \frac{4}{d^3} b c \left((dx+c)^3 \sinh(dx+c) - 3(dx+c)^2 \cosh(dx+c) + 6(dx+c) \sinh(dx+c) - 6 \cosh(dx+c) \right) + \frac{6}{d^3} b c^2 \left((dx+c)^2 \sinh(dx+c) - 2(dx+c) \cosh(dx+c) + 2 \sinh(dx+c) \right) - \frac{4}{d^3} b c^3 \left((dx+c) \sinh(dx+c) - \cosh(dx+c) \right) + a \left((dx+c) \sinh(dx+c) - \cosh(dx+c) \right) + \frac{1}{d^3} b c^4 \sinh(dx+c) - a c \sinh(dx+c) \right)$

maxima [B] time = 0.35, size = 196, normalized size = 2.09

$$-\frac{1}{20} d \left(\frac{5(d^2x^2e^c - 2dxe^c + 2e^c)ae^{(dx)}}{d^3} + \frac{5(d^2x^2 + 2dx + 2)ae^{-(dx-c)}}{d^3} + \frac{2(d^5x^5e^c - 5d^4x^4e^c + 20d^3x^3e^c - 60d^2x^2e^c - 60d^2x^2e^{-c} + 120d^2xe^{-c} - 120e^{-c})b^2e^{(dx)}}{d^6} + \frac{2(d^5x^5e^{-c} - 5d^4x^4e^{-c} + 20d^3x^3e^{-c} - 60d^2x^2e^{-c} + 120d^2xe^{-c} + 120)e^{-c}b^2e^{-(dx-c)}}{d^6} + \frac{1}{10} (2bx^5 + 5ax^2) \cosh(dx+c) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x^3+a)*cosh(d*x+c),x, algorithm="maxima")

[Out] $-\frac{1}{20} d \left(\frac{5(d^2x^2e^c - 2dxe^c + 2e^c)ae^{(dx)}}{d^3} + \frac{5(d^2x^2 + 2dx + 2)ae^{-(dx-c)}}{d^3} + \frac{2(d^5x^5e^c - 5d^4x^4e^c + 20d^3x^3e^c - 60d^2x^2e^c + 120d^2xe^c - 120e^c)b^2e^{(dx)}}{d^6} + \frac{2(d^5x^5e^{-c} - 5d^4x^4e^{-c} + 20d^3x^3e^{-c} - 60d^2x^2e^{-c} + 120d^2xe^{-c} + 120)e^{-c}b^2e^{-(dx-c)}}{d^6} + \frac{1}{10} (2bx^5 + 5ax^2) \cosh(dx+c) \right)$

mupad [B] time = 0.92, size = 92, normalized size = 0.98

$$\frac{bx^4 \sinh(c+dx) + ax \sinh(c+dx)}{d} - \frac{a \cosh(c+dx) + 4bx^3 \cosh(c+dx)}{d^2} + \frac{24b \sinh(c+dx)}{d^5} - \frac{24bx \cosh(c+dx)}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*cosh(c + d*x)*(a + b*x^3),x)`

[Out] $(b*x^4*\sinh(c + d*x) + a*x*\sinh(c + d*x))/d - (a*\cosh(c + d*x) + 4*b*x^3*\cosh(c + d*x))/d^2 + (24*b*\sinh(c + d*x))/d^5 - (24*b*x*\cosh(c + d*x))/d^4 + (12*b*x^2*\sinh(c + d*x))/d^3$

sympy [A] time = 1.92, size = 116, normalized size = 1.23

$$\left\{ \begin{array}{l} \frac{ax \sinh(c+dx)}{d} - \frac{a \cosh(c+dx)}{d^2} + \frac{bx^4 \sinh(c+dx)}{d} - \frac{4bx^3 \cosh(c+dx)}{d^2} + \frac{12bx^2 \sinh(c+dx)}{d^3} - \frac{24bx \cosh(c+dx)}{d^4} + \frac{24b \sinh(c+dx)}{d^5} \\ \left(\frac{ax^2}{2} + \frac{bx^5}{5} \right) \cosh(c) \end{array} \right.$$

for d

othe

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(b*x**3+a)*cosh(d*x+c),x)`

[Out] `Piecewise((a*x*sinh(c + d*x)/d - a*cosh(c + d*x)/d**2 + b*x**4*sinh(c + d*x)/d - 4*b*x**3*cosh(c + d*x)/d**2 + 12*b*x**2*sinh(c + d*x)/d**3 - 24*b*x*cosh(c + d*x)/d**4 + 24*b*sinh(c + d*x)/d**5, Ne(d, 0)), ((a*x**2/2 + b*x**5/5)*cosh(c), True))`

3.82 $\int (a + bx^3) \cosh(c + dx) dx$

Optimal. Leaf size=66

$$\frac{a \sinh(c + dx)}{d} - \frac{6b \cosh(c + dx)}{d^4} + \frac{6bx \sinh(c + dx)}{d^3} - \frac{3bx^2 \cosh(c + dx)}{d^2} + \frac{bx^3 \sinh(c + dx)}{d}$$

[Out] $-6*b*cosh(d*x+c)/d^4-3*b*x^2*cosh(d*x+c)/d^2+a*sinh(d*x+c)/d+6*b*x*sinh(d*x+c)/d^3+b*x^3*sinh(d*x+c)/d$

Rubi [A] time = 0.10, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {5277, 2637, 3296, 2638}

$$\frac{a \sinh(c + dx)}{d} - \frac{3bx^2 \cosh(c + dx)}{d^2} + \frac{6bx \sinh(c + dx)}{d^3} - \frac{6b \cosh(c + dx)}{d^4} + \frac{bx^3 \sinh(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] `Int[(a + b*x^3)*Cosh[c + d*x], x]`

[Out] $(-6*b*Cosh[c + d*x])/d^4 - (3*b*x^2*Cosh[c + d*x])/d^2 + (a*Sinh[c + d*x])/d + (6*b*x*Sinh[c + d*x])/d^3 + (b*x^3*Sinh[c + d*x])/d$

Rule 2637

`Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;`
`FreeQ[{c, d}, x]`

Rule 2638

`Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /;`
`FreeQ[{c, d}, x]`

Rule 3296

`Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[`
`((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[`
`e + f*x], x], x] /;`
`FreeQ[{c, d, e, f}, x] && GtQ[m, 0]`

Rule 5277

`Int[Cosh[(c_.) + (d_.)*(x_)]*((a_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] := I`
`nt[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /;`
`FreeQ[{a, b, c,`
`d, n}, x] && IGtQ[p, 0]`

Rubi steps

$$\begin{aligned}
\int (a + bx^3) \cosh(c + dx) dx &= \int (a \cosh(c + dx) + bx^3 \cosh(c + dx)) dx \\
&= a \int \cosh(c + dx) dx + b \int x^3 \cosh(c + dx) dx \\
&= \frac{a \sinh(c + dx)}{d} + \frac{bx^3 \sinh(c + dx)}{d} - \frac{(3b) \int x^2 \sinh(c + dx) dx}{d} \\
&= -\frac{3bx^2 \cosh(c + dx)}{d^2} + \frac{a \sinh(c + dx)}{d} + \frac{bx^3 \sinh(c + dx)}{d} + \frac{(6b) \int x \cosh(c + dx) dx}{d^2} \\
&= -\frac{3bx^2 \cosh(c + dx)}{d^2} + \frac{a \sinh(c + dx)}{d} + \frac{6bx \sinh(c + dx)}{d^3} + \frac{bx^3 \sinh(c + dx)}{d} \\
&= -\frac{6b \cosh(c + dx)}{d^4} - \frac{3bx^2 \cosh(c + dx)}{d^2} + \frac{a \sinh(c + dx)}{d} + \frac{6bx \sinh(c + dx)}{d^3} + \frac{bx^3 \sinh(c + dx)}{d}
\end{aligned}$$

Mathematica [A] time = 0.10, size = 49, normalized size = 0.74

$$\frac{d(ad^2 + bx(d^2x^2 + 6)) \sinh(c + dx) - 3b(d^2x^2 + 2) \cosh(c + dx)}{d^4}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^3)*Cosh[c + d*x], x]

[Out] (-3*b*(2 + d^2*x^2)*Cosh[c + d*x] + d*(a*d^2 + b*x*(6 + d^2*x^2))*Sinh[c + d*x])/d^4

fricas [A] time = 0.57, size = 53, normalized size = 0.80

$$\frac{3(bd^2x^2 + 2b) \cosh(dx + c) - (bd^3x^3 + ad^3 + 6bdx) \sinh(dx + c)}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)*cosh(d*x+c), x, algorithm="fricas")

[Out] -(3*(b*d^2*x^2 + 2*b)*cosh(d*x + c) - (b*d^3*x^3 + a*d^3 + 6*b*d*x)*sinh(d*x + c))/d^4

giac [A] time = 0.12, size = 88, normalized size = 1.33

$$\frac{(bd^3x^3 - 3bd^2x^2 + ad^3 + 6bdx - 6b)e^{(dx+c)}}{2d^4} - \frac{(bd^3x^3 + 3bd^2x^2 + ad^3 + 6bdx + 6b)e^{(-dx-c)}}{2d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)*cosh(d*x+c),x, algorithm="giac")

[Out] $\frac{1}{2}*(b*d^3*x^3 - 3*b*d^2*x^2 + a*d^3 + 6*b*d*x - 6*b)*e^{(d*x + c)}/d^4 - \frac{1}{2}*(b*d^3*x^3 + 3*b*d^2*x^2 + a*d^3 + 6*b*d*x + 6*b)*e^{(-d*x - c)}/d^4$

maple [B] time = 0.04, size = 158, normalized size = 2.39

$$\frac{b((dx+c)^3 \sinh(dx+c) - 3(dx+c)^2 \cosh(dx+c) + 6(dx+c) \sinh(dx+c) - 6 \cosh(dx+c))}{d^3} - \frac{3bc((dx+c)^2 \sinh(dx+c) - 2(dx+c) \cosh(dx+c) + 2 \sinh(dx+c))}{d^3} + \frac{3bc}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)*cosh(d*x+c),x)

[Out] $\frac{1}{d}*(\frac{1}{d^3}*b*((d*x+c)^3*\sinh(d*x+c) - 3*(d*x+c)^2*\cosh(d*x+c) + 6*(d*x+c)*\sinh(d*x+c) - 6*\cosh(d*x+c)) - \frac{3}{d^3}*b*c*((d*x+c)^2*\sinh(d*x+c) - 2*(d*x+c)*\cosh(d*x+c) + 2*\sinh(d*x+c)) + \frac{3}{d^3}*b*c^2*((d*x+c)*\sinh(d*x+c) - \cosh(d*x+c)) - \frac{1}{d^3}*b*c^3*\sinh(d*x+c) + a*\sinh(d*x+c))$

maxima [A] time = 0.34, size = 104, normalized size = 1.58

$$\frac{ae^{(dx+c)}}{2d} - \frac{ae^{(-dx-c)}}{2d} + \frac{(d^3x^3e^c - 3d^2x^2e^c + 6dxe^c - 6e^c)be^{(dx)}}{2d^4} - \frac{(d^3x^3 + 3d^2x^2 + 6dx + 6)be^{(-dx-c)}}{2d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)*cosh(d*x+c),x, algorithm="maxima")

[Out] $\frac{1}{2}*a*e^{(d*x + c)}/d - \frac{1}{2}*a*e^{(-d*x - c)}/d + \frac{1}{2}*(d^3*x^3*e^c - 3*d^2*x^2*e^c + 6*d*x*e^c - 6*e^c)*b*e^{(d*x)}/d^4 - \frac{1}{2}*(d^3*x^3 + 3*d^2*x^2 + 6*d*x + 6)*b*e^{(-d*x - c)}/d^4$

mupad [B] time = 0.10, size = 65, normalized size = 0.98

$$\frac{a \sinh(c + dx) + b x^3 \sinh(c + dx)}{d} - \frac{6 b \cosh(c + dx)}{d^4} + \frac{6 b x \sinh(c + dx)}{d^3} - \frac{3 b x^2 \cosh(c + dx)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(c + d*x)*(a + b*x^3),x)

[Out] $(a*\sinh(c + d*x) + b*x^3*\sinh(c + d*x))/d - (6*b*\cosh(c + d*x))/d^4 + (6*b*x*\sinh(c + d*x))/d^3 - (3*b*x^2*\cosh(c + d*x))/d^2$

sympy [A] time = 0.94, size = 82, normalized size = 1.24

$$\begin{cases} \frac{a \sinh(c+dx)}{d} + \frac{bx^3 \sinh(c+dx)}{d} - \frac{3bx^2 \cosh(c+dx)}{d^2} + \frac{6bx \sinh(c+dx)}{d^3} - \frac{6b \cosh(c+dx)}{d^4} & \text{for } d \neq 0 \\ \left(ax + \frac{bx^4}{4}\right) \cosh(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a)*cosh(d*x+c),x)

[Out] Piecewise((a*sinh(c + d*x)/d + b*x**3*sinh(c + d*x)/d - 3*b*x**2*cosh(c + d*x)/d**2 + 6*b*x*sinh(c + d*x)/d**3 - 6*b*cosh(c + d*x)/d**4, Ne(d, 0)), ((a*x + b*x**4/4)*cosh(c), True))

$$3.83 \quad \int \frac{(a+bx^3) \cosh(c+dx)}{x} dx$$

Optimal. Leaf size=56

$$a \cosh(c) \operatorname{Chi}(dx) + a \sinh(c) \operatorname{Shi}(dx) + \frac{2b \sinh(c+dx)}{d^3} - \frac{2bx \cosh(c+dx)}{d^2} + \frac{bx^2 \sinh(c+dx)}{d}$$

[Out] a*Chi(d*x)*cosh(c)-2*b*x*cosh(d*x+c)/d^2+a*Shi(d*x)*sinh(c)+2*b*sinh(d*x+c)/d^3+b*x^2*sinh(d*x+c)/d

Rubi [A] time = 0.13, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {5287, 3303, 3298, 3301, 3296, 2637}

$$a \cosh(c) \operatorname{Chi}(dx) + a \sinh(c) \operatorname{Shi}(dx) + \frac{2b \sinh(c+dx)}{d^3} - \frac{2bx \cosh(c+dx)}{d^2} + \frac{bx^2 \sinh(c+dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[((a + b*x^3)*Cosh[c + d*x])/x,x]

[Out] (-2*b*x*Cosh[c + d*x])/d^2 + a*Cosh[c]*CoshIntegral[d*x] + (2*b*Sinh[c + d*x])/d^3 + (b*x^2*Sinh[c + d*x])/d + a*Sinh[c]*SinhIntegral[d*x]

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_.)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3296

Int[((c_.) + (d_.)*(x_.))^(m_.)*sin[(e_.) + (f_.)*(x_.)], x_Symbol] := -Simp[((c + d*x)^m*cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m-1)*cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 3298

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x_Symbol] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]

Rule 3301

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x_Symbol] := Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}

}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]

Rule 3303

Int[sin[(e_.) + (f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x_Symbol] :> Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]

Rule 5287

Int[Cosh[(c_.) + (d_.)*(x_.)]*((e_.)*(x_.))^(m_.)*((a_.) + (b_.)*(x_.)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{(a + bx^3) \cosh(c + dx)}{x} dx &= \int \left(\frac{a \cosh(c + dx)}{x} + bx^2 \cosh(c + dx) \right) dx \\
 &= a \int \frac{\cosh(c + dx)}{x} dx + b \int x^2 \cosh(c + dx) dx \\
 &= \frac{bx^2 \sinh(c + dx)}{d} - \frac{(2b) \int x \sinh(c + dx) dx}{d} + (a \cosh(c)) \int \frac{\cosh(dx)}{x} dx + (a \sinh(c)) \operatorname{Chi}(dx) \\
 &= -\frac{2bx \cosh(c + dx)}{d^2} + a \cosh(c) \operatorname{Chi}(dx) + \frac{bx^2 \sinh(c + dx)}{d} + a \sinh(c) \operatorname{Shi}(dx) \\
 &= -\frac{2bx \cosh(c + dx)}{d^2} + a \cosh(c) \operatorname{Chi}(dx) + \frac{2b \sinh(c + dx)}{d^3} + \frac{bx^2 \sinh(c + dx)}{d} + a \sinh(c) \operatorname{Shi}(dx)
 \end{aligned}$$

Mathematica [A] time = 0.18, size = 49, normalized size = 0.88

$$a \cosh(c) \operatorname{Chi}(dx) + a \sinh(c) \operatorname{Shi}(dx) + \frac{b \left((d^2 x^2 + 2) \sinh(c + dx) - 2dx \cosh(c + dx) \right)}{d^3}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x^3)*Cosh[c + d*x])/x,x]

[Out] a*Cosh[c]*CoshIntegral[d*x] + (b*(-2*d*x*Cosh[c + d*x] + (2 + d^2*x^2)*Sinh[c + d*x]))/d^3 + a*Sinh[c]*SinhIntegral[d*x]

fricas [A] time = 0.55, size = 86, normalized size = 1.54

$$\frac{4 b d x \cosh (d x+c)-\left(a d^3 \operatorname{Ei}(d x)+a d^3 \operatorname{Ei}(-d x)\right) \cosh (c)-2\left(b d^2 x^2+2 b\right) \sinh (d x+c)-\left(a d^3 \operatorname{Ei}(d x)-a d^3 \operatorname{Ei}(-d x)\right) \sinh (c)}{2 d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)*cosh(d*x+c)/x,x, algorithm="fricas")

[Out] -1/2*(4*b*d*x*cosh(d*x + c) - (a*d^3*Ei(d*x) + a*d^3*Ei(-d*x))*cosh(c) - 2*(b*d^2*x^2 + 2*b)*sinh(d*x + c) - (a*d^3*Ei(d*x) - a*d^3*Ei(-d*x))*sinh(c)) /d^3

giac [A] time = 0.15, size = 109, normalized size = 1.95

$$\frac{b d^2 x^2 e^{(d x+c)}-b d^2 x^2 e^{(-d x-c)}+a d^3 \operatorname{Ei}(-d x) e^{(-c)}+a d^3 \operatorname{Ei}(d x) e^c-2 b d x e^{(d x+c)}-2 b d x e^{(-d x-c)}+2 b e^{(d x+c)}-2 b e^{(-d x-c)}}{2 d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)*cosh(d*x+c)/x,x, algorithm="giac")

[Out] 1/2*(b*d^2*x^2*e^(d*x + c) - b*d^2*x^2*e^(-d*x - c) + a*d^3*Ei(-d*x)*e^(-c) + a*d^3*Ei(d*x)*e^c - 2*b*d*x*e^(d*x + c) - 2*b*d*x*e^(-d*x - c) + 2*b*e^(d*x + c) - 2*b*e^(-d*x - c))/d^3

maple [A] time = 0.12, size = 113, normalized size = 2.02

$$-\frac{b e^{d x+c} x}{d^2}+\frac{b e^{d x+c} x^2}{2 d}-\frac{b e^{-d x-c} x^2}{2 d}-\frac{b e^{-d x-c} x}{d^2}-\frac{a e^c \operatorname{Ei}(1,-d x)}{2}-\frac{b e^{-d x-c}}{d^3}-\frac{a e^{-c} \operatorname{Ei}(1, d x)}{2}+\frac{b e^{d x+c}}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)*cosh(d*x+c)/x,x)

[Out] -1/d^2*b*exp(d*x+c)*x+1/2/d*b*exp(d*x+c)*x^2-1/2/d*b*exp(-d*x-c)*x^2-1/d^2*b*exp(-d*x-c)*x-1/2*a*exp(c)*Ei(1,-d*x)-1/d^3*b*exp(-d*x-c)-1/2*a*exp(-c)*Ei(1,d*x)+1/d^3*b*exp(d*x+c)

maxima [B] time = 0.40, size = 140, normalized size = 2.50

$$-\frac{1}{6} \left(b \left(\frac{\left(d^3 x^3 e^c - 3 d^2 x^2 e^c + 6 d x e^c - 6 e^c \right) e^{(d x)}}{d^4} + \frac{\left(d^3 x^3 + 3 d^2 x^2 + 6 d x + 6 \right) e^{(-d x-c)}}{d^4} \right) + \frac{2 a \cosh (d x+c) \log \left(x^3 \right)}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)*cosh(d*x+c)/x,x, algorithm="maxima")

[Out] $-1/6*(b*((d^3*x^3*e^c - 3*d^2*x^2*e^c + 6*d*x*e^c - 6*e^c)*e^{(d*x)}/d^4 + (d^3*x^3 + 3*d^2*x^2 + 6*d*x + 6)*e^{(-d*x - c)}/d^4) + 2*a*\cosh(d*x + c)*\log(x^3)/d - 3*(\text{Ei}(-d*x)*e^{-c} + \text{Ei}(d*x)*e^c)*a/d)*d + 1/3*(b*x^3 + a*\log(x^3))*\cosh(d*x + c)$

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$a \operatorname{coshint}(dx) \cosh(c) + a \operatorname{sinhint}(dx) \sinh(c) + \frac{b \left(2 \sinh(c + dx) + d^2 x^2 \sinh(c + dx) - 2 dx \cosh(c + dx) \right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((cosh(c + d*x)*(a + b*x^3))/x,x)`

[Out] $a*\operatorname{coshint}(d*x)*\cosh(c) + a*\operatorname{sinhint}(d*x)*\sinh(c) + (b*(2*\sinh(c + d*x) + d^2*x^2*\sinh(c + d*x) - 2*d*x*\cosh(c + d*x)))/d^3$

sympy [A] time = 3.91, size = 66, normalized size = 1.18

$$a \sinh(c) \operatorname{Shi}(dx) + a \cosh(c) \operatorname{Chi}(dx) + b \left(\begin{array}{l} \frac{x^2 \sinh(c+dx)}{d} - \frac{2x \cosh(c+dx)}{d^2} + \frac{2 \sinh(c+dx)}{d^3} \quad \text{for } d \neq 0 \\ \frac{x^3 \cosh(c)}{3} \quad \text{otherwise} \end{array} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**3+a)*cosh(d*x+c)/x,x)`

[Out] $a*\sinh(c)*\operatorname{Shi}(d*x) + a*\cosh(c)*\operatorname{Chi}(d*x) + b*\operatorname{Piecewise}((x**2*\sinh(c + d*x)/d - 2*x*\cosh(c + d*x)/d**2 + 2*\sinh(c + d*x)/d**3, \text{Ne}(d, 0)), (x**3*\cosh(c)/3, \text{True}))$

$$3.84 \quad \int \frac{(a+bx^3) \cosh(c+dx)}{x^2} dx$$

Optimal. Leaf size=55

$$ad \sinh(c)\text{Chi}(dx) + ad \cosh(c)\text{Shi}(dx) - \frac{a \cosh(c+dx)}{x} - \frac{b \cosh(c+dx)}{d^2} + \frac{bx \sinh(c+dx)}{d}$$

[Out] $-b*\cosh(d*x+c)/d^2-a*\cosh(d*x+c)/x+a*d*\cosh(c)*\text{Shi}(d*x)+a*d*\text{Chi}(d*x)*\sinh(c)+b*x*\sinh(d*x+c)/d$

Rubi [A] time = 0.13, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {5287, 3297, 3303, 3298, 3301, 3296, 2638}

$$ad \sinh(c)\text{Chi}(dx) + ad \cosh(c)\text{Shi}(dx) - \frac{a \cosh(c+dx)}{x} - \frac{b \cosh(c+dx)}{d^2} + \frac{bx \sinh(c+dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[((a + b*x^3)*Cosh[c + d*x])/x^2,x]

[Out] $-((b*\text{Cosh}[c + d*x])/d^2) - (a*\text{Cosh}[c + d*x])/x + a*d*\text{CoshIntegral}[d*x]*\text{Sinh}[c] + (b*x*\text{Sinh}[c + d*x])/d + a*d*\text{Cosh}[c]*\text{SinhIntegral}[d*x]$

Rule 2638

Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3296

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[((c + d*x)^m*cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m-1)*cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 3297

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((c + d*x)^(m+1)*sin[e + f*x]/(d*(m+1)), x] - Dist[f/(d*(m+1)), Int[(c + d*x)^(m+1)*cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]

Rule 3298

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f}

, fz}, x] && EqQ[d*e - c*f*fz*I, 0]

Rule 3301

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]

Rule 3303

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]

Rule 5287

Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{(a + bx^3) \cosh(c + dx)}{x^2} dx &= \int \left(\frac{a \cosh(c + dx)}{x^2} + bx \cosh(c + dx) \right) dx \\
 &= a \int \frac{\cosh(c + dx)}{x^2} dx + b \int x \cosh(c + dx) dx \\
 &= -\frac{a \cosh(c + dx)}{x} + \frac{bx \sinh(c + dx)}{d} - \frac{b \int \sinh(c + dx) dx}{d} + (ad) \int \frac{\sinh(c + dx)}{x} dx \\
 &= -\frac{b \cosh(c + dx)}{d^2} - \frac{a \cosh(c + dx)}{x} + \frac{bx \sinh(c + dx)}{d} + (ad \cosh(c)) \int \frac{\sinh(dx)}{x} dx \\
 &= -\frac{b \cosh(c + dx)}{d^2} - \frac{a \cosh(c + dx)}{x} + ad \operatorname{Chi}(dx) \sinh(c) + \frac{bx \sinh(c + dx)}{d} + ad \operatorname{Shi}(dx)
 \end{aligned}$$

Mathematica [A] time = 0.14, size = 55, normalized size = 1.00

$$ad \sinh(c) \operatorname{Chi}(dx) + ad \cosh(c) \operatorname{Shi}(dx) - \frac{a \cosh(c + dx)}{x} - \frac{b \cosh(c + dx)}{d^2} + \frac{bx \sinh(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x^3)*Cosh[c + d*x])/x^2,x]

[Out] -((b*Cosh[c + d*x])/d^2) - (a*Cosh[c + d*x])/x + a*d*CoshIntegral[d*x]*Sinh[c] + (b*x*Sinh[c + d*x])/d + a*d*Cosh[c]*SinhIntegral[d*x]

fricas [A] time = 0.59, size = 90, normalized size = 1.64

$$\frac{2 b d x^2 \sinh (d x+c)-2\left(a d^2+b x\right) \cosh (d x+c)+\left(a d^3 x \operatorname{Ei}(d x)-a d^3 x \operatorname{Ei}(-d x)\right) \cosh (c)+\left(a d^3 x \operatorname{Ei}(d x)+a d^3 x \operatorname{Ei}(-d x)\right) \sinh (c)}{2 d^2 x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)*cosh(d*x+c)/x^2,x, algorithm="fricas")

[Out] 1/2*(2*b*d*x^2*sinh(d*x + c) - 2*(a*d^2 + b*x)*cosh(d*x + c) + (a*d^3*x*Ei(d*x) - a*d^3*x*Ei(-d*x))*cosh(c) + (a*d^3*x*Ei(d*x) + a*d^3*x*Ei(-d*x))*sinh(c))/(d^2*x)

giac [B] time = 0.14, size = 111, normalized size = 2.02

$$\frac{a d^3 x \operatorname{Ei}(-d x) e^{-c}-a d^3 x \operatorname{Ei}(d x) e^c-b d x^2 e^{(d x+c)}+b d x^2 e^{-(d x-c)}+a d^2 e^{(d x+c)}+a d^2 e^{-(d x-c)}+b x e^{(d x+c)}+b x e^{-(d x-c)}}{2 d^2 x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)*cosh(d*x+c)/x^2,x, algorithm="giac")

[Out] -1/2*(a*d^3*x*Ei(-d*x)*e^(-c) - a*d^3*x*Ei(d*x)*e^c - b*d*x^2*e^(d*x + c) + b*d*x^2*e^(-d*x - c) + a*d^2*e^(d*x + c) + a*d^2*e^(-d*x - c) + b*x*e^(d*x + c) + b*x*e^(-d*x - c))/(d^2*x)

maple [A] time = 0.14, size = 110, normalized size = 2.00

$$-\frac{a e^{-d x-c}}{2 x}+\frac{d a e^{-c} \operatorname{Ei}(1, d x)}{2}-\frac{b e^{-d x-c} x}{2 d}-\frac{b e^{-d x-c}}{2 d^2}-\frac{a e^{d x+c}}{2 x}-\frac{d a e^c \operatorname{Ei}(1,-d x)}{2}+\frac{b e^{d x+c} x}{2 d}-\frac{b e^{d x+c}}{2 d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)*cosh(d*x+c)/x^2,x)

[Out] -1/2*a*exp(-d*x-c)/x+1/2*d*a*exp(-c)*Ei(1,d*x)-1/2/d*b*exp(-d*x-c)*x-1/2/d^2*b*exp(-d*x-c)-1/2*a/x*exp(d*x+c)-1/2*d*a*exp(c)*Ei(1,-d*x)+1/2/d*b*exp(d*x+c)*x-1/2/d^2*b*exp(d*x+c)

maxima [A] time = 0.40, size = 102, normalized size = 1.85

$$-\frac{1}{4}\left(2 a \operatorname{Ei}(-d x) e^{-c}-2 a \operatorname{Ei}(d x) e^c+\frac{\left(d^2 x^2 e^c-2 d x e^c+2 e^c\right) b e^{(d x)}}{d^3}+\frac{\left(d^2 x^2+2 d x+2\right) b e^{(-d x-c)}}{d^3}\right) d+\frac{1}{2}\left(b x^2-\frac{2 a}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)*cosh(d*x+c)/x^2,x, algorithm="maxima")

[Out] $-1/4*(2*a*Ei(-d*x)*e^{-c} - 2*a*Ei(d*x)*e^c + (d^2*x^2*e^c - 2*d*x*e^c + 2*e^c)*b*e^{(d*x)}/d^3 + (d^2*x^2 + 2*d*x + 2)*b*e^{(-d*x - c)}/d^3)*d + 1/2*(b*x^2 - 2*a/x)*cosh(d*x + c)$

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cosh(c + dx) (bx^3 + a)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cosh(c + d*x)*(a + b*x^3))/x^2,x)

[Out] int((cosh(c + d*x)*(a + b*x^3))/x^2, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^3) \cosh(c + dx)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a)*cosh(d*x+c)/x**2,x)

[Out] Integral((a + b*x**3)*cosh(c + d*x)/x**2, x)

$$3.85 \quad \int \frac{(a+bx^3) \cosh(c+dx)}{x^3} dx$$

Optimal. Leaf size=69

$$\frac{1}{2}ad^2 \cosh(c)\text{Chi}(dx) + \frac{1}{2}ad^2 \sinh(c)\text{Shi}(dx) - \frac{a \cosh(c+dx)}{2x^2} - \frac{ad \sinh(c+dx)}{2x} + \frac{b \sinh(c+dx)}{d}$$

[Out] $1/2*a*d^2*\text{Chi}(d*x)*\cosh(c) - 1/2*a*\cosh(d*x+c)/x^2 + 1/2*a*d^2*\text{Shi}(d*x)*\sinh(c) + b*\sinh(d*x+c)/d - 1/2*a*d*\sinh(d*x+c)/x$

Rubi [A] time = 0.14, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {5287, 2637, 3297, 3303, 3298, 3301}

$$\frac{1}{2}ad^2 \cosh(c)\text{Chi}(dx) + \frac{1}{2}ad^2 \sinh(c)\text{Shi}(dx) - \frac{a \cosh(c+dx)}{2x^2} - \frac{ad \sinh(c+dx)}{2x} + \frac{b \sinh(c+dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[((a + b*x^3)*Cosh[c + d*x])/x^3,x]

[Out] $-(a*\text{Cosh}[c + d*x])/(2*x^2) + (a*d^2*\text{Cosh}[c]*\text{CoshIntegral}[d*x])/2 + (b*\text{Sinh}[c + d*x])/d - (a*d*\text{Sinh}[c + d*x])/(2*x) + (a*d^2*\text{Sinh}[c]*\text{SinhIntegral}[d*x])/2$

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3297

Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((c + d*x)^(m + 1)*Sin[e + f*x]/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]

Rule 3298

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x]
&& EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[
Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

Rule 5287

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_.),
x_Symbol]
:> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p, x], x] /;
FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + bx^3) \cosh(c + dx)}{x^3} dx &= \int \left(b \cosh(c + dx) + \frac{a \cosh(c + dx)}{x^3} \right) dx \\
&= a \int \frac{\cosh(c + dx)}{x^3} dx + b \int \cosh(c + dx) dx \\
&= -\frac{a \cosh(c + dx)}{2x^2} + \frac{b \sinh(c + dx)}{d} + \frac{1}{2}(ad) \int \frac{\sinh(c + dx)}{x^2} dx \\
&= -\frac{a \cosh(c + dx)}{2x^2} + \frac{b \sinh(c + dx)}{d} - \frac{ad \sinh(c + dx)}{2x} + \frac{1}{2}(ad^2) \int \frac{\cosh(c + dx)}{x} dx \\
&= -\frac{a \cosh(c + dx)}{2x^2} + \frac{b \sinh(c + dx)}{d} - \frac{ad \sinh(c + dx)}{2x} + \frac{1}{2}(ad^2 \cosh(c)) \int \frac{\cosh(dx)}{x} dx \\
&= -\frac{a \cosh(c + dx)}{2x^2} + \frac{1}{2}ad^2 \cosh(c) \text{Chi}(dx) + \frac{b \sinh(c + dx)}{d} - \frac{ad \sinh(c + dx)}{2x} + \frac{1}{2}ad^2 \sinh(c) \text{Shi}(dx)
\end{aligned}$$

Mathematica [A] time = 0.15, size = 86, normalized size = 1.25

$$\frac{1}{2}ad^2(\cosh(c)\text{Chi}(dx)+\sinh(c)\text{Shi}(dx))-\frac{a \cosh(dx)(dx \sinh(c) + \cosh(c))}{2x^2}-\frac{a \sinh(dx)(dx \cosh(c) + \sinh(c))}{2x^2}+$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)*Cosh[c + d*x])/x^3,x]
```

[Out] $(b \cosh[dx] \sinh[c])/d - (a \cosh[dx] (\cosh[c] + dx \sinh[c]))/(2x^2) + (b \cosh[c] \sinh[dx])/d - (a(dx \cosh[c] + \sinh[c]) \sinh[dx])/ (2x^2) + (a d^2 (\cosh[c] \operatorname{CoshIntegral}[dx] + \sinh[c] \operatorname{SinhIntegral}[dx]))/2$

fricas [A] time = 0.45, size = 101, normalized size = 1.46

$$\frac{2 ad \cosh(dx + c) - (ad^3 x^2 \operatorname{Ei}(dx) + ad^3 x^2 \operatorname{Ei}(-dx)) \cosh(c) + 2(ad^2 x - 2bx^2) \sinh(dx + c) - (ad^3 x^2 \operatorname{Ei}(dx) - ad^3 x^2 \operatorname{Ei}(-dx)) \sinh(c)}{4 dx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a)*cosh(d*x+c)/x^3,x, algorithm="fricas")`

[Out] $-1/4*(2*a*d*\cosh(dx + c) - (a*d^3*x^2*\operatorname{Ei}(dx) + a*d^3*x^2*\operatorname{Ei}(-dx))*\cosh(c) + 2*(a*d^2*x - 2*b*x^2)*\sinh(dx + c) - (a*d^3*x^2*\operatorname{Ei}(dx) - a*d^3*x^2*\operatorname{Ei}(-dx))*\sinh(c))/(d*x^2)$

giac [A] time = 0.12, size = 118, normalized size = 1.71

$$\frac{ad^3 x^2 \operatorname{Ei}(-dx) e^{(-c)} + ad^3 x^2 \operatorname{Ei}(dx) e^c - ad^2 x e^{(dx+c)} + ad^2 x e^{(-dx-c)} + 2bx^2 e^{(dx+c)} - 2bx^2 e^{(-dx-c)} - ade^{(dx+c)} - ade^{(-dx-c)}}{4 dx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a)*cosh(d*x+c)/x^3,x, algorithm="giac")`

[Out] $1/4*(a*d^3*x^2*\operatorname{Ei}(-dx)*e^{(-c)} + a*d^3*x^2*\operatorname{Ei}(dx)*e^c - a*d^2*x*e^{(dx + c)} + a*d^2*x*e^{(-dx - c)} + 2*b*x^2*e^{(dx + c)} - 2*b*x^2*e^{(-dx - c)} - a*d*e^{(dx + c)} - a*d*e^{(-dx - c)})/(d*x^2)$

maple [A] time = 0.13, size = 114, normalized size = 1.65

$$\frac{da e^{-dx-c}}{4x} - \frac{a e^{-dx-c}}{4x^2} - \frac{d^2 a e^{-c} \operatorname{Ei}(1, dx)}{4} - \frac{b e^{-dx-c}}{2d} - \frac{a e^{dx+c}}{4x^2} - \frac{da e^{dx+c}}{4x} - \frac{d^2 a e^c \operatorname{Ei}(1, -dx)}{4} + \frac{b e^{dx+c}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^3+a)*cosh(d*x+c)/x^3,x)`

[Out] $1/4*d*a*\exp(-d*x-c)/x - 1/4*a*\exp(-d*x-c)/x^2 - 1/4*d^2*a*\exp(-c)*\operatorname{Ei}(1, d*x) - 1/2/d*b*\exp(-d*x-c) - 1/4*a/x^2*\exp(d*x+c) - 1/4*d*a/x*\exp(d*x+c) - 1/4*d^2*a*\exp(c)*\operatorname{Ei}(1, -d*x) + 1/2/d*b*\exp(d*x+c)$

maxima [A] time = 0.40, size = 87, normalized size = 1.26

$$\frac{1}{4} \left(ade^{(-c)} \Gamma(-1, dx) + ade^c \Gamma(-1, -dx) - \frac{2(dx e^c - e^c) b e^{(dx)}}{d^2} - \frac{2(dx + 1) b e^{(-dx-c)}}{d^2} \right) d + \frac{1}{2} \left(2bx - \frac{a}{x^2} \right) \cosh(dx + c)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)*cosh(d*x+c)/x^3,x, algorithm="maxima")

[Out] 1/4*(a*d*e^(-c)*gamma(-1, d*x) + a*d*e^c*gamma(-1, -d*x) - 2*(d*x*e^c - e^c)*b*e^(d*x)/d^2 - 2*(d*x + 1)*b*e^(-d*x - c)/d^2)*d + 1/2*(2*b*x - a/x^2)*cosh(d*x + c)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c + dx) (bx^3 + a)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cosh(c + d*x)*(a + b*x^3))/x^3,x)

[Out] int((cosh(c + d*x)*(a + b*x^3))/x^3, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^3) \cosh(c + dx)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a)*cosh(d*x+c)/x**3,x)

[Out] Integral((a + b*x**3)*cosh(c + d*x)/x**3, x)

$$3.86 \quad \int \frac{(a+bx^3) \cosh(c+dx)}{x^4} dx$$

Optimal. Leaf size=91

$$\frac{1}{6}ad^3 \sinh(c)\text{Chi}(dx) + \frac{1}{6}ad^3 \cosh(c)\text{Shi}(dx) - \frac{ad^2 \cosh(c+dx)}{6x} - \frac{a \cosh(c+dx)}{3x^3} - \frac{ad \sinh(c+dx)}{6x^2} + b \cosh(c)\text{Chi}(dx)$$

[Out] b*Chi(d*x)*cosh(c)-1/3*a*cosh(d*x+c)/x^3-1/6*a*d^2*cosh(d*x+c)/x+1/6*a*d^3*cosh(c)*Shi(d*x)+1/6*a*d^3*Chi(d*x)*sinh(c)+b*Shi(d*x)*sinh(c)-1/6*a*d*sinh(d*x+c)/x^2

Rubi [A] time = 0.22, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {5287, 3297, 3303, 3298, 3301}

$$\frac{1}{6}ad^3 \sinh(c)\text{Chi}(dx) + \frac{1}{6}ad^3 \cosh(c)\text{Shi}(dx) - \frac{ad^2 \cosh(c+dx)}{6x} - \frac{ad \sinh(c+dx)}{6x^2} - \frac{a \cosh(c+dx)}{3x^3} + b \cosh(c)\text{Chi}(dx)$$

Antiderivative was successfully verified.

[In] Int[((a + b*x^3)*Cosh[c + d*x])/x^4,x]

[Out] -(a*Cosh[c + d*x])/(3*x^3) - (a*d^2*Cosh[c + d*x])/(6*x) + b*Cosh[c]*CoshIntegral[d*x] + (a*d^3*CoshIntegral[d*x]*Sinh[c])/6 - (a*d*Sinh[c + d*x])/(6*x^2) + (a*d^3*Cosh[c]*SinhIntegral[d*x])/6 + b*Sinh[c]*SinhIntegral[d*x]

Rule 3297

Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]

Rule 3298

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)])/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]

Rule 3301

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)])/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

Rule 5287

```
Int[Cosh[(c_.) + (d_.)*(x_.)]*((e_.)*(x_.))^(m_.)*((a_.) + (b_.)*(x_.)^(n_.))^(p
_.), x_Symbol] := Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + bx^3) \cosh(c + dx)}{x^4} dx &= \int \left(\frac{a \cosh(c + dx)}{x^4} + \frac{b \cosh(c + dx)}{x} \right) dx \\
&= a \int \frac{\cosh(c + dx)}{x^4} dx + b \int \frac{\cosh(c + dx)}{x} dx \\
&= -\frac{a \cosh(c + dx)}{3x^3} + \frac{1}{3}(ad) \int \frac{\sinh(c + dx)}{x^3} dx + (b \cosh(c)) \int \frac{\cosh(dx)}{x} dx + (b \sinh(c)) \int \frac{\sinh(dx)}{x} dx \\
&= -\frac{a \cosh(c + dx)}{3x^3} + b \cosh(c) \text{Chi}(dx) - \frac{ad \sinh(c + dx)}{6x^2} + b \sinh(c) \text{Shi}(dx) + \frac{1}{6} ad^2 \text{Chi}(dx) \sinh(c) \\
&= -\frac{a \cosh(c + dx)}{3x^3} - \frac{ad^2 \cosh(c + dx)}{6x} + b \cosh(c) \text{Chi}(dx) - \frac{ad \sinh(c + dx)}{6x^2} + b \sinh(c) \text{Shi}(dx) \\
&= -\frac{a \cosh(c + dx)}{3x^3} - \frac{ad^2 \cosh(c + dx)}{6x} + b \cosh(c) \text{Chi}(dx) - \frac{ad \sinh(c + dx)}{6x^2} + b \sinh(c) \text{Shi}(dx) \\
&= -\frac{a \cosh(c + dx)}{3x^3} - \frac{ad^2 \cosh(c + dx)}{6x} + b \cosh(c) \text{Chi}(dx) + \frac{1}{6} ad^3 \text{Chi}(dx) \sinh(c)
\end{aligned}$$

Mathematica [A] time = 0.26, size = 73, normalized size = 0.80

$$\frac{1}{6} \left(\text{Chi}(dx) (ad^3 \sinh(c) + 6b \cosh(c)) + \text{Shi}(dx) (ad^3 \cosh(c) + 6b \sinh(c)) - \frac{a((d^2 x^2 + 2) \cosh(c + dx) + dx \sinh(c + dx))}{x^3} \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)*Cosh[c + d*x])/x^4,x]
```

```
[Out] (CoshIntegral[d*x]*(6*b*Cosh[c] + a*d^3*Sinh[c]) - (a*((2 + d^2*x^2)*Cosh[c
+ d*x] + d*x*Sinh[c + d*x]))/x^3 + (a*d^3*Cosh[c] + 6*b*Sinh[c])*SinhInteg
ral[d*x])/6
```

fricas [A] time = 0.48, size = 118, normalized size = 1.30

$$\frac{2 adx \sinh(dx + c) + 2(ad^2x^2 + 2a) \cosh(dx + c) - ((ad^3 + 6b)x^3 \operatorname{Ei}(dx) - (ad^3 - 6b)x^3 \operatorname{Ei}(-dx)) \cosh(c) - (ad^3 + 6b)x^3 \operatorname{Ei}(dx) + (ad^3 - 6b)x^3 \operatorname{Ei}(-dx) \sinh(c)}{12x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)*cosh(d*x+c)/x^4,x, algorithm="fricas")

[Out] -1/12*(2*a*d*x*sinh(d*x + c) + 2*(a*d^2*x^2 + 2*a)*cosh(d*x + c) - ((a*d^3 + 6*b)*x^3*Ei(d*x) - (a*d^3 - 6*b)*x^3*Ei(-d*x))*cosh(c) - ((a*d^3 + 6*b)*x^3*Ei(d*x) + (a*d^3 - 6*b)*x^3*Ei(-d*x))*sinh(c))/x^3

giac [A] time = 0.12, size = 141, normalized size = 1.55

$$\frac{ad^3x^3\operatorname{Ei}(-dx)e^{(-c)} - ad^3x^3\operatorname{Ei}(dx)e^c + ad^2x^2e^{(dx+c)} + ad^2x^2e^{(-dx-c)} - 6bx^3\operatorname{Ei}(-dx)e^{(-c)} - 6bx^3\operatorname{Ei}(dx)e^c + adxe^{(dx+c)}}{12x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)*cosh(d*x+c)/x^4,x, algorithm="giac")

[Out] -1/12*(a*d^3*x^3*Ei(-d*x)*e^(-c) - a*d^3*x^3*Ei(d*x)*e^c + a*d^2*x^2*e^(d*x + c) + a*d^2*x^2*e^(-d*x - c) - 6*b*x^3*Ei(-d*x)*e^(-c) - 6*b*x^3*Ei(d*x)*e^c + a*d*x*e^(d*x + c) - a*d*x*e^(-d*x - c) + 2*a*e^(d*x + c) + 2*a*e^(-d*x - c))/x^3

maple [A] time = 0.13, size = 143, normalized size = 1.57

$$-\frac{d^2 a e^{-dx-c}}{12x} + \frac{da e^{-dx-c}}{12x^2} - \frac{a e^{-dx-c}}{6x^3} + \frac{d^3 a e^{-c} \operatorname{Ei}(1, dx)}{12} - \frac{b e^{-c} \operatorname{Ei}(1, dx)}{2} - \frac{b e^c \operatorname{Ei}(1, -dx)}{2} - \frac{a e^{dx+c}}{6x^3} - \frac{da e^{dx+c}}{12x^2} - \frac{d^2 a e^{dx+c}}{12x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)*cosh(d*x+c)/x^4,x)

[Out] -1/12*d^2*a*exp(-d*x-c)/x+1/12*d*a*exp(-d*x-c)/x^2-1/6*a*exp(-d*x-c)/x^3+1/12*d^3*a*exp(-c)*Ei(1,d*x)-1/2*b*exp(-c)*Ei(1,d*x)-1/2*b*exp(c)*Ei(1,-d*x)-1/6*a/x^3*exp(d*x+c)-1/12*d*a/x^2*exp(d*x+c)-1/12*d^2*a/x*exp(d*x+c)-1/12*d^3*a*exp(c)*Ei(1,-d*x)

maxima [A] time = 0.42, size = 95, normalized size = 1.04

$$\frac{1}{6} \left((d^2 e^{(-c)} \Gamma(-2, dx) - d^2 e^c \Gamma(-2, -dx)) a - \frac{2b \cosh(dx + c) \log(x^3)}{d} + \frac{3(\operatorname{Ei}(-dx) e^{(-c)} + \operatorname{Ei}(dx) e^c) b}{d} \right) d + \frac{1}{3} (b \log(x^3) + \dots)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)*cosh(d*x+c)/x^4,x, algorithm="maxima")

[Out] 1/6*((d^2*e^(-c)*gamma(-2, d*x) - d^2*e^c*gamma(-2, -d*x))*a - 2*b*cosh(d*x + c)*log(x^3)/d + 3*(Ei(-d*x)*e^(-c) + Ei(d*x)*e^c)*b/d)*d + 1/3*(b*log(x^3) - a/x^3)*cosh(d*x + c)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c + dx) (bx^3 + a)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cosh(c + d*x)*(a + b*x^3))/x^4,x)

[Out] int((cosh(c + d*x)*(a + b*x^3))/x^4, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^3) \cosh(c + dx)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a)*cosh(d*x+c)/x**4,x)

[Out] Integral((a + b*x**3)*cosh(c + d*x)/x**4, x)

3.87 $\int x (a + bx^3)^2 \cosh(c + dx) dx$

Optimal. Leaf size=234

$$-\frac{a^2 \cosh(c + dx)}{d^2} + \frac{a^2 x \sinh(c + dx)}{d} + \frac{48ab \sinh(c + dx)}{d^5} - \frac{48abx \cosh(c + dx)}{d^4} + \frac{24abx^2 \sinh(c + dx)}{d^3} - \frac{8abx^3 \cosh(c + dx)}{d^2}$$

[Out] $-5040*b^2*\cosh(d*x+c)/d^8 - a^2*\cosh(d*x+c)/d^2 - 48*a*b*x*\cosh(d*x+c)/d^4 - 2520*b^2*x^2*\cosh(d*x+c)/d^6 - 8*a*b*x^3*\cosh(d*x+c)/d^2 - 210*b^2*x^4*\cosh(d*x+c)/d^4 - 7*b^2*x^6*\cosh(d*x+c)/d^2 + 48*a*b*\sinh(d*x+c)/d^5 + 5040*b^2*x*\sinh(d*x+c)/d^7 + a^2*x*\sinh(d*x+c)/d + 24*a*b*x^2*\sinh(d*x+c)/d^3 + 840*b^2*x^3*\sinh(d*x+c)/d^5 + 2*a*b*x^4*\sinh(d*x+c)/d + 42*b^2*x^5*\sinh(d*x+c)/d^3 + b^2*x^7*\sinh(d*x+c)/d$

Rubi [A] time = 0.40, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {5287, 3296, 2638, 2637}

$$-\frac{a^2 \cosh(c + dx)}{d^2} + \frac{a^2 x \sinh(c + dx)}{d} + \frac{24abx^2 \sinh(c + dx)}{d^3} - \frac{8abx^3 \cosh(c + dx)}{d^2} + \frac{48ab \sinh(c + dx)}{d^5} - \frac{48abx \cosh(c + dx)}{d^4}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x*(a + b*x^3)^2*\text{Cosh}[c + d*x], x]$

[Out] $(-5040*b^2*\text{Cosh}[c + d*x])/d^8 - (a^2*\text{Cosh}[c + d*x])/d^2 - (48*a*b*x*\text{Cosh}[c + d*x])/d^4 - (2520*b^2*x^2*\text{Cosh}[c + d*x])/d^6 - (8*a*b*x^3*\text{Cosh}[c + d*x])/d^2 - (210*b^2*x^4*\text{Cosh}[c + d*x])/d^4 - (7*b^2*x^6*\text{Cosh}[c + d*x])/d^2 + (48*a*b*\text{Sinh}[c + d*x])/d^5 + (5040*b^2*x*\text{Sinh}[c + d*x])/d^7 + (a^2*x*\text{Sinh}[c + d*x])/d + (24*a*b*x^2*\text{Sinh}[c + d*x])/d^3 + (840*b^2*x^3*\text{Sinh}[c + d*x])/d^5 + (2*a*b*x^4*\text{Sinh}[c + d*x])/d + (42*b^2*x^5*\text{Sinh}[c + d*x])/d^3 + (b^2*x^7*\text{Sinh}[c + d*x])/d$

Rule 2637

$\text{Int}[\sin[\text{Pi}/2 + (c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow \text{Simp}[\text{Sin}[c + d*x]/d, x] /;$
FreeQ[{c, d}, x]

Rule 2638

$\text{Int}[\sin[(c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow -\text{Simp}[\text{Cos}[c + d*x]/d, x] /;$ FreeQ[{c, d}, x]

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[
((c + d*x)^m*cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5287

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_.))^(p
_.), x_Symbol] := Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int x (a + bx^3)^2 \cosh(c + dx) dx &= \int (a^2 x \cosh(c + dx) + 2abx^4 \cosh(c + dx) + b^2 x^7 \cosh(c + dx)) dx \\
&= a^2 \int x \cosh(c + dx) dx + (2ab) \int x^4 \cosh(c + dx) dx + b^2 \int x^7 \cosh(c + dx) dx \\
&= \frac{a^2 x \sinh(c + dx)}{d} + \frac{2abx^4 \sinh(c + dx)}{d} + \frac{b^2 x^7 \sinh(c + dx)}{d} - \frac{a^2 \int \sinh(c + dx)}{d} \\
&= -\frac{a^2 \cosh(c + dx)}{d^2} - \frac{8abx^3 \cosh(c + dx)}{d^2} - \frac{7b^2 x^6 \cosh(c + dx)}{d^2} + \frac{a^2 x \sinh(c + dx)}{d} \\
&= -\frac{a^2 \cosh(c + dx)}{d^2} - \frac{8abx^3 \cosh(c + dx)}{d^2} - \frac{7b^2 x^6 \cosh(c + dx)}{d^2} + \frac{a^2 x \sinh(c + dx)}{d} \\
&= -\frac{a^2 \cosh(c + dx)}{d^2} - \frac{48abx \cosh(c + dx)}{d^4} - \frac{8abx^3 \cosh(c + dx)}{d^2} - \frac{210b^2 x^4 \cosh(c + dx)}{d^4} \\
&= -\frac{a^2 \cosh(c + dx)}{d^2} - \frac{48abx \cosh(c + dx)}{d^4} - \frac{8abx^3 \cosh(c + dx)}{d^2} - \frac{210b^2 x^4 \cosh(c + dx)}{d^4} \\
&= -\frac{a^2 \cosh(c + dx)}{d^2} - \frac{48abx \cosh(c + dx)}{d^4} - \frac{2520b^2 x^2 \cosh(c + dx)}{d^6} - \frac{8abx^3 \cosh(c + dx)}{d^2} \\
&= -\frac{a^2 \cosh(c + dx)}{d^2} - \frac{48abx \cosh(c + dx)}{d^4} - \frac{2520b^2 x^2 \cosh(c + dx)}{d^6} - \frac{8abx^3 \cosh(c + dx)}{d^2} \\
&= -\frac{5040b^2 \cosh(c + dx)}{d^8} - \frac{a^2 \cosh(c + dx)}{d^2} - \frac{48abx \cosh(c + dx)}{d^4} - \frac{2520b^2 x^2 \cosh(c + dx)}{d^6}
\end{aligned}$$

Mathematica [A] time = 0.32, size = 139, normalized size = 0.59

$$\frac{d (a^2 d^6 x + 2abd^2 (d^4 x^4 + 12d^2 x^2 + 24) + b^2 x (d^6 x^6 + 42d^4 x^4 + 840d^2 x^2 + 5040)) \sinh(c + dx) - (a^2 d^6 + 8abd^4)}{d^8}$$

Antiderivative was successfully verified.

[In] Integrate[x*(a + b*x^3)^2*Cosh[c + d*x],x]

[Out]
$$\frac{-((a^2d^6 + 8ab*d^4*x*(6 + d^2*x^2) + 7b^2*(720 + 360d^2*x^2 + 30d^4*x^4 + d^6*x^6))*\text{Cosh}[c + d*x]) + d*(a^2d^6*x + 2a*b*d^2*(24 + 12d^2*x^2 + d^4*x^4) + b^2*x*(5040 + 840d^2*x^2 + 42d^4*x^4 + d^6*x^6))*\text{Sinh}[c + d*x]}{d^8}$$

fricas [A] time = 0.47, size = 161, normalized size = 0.69

$$\frac{(7b^2d^6x^6 + 8abd^6x^3 + 210b^2d^4x^4 + a^2d^6 + 48abd^4x + 2520b^2d^2x^2 + 5040b^2)\cosh(dx + c) - (b^2d^7x^7 + 2abd^7x^4 + 42b^2d^5x^5 + 24a*b*d^5*x^2 + 840b^2*d^3*x^3 + 48a*b*d^3 + (a^2*d^7 + 5040*b^2*d)*x)*\sinh(dx + c)}{d^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x^3+a)^2*cosh(d*x+c),x, algorithm="fricas")

[Out]
$$\frac{-((7b^2d^6x^6 + 8ab*d^6*x^3 + 210b^2d^4x^4 + a^2d^6 + 48ab*d^4*x + 2520b^2d^2x^2 + 5040b^2)*\cosh(dx + c) - (b^2d^7x^7 + 2ab*d^7*x^4 + 42b^2d^5x^5 + 24a*b*d^5*x^2 + 840b^2*d^3*x^3 + 48a*b*d^3 + (a^2*d^7 + 5040*b^2*d)*x)*\sinh(dx + c))}{d^8}$$

giac [A] time = 0.12, size = 303, normalized size = 1.29

$$\frac{(b^2d^7x^7 - 7b^2d^6x^6 + 2abd^7x^4 + 42b^2d^5x^5 - 8abd^6x^3 + a^2d^7x - 210b^2d^4x^4 + 24abd^5x^2 - a^2d^6 + 840b^2d^3x^3 - 4a^2d^7x)*\cosh(dx + c) - (b^2d^7x^7 + 2abd^7x^4 + 42b^2d^5x^5 + 24abd^5x^2 - a^2d^6 + 840b^2d^3x^3 - 4a^2d^7x)*\sinh(dx + c)}{2d^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x^3+a)^2*cosh(d*x+c),x, algorithm="giac")

[Out]
$$\frac{1}{2}*(b^2d^7x^7 - 7b^2d^6x^6 + 2ab*d^7*x^4 + 42b^2d^5x^5 - 8ab*d^6*x^3 + a^2d^7*x - 210b^2d^4x^4 + 24ab*d^5*x^2 - a^2d^6 + 840b^2d^3x^3 - 48ab*d^4*x - 2520b^2d^2x^2 + 48ab*d^3 + 5040b^2d*x - 5040b^2)*e^{(dx + c)}/d^8 - \frac{1}{2}*(b^2d^7x^7 + 7b^2d^6x^6 + 2ab*d^7*x^4 + 42b^2d^5x^5 + 8ab*d^6*x^3 + a^2d^7*x + 210b^2d^4x^4 + 24ab*d^5*x^2 + a^2d^6 + 840b^2d^3x^3 + 48ab*d^4*x + 2520b^2d^2x^2 + 48ab*d^3 + 5040b^2d*x + 5040b^2)*e^{(-dx - c)}/d^8$$

maple [B] time = 0.04, size = 818, normalized size = 3.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(b*x^3+a)^2*cosh(d*x+c),x)

[Out]
$$\frac{1}{d^2}*(\frac{1}{d^6}b^2*((dx+c)^7*\sinh(dx+c) - 7*(dx+c)^6*\cosh(dx+c) + 42*(dx+c)^5*\sinh(dx+c) - 210*(dx+c)^4*\cosh(dx+c) + 840*(dx+c)^3*\sinh(dx+c) - 2520*(dx+c)^2*\cosh(dx+c) + 2520*(dx+c)*\sinh(dx+c) - 2520*\cosh(dx+c) + 2520*\sinh(dx+c) - 2520)*d^6 + a^2*(dx+c)^7*\sinh(dx+c) - 7a^2*(dx+c)^6*\cosh(dx+c) + 42a^2*(dx+c)^5*\sinh(dx+c) - 210a^2*(dx+c)^4*\cosh(dx+c) + 840a^2*(dx+c)^3*\sinh(dx+c) - 2520a^2*(dx+c)^2*\cosh(dx+c) + 2520a^2*(dx+c)*\sinh(dx+c) - 2520a^2*\cosh(dx+c) + 2520a^2*\sinh(dx+c) - 2520a^2)$$

$+c)^2 \cosh(dx+c) + 5040(dx+c) \sinh(dx+c) - 5040 \cosh(dx+c) + 7/d^6 b^2 c^6 ((dx+c) \sinh(dx+c) - \cosh(dx+c)) - 8/d^3 b^3 c^3 a ((dx+c) \sinh(dx+c) - \cosh(dx+c)) - 35/d^6 b^2 c^3 ((dx+c)^4 \sinh(dx+c) - 4(dx+c)^3 \cosh(dx+c) + 12(dx+c)^2 \sinh(dx+c) - 24(dx+c) \cosh(dx+c) + 24 \sinh(dx+c)) + 2/d^3 b^3 a ((dx+c)^4 \sinh(dx+c) - 4(dx+c)^3 \cosh(dx+c) + 12(dx+c)^2 \sinh(dx+c) - 24(dx+c) \cosh(dx+c) + 24 \sinh(dx+c)) + 35/d^6 b^2 c^4 ((dx+c)^3 \sinh(dx+c) - 3(dx+c)^2 \cosh(dx+c) + 6(dx+c) \sinh(dx+c) - 6 \cosh(dx+c)) - 21/d^6 b^2 c^5 ((dx+c)^2 \sinh(dx+c) - 2(dx+c) \cosh(dx+c) + 2 \sinh(dx+c)) - 7/d^6 b^2 c^6 ((dx+c)^6 \sinh(dx+c) - 6(dx+c)^5 \cosh(dx+c) + 30(dx+c)^4 \sinh(dx+c) - 120(dx+c)^3 \cosh(dx+c) + 360(dx+c)^2 \sinh(dx+c) - 720(dx+c) \cosh(dx+c) + 720 \sinh(dx+c)) + 21/d^6 b^2 c^2 ((dx+c)^5 \sinh(dx+c) - 5(dx+c)^4 \cosh(dx+c) + 20(dx+c)^3 \sinh(dx+c) - 60(dx+c)^2 \cosh(dx+c) + 120(dx+c) \sinh(dx+c) - 120 \cosh(dx+c)) + a^2 ((dx+c) \sinh(dx+c) - \cosh(dx+c)) + 12/d^3 b^3 c^2 a ((dx+c)^2 \sinh(dx+c) - 2(dx+c) \cosh(dx+c) + 2 \sinh(dx+c)) - 8/d^3 b^3 c^3 a ((dx+c)^3 \sinh(dx+c) - 3(dx+c)^2 \cosh(dx+c) + 6(dx+c) \sinh(dx+c) - 6 \cosh(dx+c)) - 1/d^6 b^2 c^7 \sinh(dx+c) + 2/d^3 b^3 c^4 a \sinh(dx+c) - c a^2 \sinh(dx+c)$

maxima [A] time = 0.35, size = 383, normalized size = 1.64

$$-\frac{1}{80} d \left(\frac{20(d^2 x^2 e^c - 2 dx e^c + 2 e^c) a^2 e^{(dx)}}{d^3} + \frac{20(d^2 x^2 + 2 dx + 2) a^2 e^{(-dx-c)}}{d^3} + \frac{16(d^5 x^5 e^c - 5 d^4 x^4 e^c + 20 d^3 x^3 e^c - \dots)}{d^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x^3+a)^2*cosh(d*x+c),x, algorithm="maxima")

[Out] $-1/80*d*(20*(d^2*x^2*e^c - 2*d*x*e^c + 2*e^c)*a^2*e^{(d*x)}/d^3 + 20*(d^2*x^2 + 2*d*x + 2)*a^2*e^{(-d*x - c)}/d^3 + 16*(d^5*x^5*e^c - 5*d^4*x^4*e^c + 20*d^3*x^3*e^c - 60*d^2*x^2*e^c + 120*d*x*e^c - 120*e^c)*a*b*e^{(d*x)}/d^6 + 16*(d^5*x^5 + 5*d^4*x^4 + 20*d^3*x^3 + 60*d^2*x^2 + 120*d*x + 120)*a*b*e^{(-d*x - c)}/d^6 + 5*(d^8*x^8*e^c - 8*d^7*x^7*e^c + 56*d^6*x^6*e^c - 336*d^5*x^5*e^c + 1680*d^4*x^4*e^c - 6720*d^3*x^3*e^c + 20160*d^2*x^2*e^c - 40320*d*x*e^c + 40320*e^c)*b^2*e^{(d*x)}/d^9 + 5*(d^8*x^8 + 8*d^7*x^7 + 56*d^6*x^6 + 336*d^5*x^5 + 1680*d^4*x^4 + 6720*d^3*x^3 + 20160*d^2*x^2 + 40320*d*x + 40320)*b^2*e^{(-d*x - c)}/d^9 + 1/40*(5*b^2*x^8 + 16*a*b*x^5 + 20*a^2*x^2)*cosh(d*x + c)$

mupad [B] time = 0.35, size = 301, normalized size = 1.29

$$-e^{c+dx} \left(\frac{a^2 d^6 - 48 a b d^3 + 5040 b^2}{2 d^8} - \frac{x (a^2 d^7 - 48 a b d^4 + 5040 b^2 d)}{2 d^8} - \frac{b^2 x^7}{2 d} + \frac{7 b^2 x^6}{2 d^2} - \frac{21 b^2 x^5}{d^3} + \frac{b x^4 (105 \dots)}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*cosh(c + d*x)*(a + b*x^3)^2,x)

```
[Out] - exp(c + d*x)*((5040*b^2 + a^2*d^6 - 48*a*b*d^3)/(2*d^8) - (x*(5040*b^2*d
+ a^2*d^7 - 48*a*b*d^4))/(2*d^8) - (b^2*x^7)/(2*d) + (7*b^2*x^6)/(2*d^2) -
(21*b^2*x^5)/d^3 + (b*x^4*(105*b - a*d^3))/d^4 - (4*b*x^3*(105*b - a*d^3))/
d^5 + (12*b*x^2*(105*b - a*d^3))/d^6) - exp(- c - d*x)*((5040*b^2 + a^2*d^6
+ 48*a*b*d^3)/(2*d^8) + (x*(5040*b^2*d + a^2*d^7 + 48*a*b*d^4))/(2*d^8) +
(b^2*x^7)/(2*d) + (7*b^2*x^6)/(2*d^2) + (21*b^2*x^5)/d^3 + (b*x^4*(105*b +
a*d^3))/d^4 + (4*b*x^3*(105*b + a*d^3))/d^5 + (12*b*x^2*(105*b + a*d^3))/d^
6)
```

sympy [A] time = 9.63, size = 284, normalized size = 1.21

$$\left\{ \begin{array}{l} \frac{a^2 x \sinh(c+dx)}{d} - \frac{a^2 \cosh(c+dx)}{d^2} + \frac{2abx^4 \sinh(c+dx)}{d} - \frac{8abx^3 \cosh(c+dx)}{d^2} + \frac{24abx^2 \sinh(c+dx)}{d^3} - \frac{48abx \cosh(c+dx)}{d^4} + \frac{48ab \sinh(c+dx)}{d^5} + \\ \left(\frac{a^2 x^2}{2} + \frac{2abx^5}{5} + \frac{b^2 x^8}{8} \right) \cosh(c) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3+a)**2*cosh(d*x+c),x)
```

```
[Out] Piecewise((a**2*x*sinh(c + d*x)/d - a**2*cosh(c + d*x)/d**2 + 2*a*b*x**4*si
nh(c + d*x)/d - 8*a*b*x**3*cosh(c + d*x)/d**2 + 24*a*b*x**2*sinh(c + d*x)/d
**3 - 48*a*b*x*cosh(c + d*x)/d**4 + 48*a*b*sinh(c + d*x)/d**5 + b**2*x**7*s
inh(c + d*x)/d - 7*b**2*x**6*cosh(c + d*x)/d**2 + 42*b**2*x**5*sinh(c + d*x
)/d**3 - 210*b**2*x**4*cosh(c + d*x)/d**4 + 840*b**2*x**3*sinh(c + d*x)/d**
5 - 2520*b**2*x**2*cosh(c + d*x)/d**6 + 5040*b**2*x*sinh(c + d*x)/d**7 - 50
40*b**2*cosh(c + d*x)/d**8, Ne(d, 0)), ((a**2*x**2/2 + 2*a*b*x**5/5 + b**2*
x**8/8)*cosh(c), True))
```

3.88 $\int (a + bx^3)^2 \cosh(c + dx) dx$

Optimal. Leaf size=186

$$\frac{a^2 \sinh(c + dx)}{d} - \frac{12ab \cosh(c + dx)}{d^4} + \frac{12abx \sinh(c + dx)}{d^3} - \frac{6abx^2 \cosh(c + dx)}{d^2} + \frac{2abx^3 \sinh(c + dx)}{d} + \frac{720b^2 \sinh(c + dx)}{d^7}$$

[Out] $-12*a*b*\cosh(d*x+c)/d^4-720*b^2*x*\cosh(d*x+c)/d^6-6*a*b*x^2*\cosh(d*x+c)/d^2-120*b^2*x^3*\cosh(d*x+c)/d^4-6*b^2*x^5*\cosh(d*x+c)/d^2+720*b^2*\sinh(d*x+c)/d^7+a^2*\sinh(d*x+c)/d+12*a*b*x*\sinh(d*x+c)/d^3+360*b^2*x^2*\sinh(d*x+c)/d^5+2*a*b*x^3*\sinh(d*x+c)/d+30*b^2*x^4*\sinh(d*x+c)/d^3+b^2*x^6*\sinh(d*x+c)/d$

Rubi [A] time = 0.29, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5277, 2637, 3296, 2638}

$$\frac{a^2 \sinh(c + dx)}{d} - \frac{6abx^2 \cosh(c + dx)}{d^2} + \frac{12abx \sinh(c + dx)}{d^3} - \frac{12ab \cosh(c + dx)}{d^4} + \frac{2abx^3 \sinh(c + dx)}{d} + \frac{30b^2x^4 \sinh(c + dx)}{d^3}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^3)^2*Cosh[c + d*x],x]

[Out] $(-12*a*b*\cosh[c + d*x])/d^4 - (720*b^2*x*\cosh[c + d*x])/d^6 - (6*a*b*x^2*\cosh[c + d*x])/d^2 - (120*b^2*x^3*\cosh[c + d*x])/d^4 - (6*b^2*x^5*\cosh[c + d*x])/d^2 + (720*b^2*\sinh[c + d*x])/d^7 + (a^2*\sinh[c + d*x])/d + (12*a*b*x*\sinh[c + d*x])/d^3 + (360*b^2*x^2*\sinh[c + d*x])/d^5 + (2*a*b*x^3*\sinh[c + d*x])/d + (30*b^2*x^4*\sinh[c + d*x])/d^3 + (b^2*x^6*\sinh[c + d*x])/d$

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 2638

Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3296

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[((c + d*x)^m*cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 5277

Int[Cosh[(c_.) + (d_.)*(x_.)]*((a_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[p, 0]

Rubi steps

$$\begin{aligned}
 \int (a + bx^3)^2 \cosh(c + dx) dx &= \int (a^2 \cosh(c + dx) + 2abx^3 \cosh(c + dx) + b^2x^6 \cosh(c + dx)) dx \\
 &= a^2 \int \cosh(c + dx) dx + (2ab) \int x^3 \cosh(c + dx) dx + b^2 \int x^6 \cosh(c + dx) dx \\
 &= \frac{a^2 \sinh(c + dx)}{d} + \frac{2abx^3 \sinh(c + dx)}{d} + \frac{b^2x^6 \sinh(c + dx)}{d} - \frac{(6ab) \int x^2 \sinh(c + dx) dx}{d} \\
 &= -\frac{6abx^2 \cosh(c + dx)}{d^2} - \frac{6b^2x^5 \cosh(c + dx)}{d^2} + \frac{a^2 \sinh(c + dx)}{d} + \frac{2abx^3 \sinh(c + dx)}{d} \\
 &= -\frac{6abx^2 \cosh(c + dx)}{d^2} - \frac{6b^2x^5 \cosh(c + dx)}{d^2} + \frac{a^2 \sinh(c + dx)}{d} + \frac{12abx \sinh(c + dx)}{d^3} \\
 &= -\frac{12ab \cosh(c + dx)}{d^4} - \frac{6abx^2 \cosh(c + dx)}{d^2} - \frac{120b^2x^3 \cosh(c + dx)}{d^4} - \frac{6b^2x^5 \cosh(c + dx)}{d^2} \\
 &= -\frac{12ab \cosh(c + dx)}{d^4} - \frac{6abx^2 \cosh(c + dx)}{d^2} - \frac{120b^2x^3 \cosh(c + dx)}{d^4} - \frac{6b^2x^5 \cosh(c + dx)}{d^2} \\
 &= -\frac{12ab \cosh(c + dx)}{d^4} - \frac{720b^2x \cosh(c + dx)}{d^6} - \frac{6abx^2 \cosh(c + dx)}{d^2} - \frac{120b^2x^3 \cosh(c + dx)}{d^4} \\
 &= -\frac{12ab \cosh(c + dx)}{d^4} - \frac{720b^2x \cosh(c + dx)}{d^6} - \frac{6abx^2 \cosh(c + dx)}{d^2} - \frac{120b^2x^3 \cosh(c + dx)}{d^4}
 \end{aligned}$$

Mathematica [A] time = 0.24, size = 111, normalized size = 0.60

$$\frac{(a^2d^6 + 2abd^4x(d^2x^2 + 6) + b^2(d^6x^6 + 30d^4x^4 + 360d^2x^2 + 720)) \sinh(c + dx) - 6bd(ad^2(d^2x^2 + 2) + bx(d^4x^4 + 6d^2x^2 + 6)) \cosh(c + dx)}{d^7}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^3)^2*Cosh[c + d*x],x]

[Out] (-6*b*d*(a*d^2*(2 + d^2*x^2) + b*x*(120 + 20*d^2*x^2 + d^4*x^4))*Cosh[c + d*x] + (a^2*d^6 + 2*a*b*d^4*x*(6 + d^2*x^2) + b^2*(720 + 360*d^2*x^2 + 30*d^4*x^4 + d^6*x^6))*Sinh[c + d*x])/d^7

fricas [A] time = 0.65, size = 130, normalized size = 0.70

$$\frac{6(b^2 d^5 x^5 + a b d^5 x^2 + 20 b^2 d^3 x^3 + 2 a b d^3 + 120 b^2 d x) \cosh(dx + c) - (b^2 d^6 x^6 + 2 a b d^6 x^3 + 30 b^2 d^4 x^4 + a^2 d^6 + 2 a^2 b d^4 x + 360 b^2 d^2 x^2 + 720 b^2) \sinh(dx + c)}{d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^2*cosh(d*x+c),x, algorithm="fricas")

[Out] $-(6*(b^2*d^5*x^5 + a*b*d^5*x^2 + 20*b^2*d^3*x^3 + 2*a*b*d^3 + 120*b^2*d*x)*\cosh(d*x + c) - (b^2*d^6*x^6 + 2*a*b*d^6*x^3 + 30*b^2*d^4*x^4 + a^2*d^6 + 12*a*b*d^4*x + 360*b^2*d^2*x^2 + 720*b^2)*\sinh(d*x + c))/d^7$

giac [A] time = 0.14, size = 244, normalized size = 1.31

$$\frac{(b^2 d^6 x^6 - 6 b^2 d^5 x^5 + 2 a b d^6 x^3 + 30 b^2 d^4 x^4 - 6 a b d^5 x^2 + a^2 d^6 - 120 b^2 d^3 x^3 + 12 a b d^4 x + 360 b^2 d^2 x^2 - 12 a b d^3 - 720 b^2) e^{dx+c} - (b^2 d^6 x^6 + 6 b^2 d^5 x^5 + 2 a b d^6 x^3 + 30 b^2 d^4 x^4 + 6 a b d^5 x^2 + a^2 d^6 + 120 b^2 d^3 x^3 + 12 a b d^4 x + 360 b^2 d^2 x^2 + 12 a b d^3 + 720 b^2) e^{-dx-c}}{2 d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^2*cosh(d*x+c),x, algorithm="giac")

[Out] $1/2*(b^2*d^6*x^6 - 6*b^2*d^5*x^5 + 2*a*b*d^6*x^3 + 30*b^2*d^4*x^4 - 6*a*b*d^5*x^2 + a^2*d^6 - 120*b^2*d^3*x^3 + 12*a*b*d^4*x + 360*b^2*d^2*x^2 - 12*a*b*d^3 - 720*b^2*d*x + 720*b^2)*e^{(d*x + c)}/d^7 - 1/2*(b^2*d^6*x^6 + 6*b^2*d^5*x^5 + 2*a*b*d^6*x^3 + 30*b^2*d^4*x^4 + 6*a*b*d^5*x^2 + a^2*d^6 + 120*b^2*d^3*x^3 + 12*a*b*d^4*x + 360*b^2*d^2*x^2 + 12*a*b*d^3 + 720*b^2*d*x + 720*b^2)*e^{(-d*x - c)}/d^7$

maple [B] time = 0.04, size = 592, normalized size = 3.18

$$\frac{b^2(dx+c)^6 \sinh(dx+c) - 6(dx+c)^5 \cosh(dx+c) + 30(dx+c)^4 \sinh(dx+c) - 120(dx+c)^3 \cosh(dx+c) + 360(dx+c)^2 \sinh(dx+c) - 720(dx+c) \cosh(dx+c) + 720 b^2}{d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)^2*cosh(d*x+c),x)

[Out] $1/d*(1/d^6*b^2*((d*x+c)^6*\sinh(d*x+c)-6*(d*x+c)^5*\cosh(d*x+c)+30*(d*x+c)^4*\sinh(d*x+c)-120*(d*x+c)^3*\cosh(d*x+c)+360*(d*x+c)^2*\sinh(d*x+c)-720*(d*x+c)*\cosh(d*x+c)+720*\sinh(d*x+c))+1/d^6*b^2*c^6*\sinh(d*x+c)+a^2*\sinh(d*x+c)-2/d^3*b*c^3*a*\sinh(d*x+c)-6/d^6*b^2*c^5*((d*x+c)*\sinh(d*x+c)-\cosh(d*x+c))+15/d^6*b^2*c^4*((d*x+c)^2*\sinh(d*x+c)-2*(d*x+c)*\cosh(d*x+c)+2*\sinh(d*x+c))-6/d^6*b^2*c*((d*x+c)^5*\sinh(d*x+c)-5*(d*x+c)^4*\cosh(d*x+c)+20*(d*x+c)^3*\sinh(d*x+c)-60*(d*x+c)^2*\cosh(d*x+c)+120*(d*x+c)*\sinh(d*x+c)-120*\cosh(d*x+c))+15/d^6*b^2*c^2*((d*x+c)^4*\sinh(d*x+c)-4*(d*x+c)^3*\cosh(d*x+c)+12*(d*x+c)^2*\sinh$

$$(d*x+c)-24*(d*x+c)*\cosh(d*x+c)+24*\sinh(d*x+c))-20/d^6*b^2*c^3*((d*x+c)^3*\sinh(d*x+c)-3*(d*x+c)^2*\cosh(d*x+c)+6*(d*x+c)*\sinh(d*x+c)-6*\cosh(d*x+c))+2/d^3*b*a*((d*x+c)^3*\sinh(d*x+c)-3*(d*x+c)^2*\cosh(d*x+c)+6*(d*x+c)*\sinh(d*x+c)-6*\cosh(d*x+c))-6/d^3*b*c*a*((d*x+c)^2*\sinh(d*x+c)-2*(d*x+c)*\cosh(d*x+c)+2*\sinh(d*x+c))+6/d^3*b*c^2*a*((d*x+c)*\sinh(d*x+c)-\cosh(d*x+c)))$$

maxima [A] time = 0.35, size = 243, normalized size = 1.31

$$\frac{a^2 e^{(dx+c)}}{2d} - \frac{a^2 e^{(-dx-c)}}{2d} + \frac{(d^3 x^3 e^c - 3d^2 x^2 e^c + 6dx e^c - 6e^c) a b e^{(dx)}}{d^4} - \frac{(d^3 x^3 + 3d^2 x^2 + 6dx + 6) a b e^{(-dx-c)}}{d^4} + \frac{(d^6 x^6 e^c - 6d^5 x^5 e^c + 30d^4 x^4 e^c - 120d^3 x^3 e^c + 360d^2 x^2 e^c - 720d x e^c + 720e^c) b^2 e^{(dx)}}{d^7} - \frac{(d^6 x^6 + 6d^5 x^5 + 30d^4 x^4 + 120d^3 x^3 + 360d^2 x^2 + 720d x + 720) b^2 e^{(-dx-c)}}{d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^2*cosh(d*x+c),x, algorithm="maxima")

[Out] $\frac{1}{2} a^2 e^{(dx+c)} / d - \frac{1}{2} a^2 e^{(-dx-c)} / d + \frac{(d^3 x^3 e^c - 3d^2 x^2 e^c + 6dx e^c - 6e^c) a b e^{(dx)}}{d^4} - \frac{(d^3 x^3 + 3d^2 x^2 + 6dx + 6) a b e^{(-dx-c)}}{d^4} + \frac{1}{2} (d^6 x^6 e^c - 6d^5 x^5 e^c + 30d^4 x^4 e^c - 120d^3 x^3 e^c + 360d^2 x^2 e^c - 720d x e^c + 720e^c) b^2 e^{(dx)} / d^7 - \frac{1}{2} (d^6 x^6 + 6d^5 x^5 + 30d^4 x^4 + 120d^3 x^3 + 360d^2 x^2 + 720d x + 720) b^2 e^{(-dx-c)} / d^7$

mupad [B] time = 0.24, size = 182, normalized size = 0.98

$$\frac{\sinh(c+dx) (a^2 d^6 + 720 b^2)}{d^7} - \frac{6 b^2 x^5 \cosh(c+dx)}{d^2} - \frac{120 b^2 x^3 \cosh(c+dx)}{d^4} + \frac{b^2 x^6 \sinh(c+dx)}{d} + \frac{30 b^2 x^4 \sinh(c+dx)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(c+d*x)*(a+b*x^3)^2,x)

[Out] $(\sinh(c+dx) * (720 b^2 + a^2 d^6)) / d^7 - (6 b^2 x^5 \cosh(c+dx)) / d^2 - (120 b^2 x^3 \cosh(c+dx)) / d^4 + (b^2 x^6 \sinh(c+dx)) / d + (30 b^2 x^4 \sinh(c+dx)) / d^3 + (360 b^2 x^2 \sinh(c+dx)) / d^5 - (12 a b \cosh(c+dx)) / d^4 - (720 b^2 x \cosh(c+dx)) / d^6 - (6 a b x^2 \cosh(c+dx)) / d^2 + (2 a b x^3 \sinh(c+dx)) / d + (12 a b x \sinh(c+dx)) / d^3$

sympy [A] time = 5.86, size = 226, normalized size = 1.22

$$\left\{ \begin{array}{l} \frac{a^2 \sinh(c+dx)}{d} + \frac{2abx^3 \sinh(c+dx)}{d} - \frac{6abx^2 \cosh(c+dx)}{d^2} + \frac{12abx \sinh(c+dx)}{d^3} - \frac{12ab \cosh(c+dx)}{d^4} + \frac{b^2 x^6 \sinh(c+dx)}{d} - \frac{6b^2 x^5 \cosh(c+dx)}{d^2} + \\ \left(a^2 x + \frac{abx^4}{2} + \frac{b^2 x^7}{7} \right) \cosh(c) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*cosh(d*x+c),x)
```

```
[Out] Piecewise((a**2*sinh(c + d*x)/d + 2*a*b*x**3*sinh(c + d*x)/d - 6*a*b*x**2*cosh(c + d*x)/d**2 + 12*a*b*x*sinh(c + d*x)/d**3 - 12*a*b*cosh(c + d*x)/d**4 + b**2*x**6*sinh(c + d*x)/d - 6*b**2*x**5*cosh(c + d*x)/d**2 + 30*b**2*x**4*sinh(c + d*x)/d**3 - 120*b**2*x**3*cosh(c + d*x)/d**4 + 360*b**2*x**2*sinh(c + d*x)/d**5 - 720*b**2*x*cosh(c + d*x)/d**6 + 720*b**2*sinh(c + d*x)/d**7, Ne(d, 0)), ((a**2*x + a*b*x**4/2 + b**2*x**7/7)*cosh(c), True))
```

$$3.89 \quad \int \frac{(a+bx^3)^2 \cosh(c+dx)}{x} dx$$

Optimal. Leaf size=160

$$a^2 \cosh(c)\text{Chi}(dx)+a^2 \sinh(c)\text{Shi}(dx)+\frac{4ab \sinh(c+dx)}{d^3}-\frac{4abx \cosh(c+dx)}{d^2}+\frac{2abx^2 \sinh(c+dx)}{d}-\frac{120b^2 \cosh(c)}{d^6}$$

[Out] a^2*Chi(d*x)*cosh(c)-120*b^2*cosh(d*x+c)/d^6-4*a*b*x*cosh(d*x+c)/d^2-60*b^2*x^2*cosh(d*x+c)/d^4-5*b^2*x^4*cosh(d*x+c)/d^2+a^2*Shi(d*x)*sinh(c)+4*a*b*sinh(d*x+c)/d^3+120*b^2*x*sinh(d*x+c)/d^5+2*a*b*x^2*sinh(d*x+c)/d+20*b^2*x^3*sinh(d*x+c)/d^3+b^2*x^5*sinh(d*x+c)/d

Rubi [A] time = 0.28, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {5287, 3303, 3298, 3301, 3296, 2637, 2638}

$$a^2 \cosh(c)\text{Chi}(dx)+a^2 \sinh(c)\text{Shi}(dx)+\frac{4ab \sinh(c+dx)}{d^3}-\frac{4abx \cosh(c+dx)}{d^2}+\frac{2abx^2 \sinh(c+dx)}{d}+\frac{20b^2x^3 \sinh(c)}{d^3}$$

Antiderivative was successfully verified.

[In] Int[((a + b*x^3)^2*Cosh[c + d*x])/x,x]

[Out] (-120*b^2*Cosh[c + d*x])/d^6 - (4*a*b*x*Cosh[c + d*x])/d^2 - (60*b^2*x^2*Cosh[c + d*x])/d^4 - (5*b^2*x^4*Cosh[c + d*x])/d^2 + a^2*Cosh[c]*CoshIntegral[d*x] + (4*a*b*Sinh[c + d*x])/d^3 + (120*b^2*x*Sinh[c + d*x])/d^5 + (2*a*b*x^2*Sinh[c + d*x])/d + (20*b^2*x^3*Sinh[c + d*x])/d^3 + (b^2*x^5*Sinh[c + d*x])/d + a^2*Sinh[c]*SinhIntegral[d*x]

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 2638

Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3296

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[((c + d*x)^m*cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m-1)*cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x]
&& EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x]
&& EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d],
Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

Rule 5287

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_.),
x_Symbol]
:> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
&& IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + bx^3)^2 \cosh(c + dx)}{x} dx &= \int \left(\frac{a^2 \cosh(c + dx)}{x} + 2abx^2 \cosh(c + dx) + b^2x^5 \cosh(c + dx) \right) dx \\
&= a^2 \int \frac{\cosh(c + dx)}{x} dx + (2ab) \int x^2 \cosh(c + dx) dx + b^2 \int x^5 \cosh(c + dx) dx \\
&= \frac{2abx^2 \sinh(c + dx)}{d} + \frac{b^2x^5 \sinh(c + dx)}{d} - \frac{(4ab) \int x \sinh(c + dx) dx}{d} - \frac{(5b^2) \int x^4 \sinh(c + dx) dx}{d} \\
&= -\frac{4abx \cosh(c + dx)}{d^2} - \frac{5b^2x^4 \cosh(c + dx)}{d^2} + a^2 \cosh(c) \text{Chi}(dx) + \frac{2abx^2 \sinh(c + dx)}{d} \\
&= -\frac{4abx \cosh(c + dx)}{d^2} - \frac{5b^2x^4 \cosh(c + dx)}{d^2} + a^2 \cosh(c) \text{Chi}(dx) + \frac{4ab \sinh(c + dx)}{d^3} \\
&= -\frac{4abx \cosh(c + dx)}{d^2} - \frac{60b^2x^2 \cosh(c + dx)}{d^4} - \frac{5b^2x^4 \cosh(c + dx)}{d^2} + a^2 \cosh(c) \text{Chi}(dx) \\
&= -\frac{4abx \cosh(c + dx)}{d^2} - \frac{60b^2x^2 \cosh(c + dx)}{d^4} - \frac{5b^2x^4 \cosh(c + dx)}{d^2} + a^2 \cosh(c) \text{Chi}(dx) \\
&= -\frac{120b^2 \cosh(c + dx)}{d^6} - \frac{4abx \cosh(c + dx)}{d^2} - \frac{60b^2x^2 \cosh(c + dx)}{d^4} - \frac{5b^2x^4 \cosh(c + dx)}{d^2} + a^2 \cosh(c) \text{Chi}(dx)
\end{aligned}$$

Mathematica [A] time = 0.52, size = 108, normalized size = 0.68

$$a^2 \cosh(c) \text{Chi}(dx) + a^2 \sinh(c) \text{Shi}(dx) - \frac{b(4ad^4x + 5b(d^4x^4 + 12d^2x^2 + 24)) \cosh(c + dx)}{d^6} + \frac{b(2ad^2(d^2x^2 + 2) + 12abx) \sinh(c + dx)}{d^6}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x^3)^2*Cosh[c + d*x])/x,x]

[Out] -((b*(4*a*d^4*x + 5*b*(24 + 12*d^2*x^2 + d^4*x^4))*Cosh[c + d*x])/d^6) + a^2*Cosh[c]*CoshIntegral[d*x] + (b*(2*a*d^2*(2 + d^2*x^2) + b*x*(120 + 20*d^2*x^2 + d^4*x^4))*Sinh[c + d*x])/d^5 + a^2*Sinh[c]*SinhIntegral[d*x]

fricas [A] time = 0.58, size = 161, normalized size = 1.01

$$\frac{2(5b^2d^4x^4 + 4abd^4x + 60b^2d^2x^2 + 120b^2) \cosh(dx + c) - (a^2d^6 \text{Ei}(dx) + a^2d^6 \text{Ei}(-dx)) \cosh(c) - 2(b^2d^5x^5 + 12abd^4x + 120b^2d^2x^2 + 24b^2) \sinh(c + dx)}{2d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^2*cosh(d*x+c)/x,x, algorithm="fricas")

[Out] $-1/2*(2*(5*b^2*d^4*x^4 + 4*a*b*d^4*x + 60*b^2*d^2*x^2 + 120*b^2)*\cosh(d*x + c) - (a^2*d^6*Ei(d*x) + a^2*d^6*Ei(-d*x))*\cosh(c) - 2*(b^2*d^5*x^5 + 2*a*b*d^5*x^2 + 20*b^2*d^3*x^3 + 4*a*b*d^3 + 120*b^2*d*x)*\sinh(d*x + c) - (a^2*d^6*Ei(d*x) - a^2*d^6*Ei(-d*x))*\sinh(c))/d^6$

giac [B] time = 0.15, size = 331, normalized size = 2.07

$$\frac{b^2 d^5 x^5 e^{(dx+c)} - b^2 d^5 x^5 e^{(-dx-c)} - 5 b^2 d^4 x^4 e^{(dx+c)} - 5 b^2 d^4 x^4 e^{(-dx-c)} + 2 a b d^5 x^2 e^{(dx+c)} - 2 a b d^5 x^2 e^{(-dx-c)} + a^2 d^6 Ei(-d*x)}{d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a)^2*cosh(d*x+c)/x,x, algorithm="giac")`

[Out] $1/2*(b^2*d^5*x^5*e^{(d*x + c)} - b^2*d^5*x^5*e^{(-d*x - c)} - 5*b^2*d^4*x^4*e^{(d*x + c)} - 5*b^2*d^4*x^4*e^{(-d*x - c)} + 2*a*b*d^5*x^2*e^{(d*x + c)} - 2*a*b*d^5*x^2*e^{(-d*x - c)} + a^2*d^6*Ei(-d*x)*e^{(-c)} + a^2*d^6*Ei(d*x)*e^c + 20*b^2*d^3*x^3*e^{(d*x + c)} - 20*b^2*d^3*x^3*e^{(-d*x - c)} - 4*a*b*d^4*x*e^{(d*x + c)} - 4*a*b*d^4*x*e^{(-d*x - c)} - 60*b^2*d^2*x^2*e^{(d*x + c)} - 60*b^2*d^2*x^2*e^{(-d*x - c)} + 4*a*b*d^3*e^{(d*x + c)} - 4*a*b*d^3*e^{(-d*x - c)} + 120*b^2*d*x*e^{(d*x + c)} - 120*b^2*d*x*e^{(-d*x - c)} - 120*b^2*e^{(d*x + c)} - 120*b^2*e^{(-d*x - c)})/d^6$

maple [B] time = 0.19, size = 335, normalized size = 2.09

$$\frac{30b^2e^{-dx-c}x^2}{d^4} - \frac{60b^2e^{-dx-c}x}{d^5} - \frac{2abe^{-dx-c}x}{d^2} - \frac{abe^{-dx-c}x^2}{d} - \frac{b^2e^{-dx-c}x^5}{2d} - \frac{5b^2e^{-dx-c}x^4}{2d^2} - \frac{10b^2e^{-dx-c}x^3}{d^3} + \frac{abe^{dx+c}x^2}{d} - \frac{2}{d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^3+a)^2*cosh(d*x+c)/x,x)`

[Out] $-30/d^4*b^2*\exp(-d*x-c)*x^2-60/d^5*b^2*\exp(-d*x-c)*x-2/d^2*a*b*\exp(-d*x-c)*x-1/d*a*b*\exp(-d*x-c)*x^2-1/2/d*b^2*\exp(-d*x-c)*x^5-5/2/d^2*b^2*\exp(-d*x-c)*x^4-10/d^3*b^2*\exp(-d*x-c)*x^3+1/d*a*b*\exp(d*x+c)*x^2-2/d^2*a*b*\exp(d*x+c)*x-1/2*a^2*\exp(-c)*Ei(1,d*x)-30/d^4*b^2*\exp(d*x+c)*x^2+60/d^5*b^2*\exp(d*x+c)*x-1/2*a^2*\exp(c)*Ei(1,-d*x)-60/d^6*b^2*\exp(d*x+c)-60/d^6*b^2*\exp(-d*x-c)-2/d^3*a*b*\exp(-d*x-c)+2/d^3*a*b*\exp(d*x+c)+1/2/d*b^2*\exp(d*x+c)*x^5-5/2/d^2*b^2*\exp(d*x+c)*x^4+10/d^3*b^2*\exp(d*x+c)*x^3$

maxima [A] time = 0.41, size = 289, normalized size = 1.81

$$-\frac{1}{12} \left(4ab \left(\frac{(d^3 x^3 e^c - 3 d^2 x^2 e^c + 6 dx e^c - 6 e^c) e^{(dx)}}{d^4} + \frac{(d^3 x^3 + 3 d^2 x^2 + 6 dx + 6) e^{(-dx-c)}}{d^4} \right) \right) + b^2 \left(\frac{(d^6 x^6 e^c - 6 d^5 x^5 e^c)}{d^6} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^2*cosh(d*x+c)/x,x, algorithm="maxima")

[Out] $-1/12*(4*a*b*((d^3*x^3*e^c - 3*d^2*x^2*e^c + 6*d*x*e^c - 6*e^c)*e^{(d*x)}/d^4 + (d^3*x^3 + 3*d^2*x^2 + 6*d*x + 6)*e^{(-d*x - c)}/d^4) + b^2*((d^6*x^6*e^c - 6*d^5*x^5*e^c + 30*d^4*x^4*e^c - 120*d^3*x^3*e^c + 360*d^2*x^2*e^c - 720*d*x*e^c + 720*e^c)*e^{(d*x)}/d^7 + (d^6*x^6 + 6*d^5*x^5 + 30*d^4*x^4 + 120*d^3*x^3 + 360*d^2*x^2 + 720*d*x + 720)*e^{(-d*x - c)}/d^7) + 4*a^2*cosh(d*x + c)*\log(x^3)/d - 6*(Ei(-d*x)*e^{-c} + Ei(d*x)*e^c)*a^2/d*d + 1/6*(b^2*x^6 + 4*a*b*x^3 + 2*a^2*\log(x^3))*cosh(d*x + c)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c + dx) (bx^3 + a)^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cosh(c + d*x)*(a + b*x^3)^2)/x,x)

[Out] int((cosh(c + d*x)*(a + b*x^3)^2)/x, x)

sympy [A] time = 8.10, size = 168, normalized size = 1.05

$$a^2 \sinh(c) \operatorname{Shi}(dx) + a^2 \cosh(c) \operatorname{Chi}(dx) + 2ab \begin{cases} \left(\frac{x^2 \sinh(c+dx)}{d} - \frac{2x \cosh(c+dx)}{d^2} + \frac{2 \sinh(c+dx)}{d^3} \right) & \text{for } d \neq 0 \\ \frac{x^3 \cosh(c)}{3} & \text{otherwise} \end{cases} + b^2 \begin{cases} \left(\frac{x^5 \sinh(c+dx)}{d^5} - \frac{5x^4 \cosh(c+dx)}{d^4} + \frac{10x^3 \sinh(c+dx)}{d^3} - \frac{10x^2 \cosh(c+dx)}{d^2} + \frac{5x \sinh(c+dx)}{d} - \cosh(c+dx) \right) & \text{for } d \neq 0 \\ \frac{x^6 \cosh(c)}{6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a)**2*cosh(d*x+c)/x,x)

[Out] $a**2*\sinh(c)*\operatorname{Shi}(d*x) + a**2*\cosh(c)*\operatorname{Chi}(d*x) + 2*a*b*\operatorname{Piecewise}((x**2*\sinh(c + d*x)/d - 2*x*\cosh(c + d*x)/d**2 + 2*\sinh(c + d*x)/d**3, \operatorname{Ne}(d, 0)), (x**3*\cosh(c)/3, \operatorname{True})) + b**2*\operatorname{Piecewise}((x**5*\sinh(c + d*x)/d - 5*x**4*\cosh(c + d*x)/d**2 + 20*x**3*\sinh(c + d*x)/d**3 - 60*x**2*\cosh(c + d*x)/d**4 + 120*x*\sinh(c + d*x)/d**5 - 120*\cosh(c + d*x)/d**6, \operatorname{Ne}(d, 0)), (x**6*\cosh(c)/6, \operatorname{True}))$

$$3.90 \quad \int \frac{(a+bx^3)^2 \cosh(c+dx)}{x^2} dx$$

Optimal. Leaf size=143

$$a^2 d \sinh(c) \operatorname{Chi}(dx) + a^2 d \cosh(c) \operatorname{Shi}(dx) - \frac{a^2 \cosh(c+dx)}{x} - \frac{2ab \cosh(c+dx)}{d^2} + \frac{2abx \sinh(c+dx)}{d} + \frac{24b^2 \sinh(c+dx)}{d^5}$$

[Out] $-2*a*b*\cosh(d*x+c)/d^2 - a^2*\cosh(d*x+c)/x - 24*b^2*x*\cosh(d*x+c)/d^4 - 4*b^2*x^3*\cosh(d*x+c)/d^2 + a^2*d*\cosh(c)*\operatorname{Shi}(d*x) + a^2*d*\operatorname{Chi}(d*x)*\sinh(c) + 24*b^2*\sinh(d*x+c)/d^5 + 2*a*b*x*\sinh(d*x+c)/d + 12*b^2*x^2*\sinh(d*x+c)/d^3 + b^2*x^4*\sinh(d*x+c)/d$

Rubi [A] time = 0.25, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {5287, 3297, 3303, 3298, 3301, 3296, 2638, 2637}

$$a^2 d \sinh(c) \operatorname{Chi}(dx) + a^2 d \cosh(c) \operatorname{Shi}(dx) - \frac{a^2 \cosh(c+dx)}{x} - \frac{2ab \cosh(c+dx)}{d^2} + \frac{2abx \sinh(c+dx)}{d} + \frac{12b^2 x^2 \sinh(c+dx)}{d^3}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(a + b*x^3)^2*\operatorname{Cosh}[c + d*x])/x^2, x]$

[Out] $(-2*a*b*\operatorname{Cosh}[c + d*x])/d^2 - (a^2*\operatorname{Cosh}[c + d*x])/x - (24*b^2*x*\operatorname{Cosh}[c + d*x])/d^4 - (4*b^2*x^3*\operatorname{Cosh}[c + d*x])/d^2 + a^2*d*\operatorname{CoshIntegral}[d*x]*\operatorname{Sinh}[c] + (24*b^2*\operatorname{Sinh}[c + d*x])/d^5 + (2*a*b*x*\operatorname{Sinh}[c + d*x])/d + (12*b^2*x^2*\operatorname{Sinh}[c + d*x])/d^3 + (b^2*x^4*\operatorname{Sinh}[c + d*x])/d + a^2*d*\operatorname{Cosh}[c]*\operatorname{SinhIntegral}[d*x]$

Rule 2637

$\operatorname{Int}[\sin[\operatorname{Pi}/2 + (c_.) + (d_.)*(x_)], x_Symbol] \rightarrow \operatorname{Simp}[\sin[c + d*x]/d, x] /;$
FreeQ[{c, d}, x]

Rule 2638

$\operatorname{Int}[\sin[(c_.) + (d_.)*(x_)], x_Symbol] \rightarrow -\operatorname{Simp}[\cos[c + d*x]/d, x] /;$ FreeQ[{c, d}, x]

Rule 3296

$\operatorname{Int}[(c_.) + (d_.)*(x_)]^{(m_.)}*\sin[(e_.) + (f_.)*(x_)], x_Symbol] \rightarrow -\operatorname{Simp}[(c + d*x)^m*\cos[e + f*x]/f, x] + \operatorname{Dist}[(d*m)/f, \operatorname{Int}[(c + d*x)^{(m-1)}*\cos[e + f*x], x], x] /;$ FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((c + d*x)^(m + 1)*Sin[e + f*x]/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]
```

Rule 5287

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] := Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + bx^3)^2 \cosh(c + dx)}{x^2} dx &= \int \left(\frac{a^2 \cosh(c + dx)}{x^2} + 2abx \cosh(c + dx) + b^2 x^4 \cosh(c + dx) \right) dx \\
&= a^2 \int \frac{\cosh(c + dx)}{x^2} dx + (2ab) \int x \cosh(c + dx) dx + b^2 \int x^4 \cosh(c + dx) dx \\
&= -\frac{a^2 \cosh(c + dx)}{x} + \frac{2abx \sinh(c + dx)}{d} + \frac{b^2 x^4 \sinh(c + dx)}{d} - \frac{(2ab) \int \sinh(c + dx)}{d} \\
&= -\frac{2ab \cosh(c + dx)}{d^2} - \frac{a^2 \cosh(c + dx)}{x} - \frac{4b^2 x^3 \cosh(c + dx)}{d^2} + \frac{2abx \sinh(c + dx)}{d} \\
&= -\frac{2ab \cosh(c + dx)}{d^2} - \frac{a^2 \cosh(c + dx)}{x} - \frac{4b^2 x^3 \cosh(c + dx)}{d^2} + a^2 d \operatorname{Chi}(dx) \sinh(c + dx) \\
&= -\frac{2ab \cosh(c + dx)}{d^2} - \frac{a^2 \cosh(c + dx)}{x} - \frac{24b^2 x \cosh(c + dx)}{d^4} - \frac{4b^2 x^3 \cosh(c + dx)}{d^2} \\
&= -\frac{2ab \cosh(c + dx)}{d^2} - \frac{a^2 \cosh(c + dx)}{x} - \frac{24b^2 x \cosh(c + dx)}{d^4} - \frac{4b^2 x^3 \cosh(c + dx)}{d^2}
\end{aligned}$$

Mathematica [A] time = 0.38, size = 143, normalized size = 1.00

$$a^2 d \sinh(c) \operatorname{Chi}(dx) + a^2 d \cosh(c) \operatorname{Shi}(dx) - \frac{a^2 \cosh(c + dx)}{x} - \frac{2ab \cosh(c + dx)}{d^2} + \frac{2abx \sinh(c + dx)}{d} + \frac{24b^2 \sinh(c + dx)}{d^5}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x^3)^2*Cosh[c + d*x])/x^2,x]

[Out] (-2*a*b*Cosh[c + d*x])/d^2 - (a^2*Cosh[c + d*x])/x - (24*b^2*x*Cosh[c + d*x])/d^4 - (4*b^2*x^3*Cosh[c + d*x])/d^2 + a^2*d*CoshIntegral[d*x]*Sinh[c] + (24*b^2*Sinh[c + d*x])/d^5 + (2*a*b*x*Sinh[c + d*x])/d + (12*b^2*x^2*Sinh[c + d*x])/d^3 + (b^2*x^4*Sinh[c + d*x])/d + a^2*d*Cosh[c]*SinhIntegral[d*x]

fricas [A] time = 0.55, size = 160, normalized size = 1.12

$$\frac{2(4b^2d^3x^4 + a^2d^5 + 2abd^3x + 24b^2dx^2) \cosh(dx + c) - (a^2d^6x \operatorname{Ei}(dx) - a^2d^6x \operatorname{Ei}(-dx)) \cosh(c) - 2(b^2d^4x^5 - \dots)}{2d^5x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^2*cosh(d*x+c)/x^2,x, algorithm="fricas")

[Out] -1/2*(2*(4*b^2*d^3*x^4 + a^2*d^5 + 2*a*b*d^3*x + 24*b^2*d*x^2)*cosh(d*x + c) - (a^2*d^6*x*Ei(d*x) - a^2*d^6*x*Ei(-d*x))*cosh(c) - 2*(b^2*d^4*x^5 + 2*a

$$*b*d^4*x^2 + 12*b^2*d^2*x^3 + 24*b^2*x)*\sinh(d*x + c) - (a^2*d^6*x*Ei(d*x) + a^2*d^6*x*Ei(-d*x))*\sinh(c))/(d^5*x)$$

giac [B] time = 0.12, size = 308, normalized size = 2.15

$$\frac{b^2 d^4 x^5 e^{(dx+c)} - b^2 d^4 x^5 e^{(-dx-c)} - a^2 d^6 x Ei(-dx) e^{(-c)} + a^2 d^6 x Ei(dx) e^c - 4 b^2 d^3 x^4 e^{(dx+c)} - 4 b^2 d^3 x^4 e^{(-dx-c)} + 2 a b d^4 x^2 \sinh(dx+c) - (a^2 d^6 x Ei(d*x) + a^2 d^6 x Ei(-d*x))*\sinh(c)}{d^5 x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^2*cosh(d*x+c)/x^2,x, algorithm="giac")

$$[Out] \frac{1}{2} * (b^2 * d^4 * x^5 * e^{(d*x + c)} - b^2 * d^4 * x^5 * e^{(-d*x - c)} - a^2 * d^6 * x * Ei(-d*x) * e^{(-c)} + a^2 * d^6 * x * Ei(d*x) * e^c - 4 * b^2 * d^3 * x^4 * e^{(d*x + c)} - 4 * b^2 * d^3 * x^4 * e^{(-d*x - c)} + 2 * a * b * d^4 * x^2 * e^{(d*x + c)} - 2 * a * b * d^4 * x^2 * e^{(-d*x - c)} - a^2 * d^5 * e^{(d*x + c)} + 12 * b^2 * d^2 * x^3 * e^{(d*x + c)} - a^2 * d^5 * e^{(-d*x - c)} - 12 * b^2 * d^2 * x^3 * e^{(-d*x - c)} - 2 * a * b * d^3 * x * e^{(d*x + c)} - 2 * a * b * d^3 * x * e^{(-d*x - c)} - 24 * b^2 * d * x^2 * e^{(d*x + c)} - 24 * b^2 * d * x^2 * e^{(-d*x - c)} + 24 * b^2 * x * e^{(d*x + c)} - 24 * b^2 * x * e^{(-d*x - c)}) / (d^5 * x)$$

maple [B] time = 0.21, size = 296, normalized size = 2.07

$$\frac{6b^2 e^{-dx-c} x^2}{d^3} - \frac{12b^2 e^{-dx-c} x}{d^4} - \frac{ab e^{-dx-c} x}{d} - \frac{b^2 e^{-dx-c} x^4}{2d} - \frac{2b^2 e^{-dx-c} x^3}{d^2} + \frac{d a^2 e^{-c} Ei(1, dx)}{2} - \frac{12b^2 e^{-dx-c}}{d^5} - \frac{ab e^{-dx-c}}{d^2} - \frac{a^2 e^{-c}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)^2*cosh(d*x+c)/x^2,x)

$$[Out] -6/d^3*b^2*exp(-d*x-c)*x^2-12/d^4*b^2*exp(-d*x-c)*x-1/d*a*b*exp(-d*x-c)*x-1/2/d*b^2*exp(-d*x-c)*x^4-2/d^2*b^2*exp(-d*x-c)*x^3+1/2*d*a^2*exp(-c)*Ei(1,d*x)-12/d^5*b^2*exp(-d*x-c)-1/d^2*a*b*exp(-d*x-c)-1/2*a^2*exp(-d*x-c)/x-1/2*a^2/x*exp(d*x+c)+1/d*a*b*exp(d*x+c)*x+6/d^3*b^2*exp(d*x+c)*x^2-12/d^4*b^2*exp(d*x+c)*x-1/2*d*a^2*exp(c)*Ei(1,-d*x)+12/d^5*b^2*exp(d*x+c)-1/d^2*a*b*exp(d*x+c)+1/2/d*b^2*exp(d*x+c)*x^4-2/d^2*b^2*exp(d*x+c)*x^3$$

maxima [A] time = 0.40, size = 235, normalized size = 1.64

$$-\frac{1}{10} \left(5 a^2 Ei(-dx) e^{(-c)} - 5 a^2 Ei(dx) e^c + \frac{5 (d^2 x^2 e^c - 2 dx e^c + 2 e^c) a b e^{(dx)}}{d^3} + \frac{5 (d^2 x^2 + 2 dx + 2) a b e^{(-dx-c)}}{d^3} + \frac{(d^5 x^5)}{d^5} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^2*cosh(d*x+c)/x^2,x, algorithm="maxima")

$$[Out] -1/10*(5*a^2*Ei(-d*x)*e^{(-c)} - 5*a^2*Ei(d*x)*e^c + 5*(d^2*x^2*e^c - 2*d*x*e^c + 2*e^c)*a*b*e^{(d*x)}/d^3 + 5*(d^2*x^2 + 2*d*x + 2)*a*b*e^{(-d*x - c)}/d^3 + (d^5*x^5)/d^5)$$

$(d^5 x^5 e^c - 5d^4 x^4 e^c + 20d^3 x^3 e^c - 60d^2 x^2 e^c + 120d x e^c - 120e^c) b^2 e^{dx} / d^6 + (d^5 x^5 + 5d^4 x^4 + 20d^3 x^3 + 60d^2 x^2 + 120d x + 120) b^2 e^{-dx - c} / d^6 * d + 1/5 (b^2 x^5 + 5a b x^2 - 5a^2/x) * \cosh(dx + c)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c + dx) (bx^3 + a)^2}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cosh(c + d*x)*(a + b*x^3)^2)/x^2, x)

[Out] int((cosh(c + d*x)*(a + b*x^3)^2)/x^2, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^3)^2 \cosh(c + dx)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a)**2*cosh(d*x+c)/x**2, x)

[Out] Integral((a + b*x**3)**2*cosh(c + d*x)/x**2, x)

$$3.91 \quad \int \frac{(a+bx^3)^2 \cosh(c+dx)}{x^3} dx$$

Optimal. Leaf size=141

$$\frac{1}{2}a^2d^2 \cosh(c)\text{Chi}(dx) + \frac{1}{2}a^2d^2 \sinh(c)\text{Shi}(dx) - \frac{a^2 \cosh(c+dx)}{2x^2} - \frac{a^2d \sinh(c+dx)}{2x} + \frac{2ab \sinh(c+dx)}{d} - \frac{6b^2 \cosh(c+dx)}{d^4}$$

[Out] $1/2*a^2*d^2*\text{Chi}(d*x)*\cosh(c) - 6*b^2*\cosh(d*x+c)/d^4 - 1/2*a^2*\cosh(d*x+c)/x^2 - 3*b^2*x^2*\cosh(d*x+c)/d^2 + 1/2*a^2*d^2*\text{Shi}(d*x)*\sinh(c) + 2*a*b*\sinh(d*x+c)/d - 1/2*a^2*d*\sinh(d*x+c)/x + 6*b^2*x*\sinh(d*x+c)/d^3 + b^2*x^3*\sinh(d*x+c)/d$

Rubi [A] time = 0.23, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {5287, 2637, 3297, 3303, 3298, 3301, 3296, 2638}

$$\frac{1}{2}a^2d^2 \cosh(c)\text{Chi}(dx) + \frac{1}{2}a^2d^2 \sinh(c)\text{Shi}(dx) - \frac{a^2 \cosh(c+dx)}{2x^2} - \frac{a^2d \sinh(c+dx)}{2x} + \frac{2ab \sinh(c+dx)}{d} - \frac{3b^2x^2 \cosh(c+dx)}{d^2}$$

Antiderivative was successfully verified.

[In] Int[((a + b*x^3)^2*Cosh[c + d*x])/x^3,x]

[Out] $(-6*b^2*\text{Cosh}[c + d*x])/d^4 - (a^2*\text{Cosh}[c + d*x])/(2*x^2) - (3*b^2*x^2*\text{Cosh}[c + d*x])/d^2 + (a^2*d^2*\text{Cosh}[c]*\text{CoshIntegral}[d*x])/2 + (2*a*b*\text{Sinh}[c + d*x])/d - (a^2*d*\text{Sinh}[c + d*x])/(2*x) + (6*b^2*x*\text{Sinh}[c + d*x])/d^3 + (b^2*x^3*\text{Sinh}[c + d*x])/d + (a^2*d^2*\text{Sinh}[c]*\text{SinhIntegral}[d*x])/2$

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 2638

Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3296

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m-1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]
]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]
```

Rule 5287

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] := Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + bx^3)^2 \cosh(c + dx)}{x^3} dx &= \int \left(2ab \cosh(c + dx) + \frac{a^2 \cosh(c + dx)}{x^3} + b^2 x^3 \cosh(c + dx) \right) dx \\
&= a^2 \int \frac{\cosh(c + dx)}{x^3} dx + (2ab) \int \cosh(c + dx) dx + b^2 \int x^3 \cosh(c + dx) dx \\
&= -\frac{a^2 \cosh(c + dx)}{2x^2} + \frac{2ab \sinh(c + dx)}{d} + \frac{b^2 x^3 \sinh(c + dx)}{d} - \frac{(3b^2) \int x^2 \sinh(c + dx)}{d} \\
&= -\frac{a^2 \cosh(c + dx)}{2x^2} - \frac{3b^2 x^2 \cosh(c + dx)}{d^2} + \frac{2ab \sinh(c + dx)}{d} - \frac{a^2 d \sinh(c + dx)}{2x} \\
&= -\frac{a^2 \cosh(c + dx)}{2x^2} - \frac{3b^2 x^2 \cosh(c + dx)}{d^2} + \frac{2ab \sinh(c + dx)}{d} - \frac{a^2 d \sinh(c + dx)}{2x} \\
&= -\frac{6b^2 \cosh(c + dx)}{d^4} - \frac{a^2 \cosh(c + dx)}{2x^2} - \frac{3b^2 x^2 \cosh(c + dx)}{d^2} + \frac{1}{2} a^2 d^2 \cosh(c) \text{Chi}(dx) \\
&\quad + a^2 d^2 \sinh(c) \text{Shi}(dx) - \frac{a^2 \cosh(c + dx)}{x^2} - \frac{a^2 d \sinh(c + dx)}{x} + \frac{4ab \sinh(c + dx)}{d} - \frac{12b^2}{d}
\end{aligned}$$

Mathematica [A] time = 0.38, size = 136, normalized size = 0.96

$$\frac{1}{2} \left(a^2 d^2 \cosh(c) \text{Chi}(dx) + a^2 d^2 \sinh(c) \text{Shi}(dx) - \frac{a^2 \cosh(c + dx)}{x^2} - \frac{a^2 d \sinh(c + dx)}{x} + \frac{4ab \sinh(c + dx)}{d} - \frac{12b^2}{d} \right)$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x^3)^2*Cosh[c + d*x])/x^3,x]

[Out] ((-12*b^2*Cosh[c + d*x])/d^4 - (a^2*Cosh[c + d*x])/x^2 - (6*b^2*x^2*Cosh[c + d*x])/d^2 + a^2*d^2*Cosh[c]*CoshIntegral[d*x] + (4*a*b*Sinh[c + d*x])/d - (a^2*d*Sinh[c + d*x])/x + (12*b^2*x*Sinh[c + d*x])/d^3 + (2*b^2*x^3*Sinh[c + d*x])/d + a^2*d^2*Sinh[c]*SinhIntegral[d*x])/2

fricas [A] time = 0.44, size = 161, normalized size = 1.14

$$\frac{2 \left(6b^2 d^2 x^4 + a^2 d^4 + 12b^2 x^2 \right) \cosh(dx + c) - \left(a^2 d^6 x^2 \text{Ei}(dx) + a^2 d^6 x^2 \text{Ei}(-dx) \right) \cosh(c) - 2 \left(2b^2 d^3 x^5 - a^2 d^5 x + 4ab d^3 x^2 + 12b^2 d^3 x^3 \right) \sinh(dx + c) - \left(a^2 d^6 x^2 \text{Ei}(dx) - a^2 d^6 x^2 \text{Ei}(-dx) \right) \sinh(c)}{4d^4 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^2*cosh(d*x+c)/x^3,x, algorithm="fricas")

[Out] -1/4*(2*(6*b^2*d^2*x^4 + a^2*d^4 + 12*b^2*x^2)*cosh(d*x + c) - (a^2*d^6*x^2*Ei(d*x) + a^2*d^6*x^2*Ei(-d*x))*cosh(c) - 2*(2*b^2*d^3*x^5 - a^2*d^5*x + 4*a*b*d^3*x^2 + 12*b^2*d^3*x^3)*sinh(d*x + c) - (a^2*d^6*x^2*Ei(d*x) - a^2*d^6*x^2*Ei(-d*x))*sinh(c))/(d^4*x^2)

giac [B] time = 0.13, size = 280, normalized size = 1.99

$$\frac{a^2 d^6 x^2 \operatorname{Ei}(-dx) e^{(-c)} + a^2 d^6 x^2 \operatorname{Ei}(dx) e^c + 2 b^2 d^3 x^5 e^{(dx+c)} - 2 b^2 d^3 x^5 e^{(-dx-c)} - a^2 d^5 x e^{(dx+c)} - 6 b^2 d^2 x^4 e^{(dx+c)} + a^2 d^5 x e^{(-dx-c)}}{d^4 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^2*cosh(d*x+c)/x^3,x, algorithm="giac")

[Out] $\frac{1}{4} * (a^2 * d^6 * x^2 * \operatorname{Ei}(-d * x) * e^{(-c)} + a^2 * d^6 * x^2 * \operatorname{Ei}(d * x) * e^c + 2 * b^2 * d^3 * x^5 * e^{(d * x + c)} - 2 * b^2 * d^3 * x^5 * e^{(-d * x - c)} - a^2 * d^5 * x * e^{(d * x + c)} - 6 * b^2 * d^2 * x^4 * e^{(d * x + c)} + a^2 * d^5 * x * e^{(-d * x - c)} - 6 * b^2 * d^2 * x^4 * e^{(-d * x - c)} + 4 * a * b * d^3 * x^2 * e^{(d * x + c)} - 4 * a * b * d^3 * x^2 * e^{(-d * x - c)} - a^2 * d^4 * e^{(d * x + c)} + 12 * b^2 * d * x^3 * e^{(d * x + c)} - a^2 * d^4 * e^{(-d * x - c)} - 12 * b^2 * d * x^3 * e^{(-d * x - c)} - 12 * b^2 * x^2 * e^{(d * x + c)} - 12 * b^2 * x^2 * e^{(-d * x - c)}) / (d^4 * x^2)$

maple [A] time = 0.22, size = 265, normalized size = 1.88

$$\frac{d a^2 e^{-dx-c}}{4x} - \frac{a^2 e^{-dx-c}}{4x^2} - \frac{3b^2 e^{-dx-c} x^2}{2d^2} - \frac{3b^2 e^{-dx-c} x}{d^3} - \frac{b^2 e^{-dx-c} x^3}{2d} - \frac{d^2 a^2 e^{-c} \operatorname{Ei}(1, dx)}{4} - \frac{3b^2 e^{-dx-c}}{d^4} - \frac{ab e^{-dx-c}}{d} - \frac{a^2 e^{dx+c}}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)^2*cosh(d*x+c)/x^3,x)

[Out] $\frac{1}{4} * d * a^2 * \exp(-d * x - c) / x - \frac{1}{4} * a^2 * \exp(-d * x - c) / x^2 - \frac{3}{2} * d^2 * b^2 * \exp(-d * x - c) * x^2 - \frac{3}{d^3} * b^2 * \exp(-d * x - c) * x - \frac{1}{2} * d * b^2 * \exp(-d * x - c) * x^3 - \frac{1}{4} * d^2 * a^2 * \exp(-c) * \operatorname{Ei}(1, d * x) - \frac{3}{d^4} * b^2 * \exp(-d * x - c) - \frac{1}{d} * a * b * \exp(-d * x - c) - \frac{1}{4} * a^2 / x^2 * \exp(d * x + c) - \frac{1}{4} * d * a^2 / x * \exp(d * x + c) - \frac{3}{2} * d^2 * b^2 * \exp(d * x + c) * x^2 + \frac{3}{d^3} * b^2 * \exp(d * x + c) * x - \frac{1}{4} * d^2 * a^2 * \exp(c) * \operatorname{Ei}(1, -d * x) - \frac{3}{d^4} * b^2 * \exp(d * x + c) + \frac{1}{d} * a * b * \exp(d * x + c) + \frac{1}{2} * d * b^2 * \exp(d * x + c) * x^3$

maxima [A] time = 0.39, size = 203, normalized size = 1.44

$$\frac{1}{8} \left(2 a^2 d e^{(-c)} \Gamma(-1, dx) + 2 a^2 d e^c \Gamma(-1, -dx) - \frac{8(dx e^c - e^c) a b e^{(dx)}}{d^2} - \frac{8(dx + 1) a b e^{(-dx-c)}}{d^2} - \frac{(d^4 x^4 e^c - 4 d^3 x^3 e^c + 12 d^2 x^2 e^c - 4 d x e^c + e^c) a^2}{d^4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^2*cosh(d*x+c)/x^3,x, algorithm="maxima")

[Out] $\frac{1}{8} * (2 * a^2 * d * e^{(-c)} * \operatorname{gamma}(-1, d * x) + 2 * a^2 * d * e^c * \operatorname{gamma}(-1, -d * x) - 8 * (d * x * e^c - e^c) * a * b * e^{(d * x)} / d^2 - 8 * (d * x + 1) * a * b * e^{(-d * x - c)} / d^2 - (d^4 * x^4 * e^c - 4 * d^3 * x^3 * e^c + 12 * d^2 * x^2 * e^c - 4 * d * x * e^c + e^c) * a^2 / d^4 - 4 * d^3 * x^3 * e^c + 12 * d^2 * x^2 * e^c - 24 * d * x * e^c + 24 * e^c) * b^2 * e^{(d * x)} / d^5 - (d^4 * x^4 + 4 * d^3 * x^3 + 12 * d^2 * x^2 + 24 * d * x + 24) * b^2 * e^{(-d * x - c)} / d^5) * d + \frac{1}{4} * (b^2 * x^4 + 8 * a * b * x - 2 * a^2 / x^2) * \cosh(d * x + c)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c + dx) (bx^3 + a)^2}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cosh(c + d*x)*(a + b*x^3)^2)/x^3, x)

[Out] int((cosh(c + d*x)*(a + b*x^3)^2)/x^3, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^3)^2 \cosh(c + dx)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a)**2*cosh(d*x+c)/x**3, x)

[Out] Integral((a + b*x**3)**2*cosh(c + d*x)/x**3, x)

$$3.92 \quad \int \frac{(a+bx^3)^2 \cosh(c+dx)}{x^4} dx$$

Optimal. Leaf size=150

$$\frac{1}{6}a^2d^3 \sinh(c)\text{Chi}(dx) + \frac{1}{6}a^2d^3 \cosh(c)\text{Shi}(dx) - \frac{a^2d^2 \cosh(c+dx)}{6x} - \frac{a^2 \cosh(c+dx)}{3x^3} - \frac{a^2d \sinh(c+dx)}{6x^2} + 2ab \cosh(c)$$

[Out] $2*a*b*\text{Chi}(d*x)*\cosh(c) - 1/3*a^2*\cosh(d*x+c)/x^3 - 1/6*a^2*d^2*\cosh(d*x+c)/x - 2*b^2*x*\cosh(d*x+c)/d^2 + 1/6*a^2*d^3*\cosh(c)*\text{Shi}(d*x) + 1/6*a^2*d^3*\text{Chi}(d*x)*\sinh(c) + 2*a*b*\text{Shi}(d*x)*\sinh(c) + 2*b^2*\sinh(d*x+c)/d^3 - 1/6*a^2*d*\sinh(d*x+c)/x^2 + b^2*x^2*\sinh(d*x+c)/d$

Rubi [A] time = 0.28, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {5287, 3297, 3303, 3298, 3301, 3296, 2637}

$$\frac{1}{6}a^2d^3 \sinh(c)\text{Chi}(dx) + \frac{1}{6}a^2d^3 \cosh(c)\text{Shi}(dx) - \frac{a^2d^2 \cosh(c+dx)}{6x} - \frac{a^2d \sinh(c+dx)}{6x^2} - \frac{a^2 \cosh(c+dx)}{3x^3} + 2ab \cosh(c)$$

Antiderivative was successfully verified.

[In] $\text{Int}[\frac{(a + b*x^3)^2*\text{Cosh}[c + d*x]}{x^4}, x]$

[Out] $-(a^2*\text{Cosh}[c + d*x])/(3*x^3) - (a^2*d^2*\text{Cosh}[c + d*x])/(6*x) - (2*b^2*x*\text{Cosh}[c + d*x])/d^2 + 2*a*b*\text{Cosh}[c]*\text{CoshIntegral}[d*x] + (a^2*d^3*\text{CoshIntegral}[d*x]*\text{Sinh}[c])/6 + (2*b^2*\text{Sinh}[c + d*x])/d^3 - (a^2*d*\text{Sinh}[c + d*x])/(6*x^2) + (b^2*x^2*\text{Sinh}[c + d*x])/d + (a^2*d^3*\text{Cosh}[c]*\text{SinhIntegral}[d*x])/6 + 2*a*b*\text{Sinh}[c]*\text{SinhIntegral}[d*x]$

Rule 2637

$\text{Int}[\sin[\text{Pi}/2 + (c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow \text{Simp}[\sin[c + d*x]/d, x] /;$
FreeQ[{c, d}, x]

Rule 3296

$\text{Int}[\frac{((c_.) + (d_.)*(x_.))^{(m_.)}*\sin[(e_.) + (f_.)*(x_.)]}{(c + d*x)^m*\cos[e + f*x]}/f, x] + \text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{(m-1)}*\cos[e + f*x], x], x] /;$ FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 3297

$\text{Int}[\frac{((c_.) + (d_.)*(x_.))^{(m_.)}*\sin[(e_.) + (f_.)*(x_.)]}{(c + d*x)^{(m+1)}*\sin[e + f*x]}/(d*(m+1)), x] - \text{Dist}[f/(d*(m+1)), \text{Int}[(c + d*x)^{(m+1)}*\cos[e + f*x], x], x] /;$ FreeQ[{c, d, e, f}, x] && LtQ[m, -1]

]

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x]
&& EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x]
&& EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*e - c*f)/d],
Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d],
Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

Rule 5287

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_.),
x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + bx^3)^2 \cosh(c + dx)}{x^4} dx &= \int \left(\frac{a^2 \cosh(c + dx)}{x^4} + \frac{2ab \cosh(c + dx)}{x} + b^2 x^2 \cosh(c + dx) \right) dx \\
&= a^2 \int \frac{\cosh(c + dx)}{x^4} dx + (2ab) \int \frac{\cosh(c + dx)}{x} dx + b^2 \int x^2 \cosh(c + dx) dx \\
&= -\frac{a^2 \cosh(c + dx)}{3x^3} + \frac{b^2 x^2 \sinh(c + dx)}{d} - \frac{(2b^2) \int x \sinh(c + dx) dx}{d} + \frac{1}{3} (a^2 d) \\
&= -\frac{a^2 \cosh(c + dx)}{3x^3} - \frac{2b^2 x \cosh(c + dx)}{d^2} + 2ab \cosh(c) \text{Chi}(dx) - \frac{a^2 d \sinh(c + dx)}{6x^2} \\
&= -\frac{a^2 \cosh(c + dx)}{3x^3} - \frac{a^2 d^2 \cosh(c + dx)}{6x} - \frac{2b^2 x \cosh(c + dx)}{d^2} + 2ab \cosh(c) \text{Chi}(dx) \\
&= -\frac{a^2 \cosh(c + dx)}{3x^3} - \frac{a^2 d^2 \cosh(c + dx)}{6x} - \frac{2b^2 x \cosh(c + dx)}{d^2} + 2ab \cosh(c) \text{Chi}(dx) \\
&= -\frac{a^2 \cosh(c + dx)}{3x^3} - \frac{a^2 d^2 \cosh(c + dx)}{6x} - \frac{2b^2 x \cosh(c + dx)}{d^2} + 2ab \cosh(c) \text{Chi}(dx)
\end{aligned}$$

Mathematica [A] time = 0.63, size = 135, normalized size = 0.90

$$\frac{1}{6} \left(-\frac{a^2 d^2 \cosh(c + dx)}{x} - \frac{2a^2 \cosh(c + dx)}{x^3} - \frac{a^2 d \sinh(c + dx)}{x^2} + a \text{Chi}(dx) (ad^3 \sinh(c) + 12b \cosh(c)) + a \text{Shi}(dx) (ad^3 \cosh(c) + 12b \sinh(c)) \right)$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x^3)^2*Cosh[c + d*x])/x^4,x]

[Out] ((-2*a^2*Cosh[c + d*x])/x^3 - (a^2*d^2*Cosh[c + d*x])/x - (12*b^2*x*Cosh[c + d*x])/d^2 + a*CoshIntegral[d*x]*(12*b*Cosh[c] + a*d^3*Sinh[c]) + (12*b^2*Sinh[c + d*x])/d^3 - (a^2*d*Sinh[c + d*x])/x^2 + (6*b^2*x^2*Sinh[c + d*x])/d + a*(a*d^3*Cosh[c] + 12*b*Sinh[c])*SinhIntegral[d*x])/6

fricas [A] time = 0.45, size = 187, normalized size = 1.25

$$\frac{2(a^2 d^5 x^2 + 12 b^2 d x^4 + 2 a^2 d^3) \cosh(dx + c) - ((a^2 d^6 + 12 a b d^3) x^3 \text{Ei}(dx) - (a^2 d^6 - 12 a b d^3) x^3 \text{Ei}(-dx)) \cosh(c) - ((a^2 d^6 + 12 a b d^3) x^3 \text{Ei}(dx) - (a^2 d^6 - 12 a b d^3) x^3 \text{Ei}(-dx)) \sinh(c)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^2*cosh(d*x+c)/x^4,x, algorithm="fricas")

[Out] -1/12*(2*(a^2*d^5*x^2 + 12*b^2*d*x^4 + 2*a^2*d^3)*cosh(d*x + c) - ((a^2*d^6 + 12*a*b*d^3)*x^3*Ei(d*x) - (a^2*d^6 - 12*a*b*d^3)*x^3*Ei(-d*x))*cosh(c) - 2*(6*b^2*d^2*x^5 - a^2*d^4*x + 12*b^2*x^3)*sinh(d*x + c) - ((a^2*d^6 + 12*a*b*d^3)*x^3*Ei(d*x) - (a^2*d^6 - 12*a*b*d^3)*x^3*Ei(-d*x))*sinh(c))

$$a*b*d^3)*x^3*Ei(d*x) + (a^2*d^6 - 12*a*b*d^3)*x^3*Ei(-d*x))*sinh(c))/(d^3*x^3)$$

giac [A] time = 0.13, size = 279, normalized size = 1.86

$$\frac{a^2 d^6 x^3 Ei(-dx) e^{(-c)} - a^2 d^6 x^3 Ei(dx) e^c + a^2 d^5 x^2 e^{(dx+c)} - 6 b^2 d^2 x^5 e^{(dx+c)} + a^2 d^5 x^2 e^{(-dx-c)} + 6 b^2 d^2 x^5 e^{(-dx-c)} - 12 a b d^3 x^3 Ei(d x) e^c + a^2 d^4 x^2 e^{(dx+c)} + 12 b^2 d x^4 e^{(dx+c)} - a^2 d^4 x^2 e^{(-dx-c)} + 12 b^2 d x^4 e^{(-dx-c)} + 2 a^2 d^3 e^{(dx+c)} - 12 b^2 x^3 e^{(dx+c)} + 2 a^2 d^3 e^{(-dx-c)} + 12 b^2 x^3 e^{(-dx-c)}}{(d^3 x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^2*cosh(d*x+c)/x^4,x, algorithm="giac")

$$[Out] -1/12*(a^2*d^6*x^3*Ei(-d*x)*e^{(-c)} - a^2*d^6*x^3*Ei(d*x)*e^c + a^2*d^5*x^2*e^{(d*x+c)} - 6*b^2*d^2*x^5*e^{(d*x+c)} + a^2*d^5*x^2*e^{(-d*x-c)} + 6*b^2*d^2*x^5*e^{(-d*x-c)} - 12*a*b*d^3*x^3*Ei(-d*x)*e^{(-c)} - 12*a*b*d^3*x^3*Ei(d*x)*e^c + a^2*d^4*x^2*e^{(d*x+c)} + 12*b^2*d*x^4*e^{(d*x+c)} - a^2*d^4*x^2*e^{(-d*x-c)} + 12*b^2*d*x^4*e^{(-d*x-c)} + 2*a^2*d^3*e^{(d*x+c)} - 12*b^2*x^3*e^{(d*x+c)} + 2*a^2*d^3*e^{(-d*x-c)} + 12*b^2*x^3*e^{(-d*x-c)}))/(d^3*x^3)$$

maple [A] time = 0.24, size = 261, normalized size = 1.74

$$\frac{b^2 e^{-dx-c} x^2}{2d} - \frac{b^2 e^{-dx-c} x}{d^2} + \frac{d^3 a^2 e^{-c} Ei(1, dx)}{12} - \frac{b^2 e^{-dx-c}}{d^3} - \frac{d^2 a^2 e^{-dx-c}}{12x} + \frac{d a^2 e^{-dx-c}}{12x^2} - \frac{a^2 e^{-dx-c}}{6x^3} - a b e^{-c} Ei(1, dx) - \frac{a^2 e^{dx+c}}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)^2*cosh(d*x+c)/x^4,x)

$$[Out] -1/2/d*b^2*exp(-d*x-c)*x^2-1/d^2*b^2*exp(-d*x-c)*x+1/12*d^3*a^2*exp(-c)*Ei(1,d*x)-1/d^3*b^2*exp(-d*x-c)-1/12*d^2*a^2*exp(-d*x-c)/x+1/12*d*a^2*exp(-d*x-c)/x^2-1/6*a^2*exp(-d*x-c)/x^3-a*b*exp(-c)*Ei(1,d*x)-1/6*a^2/x^3*exp(d*x+c)+1/2/d*b^2*exp(d*x+c)*x^2-1/d^2*b^2*exp(d*x+c)*x-1/12*d^3*a^2*exp(c)*Ei(1,-d*x)+1/d^3*b^2*exp(d*x+c)-1/12*d*a^2/x^2*exp(d*x+c)-1/12*d^2*a^2/x*exp(d*x+c)-a*b*exp(c)*Ei(1,-d*x)$$

maxima [A] time = 0.44, size = 188, normalized size = 1.25

$$\frac{1}{6} \left((d^2 e^{(-c)} \Gamma(-2, dx) - d^2 e^c \Gamma(-2, -dx)) a^2 - b^2 \left(\frac{(d^3 x^3 e^c - 3 d^2 x^2 e^c + 6 dx e^c - 6 e^c) e^{(dx)}}{d^4} + \frac{(d^3 x^3 + 3 d^2 x^2 + 6 dx + 6 e^{(-c)}) e^{(-dx-c)}}{d^4} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^2*cosh(d*x+c)/x^4,x, algorithm="maxima")

$$[Out] 1/6*((d^2*e^{(-c)}*gamma(-2, d*x) - d^2*e^c*gamma(-2, -d*x))*a^2 - b^2*((d^3*x^3*e^c - 3*d^2*x^2*e^c + 6*d*x*e^c - 6*e^c)*e^{(d*x)}/d^4 + (d^3*x^3 + 3*d^2*x^2 + 6*d*x + 6)*e^{(-d*x-c)}/d^4) - 4*a*b*cosh(d*x+c)*log(x^3)/d + 6*(E$$

$i(-d*x)*e^{-c} + Ei(d*x)*e^c)*a*b/d)*d + 1/3*(b^2*x^3 + 2*a*b*log(x^3) - a^2/x^3)*cosh(d*x + c)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c + dx) (bx^3 + a)^2}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cosh(c + d*x)*(a + b*x^3)^2)/x^4, x)

[Out] int((cosh(c + d*x)*(a + b*x^3)^2)/x^4, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^3)^2 \cosh(c + dx)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a)**2*cosh(d*x+c)/x**4, x)

[Out] Integral((a + b*x**3)**2*cosh(c + d*x)/x**4, x)

$$3.93 \quad \int \frac{(a+bx^3)^2 \cosh(c+dx)}{x^5} dx$$

Optimal. Leaf size=167

$$\frac{1}{24}a^2d^4 \cosh(c)\text{Chi}(dx) + \frac{1}{24}a^2d^4 \sinh(c)\text{Shi}(dx) - \frac{a^2d^3 \sinh(c+dx)}{24x} - \frac{a^2d^2 \cosh(c+dx)}{24x^2} - \frac{a^2 \cosh(c+dx)}{4x^4} - \frac{a^2d \sinh(c+dx)}{12x^3}$$

[Out] $1/24*a^2*d^4*\text{Chi}(d*x)*\cosh(c) - b^2*\cosh(d*x+c)/d^2 - 1/4*a^2*\cosh(d*x+c)/x^4 - 1/24*a^2*d^2*\cosh(d*x+c)/x^2 - 2*a*b*\cosh(d*x+c)/x + 2*a*b*d*\cosh(c)*\text{Shi}(d*x) + 2*a*b*d*\text{Chi}(d*x)*\sinh(c) + 1/24*a^2*d^4*\text{Shi}(d*x)*\sinh(c) - 1/12*a^2*d*\sinh(d*x+c)/x^3 - 1/24*a^2*d^3*\sinh(d*x+c)/x + b^2*x*\sinh(d*x+c)/d$

Rubi [A] time = 0.31, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {5287, 3297, 3303, 3298, 3301, 3296, 2638}

$$\frac{1}{24}a^2d^4 \cosh(c)\text{Chi}(dx) + \frac{1}{24}a^2d^4 \sinh(c)\text{Shi}(dx) - \frac{a^2d^2 \cosh(c+dx)}{24x^2} - \frac{a^2d^3 \sinh(c+dx)}{24x} - \frac{a^2d \sinh(c+dx)}{12x^3} - \frac{a^2 \cosh(c+dx)}{4x^4}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + b*x^3)^2*\text{Cosh}[c + d*x])/x^5, x]$

[Out] $-(b^2*\text{Cosh}[c + d*x])/d^2 - (a^2*\text{Cosh}[c + d*x])/(4*x^4) - (a^2*d^2*\text{Cosh}[c + d*x])/(24*x^2) - (2*a*b*\text{Cosh}[c + d*x])/x + (a^2*d^4*\text{Cosh}[c]*\text{CoshIntegral}[d*x])/24 + 2*a*b*d*\text{CoshIntegral}[d*x]*\text{Sinh}[c] - (a^2*d*\text{Sinh}[c + d*x])/(12*x^3) - (a^2*d^3*\text{Sinh}[c + d*x])/(24*x) + (b^2*x*\text{Sinh}[c + d*x])/d + 2*a*b*d*\text{Cosh}[c]*\text{SinhIntegral}[d*x] + (a^2*d^4*\text{Sinh}[c]*\text{SinhIntegral}[d*x])/24$

Rule 2638

$\text{Int}[\sin[(c_.) + (d_.)*(x_.)], x_Symbol] := -\text{Simp}[\text{Cos}[c + d*x]/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 3296

$\text{Int}[(c_. + (d_.)*(x_.))^{(m_.)}*\sin[(e_.) + (f_.)*(x_.)], x_Symbol] := -\text{Simp}[(c + d*x)^m*\text{Cos}[e + f*x]/f, x] + \text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{(m-1)}*\text{Cos}[e + f*x], x], x] /; \text{FreeQ}\{c, d, e, f\}, x \ \&\& \ \text{GtQ}[m, 0]$

Rule 3297

$\text{Int}[(c_. + (d_.)*(x_.))^{(m_.)}*\sin[(e_.) + (f_.)*(x_.)], x_Symbol] := \text{Simp}[(c + d*x)^{(m+1)}*\text{Sin}[e + f*x]/(d*(m+1)), x] - \text{Dist}[f/(d*(m+1)), \text{Int}[(c + d*x)^{(m+1)}*\text{Cos}[e + f*x], x], x] /; \text{FreeQ}\{c, d, e, f\}, x \ \&\& \ \text{LtQ}[m, -1]$

]

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f,
fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz},
x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

Rule 5287

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + bx^3)^2 \cosh(c + dx)}{x^5} dx &= \int \left(\frac{a^2 \cosh(c + dx)}{x^5} + \frac{2ab \cosh(c + dx)}{x^2} + b^2 x \cosh(c + dx) \right) dx \\
&= a^2 \int \frac{\cosh(c + dx)}{x^5} dx + (2ab) \int \frac{\cosh(c + dx)}{x^2} dx + b^2 \int x \cosh(c + dx) dx \\
&= -\frac{a^2 \cosh(c + dx)}{4x^4} - \frac{2ab \cosh(c + dx)}{x} + \frac{b^2 x \sinh(c + dx)}{d} - \frac{b^2 \int \sinh(c + dx) dx}{d} \\
&= -\frac{b^2 \cosh(c + dx)}{d^2} - \frac{a^2 \cosh(c + dx)}{4x^4} - \frac{2ab \cosh(c + dx)}{x} - \frac{a^2 d \sinh(c + dx)}{12x^3} + \frac{b^2}{12} \frac{\sinh^2(c + dx)}{x^3} \\
&= -\frac{b^2 \cosh(c + dx)}{d^2} - \frac{a^2 \cosh(c + dx)}{4x^4} - \frac{a^2 d^2 \cosh(c + dx)}{24x^2} - \frac{2ab \cosh(c + dx)}{x} + \frac{b^2}{12} \frac{\sinh^2(c + dx)}{x^3} \\
&= -\frac{b^2 \cosh(c + dx)}{d^2} - \frac{a^2 \cosh(c + dx)}{4x^4} - \frac{a^2 d^2 \cosh(c + dx)}{24x^2} - \frac{2ab \cosh(c + dx)}{x} + \frac{b^2}{12} \frac{\sinh^2(c + dx)}{x^3} \\
&= -\frac{b^2 \cosh(c + dx)}{d^2} - \frac{a^2 \cosh(c + dx)}{4x^4} - \frac{a^2 d^2 \cosh(c + dx)}{24x^2} - \frac{2ab \cosh(c + dx)}{x} + \frac{b^2}{12} \frac{\sinh^2(c + dx)}{x^3} \\
&= -\frac{b^2 \cosh(c + dx)}{d^2} - \frac{a^2 \cosh(c + dx)}{4x^4} - \frac{a^2 d^2 \cosh(c + dx)}{24x^2} - \frac{2ab \cosh(c + dx)}{x} + \frac{b^2}{12} \frac{\sinh^2(c + dx)}{x^3}
\end{aligned}$$

Mathematica [A] time = 0.58, size = 150, normalized size = 0.90

$$\frac{1}{24} \left(-\frac{a^2 d^3 \sinh(c + dx)}{x} - \frac{a^2 d^2 \cosh(c + dx)}{x^2} - \frac{6a^2 \cosh(c + dx)}{x^4} - \frac{2a^2 d \sinh(c + dx)}{x^3} + ad \operatorname{Chi}(dx) (ad^3 \cosh(c) + \sinh^2(c)) \right)$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x^3)^2*Cosh[c + d*x])/x^5,x]

[Out] ((-24*b^2*Cosh[c + d*x])/d^2 - (6*a^2*Cosh[c + d*x])/x^4 - (a^2*d^2*Cosh[c + d*x])/x^2 - (48*a*b*Cosh[c + d*x])/x + a*d*CoshIntegral[d*x]*(a*d^3*Cosh[c] + 48*b*Sinh[c]) - (2*a^2*d*Sinh[c + d*x])/x^3 - (a^2*d^3*Sinh[c + d*x])/x + (24*b^2*x*Sinh[c + d*x])/d + a*d*(48*b*Cosh[c] + a*d^3*Sinh[c])*SinhIntegral[d*x])/24

fricas [A] time = 0.40, size = 196, normalized size = 1.17

$$-\frac{2(a^2 d^4 x^2 + 48 a b d^2 x^3 + 24 b^2 x^4 + 6 a^2 d^2) \cosh(dx + c) - ((a^2 d^6 + 48 a b d^3) x^4 \operatorname{Ei}(dx) + (a^2 d^6 - 48 a b d^3) x^4 \operatorname{Ei}(-dx))}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^2*cosh(d*x+c)/x^5,x, algorithm="fricas")

[Out] $-1/48*(2*(a^2*d^4*x^2 + 48*a*b*d^2*x^3 + 24*b^2*x^4 + 6*a^2*d^2)*\cosh(d*x + c) - ((a^2*d^6 + 48*a*b*d^3)*x^4*Ei(d*x) + (a^2*d^6 - 48*a*b*d^3)*x^4*Ei(-d*x))*\cosh(c) + 2*(a^2*d^5*x^3 - 24*b^2*d*x^5 + 2*a^2*d^3*x)*\sinh(d*x + c) - ((a^2*d^6 + 48*a*b*d^3)*x^4*Ei(d*x) - (a^2*d^6 - 48*a*b*d^3)*x^4*Ei(-d*x))*\sinh(c))/(d^2*x^4)$

giac [B] time = 0.14, size = 315, normalized size = 1.89

$$\frac{a^2 d^6 x^4 Ei(-dx) e^{(-c)} + a^2 d^6 x^4 Ei(dx) e^c - a^2 d^5 x^3 e^{(dx+c)} + a^2 d^5 x^3 e^{(-dx-c)} - 48 abd^3 x^4 Ei(-dx) e^{(-c)} + 48 abd^3 x^4 Ei(dx) e^c}{d^2 x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a)^2*cosh(d*x+c)/x^5,x, algorithm="giac")`

[Out] $1/48*(a^2*d^6*x^4*Ei(-d*x)*e^{(-c)} + a^2*d^6*x^4*Ei(d*x)*e^c - a^2*d^5*x^3*e^{(d*x + c)} + a^2*d^5*x^3*e^{(-d*x - c)} - 48*a*b*d^3*x^4*Ei(-d*x)*e^{(-c)} + 48*a*b*d^3*x^4*Ei(d*x)*e^c - a^2*d^4*x^2*e^{(d*x + c)} + 24*b^2*d*x^5*e^{(d*x + c)} - a^2*d^4*x^2*e^{(-d*x - c)} - 24*b^2*d*x^5*e^{(-d*x - c)} - 48*a*b*d^2*x^3*e^{(d*x + c)} - 48*a*b*d^2*x^3*e^{(-d*x - c)} - 2*a^2*d^3*x*e^{(d*x + c)} - 24*b^2*x^4*e^{(d*x + c)} + 2*a^2*d^3*x*e^{(-d*x - c)} - 24*b^2*x^4*e^{(-d*x - c)} - 6*a^2*d^2*e^{(d*x + c)} - 6*a^2*d^2*e^{(-d*x - c)})/(d^2*x^4)$

maple [A] time = 0.26, size = 292, normalized size = 1.75

$$\frac{d^2 a^2 e^{-dx-c}}{48x^2} + \frac{d a^2 e^{-dx-c}}{24x^3} - \frac{a^2 e^{-dx-c}}{8x^4} - \frac{b^2 e^{-dx-c} x}{2d} - \frac{b^2 e^{-dx-c}}{2d^2} - \frac{d^4 a^2 e^{-c} Ei(1, dx)}{48} + \frac{d^3 a^2 e^{-dx-c}}{48x} - \frac{ab e^{-dx-c}}{x} + dab e^{-c} Ei(1, dx)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^3+a)^2*cosh(d*x+c)/x^5,x)`

[Out] $-1/48*d^2*a^2*\exp(-d*x-c)/x^2+1/24*d*a^2*\exp(-d*x-c)/x^3-1/8*a^2*\exp(-d*x-c)/x^4-1/2/d*b^2*\exp(-d*x-c)*x-1/2/d^2*b^2*\exp(-d*x-c)-1/48*d^4*a^2*\exp(-c)*Ei(1,d*x)+1/48*d^3*a^2*\exp(-d*x-c)/x-a*b*\exp(-d*x-c)/x+d*a*b*\exp(-c)*Ei(1,d*x)+1/2/d*b^2*\exp(d*x+c)*x-1/48*d^4*a^2*\exp(c)*Ei(1,-d*x)-1/2/d^2*b^2*\exp(d*x+c)-1/24*d*a^2/x^3*\exp(d*x+c)-1/48*d^2*a^2/x^2*\exp(d*x+c)-1/48*d^3*a^2/x*\exp(d*x+c)-1/8*a^2/x^4*\exp(d*x+c)-a*b/x*\exp(d*x+c)-d*a*b*\exp(c)*Ei(1,-d*x)$

maxima [A] time = 0.46, size = 154, normalized size = 0.92

$$\frac{1}{8} \left(a^2 d^3 e^{(-c)} \Gamma(-3, dx) + a^2 d^3 e^c \Gamma(-3, -dx) - 8 ab Ei(-dx) e^{(-c)} + 8 ab Ei(dx) e^c - \frac{2(d^2 x^2 e^c - 2 dx e^c + 2 e^c) b^2 e^{(dx)}}{d^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a)^2*cosh(d*x+c)/x^5,x, algorithm="maxima")`

[Out] $\frac{1}{8}(a^2 d^3 e^{-c}) \gamma(-3, dx) + a^2 d^3 e^c \gamma(-3, -dx) - 8ab \operatorname{Ei}(-dx) e^{-c} + 8ab \operatorname{Ei}(dx) e^c - 2(d^2 x^2 e^c - 2dx e^c + 2e^c) b^2 e^{dx} / d^3 - 2(d^2 x^2 + 2dx + 2) b^2 e^{-(dx+c)} / d^3 * d + \frac{1}{4}(2b^2 x^2 - (8abx^3 + a^2) / x^4) \cosh(dx+c)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c+dx) (bx^3+a)^2}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((cosh(c + d*x)*(a + b*x^3)^2)/x^5, x)`

[Out] `int((cosh(c + d*x)*(a + b*x^3)^2)/x^5, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^3)^2 \cosh(c + dx)}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**3+a)**2*cosh(d*x+c)/x**5, x)`

[Out] `Integral((a + b*x**3)**2*cosh(c + d*x)/x**5, x)`

3.94 $\int \frac{x^4 \cosh(c+dx)}{a+bx^3} dx$

Optimal. Leaf size=373

$$\frac{(-1)^{2/3} a^{2/3} \cosh\left(\frac{\sqrt[3]{-1} \sqrt[3]{ad}}{\sqrt[3]{b}} + c\right) \text{Chi}\left(\frac{\sqrt[3]{-1} \sqrt[3]{ad}}{\sqrt[3]{b}} - dx\right)}{3b^{5/3}} - \frac{\sqrt[3]{-1} a^{2/3} \cosh\left(c - \frac{(-1)^{2/3} \sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(-xd - \frac{(-1)^{2/3} \sqrt[3]{ad}}{\sqrt[3]{b}}\right)}{3b^{5/3}} + a^{2/3}$$

[Out] $\frac{1}{3} a^{2/3} \text{Chi}\left(\frac{a^{1/3} d}{b^{1/3}} + dx\right) \cosh\left(c - \frac{a^{1/3} d}{b^{1/3}}\right) / b^{5/3} + \frac{1}{3} (-1)^{2/3} a^{2/3} \text{Chi}\left(\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - dx\right) \cosh\left(c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right) / b^{5/3} - \frac{1}{3} (-1)^{1/3} a^{2/3} \text{Chi}\left(\frac{-(-1)^{2/3} a^{1/3} d}{b^{1/3}} - dx\right) \cosh\left(c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right) / b^{5/3} - \frac{\cosh(dx+c)}{b/d^2} + \frac{1}{3} a^{2/3} \text{Shi}\left(\frac{a^{1/3} d}{b^{1/3}} + dx\right) \sinh\left(c - \frac{a^{1/3} d}{b^{1/3}}\right) / b^{5/3} + \frac{1}{3} (-1)^{2/3} a^{2/3} \text{Shi}\left(\frac{-(-1)^{1/3} a^{1/3} d}{b^{1/3}} + dx\right) \sinh\left(c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right) / b^{5/3} - \frac{1}{3} (-1)^{1/3} a^{2/3} \text{Shi}\left(\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + dx\right) \sinh\left(c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right) / b^{5/3} + x \sinh(dx+c) / b/d$

Rubi [A] time = 0.94, antiderivative size = 373, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {5293, 3296, 2638, 3303, 3298, 3301}

$$\frac{(-1)^{2/3} a^{2/3} \cosh\left(\frac{\sqrt[3]{-1} \sqrt[3]{ad}}{\sqrt[3]{b}} + c\right) \text{Chi}\left(\frac{\sqrt[3]{-1} \sqrt[3]{ad}}{\sqrt[3]{b}} - dx\right)}{3b^{5/3}} - \frac{\sqrt[3]{-1} a^{2/3} \cosh\left(c - \frac{(-1)^{2/3} \sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(-xd - \frac{(-1)^{2/3} \sqrt[3]{ad}}{\sqrt[3]{b}}\right)}{3b^{5/3}} + a^{2/3}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(x^4 \text{Cosh}[c + dx]) / (a + b x^3), x]$

[Out] $-\frac{\text{Cosh}[c + dx]}{b d^2} + \frac{(-1)^{2/3} a^{2/3} \text{Cosh}\left[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right] \text{CoshIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - dx\right]}{3 b^{5/3}} - \frac{(-1)^{1/3} a^{2/3} \text{Cosh}\left[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right] \text{CoshIntegral}\left[-\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} - dx\right]}{3 b^{5/3}} + \frac{a^{2/3} \text{Cosh}\left[c - \frac{a^{1/3} d}{b^{1/3}}\right] \text{CoshIntegral}\left[\frac{a^{1/3} d}{b^{1/3}} + dx\right]}{3 b^{5/3}} + \frac{x \sinh[c + dx]}{b d} - \frac{(-1)^{2/3} a^{2/3} \sinh\left[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right] \text{SinhIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - dx\right]}{3 b^{5/3}} + \frac{a^{2/3} \sinh\left[c - \frac{a^{1/3} d}{b^{1/3}}\right] \text{SinhIntegral}\left[\frac{a^{1/3} d}{b^{1/3}} + dx\right]}{3 b^{5/3}} - \frac{(-1)^{1/3} a^{2/3} \sinh\left[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right] \text{SinhIntegral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + dx\right]}{3 b^{5/3}}$

Rule 2638

$\text{Int}[\sin[(c_.) + (d_.)(x_.)], x_Symbol] \rightarrow -\text{Simp}[\text{Cos}[c + dx]/d, x] /;$ FreeQ[{c, d}, x]

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[
((c + d*x)^m*cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :=
Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] &&
EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :=
Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] &&
EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

Rule 5293

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :=
Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d}, x] &&
ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4 \cosh(c + dx)}{a + bx^3} dx &= \int \left(\frac{x \cosh(c + dx)}{b} - \frac{ax \cosh(c + dx)}{b(a + bx^3)} \right) dx \\
&= \frac{\int x \cosh(c + dx) dx}{b} - \frac{a \int \frac{x \cosh(c + dx)}{a + bx^3} dx}{b} \\
&= \frac{x \sinh(c + dx)}{bd} - \frac{a \int \left(-\frac{\cosh(c + dx)}{3 \sqrt[3]{a} \sqrt[3]{b} (\sqrt[3]{a} + \sqrt[3]{b} x)} - \frac{(-1)^{2/3} \cosh(c + dx)}{3 \sqrt[3]{a} \sqrt[3]{b} (\sqrt[3]{a} - \sqrt[3]{-1} \sqrt[3]{b} x)} + \frac{\sqrt[3]{-1} \cosh(c + dx)}{3 \sqrt[3]{a} \sqrt[3]{b} (\sqrt[3]{a} + (-1)^{2/3} \sqrt[3]{b} x)} \right) dx}{b} \\
&= -\frac{\cosh(c + dx)}{bd^2} + \frac{x \sinh(c + dx)}{bd} + \frac{a^{2/3} \int \frac{\cosh(c + dx)}{\sqrt[3]{a} + \sqrt[3]{b} x} dx}{3b^{4/3}} - \frac{(\sqrt[3]{-1} a^{2/3}) \int \frac{\cosh(c + dx)}{\sqrt[3]{a} + (-1)^{2/3} \sqrt[3]{b} x} dx}{3b^{4/3}} \\
&= -\frac{\cosh(c + dx)}{bd^2} + \frac{x \sinh(c + dx)}{bd} + \frac{\left(a^{2/3} \cosh \left(c - \frac{\sqrt[3]{a} d}{\sqrt[3]{b}} \right) \int \frac{\cosh \left(\frac{\sqrt[3]{a} d}{\sqrt[3]{b}} + dx \right)}{\sqrt[3]{a} + \sqrt[3]{b} x} dx \right)}{3b^{4/3}} - \frac{\left(\sqrt[3]{-1} a^{2/3} \cosh \left(c + \frac{\sqrt[3]{-1} \sqrt[3]{a} d}{\sqrt[3]{b}} \right) \text{Chi} \left(\frac{\sqrt[3]{-1} \sqrt[3]{a} d}{\sqrt[3]{b}} - dx \right) \right)}{3b^{5/3}} - \frac{\sqrt[3]{-1} a^{2/3} \cosh(c + dx)}{3b^{5/3}}
\end{aligned}$$

Mathematica [C] time = 0.51, size = 213, normalized size = 0.57

$$ad^2 \text{RootSum} \left[\#1^3 b + a \&, \frac{-\sinh(\#1 d + c) \text{Chi}(d(x - \#1)) + \cosh(\#1 d + c) \text{Chi}(d(x - \#1)) + \sinh(\#1 d + c) \text{Shi}(d(x - \#1)) - \cosh(\#1 d + c) \text{Shi}(d(x - \#1))}{\#1} \right]$$

Antiderivative was successfully verified.

[In] Integrate[(x^4*Cosh[c + d*x])/(a + b*x^3), x]

[Out] -1/6*(a*d^2*RootSum[a + b*#1^3 &, (Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] - CoshIntegral[d*(x - #1)]*Sinh[c + d*#1] - Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] + Sinh[c + d*#1]*SinhIntegral[d*(x - #1)])/#1 &] + a*d^2*RootSum[a + b*#1^3 &, (Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] + CoshIntegral[d*(x - #1)]*Sinh[c + d*#1] + Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] + Sinh[c + d*#1]*SinhIntegral[d*(x - #1)])/#1 &] + 6*b*(Cosh[c + d*x] - d*x*Sinh[c + d*x])/(b^2*d^2)

fricas [B] time = 0.53, size = 989, normalized size = 2.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*cosh(d*x+c)/(b*x^3+a),x, algorithm="fricas")

[Out] $\frac{1}{12} \left(\left(\frac{a d^3}{b} \right)^{2/3} \left((\sqrt{-3} - 1) \cosh(dx + c)^2 - (\sqrt{-3} - 1) \sinh(dx + c)^2 \right) \operatorname{Ei} \left(dx - \frac{1}{2} \left(\frac{a d^3}{b} \right)^{1/3} (\sqrt{-3} + 1) \right) \cosh \left(\frac{1}{2} \left(\frac{a d^3}{b} \right)^{1/3} (\sqrt{-3} + 1) + c \right) + \left(-\frac{a d^3}{b} \right)^{2/3} \left((\sqrt{-3} - 1) \cosh(dx + c)^2 - (\sqrt{-3} - 1) \sinh(dx + c)^2 \right) \operatorname{Ei} \left(-dx - \frac{1}{2} \left(-\frac{a d^3}{b} \right)^{1/3} (\sqrt{-3} + 1) \right) \cosh \left(\frac{1}{2} \left(-\frac{a d^3}{b} \right)^{1/3} (\sqrt{-3} + 1) - c \right) - \left(\frac{a d^3}{b} \right)^{2/3} \left((\sqrt{-3} + 1) \cosh(dx + c)^2 - (\sqrt{-3} + 1) \sinh(dx + c)^2 \right) \operatorname{Ei} \left(dx + \frac{1}{2} \left(\frac{a d^3}{b} \right)^{1/3} (\sqrt{-3} - 1) \right) \cosh \left(\frac{1}{2} \left(\frac{a d^3}{b} \right)^{1/3} (\sqrt{-3} - 1) - c \right) - \left(-\frac{a d^3}{b} \right)^{2/3} \left((\sqrt{-3} + 1) \cosh(dx + c)^2 - (\sqrt{-3} + 1) \sinh(dx + c)^2 \right) \operatorname{Ei} \left(-dx + \frac{1}{2} \left(-\frac{a d^3}{b} \right)^{1/3} (\sqrt{-3} - 1) \right) \cosh \left(\frac{1}{2} \left(-\frac{a d^3}{b} \right)^{1/3} (\sqrt{-3} - 1) + c \right) + 2 \left(-\frac{a d^3}{b} \right)^{2/3} \left(\cosh(dx + c)^2 - \sinh(dx + c)^2 \right) \operatorname{Ei} \left(-dx + \left(-\frac{a d^3}{b} \right)^{1/3} \right) \cosh \left(c + \left(-\frac{a d^3}{b} \right)^{1/3} \right) + 2 \left(\frac{a d^3}{b} \right)^{2/3} \left(\cosh(dx + c)^2 - \sinh(dx + c)^2 \right) \operatorname{Ei} \left(dx + \left(\frac{a d^3}{b} \right)^{1/3} \right) \cosh \left(-c + \left(\frac{a d^3}{b} \right)^{1/3} \right) + \left(\frac{a d^3}{b} \right)^{2/3} \left((\sqrt{-3} - 1) \cosh(dx + c)^2 - (\sqrt{-3} - 1) \sinh(dx + c)^2 \right) \operatorname{Ei} \left(dx - \frac{1}{2} \left(\frac{a d^3}{b} \right)^{1/3} (\sqrt{-3} + 1) \right) \sinh \left(\frac{1}{2} \left(\frac{a d^3}{b} \right)^{1/3} (\sqrt{-3} + 1) + c \right) + \left(-\frac{a d^3}{b} \right)^{2/3} \left((\sqrt{-3} - 1) \cosh(dx + c)^2 - (\sqrt{-3} - 1) \sinh(dx + c)^2 \right) \operatorname{Ei} \left(-dx - \frac{1}{2} \left(-\frac{a d^3}{b} \right)^{1/3} (\sqrt{-3} + 1) \right) \sinh \left(\frac{1}{2} \left(-\frac{a d^3}{b} \right)^{1/3} (\sqrt{-3} + 1) - c \right) + \left(\frac{a d^3}{b} \right)^{2/3} \left((\sqrt{-3} + 1) \cosh(dx + c)^2 - (\sqrt{-3} + 1) \sinh(dx + c)^2 \right) \operatorname{Ei} \left(dx + \frac{1}{2} \left(\frac{a d^3}{b} \right)^{1/3} (\sqrt{-3} - 1) \right) \sinh \left(\frac{1}{2} \left(\frac{a d^3}{b} \right)^{1/3} (\sqrt{-3} - 1) - c \right) + \left(-\frac{a d^3}{b} \right)^{2/3} \left((\sqrt{-3} + 1) \cosh(dx + c)^2 - (\sqrt{-3} + 1) \sinh(dx + c)^2 \right) \operatorname{Ei} \left(-dx + \frac{1}{2} \left(-\frac{a d^3}{b} \right)^{1/3} (\sqrt{-3} - 1) \right) \sinh \left(\frac{1}{2} \left(-\frac{a d^3}{b} \right)^{1/3} (\sqrt{-3} - 1) + c \right) - 2 \left(-\frac{a d^3}{b} \right)^{2/3} \left(\cosh(dx + c)^2 - \sinh(dx + c)^2 \right) \operatorname{Ei} \left(-dx + \left(-\frac{a d^3}{b} \right)^{1/3} \right) \sinh \left(c + \left(-\frac{a d^3}{b} \right)^{1/3} \right) - 2 \left(\frac{a d^3}{b} \right)^{2/3} \left(\cosh(dx + c)^2 - \sinh(dx + c)^2 \right) \operatorname{Ei} \left(dx + \left(\frac{a d^3}{b} \right)^{1/3} \right) \sinh \left(-c + \left(\frac{a d^3}{b} \right)^{1/3} \right) + 12 dx \sinh(dx + c) - 12 \cosh(dx + c) \right) / (b d^2 \cosh(dx + c)^2 - b d^2 \sinh(dx + c)^2)$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 \cosh(dx + c)}{bx^3 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*cosh(d*x+c)/(b*x^3+a),x, algorithm="giac")

[Out] integrate(x^4*cosh(d*x + c)/(b*x^3 + a), x)

maple [C] time = 0.19, size = 925, normalized size = 2.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4*cosh(d*x+c)/(b*x^3+a),x)


```
[Out] -1/2/d*exp(-d*x-c)/b*x-1/2/d^2*exp(-d*x-c)/b-1/6/d^2/b^2*sum((6*_R1^2*b*c^2
-_R1*a*d^3-8*_R1*b*c^3-3*a*c*d^3+3*b*c^4)/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(
1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/6/d^2*
c^4/b*sum(1/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b
-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+2/3/d^2*c^3/b*sum(_R1/(_R1^2-2*_R1*c+c
^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3
-b*c^3))-1/d^2*c^2/b*sum(_R1^2/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c
),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+2/3/d^2*c/b^2*sum((
3*_R1^2*b*c-3*_R1*b*c^2-a*d^3+b*c^3)/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x
-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/2/d/b*exp(d
*x+c)*x-1/2/d^2/b*exp(d*x+c)-1/6/d^2/b^2*sum((6*_R1^2*b*c^2-_R1*a*d^3-8*_R1
*b*c^3-3*a*c*d^3+3*b*c^4)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1
=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/6/d^2*c^4/b*sum(1/(_R1
^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z
*b*c^2+a*d^3-b*c^3))+2/3/d^2*c^3/b*sum(_R1/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(
1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/d^2*c
^2/b*sum(_R1^2/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z
^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+2/3/d^2*c/b^2*sum((3*_R1^2*b*c-3*_R
1*b*c^2-a*d^3+b*c^3)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=Root
Of(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*cosh(d*x+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4 \cosh(c + dx)}{bx^3 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*cosh(c + d*x))/(a + b*x^3),x)
```

```
[Out] int((x^4*cosh(c + d*x))/(a + b*x^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 \cosh(c + dx)}{a + bx^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*cosh(d*x+c)/(b*x**3+a),x)
```

```
[Out] Integral(x**4*cosh(c + d*x)/(a + b*x**3), x)
```

3.95 $\int \frac{x^3 \cosh(c+dx)}{a+bx^3} dx$

Optimal. Leaf size=358

$$\frac{\sqrt[3]{-1} \sqrt[3]{a} \cosh\left(\frac{\sqrt[3]{-1} \sqrt[3]{ad}}{\sqrt[3]{b}} + c\right) \text{Chi}\left(\frac{\sqrt[3]{-1} \sqrt[3]{ad}}{\sqrt[3]{b}} - dx\right)}{3b^{4/3}} - \frac{(-1)^{2/3} \sqrt[3]{a} \cosh\left(c - \frac{(-1)^{2/3} \sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(-xd - \frac{(-1)^{2/3} \sqrt[3]{ad}}{\sqrt[3]{b}}\right)}{3b^{4/3}} - \frac{\sqrt[3]{a}}{\sqrt[3]{b}}$$

[Out] $-1/3*a^{(1/3)}*\text{Chi}(a^{(1/3)}*d/b^{(1/3)}+d*x)*\cosh(c-a^{(1/3)}*d/b^{(1/3)})/b^{(4/3)}+1/3*(-1)^{(1/3)}*a^{(1/3)}*\text{Chi}((-1)^{(1/3)}*a^{(1/3)}*d/b^{(1/3)}-d*x)*\cosh(c+(-1)^{(1/3)}*a^{(1/3)}*d/b^{(1/3)})/b^{(4/3)}-1/3*(-1)^{(2/3)}*a^{(1/3)}*\text{Chi}(-(-1)^{(2/3)}*a^{(1/3)}*d/b^{(1/3)}-d*x)*\cosh(c-(-1)^{(2/3)}*a^{(1/3)}*d/b^{(1/3)})/b^{(4/3)}-1/3*a^{(1/3)}*\text{Shi}(a^{(1/3)}*d/b^{(1/3)}+d*x)*\sinh(c-a^{(1/3)}*d/b^{(1/3)})/b^{(4/3)}+1/3*(-1)^{(1/3)}*a^{(1/3)}*\text{Shi}(-(-1)^{(1/3)}*a^{(1/3)}*d/b^{(1/3)}+d*x)*\sinh(c+(-1)^{(1/3)}*a^{(1/3)}*d/b^{(1/3)})/b^{(4/3)}-1/3*(-1)^{(2/3)}*a^{(1/3)}*\text{Shi}((-1)^{(2/3)}*a^{(1/3)}*d/b^{(1/3)}+d*x)*\sinh(c-(-1)^{(2/3)}*a^{(1/3)}*d/b^{(1/3)})/b^{(4/3)}+\sinh(d*x+c)/b/d$

Rubi [A] time = 0.65, antiderivative size = 358, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {5293, 2637, 5281, 3303, 3298, 3301}

$$\frac{\sqrt[3]{-1} \sqrt[3]{a} \cosh\left(\frac{\sqrt[3]{-1} \sqrt[3]{ad}}{\sqrt[3]{b}} + c\right) \text{Chi}\left(\frac{\sqrt[3]{-1} \sqrt[3]{ad}}{\sqrt[3]{b}} - dx\right)}{3b^{4/3}} - \frac{(-1)^{2/3} \sqrt[3]{a} \cosh\left(c - \frac{(-1)^{2/3} \sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(-xd - \frac{(-1)^{2/3} \sqrt[3]{ad}}{\sqrt[3]{b}}\right)}{3b^{4/3}} - \frac{\sqrt[3]{a}}{\sqrt[3]{b}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(x^3*\text{Cosh}[c + d*x])/(a + b*x^3), x]$

[Out] $((-1)^{(1/3)}*a^{(1/3)}*\text{Cosh}[c + ((-1)^{(1/3)}*a^{(1/3)}*d)/b^{(1/3)}]*\text{CoshIntegral}[((-1)^{(1/3)}*a^{(1/3)}*d)/b^{(1/3)} - d*x])/(3*b^{(4/3)}) - ((-1)^{(2/3)}*a^{(1/3)}*\text{Cosh}[c - ((-1)^{(2/3)}*a^{(1/3)}*d)/b^{(1/3)}]*\text{CoshIntegral}[-(((-1)^{(2/3)}*a^{(1/3)}*d)/b^{(1/3)} - d*x])/(3*b^{(4/3)}) - (a^{(1/3)}*\text{Cosh}[c - (a^{(1/3)}*d)/b^{(1/3)}]*\text{CoshIntegral}[(a^{(1/3)}*d)/b^{(1/3)} + d*x])/(3*b^{(4/3)}) + \text{Sinh}[c + d*x]/(b*d) - ((-1)^{(1/3)}*a^{(1/3)}*\text{Sinh}[c + ((-1)^{(1/3)}*a^{(1/3)}*d)/b^{(1/3)}]*\text{SinhIntegral}[((-1)^{(1/3)}*a^{(1/3)}*d)/b^{(1/3)} - d*x])/(3*b^{(4/3)}) - (a^{(1/3)}*\text{Sinh}[c - (a^{(1/3)}*d)/b^{(1/3)}]*\text{SinhIntegral}[(a^{(1/3)}*d)/b^{(1/3)} + d*x])/(3*b^{(4/3)}) - ((-1)^{(2/3)}*a^{(1/3)}*\text{Sinh}[c - ((-1)^{(2/3)}*a^{(1/3)}*d)/b^{(1/3)}]*\text{SinhIntegral}[((-1)^{(2/3)}*a^{(1/3)}*d)/b^{(1/3)} + d*x])/(3*b^{(4/3)})$

Rule 2637

$\text{Int}[\sin[\text{Pi}/2 + (c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow \text{Simp}[\sin[c + d*x]/d, x] /;$
FreeQ[{c, d}, x]

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x]
&& EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x]
&& EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*e - c*f)/d],
Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d],
Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

Rule 5281

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x],
(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 5293

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d}, x]
&& ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3 \cosh(c + dx)}{a + bx^3} dx &= \int \left(\frac{\cosh(c + dx)}{b} - \frac{a \cosh(c + dx)}{b(a + bx^3)} \right) dx \\
&= \frac{\int \cosh(c + dx) dx}{b} - \frac{a \int \frac{\cosh(c + dx)}{a + bx^3} dx}{b} \\
&= \frac{\sinh(c + dx)}{bd} - \frac{a \int \left(-\frac{\cosh(c + dx)}{3a^{2/3}(-\sqrt[3]{a} - \sqrt[3]{b}x)} - \frac{\cosh(c + dx)}{3a^{2/3}(-\sqrt[3]{a} + \sqrt[3]{-1}\sqrt[3]{b}x)} - \frac{\cosh(c + dx)}{3a^{2/3}(-\sqrt[3]{a} - (-1)^{2/3}\sqrt[3]{b}x)} \right) dx}{b} \\
&= \frac{\sinh(c + dx)}{bd} + \frac{\sqrt[3]{a} \int \frac{\cosh(c + dx)}{-\sqrt[3]{a} - \sqrt[3]{b}x} dx}{3b} + \frac{\sqrt[3]{a} \int \frac{\cosh(c + dx)}{-\sqrt[3]{a} + \sqrt[3]{-1}\sqrt[3]{b}x} dx}{3b} + \frac{\sqrt[3]{a} \int \frac{\cosh(c + dx)}{-\sqrt[3]{a} - (-1)^{2/3}\sqrt[3]{b}x} dx}{3b} \\
&= \frac{\sinh(c + dx)}{bd} + \frac{\left(\sqrt[3]{a} \cosh\left(c - \frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right) \right) \int \frac{\cosh\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}} + dx\right)}{-\sqrt[3]{a} - \sqrt[3]{b}x} dx + \left(\sqrt[3]{a} \cosh\left(c + \frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right) \right) \int \frac{\cosh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}} + dx\right)}{-\sqrt[3]{a} + \sqrt[3]{-1}\sqrt[3]{b}x} dx}{3b} \\
&= \frac{\sqrt[3]{-1}\sqrt[3]{a} \cosh\left(c + \frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right) \text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}} - dx\right) - (-1)^{2/3}\sqrt[3]{a} \cosh\left(c - \frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right) \text{Chi}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}} - dx\right)}{3b^{4/3}}
\end{aligned}$$

Mathematica [C] time = 0.28, size = 198, normalized size = 0.55

$$adRootSum \left[\#1^3 b + a \&, \frac{-\sinh(\#1 d + c) \text{Chi}(d(x - \#1)) + \cosh(\#1 d + c) \text{Chi}(d(x - \#1)) + \sinh(\#1 d + c) \text{Shi}(d(x - \#1)) - \cosh(\#1 d + c) \text{Shi}(d(x - \#1))}{\#1^2} \right] \&$$

Antiderivative was successfully verified.

[In] Integrate[(x^3*Cosh[c + d*x])/(a + b*x^3),x]

[Out] -1/6*(a*d*RootSum[a + b*#1^3 &, (Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] - CoshIntegral[d*(x - #1)]*Sinh[c + d*#1] - Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] + Sinh[c + d*#1]*SinhIntegral[d*(x - #1)])/#1^2 &] + a*d*RootSum[a + b*#1^3 &, (Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] + CoshIntegral[d*(x - #1)]*Sinh[c + d*#1] + Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] + Sinh[c + d*#1]*SinhIntegral[d*(x - #1)])/#1^2 &] - 6*b*Sinh[c + d*x])/(b^2*d)

fricas [B] time = 0.59, size = 977, normalized size = 2.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*cosh(d*x+c)/(b*x^3+a),x, algorithm="fricas")

[Out] $\frac{1}{12} \left(\left(\frac{a d^3}{b} \right)^{1/3} \left((\sqrt{-3} + 1) \cosh(dx + c)^2 - (\sqrt{-3} + 1) \sinh(dx + c)^2 \right) \operatorname{Ei} \left(dx - \frac{1}{2} \left(\frac{a d^3}{b} \right)^{1/3} (\sqrt{-3} + 1) \right) \cosh \left(\frac{1}{2} \left(\frac{a d^3}{b} \right)^{1/3} (\sqrt{-3} + 1) + c \right) - \left(-\frac{a d^3}{b} \right)^{1/3} \left((\sqrt{-3} + 1) \cosh(dx + c)^2 - (\sqrt{-3} + 1) \sinh(dx + c)^2 \right) \operatorname{Ei} \left(-dx - \frac{1}{2} \left(-\frac{a d^3}{b} \right)^{1/3} (\sqrt{-3} + 1) \right) \cosh \left(\frac{1}{2} \left(-\frac{a d^3}{b} \right)^{1/3} (\sqrt{-3} + 1) - c \right) - \left(\frac{a d^3}{b} \right)^{1/3} \left((\sqrt{-3} - 1) \cosh(dx + c)^2 - (\sqrt{-3} - 1) \sinh(dx + c)^2 \right) \operatorname{Ei} \left(dx + \frac{1}{2} \left(\frac{a d^3}{b} \right)^{1/3} (\sqrt{-3} - 1) \right) \cosh \left(\frac{1}{2} \left(\frac{a d^3}{b} \right)^{1/3} (\sqrt{-3} - 1) - c \right) + \left(-\frac{a d^3}{b} \right)^{1/3} \left((\sqrt{-3} - 1) \cosh(dx + c)^2 - (\sqrt{-3} - 1) \sinh(dx + c)^2 \right) \operatorname{Ei} \left(-dx + \frac{1}{2} \left(-\frac{a d^3}{b} \right)^{1/3} (\sqrt{-3} - 1) \right) \cosh \left(\frac{1}{2} \left(-\frac{a d^3}{b} \right)^{1/3} (\sqrt{-3} - 1) + c \right) + 2 \left(-\frac{a d^3}{b} \right)^{1/3} \left(\cosh(dx + c)^2 - \sinh(dx + c)^2 \right) \operatorname{Ei} \left(-dx + \left(-\frac{a d^3}{b} \right)^{1/3} \right) \cosh \left(c + \left(-\frac{a d^3}{b} \right)^{1/3} \right) - 2 \left(\frac{a d^3}{b} \right)^{1/3} \left(\cosh(dx + c)^2 - \sinh(dx + c)^2 \right) \operatorname{Ei} \left(dx + \left(\frac{a d^3}{b} \right)^{1/3} \right) \cosh \left(-c + \left(\frac{a d^3}{b} \right)^{1/3} \right) + \left(\frac{a d^3}{b} \right)^{1/3} \left((\sqrt{-3} + 1) \cosh(dx + c)^2 - (\sqrt{-3} + 1) \sinh(dx + c)^2 \right) \operatorname{Ei} \left(dx - \frac{1}{2} \left(\frac{a d^3}{b} \right)^{1/3} (\sqrt{-3} + 1) \right) \sinh \left(\frac{1}{2} \left(\frac{a d^3}{b} \right)^{1/3} (\sqrt{-3} + 1) + c \right) - \left(-\frac{a d^3}{b} \right)^{1/3} \left((\sqrt{-3} + 1) \cosh(dx + c)^2 - (\sqrt{-3} + 1) \sinh(dx + c)^2 \right) \operatorname{Ei} \left(-dx - \frac{1}{2} \left(-\frac{a d^3}{b} \right)^{1/3} (\sqrt{-3} + 1) \right) \sinh \left(\frac{1}{2} \left(-\frac{a d^3}{b} \right)^{1/3} (\sqrt{-3} + 1) - c \right) + \left(\frac{a d^3}{b} \right)^{1/3} \left((\sqrt{-3} - 1) \cosh(dx + c)^2 - (\sqrt{-3} - 1) \sinh(dx + c)^2 \right) \operatorname{Ei} \left(dx + \frac{1}{2} \left(\frac{a d^3}{b} \right)^{1/3} (\sqrt{-3} - 1) \right) \sinh \left(\frac{1}{2} \left(\frac{a d^3}{b} \right)^{1/3} (\sqrt{-3} - 1) - c \right) - \left(-\frac{a d^3}{b} \right)^{1/3} \left((\sqrt{-3} - 1) \cosh(dx + c)^2 - (\sqrt{-3} - 1) \sinh(dx + c)^2 \right) \operatorname{Ei} \left(-dx + \frac{1}{2} \left(-\frac{a d^3}{b} \right)^{1/3} (\sqrt{-3} - 1) \right) \sinh \left(\frac{1}{2} \left(-\frac{a d^3}{b} \right)^{1/3} (\sqrt{-3} - 1) + c \right) - 2 \left(-\frac{a d^3}{b} \right)^{1/3} \left(\cosh(dx + c)^2 - \sinh(dx + c)^2 \right) \operatorname{Ei} \left(-dx + \left(-\frac{a d^3}{b} \right)^{1/3} \right) \sinh \left(c + \left(-\frac{a d^3}{b} \right)^{1/3} \right) + 2 \left(\frac{a d^3}{b} \right)^{1/3} \left(\cosh(dx + c)^2 - \sinh(dx + c)^2 \right) \operatorname{Ei} \left(dx + \left(\frac{a d^3}{b} \right)^{1/3} \right) \sinh \left(-c + \left(\frac{a d^3}{b} \right)^{1/3} \right) + 12 \sinh(dx + c) \right) / (b d \cosh(dx + c)^2 - b d \sinh(dx + c)^2)$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \cosh(dx + c)}{bx^3 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*cosh(d*x+c)/(b*x^3+a),x, algorithm="giac")

[Out] integrate(x^3*cosh(d*x + c)/(b*x^3 + a), x)

maple [C] time = 0.14, size = 671, normalized size = 1.87

$$\frac{e^{-dx-c} \sum_{_R1=\text{RootOf}(b_Z^3-3cb_Z^2+3bc^2_Z+a d^3-b c^3)} \frac{(3_R1^2 bc-3_R1 b c^2-a d^3+b c^3) e^{-R1} \operatorname{Ei}(1, dx-_R1+c)}{_R1^2-2_R1 c+c^2}}{2db} + \frac{c^3 \left(\sum_{_R1=\text{RootOf}(b_Z^3-3cb_Z^2+3bc^2_Z+a d^3-b c^3)} \right)}{6db^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*cosh(d*x+c)/(b*x^3+a),x)`

[Out]
$$-1/2/d*\exp(-d*x-c)/b-1/6/d/b^2*\sum((3*_R1^2*b*c-3*_R1*b*c^2-a*d^3+b*c^3)/(_R1^2-2*_R1*c+c^2)*\exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/6/d*c^3/b*\sum(1/(_R1^2-2*_R1*c+c^2)*\exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/2/d*c^2/b*\sum(_R1/(_R1^2-2*_R1*c+c^2)*\exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/2/d*c/b*\sum(_R1^2/(_R1^2-2*_R1*c+c^2)*\exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/2/d/b*\exp(d*x+c)-1/6/d/b^2*\sum((3*_R1^2*b*c-3*_R1*b*c^2-a*d^3+b*c^3)/(_R1^2-2*_R1*c+c^2)*\exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/6/d*c^3/b*\sum(1/(_R1^2-2*_R1*c+c^2)*\exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/2/d*c^2/b*\sum(_R1/(_R1^2-2*_R1*c+c^2)*\exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/2/d*c/b*\sum(_R1^2/(_R1^2-2*_R1*c+c^2)*\exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))$$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*cosh(d*x+c)/(b*x^3+a),x, algorithm="maxima")`

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3 \cosh(c + dx)}{bx^3 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^3*cosh(c + d*x))/(a + b*x^3),x)`

[Out] `int((x^3*cosh(c + d*x))/(a + b*x^3), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \cosh(c + dx)}{a + bx^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*cosh(d*x+c)/(b*x**3+a),x)`

[Out] `Integral(x**3*cosh(c + d*x)/(a + b*x**3), x)`

$$3.96 \quad \int \frac{x^2 \cosh(c+dx)}{a+bx^3} dx$$

Optimal. Leaf size=283

$$\frac{\cosh\left(\frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}} + c\right) \text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}} - dx\right)}{3b} + \frac{\cosh\left(c - \frac{(-1)^{2/3}\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(-xd - \frac{(-1)^{2/3}\sqrt[3]{ad}}{\sqrt[3]{b}}\right)}{3b} + \frac{\cosh\left(c - \frac{\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(xd - \frac{\sqrt[3]{ad}}{\sqrt[3]{b}}\right)}{3b}$$

[Out] $1/3*\text{Chi}(a^{(1/3)*d/b^{(1/3)}+d*x)*\cosh(c-a^{(1/3)*d/b^{(1/3)}})/b+1/3*\text{Chi}((-1)^{(1/3)}*a^{(1/3)*d/b^{(1/3)}-d*x)*\cosh(c+(-1)^{(1/3)}*a^{(1/3)*d/b^{(1/3)}})/b+1/3*\text{Chi}((-1)^{(2/3)}*a^{(1/3)*d/b^{(1/3)}-d*x)*\cosh(c-(-1)^{(2/3)}*a^{(1/3)*d/b^{(1/3)}})/b+1/3*\text{Shi}(a^{(1/3)*d/b^{(1/3)}+d*x)*\sinh(c-a^{(1/3)*d/b^{(1/3)}})/b+1/3*\text{Shi}((-1)^{(1/3)}*a^{(1/3)*d/b^{(1/3)}+d*x)*\sinh(c+(-1)^{(1/3)}*a^{(1/3)*d/b^{(1/3)}})/b+1/3*\text{Shi}((-1)^{(2/3)}*a^{(1/3)*d/b^{(1/3)}+d*x)*\sinh(c-(-1)^{(2/3)}*a^{(1/3)*d/b^{(1/3)}})/b$

Rubi [A] time = 0.46, antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {5293, 3303, 3298, 3301}

$$\frac{\cosh\left(\frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}} + c\right) \text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}} - dx\right)}{3b} + \frac{\cosh\left(c - \frac{(-1)^{2/3}\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(-xd - \frac{(-1)^{2/3}\sqrt[3]{ad}}{\sqrt[3]{b}}\right)}{3b} + \frac{\cosh\left(c - \frac{\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(xd - \frac{\sqrt[3]{ad}}{\sqrt[3]{b}}\right)}{3b}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(x^2*\text{Cosh}[c + d*x])/(a + b*x^3), x]$

[Out] $(\text{Cosh}[c + ((-1)^{(1/3)}*a^{(1/3)*d}/b^{(1/3)})]*\text{CoshIntegral}[((-1)^{(1/3)}*a^{(1/3)*d}/b^{(1/3)} - d*x])/(3*b) + (\text{Cosh}[c - ((-1)^{(2/3)}*a^{(1/3)*d}/b^{(1/3)})]*\text{CoshIntegral}[-(((-1)^{(2/3)}*a^{(1/3)*d}/b^{(1/3)}) - d*x])/(3*b) + (\text{Cosh}[c - (a^{(1/3)*d}/b^{(1/3)})]*\text{CoshIntegral}[(a^{(1/3)*d}/b^{(1/3)} + d*x])/(3*b) - (\text{Sinh}[c + ((-1)^{(1/3)}*a^{(1/3)*d}/b^{(1/3)})]*\text{SinhIntegral}[((-1)^{(1/3)}*a^{(1/3)*d}/b^{(1/3)} - d*x])/(3*b) + (\text{Sinh}[c - (a^{(1/3)*d}/b^{(1/3)})]*\text{SinhIntegral}[(a^{(1/3)*d}/b^{(1/3)} + d*x])/(3*b) + (\text{Sinh}[c - ((-1)^{(2/3)}*a^{(1/3)*d}/b^{(1/3)})]*\text{SinhIntegral}[((-1)^{(2/3)}*a^{(1/3)*d}/b^{(1/3)} + d*x])/(3*b)$

Rule 3298

$\text{Int}[\sin[(e_.) + (\text{Complex}[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] \rightarrow \text{Simp}[(\text{I}*\text{SinhIntegral}[(c*f*fz)/d + f*fz*x])/d, x] /; \text{FreeQ}\{c, d, e, f, fz\}, x] \ \&\& \ \text{EqQ}[d*e - c*f*fz*I, 0]$

Rule 3301

$\text{Int}[\sin[(e_.) + (\text{Complex}[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] \rightarrow \text{Simp}[\text{CoshIntegral}[(c*f*fz)/d + f*fz*x]/d, x] /; \text{FreeQ}\{c, d, e, f, fz$

}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]

Rule 3303

Int[sin[(e_.) + (f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x_Symbol] :> Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]

Rule 5293

Int[Cosh[(c_.) + (d_.)*(x_.)]*(x_.)^(m_.)*((a_.) + (b_.)*(x_.)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])

Rubi steps

$$\begin{aligned} \int \frac{x^2 \cosh(c + dx)}{a + bx^3} dx &= \int \left(\frac{\cosh(c + dx)}{3b^{2/3} (\sqrt[3]{a} + \sqrt[3]{b}x)} + \frac{\cosh(c + dx)}{3b^{2/3} (-\sqrt[3]{-1} \sqrt[3]{a} + \sqrt[3]{b}x)} + \frac{\cosh(c + dx)}{3b^{2/3} ((-1)^{2/3} \sqrt[3]{a} + \sqrt[3]{b}x)} \right) dx \\ &= \frac{\int \frac{\cosh(c+dx)}{\sqrt[3]{a} + \sqrt[3]{b}x} dx}{3b^{2/3}} + \frac{\int \frac{\cosh(c+dx)}{-\sqrt[3]{-1} \sqrt[3]{a} + \sqrt[3]{b}x} dx}{3b^{2/3}} + \frac{\int \frac{\cosh(c+dx)}{(-1)^{2/3} \sqrt[3]{a} + \sqrt[3]{b}x} dx}{3b^{2/3}} \\ &= \frac{\cosh\left(c - \frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right) \int \frac{\cosh\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}} + dx\right)}{\sqrt[3]{a} + \sqrt[3]{b}x} dx}{3b^{2/3}} + \frac{\cosh\left(c + \frac{\sqrt[3]{-1} \sqrt[3]{a}d}{\sqrt[3]{b}}\right) \int \frac{\cos\left(\frac{(-1)^{5/6} \sqrt[3]{a}d}{\sqrt[3]{b}} - idx\right)}{-\sqrt[3]{-1} \sqrt[3]{a} + \sqrt[3]{b}x} dx}{3b^{2/3}} + \frac{\cosh\left(c + \frac{(-1)^{2/3} \sqrt[3]{a}d}{\sqrt[3]{b}}\right) \int \frac{\cos\left(\frac{(-1)^{2/3} \sqrt[3]{a}d}{\sqrt[3]{b}} - idx\right)}{(-1)^{2/3} \sqrt[3]{a} + \sqrt[3]{b}x} dx}{3b^{2/3}} \\ &= \frac{\cosh\left(c + \frac{\sqrt[3]{-1} \sqrt[3]{a}d}{\sqrt[3]{b}}\right) \text{Chi}\left(\frac{\sqrt[3]{-1} \sqrt[3]{a}d}{\sqrt[3]{b}} - dx\right)}{3b} + \frac{\cosh\left(c - \frac{(-1)^{2/3} \sqrt[3]{a}d}{\sqrt[3]{b}}\right) \text{Chi}\left(-\frac{(-1)^{2/3} \sqrt[3]{a}d}{\sqrt[3]{b}} - dx\right)}{3b} \end{aligned}$$

Mathematica [C] time = 0.24, size = 170, normalized size = 0.60

RootSum[#1^3 b + a &, -sinh(#1 d + c) Chi(d(x - #1)) + cosh(#1 d + c) Chi(d(x - #1)) + sinh(#1 d + c) Shi(d(x - #1))]

Antiderivative was successfully verified.

[In] Integrate[(x^2*Cosh[c + d*x])/(a + b*x^3), x]

[Out] (RootSum[a + b*#1^3 &, Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] - CoshIntegral[d*(x - #1)]*Sinh[c + d*#1] - Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] +

$\text{Sinh}[c + d\#1] * \text{SinhIntegral}[d*(x - \#1)] \&] + \text{RootSum}[a + b*\#1^3 \& , \text{Cosh}[c + d*\#1] * \text{CoshIntegral}[d*(x - \#1)] + \text{CoshIntegral}[d*(x - \#1)] * \text{Sinh}[c + d*\#1] + \text{Cosh}[c + d*\#1] * \text{SinhIntegral}[d*(x - \#1)] + \text{Sinh}[c + d*\#1] * \text{SinhIntegral}[d*(x - \#1)] \&])/(6*b)$

fricas [B] time = 0.45, size = 500, normalized size = 1.77

$$\frac{\text{Ei}\left(dx - \frac{1}{2}\left(\frac{ad^3}{b}\right)^{\frac{1}{3}}(\sqrt{-3} + 1)\right) \cosh\left(\frac{1}{2}\left(\frac{ad^3}{b}\right)^{\frac{1}{3}}(\sqrt{-3} + 1) + c\right) + \text{Ei}\left(-dx - \frac{1}{2}\left(-\frac{ad^3}{b}\right)^{\frac{1}{3}}(\sqrt{-3} + 1)\right) \cosh\left(\frac{1}{2}\left(-\frac{ad^3}{b}\right)^{\frac{1}{3}}(\sqrt{-3} + 1) + c\right)}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*cosh(d*x+c)/(b*x^3+a),x, algorithm="fricas")

[Out] $\frac{1}{6} * (\text{Ei}(d*x - \frac{1}{2} * (a*d^3/b)^{(1/3)} * (\text{sqrt}(-3) + 1)) * \cosh(\frac{1}{2} * (a*d^3/b)^{(1/3)} * (\text{sqrt}(-3) + 1) + c) + \text{Ei}(-d*x - \frac{1}{2} * (-a*d^3/b)^{(1/3)} * (\text{sqrt}(-3) + 1)) * \cosh(\frac{1}{2} * (-a*d^3/b)^{(1/3)} * (\text{sqrt}(-3) + 1) - c) + \text{Ei}(d*x + \frac{1}{2} * (a*d^3/b)^{(1/3)} * (\text{sqrt}(-3) - 1)) * \cosh(\frac{1}{2} * (a*d^3/b)^{(1/3)} * (\text{sqrt}(-3) - 1) - c) + \text{Ei}(-d*x + \frac{1}{2} * (-a*d^3/b)^{(1/3)} * (\text{sqrt}(-3) - 1)) * \cosh(\frac{1}{2} * (-a*d^3/b)^{(1/3)} * (\text{sqrt}(-3) - 1) + c) + \text{Ei}(-d*x + (-a*d^3/b)^{(1/3)}) * \cosh(c + (-a*d^3/b)^{(1/3)}) + \text{Ei}(d*x + (a*d^3/b)^{(1/3)}) * \cosh(-c + (a*d^3/b)^{(1/3)}) + \text{Ei}(d*x - \frac{1}{2} * (a*d^3/b)^{(1/3)} * (\text{sqrt}(-3) + 1)) * \sinh(\frac{1}{2} * (a*d^3/b)^{(1/3)} * (\text{sqrt}(-3) + 1) + c) + \text{Ei}(-d*x - \frac{1}{2} * (-a*d^3/b)^{(1/3)} * (\text{sqrt}(-3) + 1)) * \sinh(\frac{1}{2} * (-a*d^3/b)^{(1/3)} * (\text{sqrt}(-3) + 1) - c) - \text{Ei}(d*x + \frac{1}{2} * (a*d^3/b)^{(1/3)} * (\text{sqrt}(-3) - 1)) * \sinh(\frac{1}{2} * (a*d^3/b)^{(1/3)} * (\text{sqrt}(-3) - 1) - c) - \text{Ei}(-d*x + \frac{1}{2} * (-a*d^3/b)^{(1/3)} * (\text{sqrt}(-3) - 1)) * \sinh(\frac{1}{2} * (-a*d^3/b)^{(1/3)} * (\text{sqrt}(-3) - 1) + c) - \text{Ei}(-d*x + (-a*d^3/b)^{(1/3)}) * \sinh(c + (-a*d^3/b)^{(1/3)}) - \text{Ei}(d*x + (a*d^3/b)^{(1/3)}) * \sinh(-c + (a*d^3/b)^{(1/3)})) / b$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \cosh(dx + c)}{bx^3 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*cosh(d*x+c)/(b*x^3+a),x, algorithm="giac")

[Out] integrate(x^2*cosh(d*x + c)/(b*x^3 + a), x)

maple [C] time = 0.12, size = 423, normalized size = 1.49

$$\frac{\sum_{-R1=\text{RootOf}(b_Z^3-3cb_Z^2+3bc^2_Z+a d^3-b c^3)} \frac{-R1^2 e^{-R1} \text{Ei}(1, dx - R1 + c)}{-R1^2 - 2_R1 c + c^2}}{6b} c^2 \left(\sum_{-R1=\text{RootOf}(b_Z^3-3cb_Z^2+3bc^2_Z+a d^3-b c^3)} \frac{e^{-R1} \text{Ei}(1, dx - R1 + c)}{-R1^2 - 2_R1 c + c^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*cosh(d*x+c)/(b*x^3+a),x)`

[Out]
$$-1/6/b*\sum(_R1^2/(_R1^2-2*_R1*c+c^2)*\exp(-_R1)*\text{Ei}(1,d*x-_R1+c),_R1=\text{RootOf}(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/6*c^2/b*\sum(1/(_R1^2-2*_R1*c+c^2)*\exp(-_R1)*\text{Ei}(1,d*x-_R1+c),_R1=\text{RootOf}(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/3*c/b*\sum(_R1/(_R1^2-2*_R1*c+c^2)*\exp(-_R1)*\text{Ei}(1,d*x-_R1+c),_R1=\text{RootOf}(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/6/b*\sum(_R1^2/(_R1^2-2*_R1*c+c^2)*\exp(_R1)*\text{Ei}(1,-d*x+_R1-c),_R1=\text{RootOf}(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/6*c^2/b*\sum(1/(_R1^2-2*_R1*c+c^2)*\exp(_R1)*\text{Ei}(1,-d*x+_R1-c),_R1=\text{RootOf}(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/3*c/b*\sum(_R1/(_R1^2-2*_R1*c+c^2)*\exp(_R1)*\text{Ei}(1,-d*x+_R1-c),_R1=\text{RootOf}(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{(dx^2e^{2c} + xe^{2c})e^{dx} - (dx^2 - x)e^{-dx}}{2(bd^2x^3e^c + ad^2e^c)} + \frac{1}{2} \int \frac{(2bx^3e^c - 3adx e^c - ae^c)e^{dx}}{b^2d^2x^6 + 2abd^2x^3 + a^2d^2} dx + \frac{1}{2} \int \frac{(2bx^3 + 3adx - a)e^{-dx}}{b^2d^2x^6e^c + 2abd^2x^3e^c + a^2d^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*cosh(d*x+c)/(b*x^3+a),x, algorithm="maxima")`

[Out]
$$1/2*((d*x^2*e^{(2*c)} + x*e^{(2*c)})*e^{(d*x)} - (d*x^2 - x)*e^{(-d*x)})/(b*d^2*x^3*e^c + a*d^2*e^c) + 1/2*integrate((2*b*x^3*e^c - 3*a*d*x*e^c - a*e^c)*e^{(d*x)}/(b^2*d^2*x^6 + 2*a*b*d^2*x^3 + a^2*d^2), x) + 1/2*integrate((2*b*x^3 + 3*a*d*x - a)*e^{(-d*x)}/(b^2*d^2*x^6*e^c + 2*a*b*d^2*x^3*e^c + a^2*d^2*e^c), x)$$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2 \cosh(c + dx)}{bx^3 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2*cosh(c + d*x))/(a + b*x^3),x)`

[Out] `int((x^2*cosh(c + d*x))/(a + b*x^3), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \cosh(c + dx)}{a + bx^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cosh(d*x+c)/(b*x**3+a),x)
```

```
[Out] Integral(x**2*cosh(c + d*x)/(a + b*x**3), x)
```

$$3.97 \quad \int \frac{x \cosh(c+dx)}{a+bx^3} dx$$

Optimal. Leaf size=345

$$\frac{(-1)^{2/3} \cosh\left(\frac{\sqrt[3]{-1} \sqrt[3]{ad}}{\sqrt[3]{b}} + c\right) \text{Chi}\left(\frac{\sqrt[3]{-1} \sqrt[3]{ad}}{\sqrt[3]{b}} - dx\right)}{3\sqrt[3]{a} b^{2/3}} + \frac{\sqrt[3]{-1} \cosh\left(c - \frac{(-1)^{2/3} \sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(-xd - \frac{(-1)^{2/3} \sqrt[3]{ad}}{\sqrt[3]{b}}\right) \cosh\left(c - \frac{(-1)^{2/3} \sqrt[3]{ad}}{\sqrt[3]{b}}\right)}{3\sqrt[3]{a} b^{2/3}}$$

[Out] $-1/3*\text{Chi}(a^{1/3}*d/b^{1/3}+d*x)*\cosh(c-a^{1/3}*d/b^{1/3})/a^{1/3}/b^{2/3}-1/3*(-1)^{2/3}*\text{Chi}((-1)^{1/3}*a^{1/3}*d/b^{1/3}-d*x)*\cosh(c+(-1)^{1/3}*a^{1/3}*d/b^{1/3})/a^{1/3}/b^{2/3}+1/3*(-1)^{1/3}*\text{Chi}(-(-1)^{2/3}*a^{1/3}*d/b^{1/3}-d*x)*\cosh(c-(-1)^{2/3}*a^{1/3}*d/b^{1/3})/a^{1/3}/b^{2/3}-1/3*\text{Shi}(a^{1/3}*d/b^{1/3}+d*x)*\sinh(c-a^{1/3}*d/b^{1/3})/a^{1/3}/b^{2/3}-1/3*(-1)^{2/3}*\text{Shi}(-(-1)^{1/3}*a^{1/3}*d/b^{1/3}+d*x)*\sinh(c+(-1)^{1/3}*a^{1/3}*d/b^{1/3})/a^{1/3}/b^{2/3}+1/3*(-1)^{1/3}*\text{Shi}((-1)^{2/3}*a^{1/3}*d/b^{1/3}+d*x)*\sinh(c-(-1)^{2/3}*a^{1/3}*d/b^{1/3})/a^{1/3}/b^{2/3}$

Rubi [A] time = 0.41, antiderivative size = 345, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {5293, 3303, 3298, 3301}

$$\frac{(-1)^{2/3} \cosh\left(\frac{\sqrt[3]{-1} \sqrt[3]{ad}}{\sqrt[3]{b}} + c\right) \text{Chi}\left(\frac{\sqrt[3]{-1} \sqrt[3]{ad}}{\sqrt[3]{b}} - dx\right)}{3\sqrt[3]{a} b^{2/3}} + \frac{\sqrt[3]{-1} \cosh\left(c - \frac{(-1)^{2/3} \sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(-xd - \frac{(-1)^{2/3} \sqrt[3]{ad}}{\sqrt[3]{b}}\right) \cosh\left(c - \frac{(-1)^{2/3} \sqrt[3]{ad}}{\sqrt[3]{b}}\right)}{3\sqrt[3]{a} b^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[(x*Cosh[c + d*x])/(a + b*x^3), x]

[Out] $-((-1)^{2/3}*\text{Cosh}[c + ((-1)^{1/3}*a^{1/3}*d)/b^{1/3}]*\text{CoshIntegral}[((-1)^{1/3}*a^{1/3}*d)/b^{1/3} - d*x])/(3*a^{1/3}*b^{2/3}) + ((-1)^{1/3}*\text{Cosh}[c - ((-1)^{2/3}*a^{1/3}*d)/b^{1/3}]*\text{CoshIntegral}[(-((-1)^{2/3}*a^{1/3}*d)/b^{1/3}) - d*x])/(3*a^{1/3}*b^{2/3}) - (\text{Cosh}[c - (a^{1/3}*d)/b^{1/3}]*\text{CoshIntegral}[(a^{1/3}*d)/b^{1/3} + d*x])/(3*a^{1/3}*b^{2/3}) + ((-1)^{2/3}*\text{Sinh}[c + ((-1)^{1/3}*a^{1/3}*d)/b^{1/3}]*\text{SinhIntegral}[((-1)^{1/3}*a^{1/3}*d)/b^{1/3} - d*x])/(3*a^{1/3}*b^{2/3}) - (\text{Sinh}[c - (a^{1/3}*d)/b^{1/3}]*\text{SinhIntegral}[(a^{1/3}*d)/b^{1/3} + d*x])/(3*a^{1/3}*b^{2/3}) + ((-1)^{1/3}*\text{Sinh}[c - ((-1)^{2/3}*a^{1/3}*d)/b^{1/3}]*\text{SinhIntegral}[((-1)^{2/3}*a^{1/3}*d)/b^{1/3} + d*x])/(3*a^{1/3}*b^{2/3})$

Rule 3298

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f

, fz}, x] && EqQ[d*e - c*f*fz*I, 0]

Rule 3301

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]

Rule 3303

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]

Rule 5293

Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])

Rubi steps

$$\begin{aligned}
 \int \frac{x \cosh(c + dx)}{a + bx^3} dx &= \int \left(-\frac{\cosh(c + dx)}{3\sqrt[3]{a}\sqrt[3]{b}(\sqrt[3]{a} + \sqrt[3]{b}x)} - \frac{(-1)^{2/3} \cosh(c + dx)}{3\sqrt[3]{a}\sqrt[3]{b}(\sqrt[3]{a} - \sqrt[3]{-1}\sqrt[3]{b}x)} + \frac{\sqrt[3]{-1} \cosh(c + dx)}{3\sqrt[3]{a}\sqrt[3]{b}(\sqrt[3]{a} + (-1)^{2/3}\sqrt[3]{b}x)} \right) dx \\
 &= -\frac{\int \frac{\cosh(c+dx)}{\sqrt[3]{a} + \sqrt[3]{b}x} dx}{3\sqrt[3]{a}\sqrt[3]{b}} + \frac{\sqrt[3]{-1} \int \frac{\cosh(c+dx)}{\sqrt[3]{a} + (-1)^{2/3}\sqrt[3]{b}x} dx}{3\sqrt[3]{a}\sqrt[3]{b}} - \frac{(-1)^{2/3} \int \frac{\cosh(c+dx)}{\sqrt[3]{a} - \sqrt[3]{-1}\sqrt[3]{b}x} dx}{3\sqrt[3]{a}\sqrt[3]{b}} \\
 &= -\frac{\cosh\left(c - \frac{\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \int \frac{\cosh\left(\frac{\sqrt[3]{ad}}{\sqrt[3]{b}} + dx\right)}{\sqrt[3]{a} + \sqrt[3]{b}x} dx}{3\sqrt[3]{a}\sqrt[3]{b}} + \frac{\left(\sqrt[3]{-1} \cosh\left(c + \frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}}\right)\right) \int \frac{\cosh\left(\frac{(-1)^{5/6}\sqrt[3]{ad} - idx}{\sqrt[3]{b}}\right)}{\sqrt[3]{a} + (-1)^{2/3}\sqrt[3]{b}x} dx}{3\sqrt[3]{a}\sqrt[3]{b}} \\
 &= -\frac{(-1)^{2/3} \cosh\left(c + \frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \operatorname{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}} - dx\right)}{3\sqrt[3]{a}b^{2/3}} + \frac{\sqrt[3]{-1} \cosh\left(c - \frac{(-1)^{2/3}\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \operatorname{Chi}\left(-\frac{(-1)}{\sqrt[3]{b}}\right)}{3\sqrt[3]{a}b^{2/3}}
 \end{aligned}$$

Mathematica [C] time = 0.22, size = 180, normalized size = 0.52

$$\text{RootSum}\left[\#1^3 b + a \&, \frac{-\sinh(\#1 d + c) \text{Chi}(d(x - \#1)) + \cosh(\#1 d + c) \text{Chi}(d(x - \#1)) + \sinh(\#1 d + c) \text{Shi}(d(x - \#1)) - \cosh(\#1 d + c) \text{Shi}(d(x - \#1))}{\#1} \&\right] +$$

$$6b$$

Antiderivative was successfully verified.

[In] Integrate[(x*Cosh[c + d*x])/(a + b*x^3), x]

[Out] (RootSum[a + b*#1^3 &, (Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] - CoshIntegral[d*(x - #1)]*Sinh[c + d*#1] - Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] + Sinh[c + d*#1]*SinhIntegral[d*(x - #1)])/#1 &] + RootSum[a + b*#1^3 &, (Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] + CoshIntegral[d*(x - #1)]*Sinh[c + d*#1] + Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] + Sinh[c + d*#1]*SinhIntegral[d*(x - #1)])/#1 &])/(6*b)

fricas [B] time = 0.57, size = 671, normalized size = 1.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cosh(d*x+c)/(b*x^3+a), x, algorithm="fricas")

[Out] -1/12*((a*d^3/b)^(2/3)*(sqrt(-3) - 1)*Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1) + 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1) + c) + (-a*d^3/b)^(2/3)*(sqrt(-3) - 1)*Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1) - c) - (a*d^3/b)^(2/3)*(sqrt(-3) + 1)*Ei(d*x + 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1) - c) - (-a*d^3/b)^(2/3)*(sqrt(-3) + 1)*Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c) + (a*d^3/b)^(2/3)*(sqrt(-3) - 1)*Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1) + c) + (-a*d^3/b)^(2/3)*(sqrt(-3) - 1)*Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1) - c) + (a*d^3/b)^(2/3)*(sqrt(-3) + 1)*Ei(d*x + 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1))*sinh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1) - c) + (-a*d^3/b)^(2/3)*(sqrt(-3) + 1)*Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1))*sinh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c) + 2*(-a*d^3/b)^(2/3)*Ei(-d*x + (-a*d^3/b)^(1/3))*cosh(c + (-a*d^3/b)^(1/3)) + 2*(a*d^3/b)^(2/3)*Ei(d*x + (a*d^3/b)^(1/3))*cosh(-c + (a*d^3/b)^(1/3)) - 2*(-a*d^3/b)^(2/3)*Ei(-d*x + (-a*d^3/b)^(1/3))*sinh(c + (-a*d^3/b)^(1/3)) - 2*(a*d^3/b)^(2/3)*Ei(d*x + (a*d^3/b)^(1/3))*sinh(-c + (a*d^3/b)^(1/3)))/(a*d^2)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \cosh(dx + c)}{bx^3 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cosh(d*x+c)/(b*x^3+a),x, algorithm="giac")

[Out] integrate(x*cosh(d*x + c)/(b*x^3 + a), x)

maple [C] time = 0.12, size = 280, normalized size = 0.81

$$\frac{d \left(\sum_{R1=\text{RootOf}(b_Z^3-3cb_Z^2+3bc^2_Z+a d^3-bc^3)} \frac{R1 e^{-R1} \text{Ei}(1,dx-R1+c)}{R1^2-2R1c+c^2} \right)}{6b} + \frac{dc \left(\sum_{R1=\text{RootOf}(b_Z^3-3cb_Z^2+3bc^2_Z+a d^3-bc^3)} \frac{e^{-R1}}{R1} \right)}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*cosh(d*x+c)/(b*x^3+a),x)

[Out] -1/6*d/b*sum(_R1/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/6*d*c/b*sum(1/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/6*d/b*sum(_R1/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/6*d*c/b*sum(1/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cosh(d*x+c)/(b*x^3+a),x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x \cosh(c + dx)}{bx^3 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x*cosh(c + d*x))/(a + b*x^3),x)

[Out] int((x*cosh(c + d*x))/(a + b*x^3), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \cosh(c + dx)}{a + bx^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(d*x+c)/(b*x**3+a),x)
```

```
[Out] Integral(x*cosh(c + d*x)/(a + b*x**3), x)
```

3.98 $\int \frac{\cosh(c+dx)}{a+bx^3} dx$

Optimal. Leaf size=345

$$\frac{\sqrt[3]{-1} \cosh\left(\frac{\sqrt[3]{-1} \sqrt[3]{ad}}{\sqrt[3]{b}} + c\right) \text{Chi}\left(\frac{\sqrt[3]{-1} \sqrt[3]{ad}}{\sqrt[3]{b}} - dx\right)}{3a^{2/3} \sqrt[3]{b}} + \frac{(-1)^{2/3} \cosh\left(c - \frac{(-1)^{2/3} \sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(-xd - \frac{(-1)^{2/3} \sqrt[3]{ad}}{\sqrt[3]{b}}\right)}{3a^{2/3} \sqrt[3]{b}} + \frac{\cosh\left(c - \frac{(-1)^{2/3} \sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(-xd - \frac{(-1)^{2/3} \sqrt[3]{ad}}{\sqrt[3]{b}}\right)}{3a^{2/3} \sqrt[3]{b}}$$

[Out] $\frac{1}{3} \text{Chi}\left(\frac{a^{1/3} d}{b^{1/3}} + dx\right) \cosh\left(c - \frac{a^{1/3} d}{b^{1/3}}\right) / \frac{a^{2/3}}{b^{1/3}} - \frac{1}{3} (-1)^{1/3} \text{Chi}\left(\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - dx\right) \cosh\left(c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right) / \frac{a^{2/3}}{b^{1/3}} + \frac{1}{3} (-1)^{2/3} \text{Chi}\left(\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} - dx\right) \cosh\left(c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right) / \frac{a^{2/3}}{b^{1/3}} + \frac{1}{3} \text{Shi}\left(\frac{a^{1/3} d}{b^{1/3}} + dx\right) \sinh\left(c - \frac{a^{1/3} d}{b^{1/3}}\right) / \frac{a^{2/3}}{b^{1/3}} - \frac{1}{3} (-1)^{1/3} \text{Shi}\left(\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} + dx\right) \sinh\left(c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right) / \frac{a^{2/3}}{b^{1/3}} + \frac{1}{3} (-1)^{2/3} \text{Shi}\left(\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + dx\right) \sinh\left(c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right) / \frac{a^{2/3}}{b^{1/3}}$

Rubi [A] time = 0.40, antiderivative size = 345, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5281, 3303, 3298, 3301}

$$\frac{\sqrt[3]{-1} \cosh\left(\frac{\sqrt[3]{-1} \sqrt[3]{ad}}{\sqrt[3]{b}} + c\right) \text{Chi}\left(\frac{\sqrt[3]{-1} \sqrt[3]{ad}}{\sqrt[3]{b}} - dx\right)}{3a^{2/3} \sqrt[3]{b}} + \frac{(-1)^{2/3} \cosh\left(c - \frac{(-1)^{2/3} \sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(-xd - \frac{(-1)^{2/3} \sqrt[3]{ad}}{\sqrt[3]{b}}\right)}{3a^{2/3} \sqrt[3]{b}} + \frac{\cosh\left(c - \frac{(-1)^{2/3} \sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(-xd - \frac{(-1)^{2/3} \sqrt[3]{ad}}{\sqrt[3]{b}}\right)}{3a^{2/3} \sqrt[3]{b}}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]/(a + b*x^3), x]

[Out] $-\frac{(-1)^{1/3} \text{Cosh}\left[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right] \text{CoshIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - dx\right]}{3a^{2/3} b^{1/3}} + \frac{(-1)^{2/3} \text{Cosh}\left[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right] \text{CoshIntegral}\left[-\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} - dx\right]}{3a^{2/3} b^{1/3}} + \frac{\text{Cosh}\left[c - \frac{a^{1/3} d}{b^{1/3}}\right] \text{CoshIntegral}\left[\frac{a^{1/3} d}{b^{1/3}} + dx\right]}{3a^{2/3} b^{1/3}} + \frac{(-1)^{1/3} \text{Sinh}\left[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right] \text{SinhIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - dx\right]}{3a^{2/3} b^{1/3}} + \frac{\text{Sinh}\left[c - \frac{a^{1/3} d}{b^{1/3}}\right] \text{SinhIntegral}\left[\frac{a^{1/3} d}{b^{1/3}} + dx\right]}{3a^{2/3} b^{1/3}} + \frac{(-1)^{2/3} \text{Sinh}\left[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right] \text{SinhIntegral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + dx\right]}{3a^{2/3} b^{1/3}}$

Rule 3298

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f

, fz}, x] && EqQ[d*e - c*f*fz*I, 0]

Rule 3301

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]

Rule 3303

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]

Rule 5281

Int[Cosh[(c_.) + (d_.)*(x_)]*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])

Rubi steps

$$\begin{aligned} \int \frac{\cosh(c+dx)}{a+bx^3} dx &= \int \left(\frac{\cosh(c+dx)}{3a^{2/3}(-\sqrt[3]{a}-\sqrt[3]{b}x)} - \frac{\cosh(c+dx)}{3a^{2/3}(-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x)} - \frac{\cosh(c+dx)}{3a^{2/3}(-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x)} \right) dx \\ &= -\frac{\int \frac{\cosh(c+dx)}{-\sqrt[3]{a}-\sqrt[3]{b}x} dx}{3a^{2/3}} - \frac{\int \frac{\cosh(c+dx)}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x} dx}{3a^{2/3}} - \frac{\int \frac{\cosh(c+dx)}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x} dx}{3a^{2/3}} \\ &= -\frac{\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right) \int \frac{\cosh\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{-\sqrt[3]{a}-\sqrt[3]{b}x} dx}{3a^{2/3}} - \frac{\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right) \int \frac{\cos\left(\frac{(-1)^{5/6}\sqrt[3]{a}d}{\sqrt[3]{b}}-idx\right)}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x} dx}{3a^{2/3}} - \frac{\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right) \int \frac{\cosh\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x} dx}{3a^{2/3}} \\ &= -\frac{\sqrt[3]{-1} \cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right) \text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{2/3}\sqrt[3]{b}} + \frac{(-1)^{2/3} \cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right) \text{Chi}\left(-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{2/3}\sqrt[3]{b}} \end{aligned}$$

Mathematica [C] time = 0.14, size = 180, normalized size = 0.52

$$\text{RootSum}\left[\#1^3b+a\&, \frac{-\sinh(\#1d+c)\text{Chi}(d(x-\#1))+\cosh(\#1d+c)\text{Chi}(d(x-\#1))+\sinh(\#1d+c)\text{Shi}(d(x-\#1))-\cosh(\#1d+c)\text{Shi}(d(x-\#1))}{\#1^2}\& \right] +$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]/(a + b*x^3),x]
```

```
[Out] (RootSum[a + b*#1^3 & , (Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] - CoshInte
gral[d*(x - #1)]*Sinh[c + d*#1] - Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] +
Sinh[c + d*#1]*SinhIntegral[d*(x - #1)])/#1^2 & ] + RootSum[a + b*#1^3 & ,
(Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] + CoshIntegral[d*(x - #1)]*Sinh[c
+ d*#1] + Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] + Sinh[c + d*#1]*SinhInt
egral[d*(x - #1)])/#1^2 & ])/(6*b)
```

```
fricas [B] time = 0.65, size = 673, normalized size = 1.95
```

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/12*((a*d^3/b)^(1/3)*(sqrt(-3) + 1)*Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt(-3)
) + 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1) + c) - (-a*d^3/b)^(1/3)*(sq
rt(-3) + 1)*Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(-a*d^3
/b)^(1/3)*(sqrt(-3) + 1) - c) - (a*d^3/b)^(1/3)*(sqrt(-3) - 1)*Ei(d*x + 1/2
*(a*d^3/b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1) -
c) + (-a*d^3/b)^(1/3)*(sqrt(-3) - 1)*Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqrt(-
3) - 1))*cosh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c) + (a*d^3/b)^(1/3)*(s
qrt(-3) + 1)*Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(a*d^3/b
)^(1/3)*(sqrt(-3) + 1) + c) - (-a*d^3/b)^(1/3)*(sqrt(-3) + 1)*Ei(-d*x - 1/2
*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1)
- c) + (a*d^3/b)^(1/3)*(sqrt(-3) - 1)*Ei(d*x + 1/2*(a*d^3/b)^(1/3)*(sqrt(-3)
) - 1))*sinh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1) - c) - (-a*d^3/b)^(1/3)*(sq
rt(-3) - 1)*Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1))*sinh(1/2*(-a*d^3
/b)^(1/3)*(sqrt(-3) - 1) + c) + 2*(-a*d^3/b)^(1/3)*Ei(-d*x + (-a*d^3/b)^(1/
3))*cosh(c + (-a*d^3/b)^(1/3)) - 2*(a*d^3/b)^(1/3)*Ei(d*x + (a*d^3/b)^(1/3)
)*cosh(-c + (a*d^3/b)^(1/3)) - 2*(-a*d^3/b)^(1/3)*Ei(-d*x + (-a*d^3/b)^(1/3
))*sinh(c + (-a*d^3/b)^(1/3)) + 2*(a*d^3/b)^(1/3)*Ei(d*x + (a*d^3/b)^(1/3))
*sinh(-c + (a*d^3/b)^(1/3)))/(a*d)
```

```
giac [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int \frac{\cosh(dx + c)}{bx^3 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(b*x^3+a),x, algorithm="giac")
```

[Out] integrate(cosh(d*x + c)/(b*x^3 + a), x)

maple [C] time = 0.09, size = 143, normalized size = 0.41

$$\frac{d^2 \left(\sum_{_R1=\text{RootOf}(b_Z^3-3cb_Z^2+3bc^2_Z+ad^3-bc^3)} \frac{e^{-R1} \text{Ei}(1, dx - R1 + c)}{-R1^2 - 2R1c + c^2} \right)}{6b} - \frac{d^2 \left(\sum_{_R1=\text{RootOf}(b_Z^3-3cb_Z^2+3bc^2_Z+ad^3-bc^3)} \frac{e^{-R1} \text{Ei}(1, -dx + R1 - c)}{-R1^2 - 2R1c + c^2} \right)}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d*x+c)/(b*x^3+a), x)

[Out] $-1/6*d^2/b*\text{sum}(1/(_R1^2-2*_R1*c+c^2)*\exp(-_R1)*\text{Ei}(1, d*x-_R1+c), _R1=\text{RootOf}(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/6*d^2/b*\text{sum}(1/(_R1^2-2*_R1*c+c^2)*\exp(_R1)*\text{Ei}(1, -d*x+_R1-c), _R1=\text{RootOf}(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/(b*x^3+a), x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(c + dx)}{bx^3 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(c + d*x)/(a + b*x^3), x)

[Out] int(cosh(c + d*x)/(a + b*x^3), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(c + dx)}{a + bx^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/(b*x**3+a), x)

[Out] Integral(cosh(c + d*x)/(a + b*x**3), x)

$$3.99 \quad \int \frac{\cosh(c+dx)}{x(a+bx^3)} dx$$

Optimal. Leaf size=303

$$\frac{\cosh\left(\frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}} + c\right) \text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}} - dx\right)}{3a} - \frac{\cosh\left(c - \frac{(-1)^{2/3}\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(-xd - \frac{(-1)^{2/3}\sqrt[3]{ad}}{\sqrt[3]{b}}\right)}{3a} - \frac{\cosh\left(c - \frac{\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(xd - \frac{\sqrt[3]{ad}}{\sqrt[3]{b}}\right)}{3a}$$

[Out] Chi(d*x)*cosh(c)/a-1/3*Chi(a^(1/3)*d/b^(1/3)+d*x)*cosh(c-a^(1/3)*d/b^(1/3))
/a-1/3*Chi((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*cosh(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))
/a-1/3*Chi(-(-1)^(2/3)*a^(1/3)*d/b^(1/3)-d*x)*cosh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))
/a+Shi(d*x)*sinh(c)/a-1/3*Shi(a^(1/3)*d/b^(1/3)+d*x)*sinh(c-a^(1/3)*d/b^(1/3))
/a-1/3*Shi(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)*sinh(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))
/a-1/3*Shi((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)*sinh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a

Rubi [A] time = 0.52, antiderivative size = 303, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {5293, 3303, 3298, 3301}

$$\frac{\cosh\left(\frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}} + c\right) \text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}} - dx\right)}{3a} - \frac{\cosh\left(c - \frac{(-1)^{2/3}\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(-xd - \frac{(-1)^{2/3}\sqrt[3]{ad}}{\sqrt[3]{b}}\right)}{3a} - \frac{\cosh\left(c - \frac{\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(xd - \frac{\sqrt[3]{ad}}{\sqrt[3]{b}}\right)}{3a}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]/(x*(a + b*x^3)),x]

[Out] (Cosh[c]*CoshIntegral[d*x])/a - (Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]/(3*a) - (Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3) - d*x]/(3*a) - (Cosh[c - (a^(1/3)*d)/b^(1/3)]*CoshIntegral[(a^(1/3)*d)/b^(1/3) + d*x]/(3*a) + (Sinh[c]*SinhIntegral[d*x])/a + (Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]/(3*a) - (Sinh[c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x]/(3*a) - (Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]/(3*a)

Rule 3298

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x]
&& EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[
Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

Rule 5293

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d}, x]
&& ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh(c+dx)}{x(a+bx^3)} dx &= \int \left(\frac{\cosh(c+dx)}{ax} - \frac{bx^2 \cosh(c+dx)}{a(a+bx^3)} \right) dx \\
&= \frac{\int \frac{\cosh(c+dx)}{x} dx}{a} - \frac{b \int \frac{x^2 \cosh(c+dx)}{a+bx^3} dx}{a} \\
&= -\frac{b \int \left(\frac{\cosh(c+dx)}{3b^{2/3}(\sqrt[3]{a} + \sqrt[3]{b}x)} + \frac{\cosh(c+dx)}{3b^{2/3}(-\sqrt[3]{-1}\sqrt[3]{a} + \sqrt[3]{b}x)} + \frac{\cosh(c+dx)}{3b^{2/3}((-1)^{2/3}\sqrt[3]{a} + \sqrt[3]{b}x)} \right) dx}{a} + \frac{\cosh(c) \int \frac{\cosh(dx)}{x} dx}{a} \\
&= \frac{\cosh(c)\text{Chi}(dx)}{a} + \frac{\sinh(c)\text{Shi}(dx)}{a} - \frac{\sqrt[3]{b} \int \frac{\cosh(c+dx)}{\sqrt[3]{a} + \sqrt[3]{b}x} dx}{3a} - \frac{\sqrt[3]{b} \int \frac{\cosh(c+dx)}{-\sqrt[3]{-1}\sqrt[3]{a} + \sqrt[3]{b}x} dx}{3a} - \frac{\sqrt[3]{b} \int \frac{\cosh(c+dx)}{(-1)^{2/3}\sqrt[3]{a} + \sqrt[3]{b}x} dx}{3a} \\
&= \frac{\cosh(c)\text{Chi}(dx)}{a} + \frac{\sinh(c)\text{Shi}(dx)}{a} - \frac{\left(\sqrt[3]{b} \cosh\left(c - \frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right) \int \frac{\cosh\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}} + dx\right)}{\sqrt[3]{a} + \sqrt[3]{b}x} dx \right)}{3a} - \frac{\left(\sqrt[3]{b} \cosh\left(c - \frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right) \int \frac{\cosh\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}} + dx\right)}{\sqrt[3]{a} + \sqrt[3]{b}x} dx \right)}{3a} \\
&= \frac{\cosh(c)\text{Chi}(dx)}{a} - \frac{\cosh\left(c + \frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}} - dx\right)}{3a} - \frac{\cosh\left(c - \frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}} + dx\right)}{3a}
\end{aligned}$$

Mathematica [C] time = 0.28, size = 186, normalized size = 0.61

RootSum[#1^3 b + a &, -sinh(#1 d + c) Chi(d(x - #1)) + cosh(#1 d + c) Chi(d(x - #1)) + sinh(#1 d + c) Shi(d(x - #1)) &]

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d*x]/(x*(a + b*x^3)), x]

[Out] -1/6*(-6*Cosh[c]*CoshIntegral[d*x] + RootSum[a + b*#1^3 &, Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] - CoshIntegral[d*(x - #1)]*Sinh[c + d*#1] - Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] + Sinh[c + d*#1]*SinhIntegral[d*(x - #1)] &] + RootSum[a + b*#1^3 &, Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] + CoshIntegral[d*(x - #1)]*Sinh[c + d*#1] + Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] + Sinh[c + d*#1]*SinhIntegral[d*(x - #1)] &] - 6*Sinh[c]*SinhIntegral[d*x])/a

fricas [B] time = 0.52, size = 530, normalized size = 1.75

$$\text{Ei}\left(dx - \frac{1}{2}\left(\frac{ad^3}{b}\right)^{\frac{1}{3}}(\sqrt{-3} + 1)\right)\cosh\left(\frac{1}{2}\left(\frac{ad^3}{b}\right)^{\frac{1}{3}}(\sqrt{-3} + 1) + c\right) + \text{Ei}\left(-dx - \frac{1}{2}\left(-\frac{ad^3}{b}\right)^{\frac{1}{3}}(\sqrt{-3} + 1)\right)\cosh\left(\frac{1}{2}\left(-\frac{ad^3}{b}\right)^{\frac{1}{3}}(\sqrt{-3} + 1) - c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/x/(b*x^3+a), x, algorithm="fricas")

[Out] -1/6*(Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1) + c) + Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1) - c) + Ei(d*x + 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1) - c) + Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c) + Ei(-d*x + (-a*d^3/b)^(1/3))*cosh(c + (-a*d^3/b)^(1/3)) - 3*(Ei(d*x) + Ei(-d*x))*cosh(c) + Ei(d*x + (a*d^3/b)^(1/3))*cosh(-c + (a*d^3/b)^(1/3)) + Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1) + c) + Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1) - c) - Ei(d*x + 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1))*sinh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1) - c) - Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1))*sinh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c) - Ei(-d*x + (-a*d^3/b)^(1/3))*sinh(c + (-a*d^3/b)^(1/3)) - 3*(Ei(d*x) - Ei(-d*x))*sinh(c) - Ei(d*x + (a*d^3/b)^(1/3))*sinh(-c + (a*d^3/b)^(1/3)))/a

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(dx + c)}{(bx^3 + a)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/x/(b*x^3+a),x, algorithm="giac")

[Out] integrate(cosh(d*x + c)/((b*x^3 + a)*x), x)

maple [C] time = 0.11, size = 138, normalized size = 0.46

$$\frac{\sum_{_R1=\text{RootOf}(b_Z^3-3cb_Z^2+3bc^2_Z+ad^3-bc^3)} e^{-R1} \text{Ei}(1, dx - _R1 + c)}{6a} - \frac{e^{-c} \text{Ei}(1, dx)}{2a} + \frac{\sum_{_R1=\text{RootOf}(b_Z^3-3cb_Z^2+3bc^2_Z+ad^3-bc^3)} e^{R1} \text{Ei}(1, -dx + _R1 - c)}{6a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d*x+c)/x/(b*x^3+a),x)

[Out] 1/6/a*sum(exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/2/a*exp(-c)*Ei(1,d*x)+1/6/a*sum(exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/2/a*exp(c)*Ei(1,-d*x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(dx + c)}{(bx^3 + a)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/x/(b*x^3+a),x, algorithm="maxima")

[Out] integrate(cosh(d*x + c)/((b*x^3 + a)*x), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(c + dx)}{x(bx^3 + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(c + d*x)/(x*(a + b*x^3)),x)

[Out] int(cosh(c + d*x)/(x*(a + b*x^3)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(c + dx)}{x(a + bx^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x/(b*x**3+a),x)
```

```
[Out] Integral(cosh(c + d*x)/(x*(a + b*x**3)), x)
```

$$3.100 \quad \int \frac{\cosh(c+dx)}{x^2(a+bx^3)} dx$$

Optimal. Leaf size=381

$$\frac{(-1)^{2/3} \sqrt[3]{b} \cosh\left(\frac{\sqrt[3]{-1} \sqrt[3]{ad}}{\sqrt[3]{b}} + c\right) \text{Chi}\left(\frac{\sqrt[3]{-1} \sqrt[3]{ad}}{\sqrt[3]{b}} - dx\right)}{3a^{4/3}} - \frac{\sqrt[3]{-1} \sqrt[3]{b} \cosh\left(c - \frac{(-1)^{2/3} \sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(-xd - \frac{(-1)^{2/3} \sqrt[3]{ad}}{\sqrt[3]{b}}\right)}{3a^{4/3}} + \frac{\sqrt[3]{b}}{x}$$

[Out] $\frac{1}{3} b^{1/3} \text{Chi}(a^{1/3} d/b^{1/3} + dx) \cosh(c - a^{1/3} d/b^{1/3})/a^{4/3} + \frac{1}{3} (-1)^{2/3} b^{1/3} \text{Chi}((-1)^{1/3} a^{1/3} d/b^{1/3} - dx) \cosh(c + (-1)^{1/3} a^{1/3} d/b^{1/3})/a^{4/3} - \frac{1}{3} (-1)^{1/3} b^{1/3} \text{Chi}(-(-1)^{2/3} a^{1/3} d/b^{1/3} - dx) \cosh(c - (-1)^{2/3} a^{1/3} d/b^{1/3})/a^{4/3} - \frac{\cosh(dx+c)}{a} + \frac{\cosh(c) \text{Shi}(dx)}{a} + \frac{\text{Chi}(dx) \sinh(c)}{a} + \frac{1}{3} b^{1/3} \text{Shi}(a^{1/3} d/b^{1/3} + dx) \sinh(c - a^{1/3} d/b^{1/3})/a^{4/3} + \frac{1}{3} (-1)^{2/3} b^{1/3} \text{Shi}(-(-1)^{1/3} a^{1/3} d/b^{1/3} + dx) \sinh(c + (-1)^{1/3} a^{1/3} d/b^{1/3})/a^{4/3} - \frac{1}{3} (-1)^{1/3} b^{1/3} \text{Shi}((-1)^{2/3} a^{1/3} d/b^{1/3} + dx) \sinh(c - (-1)^{2/3} a^{1/3} d/b^{1/3})/a^{4/3}$

Rubi [A] time = 0.60, antiderivative size = 381, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {5293, 3297, 3303, 3298, 3301}

$$\frac{(-1)^{2/3} \sqrt[3]{b} \cosh\left(\frac{\sqrt[3]{-1} \sqrt[3]{ad}}{\sqrt[3]{b}} + c\right) \text{Chi}\left(\frac{\sqrt[3]{-1} \sqrt[3]{ad}}{\sqrt[3]{b}} - dx\right)}{3a^{4/3}} - \frac{\sqrt[3]{-1} \sqrt[3]{b} \cosh\left(c - \frac{(-1)^{2/3} \sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(-xd - \frac{(-1)^{2/3} \sqrt[3]{ad}}{\sqrt[3]{b}}\right)}{3a^{4/3}} + \frac{\sqrt[3]{b}}{x}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]/(x^2*(a + b*x^3)), x]

[Out] $-\frac{\cosh(c + dx)}{ax} + \frac{(-1)^{2/3} b^{1/3} \cosh\left[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right] \text{CoshIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - dx\right]}{3a^{4/3}} - \frac{(-1)^{1/3} b^{1/3} \cosh\left[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right] \text{CoshIntegral}\left[-\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} - dx\right]}{3a^{4/3}} + \frac{b^{1/3} \cosh\left[c - \frac{a^{1/3} d}{b^{1/3}}\right] \text{CoshIntegral}\left[\frac{a^{1/3} d}{b^{1/3}} + dx\right]}{3a^{4/3}} + \frac{d \text{CoshIntegral}[dx] \sinh(c)}{a} + \frac{d \cosh(c) \text{SinhIntegral}[dx]}{a} - \frac{(-1)^{2/3} b^{1/3} \sinh\left[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right] \text{SinhIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - dx\right]}{3a^{4/3}} + \frac{b^{1/3} \sinh\left[c - \frac{a^{1/3} d}{b^{1/3}}\right] \text{SinhIntegral}\left[\frac{a^{1/3} d}{b^{1/3}} + dx\right]}{3a^{4/3}} - \frac{(-1)^{1/3} b^{1/3} \sinh\left[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right] \text{SinhIntegral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + dx\right]}{3a^{4/3}}$

Rule 3297

Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c

```
+ d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x]
&& EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x]
&& EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*e - c*f)/d],
Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d],
Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

Rule 5293

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d}, x]
&& ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh(c+dx)}{x^2(a+bx^3)} dx &= \int \left(\frac{\cosh(c+dx)}{ax^2} - \frac{bx \cosh(c+dx)}{a(a+bx^3)} \right) dx \\
&= \frac{\int \frac{\cosh(c+dx)}{x^2} dx}{a} - \frac{b \int \frac{x \cosh(c+dx)}{a+bx^3} dx}{a} \\
&= -\frac{\cosh(c+dx)}{ax} - \frac{b \int \left(-\frac{\cosh(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}(\sqrt[3]{a}+\sqrt[3]{b}x)} - \frac{(-1)^{2/3}\cosh(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x)} + \frac{\sqrt[3]{-1}\cosh(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x)} \right) dx}{a} \\
&= -\frac{\cosh(c+dx)}{ax} + \frac{b^{2/3} \int \frac{\cosh(c+dx)}{\sqrt[3]{a}+\sqrt[3]{b}x} dx}{3a^{4/3}} - \frac{(\sqrt[3]{-1}b^{2/3}) \int \frac{\cosh(c+dx)}{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x} dx}{3a^{4/3}} + \frac{((-1)^{2/3}b^{2/3}) \int \frac{\cosh(c+dx)}{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x} dx}{3a^{4/3}} \\
&= -\frac{\cosh(c+dx)}{ax} + \frac{d\text{Chi}(dx)\sinh(c)}{a} + \frac{d\cosh(c)\text{Shi}(dx)}{a} + \frac{\left(b^{2/3}\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right) \int \frac{\cosh\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{\sqrt[3]{a}+\sqrt[3]{b}x} dx}{3a^{4/3}} \\
&= -\frac{\cosh(c+dx)}{ax} + \frac{(-1)^{2/3}\sqrt[3]{b}\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right) - \sqrt[3]{-1}\sqrt[3]{b}\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{4/3}}
\end{aligned}$$

Mathematica [C] time = 0.43, size = 215, normalized size = 0.56

$$x\text{RootSum}\left[\#1^3b + a\&, \frac{-\sinh(\#1d+c)\text{Chi}(d(x-\#1))+\cosh(\#1d+c)\text{Chi}(d(x-\#1))+\sinh(\#1d+c)\text{Shi}(d(x-\#1))-\cosh(\#1d+c)\text{Shi}(d(x-\#1))}{\#1}\&]$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d*x]/(x^2*(a + b*x^3)), x]

[Out] -1/6*(6*Cosh[c + d*x] + x*RootSum[a + b*#1^3 &, (Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] - CoshIntegral[d*(x - #1)]*Sinh[c + d*#1] - Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] + Sinh[c + d*#1]*SinhIntegral[d*(x - #1)])/#1 &] + x*RootSum[a + b*#1^3 &, (Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] + CoshIntegral[d*(x - #1)]*Sinh[c + d*#1] + Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] + Sinh[c + d*#1]*SinhIntegral[d*(x - #1)])/#1 &] - 6*d*x*CoshIntegral[d*x]*Sinh[c] - 6*d*x*Cosh[c]*SinhIntegral[d*x])/(a*x)

fricas [B] time = 0.69, size = 1154, normalized size = 3.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/x^2/(b*x^3+a),x, algorithm="fricas")

[Out]
$$\begin{aligned} & -1/12*(12*a*d^2*cosh(d*x + c) - (a*d^3/b)^{(2/3)}*((sqrt(-3)*b*x - b*x)*cosh(d*x + c)^2 - (sqrt(-3)*b*x - b*x)*sinh(d*x + c)^2)*Ei(d*x - 1/2*(a*d^3/b)^{(1/3)}*(sqrt(-3) + 1))*cosh(1/2*(a*d^3/b)^{(1/3)}*(sqrt(-3) + 1) + c) - (-a*d^3/b)^{(2/3)}*((sqrt(-3)*b*x - b*x)*cosh(d*x + c)^2 - (sqrt(-3)*b*x - b*x)*sinh(d*x + c)^2)*Ei(-d*x - 1/2*(-a*d^3/b)^{(1/3)}*(sqrt(-3) + 1))*cosh(1/2*(-a*d^3/b)^{(1/3)}*(sqrt(-3) + 1) - c) + (a*d^3/b)^{(2/3)}*((sqrt(-3)*b*x + b*x)*cosh(d*x + c)^2 - (sqrt(-3)*b*x + b*x)*sinh(d*x + c)^2)*Ei(d*x + 1/2*(a*d^3/b)^{(1/3)}*(sqrt(-3) - 1))*cosh(1/2*(a*d^3/b)^{(1/3)}*(sqrt(-3) - 1) - c) + (-a*d^3/b)^{(2/3)}*((sqrt(-3)*b*x + b*x)*cosh(d*x + c)^2 - (sqrt(-3)*b*x + b*x)*sinh(d*x + c)^2)*Ei(-d*x + 1/2*(-a*d^3/b)^{(1/3)}*(sqrt(-3) - 1))*cosh(1/2*(-a*d^3/b)^{(1/3)}*(sqrt(-3) - 1) + c) - 2*(b*x*cosh(d*x + c)^2 - b*x*sinh(d*x + c)^2)*(-a*d^3/b)^{(2/3)}*Ei(-d*x + (-a*d^3/b)^{(1/3)})*cosh(c + (-a*d^3/b)^{(1/3)}) - 2*(b*x*cosh(d*x + c)^2 - b*x*sinh(d*x + c)^2)*(a*d^3/b)^{(2/3)}*Ei(d*x + (a*d^3/b)^{(1/3)})*cosh(-c + (a*d^3/b)^{(1/3)}) - (a*d^3/b)^{(2/3)}*((sqrt(-3)*b*x - b*x)*cosh(d*x + c)^2 - (sqrt(-3)*b*x - b*x)*sinh(d*x + c)^2)*Ei(d*x - 1/2*(a*d^3/b)^{(1/3)}*(sqrt(-3) + 1))*sinh(1/2*(a*d^3/b)^{(1/3)}*(sqrt(-3) + 1) + c) - (-a*d^3/b)^{(2/3)}*((sqrt(-3)*b*x - b*x)*cosh(d*x + c)^2 - (sqrt(-3)*b*x - b*x)*sinh(d*x + c)^2)*Ei(-d*x - 1/2*(-a*d^3/b)^{(1/3)}*(sqrt(-3) + 1))*sinh(1/2*(-a*d^3/b)^{(1/3)}*(sqrt(-3) + 1) - c) - (a*d^3/b)^{(2/3)}*((sqrt(-3)*b*x + b*x)*cosh(d*x + c)^2 - (sqrt(-3)*b*x + b*x)*sinh(d*x + c)^2)*Ei(d*x + 1/2*(a*d^3/b)^{(1/3)}*(sqrt(-3) - 1))*sinh(1/2*(a*d^3/b)^{(1/3)}*(sqrt(-3) - 1) - c) - (-a*d^3/b)^{(2/3)}*((sqrt(-3)*b*x + b*x)*cosh(d*x + c)^2 - (sqrt(-3)*b*x + b*x)*sinh(d*x + c)^2)*Ei(-d*x + 1/2*(-a*d^3/b)^{(1/3)}*(sqrt(-3) - 1))*sinh(1/2*(-a*d^3/b)^{(1/3)}*(sqrt(-3) - 1) + c) + 2*(b*x*cosh(d*x + c)^2 - b*x*sinh(d*x + c)^2)*(-a*d^3/b)^{(2/3)}*Ei(-d*x + (-a*d^3/b)^{(1/3)})*sinh(c + (-a*d^3/b)^{(1/3)}) + 2*(b*x*cosh(d*x + c)^2 - b*x*sinh(d*x + c)^2)*(a*d^3/b)^{(2/3)}*Ei(d*x + (a*d^3/b)^{(1/3)})*sinh(-c + (a*d^3/b)^{(1/3)}) - 6*(a*d^3*x*Ei(d*x) - a*d^3*x*Ei(-d*x))*cosh(c) - 6*(a*d^3*x*Ei(d*x) + a*d^3*x*Ei(-d*x))*sinh(c))/(a^2*d^2*x*cosh(d*x + c)^2 - a^2*d^2*x*sinh(d*x + c)^2) \end{aligned}$$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(dx + c)}{(bx^3 + a)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/x^2/(b*x^3+a),x, algorithm="giac")

[Out] integrate(cosh(d*x + c)/((b*x^3 + a)*x^2), x)

maple [C] time = 0.13, size = 187, normalized size = 0.49

$$-\frac{e^{-dx-c}}{2ax} + \frac{d \left(\sum_{_R1=\text{RootOf}(b_Z^3-3cb_Z^2+3bc^2_Z+a d^3-bc^3)} \frac{e^{-_R1} \text{Ei}(1, dx-_R1+c)}{-_R1-c} \right)}{6a} + \frac{d e^{-c} \text{Ei}(1, dx)}{2a} - \frac{e^{dx+c}}{2ax} + \frac{d \left(\sum_{_R1=\text{RootOf}(b_Z^3-3cb_Z^2+3bc^2_Z+a d^3-bc^3)} \frac{e^{_R1} \text{Ei}(1, -dx+_R1-c)}{_R1-c} \right)}{6a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d*x+c)/x^2/(b*x^3+a), x)

[Out] $-1/2*\exp(-d*x-c)/a/x+1/6*d/a*\sum(1/(_R1-c)*\exp(-_R1)*\text{Ei}(1, d*x-_R1+c), _R1=\text{RootOf}(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/2*d/a*\exp(-c)*\text{Ei}(1, d*x)-1/2*\exp(d*x+c)/a/x+1/6*d/a*\sum(1/(_R1-c)*\exp(_R1)*\text{Ei}(1, -d*x+_R1-c), _R1=\text{RootOf}(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/2*d/a*\exp(c)*\text{Ei}(1, -d*x)$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/x^2/(b*x^3+a), x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(c + dx)}{x^2 (bx^3 + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(c + d*x)/(x^2*(a + b*x^3)), x)

[Out] int(cosh(c + d*x)/(x^2*(a + b*x^3)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/x**2/(b*x**3+a), x)

[Out] Timed out

$$3.101 \quad \int \frac{\cosh(c+dx)}{x^3(a+bx^3)} dx$$

Optimal. Leaf size=410

$$\frac{\sqrt[3]{-1} b^{2/3} \cosh\left(\frac{\sqrt[3]{-1} \sqrt[3]{ad}}{\sqrt[3]{b}} + c\right) \text{Chi}\left(\frac{\sqrt[3]{-1} \sqrt[3]{ad}}{\sqrt[3]{b}} - dx\right)}{3a^{5/3}} - \frac{(-1)^{2/3} b^{2/3} \cosh\left(c - \frac{(-1)^{2/3} \sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(-xd - \frac{(-1)^{2/3} \sqrt[3]{ad}}{\sqrt[3]{b}}\right)}{3a^{5/3}} - b^{2/3} c$$

[Out] $\frac{1}{2}d^2 \text{Chi}(dx) \cosh(c) / a - 1/3 b^{(2/3)} \text{Chi}(a^{(1/3)} d / b^{(1/3)} + dx) \cosh(c - a^{(1/3)} d / b^{(1/3)}) / a^{(5/3)} + 1/3 (-1)^{(1/3)} b^{(2/3)} \text{Chi}((-1)^{(1/3)} a^{(1/3)} d / b^{(1/3)} - dx) \cosh(c + (-1)^{(1/3)} a^{(1/3)} d / b^{(1/3)}) / a^{(5/3)} - 1/3 (-1)^{(2/3)} b^{(2/3)} \text{Chi}(-(-1)^{(2/3)} a^{(1/3)} d / b^{(1/3)} - dx) \cosh(c - (-1)^{(2/3)} a^{(1/3)} d / b^{(1/3)}) / a^{(5/3)} - 1/2 \cosh(dx+c) / a / x^2 + 1/2 d^2 \text{Shi}(dx) \sinh(c) / a - 1/3 b^{(2/3)} \text{Shi}(a^{(1/3)} d / b^{(1/3)} + dx) \sinh(c - a^{(1/3)} d / b^{(1/3)}) / a^{(5/3)} + 1/3 (-1)^{(1/3)} b^{(2/3)} \text{Shi}(-(-1)^{(1/3)} a^{(1/3)} d / b^{(1/3)} + dx) \sinh(c + (-1)^{(1/3)} a^{(1/3)} d / b^{(1/3)}) / a^{(5/3)} - 1/3 (-1)^{(2/3)} b^{(2/3)} \text{Shi}((-1)^{(2/3)} a^{(1/3)} d / b^{(1/3)} + dx) \sinh(c - (-1)^{(2/3)} a^{(1/3)} d / b^{(1/3)}) / a^{(5/3)} - 1/2 d \sinh(dx+c) / a / x$

Rubi [A] time = 0.64, antiderivative size = 410, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {5293, 3297, 3303, 3298, 3301, 5281}

$$\frac{\sqrt[3]{-1} b^{2/3} \cosh\left(\frac{\sqrt[3]{-1} \sqrt[3]{ad}}{\sqrt[3]{b}} + c\right) \text{Chi}\left(\frac{\sqrt[3]{-1} \sqrt[3]{ad}}{\sqrt[3]{b}} - dx\right)}{3a^{5/3}} - \frac{(-1)^{2/3} b^{2/3} \cosh\left(c - \frac{(-1)^{2/3} \sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(-xd - \frac{(-1)^{2/3} \sqrt[3]{ad}}{\sqrt[3]{b}}\right)}{3a^{5/3}} - b^{2/3} c$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]/(x^3*(a + b*x^3)),x]

[Out] $-\text{Cosh}[c + dx] / (2*a*x^2) + (d^2 * \text{Cosh}[c] * \text{CoshIntegral}[dx]) / (2*a) + ((-1)^{(1/3)} * b^{(2/3)} * \text{Cosh}[c + ((-1)^{(1/3)} * a^{(1/3)} * d) / b^{(1/3)}] * \text{CoshIntegral}[(((-1)^{(1/3)} * a^{(1/3)} * d) / b^{(1/3)} - dx)]) / (3*a^{(5/3)}) - ((-1)^{(2/3)} * b^{(2/3)} * \text{Cosh}[c - ((-1)^{(2/3)} * a^{(1/3)} * d) / b^{(1/3)}] * \text{CoshIntegral}[(-(((-1)^{(2/3)} * a^{(1/3)} * d) / b^{(1/3)} - dx)]) / (3*a^{(5/3)}) - (b^{(2/3)} * \text{Cosh}[c - (a^{(1/3)} * d) / b^{(1/3)}] * \text{CoshIntegral}[(a^{(1/3)} * d) / b^{(1/3)} + dx]) / (3*a^{(5/3)}) - (d * \text{Sinh}[c + dx]) / (2*a*x) + (d^2 * \text{Sinh}[c] * \text{SinhIntegral}[dx]) / (2*a) - ((-1)^{(1/3)} * b^{(2/3)} * \text{Sinh}[c + ((-1)^{(1/3)} * a^{(1/3)} * d) / b^{(1/3)}] * \text{SinhIntegral}[(((-1)^{(1/3)} * a^{(1/3)} * d) / b^{(1/3)} - dx)]) / (3*a^{(5/3)}) - (b^{(2/3)} * \text{Sinh}[c - (a^{(1/3)} * d) / b^{(1/3)}] * \text{SinhIntegral}[(a^{(1/3)} * d) / b^{(1/3)} + dx]) / (3*a^{(5/3)}) - ((-1)^{(2/3)} * b^{(2/3)} * \text{Sinh}[c - ((-1)^{(2/3)} * a^{(1/3)} * d) / b^{(1/3)}] * \text{SinhIntegral}[(((-1)^{(2/3)} * a^{(1/3)} * d) / b^{(1/3)} + dx)]) / (3*a^{(5/3)})$

Rule 3297


```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]
```

Rule 5281

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Int[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 5293

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh(c+dx)}{x^3(a+bx^3)} dx &= \int \left(\frac{\cosh(c+dx)}{ax^3} - \frac{b \cosh(c+dx)}{a(a+bx^3)} \right) dx \\
&= \frac{\int \frac{\cosh(c+dx)}{x^3} dx}{a} - \frac{b \int \frac{\cosh(c+dx)}{a+bx^3} dx}{a} \\
&= -\frac{\cosh(c+dx)}{2ax^2} - \frac{b \int \left(-\frac{\cosh(c+dx)}{3a^{2/3}(-\sqrt[3]{a}-\sqrt[3]{b}x)} - \frac{\cosh(c+dx)}{3a^{2/3}(-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x)} - \frac{\cosh(c+dx)}{3a^{2/3}(-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x)} \right) dx}{a} + \dots \\
&= -\frac{\cosh(c+dx)}{2ax^2} - \frac{d \sinh(c+dx)}{2ax} + \frac{b \int \frac{\cosh(c+dx)}{-\sqrt[3]{a}-\sqrt[3]{b}x} dx}{3a^{5/3}} + \frac{b \int \frac{\cosh(c+dx)}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x} dx}{3a^{5/3}} + \frac{b \int \frac{\cosh(c+dx)}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x} dx}{3a^{5/3}} + \dots \\
&= -\frac{\cosh(c+dx)}{2ax^2} - \frac{d \sinh(c+dx)}{2ax} + \frac{(d^2 \cosh(c)) \int \frac{\cosh(dx)}{x} dx}{2a} + \frac{\left(b \cosh\left(c - \frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right) \right) \int \frac{\cosh\left(\frac{dx}{-\sqrt[3]{a}}\right)}{-\sqrt[3]{a}}}{3a^{5/3}} + \dots \\
&= -\frac{\cosh(c+dx)}{2ax^2} + \frac{d^2 \cosh(c) \text{Chi}(dx)}{2a} + \frac{\sqrt[3]{-1} b^{2/3} \cosh\left(c + \frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right) \text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}} - dx\right)}{3a^{5/3}} + \dots
\end{aligned}$$

Mathematica [C] time = 0.41, size = 237, normalized size = 0.58

$$x^2 \text{RootSum}\left[\#1^3 b + a \&, \frac{-\sinh(\#1 d + c) \text{Chi}(d(x - \#1)) + \cosh(\#1 d + c) \text{Chi}(d(x - \#1)) + \sinh(\#1 d + c) \text{Shi}(d(x - \#1)) - \cosh(\#1 d + c) \text{Shi}(d(x - \#1))}{\#1^2} \& \right]$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d*x]/(x^3*(a + b*x^3)), x]

[Out] -1/6*(3*Cosh[c + d*x] - 3*d^2*x^2*Cosh[c]*CoshIntegral[d*x] + x^2*RootSum[a + b*#1^3 &, (Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] - CoshIntegral[d*(x - #1)]*Sinh[c + d*#1] - Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] + Sinh[c + d*#1]*SinhIntegral[d*(x - #1)])/#1^2 &] + x^2*RootSum[a + b*#1^3 &, (Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] + CoshIntegral[d*(x - #1)]*Sinh[c + d*#1] + Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] + Sinh[c + d*#1]*SinhIntegral[d*(x - #1)])/#1^2 &] + 3*d*x*Sinh[c + d*x] - 3*d^2*x^2*Sinh[c]*SinhIntegral[d*x]/(a*x^2)

fricas [B] time = 0.57, size = 1251, normalized size = 3.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/x^3/(b*x^3+a),x, algorithm="fricas")

[Out]
$$-1/12*(6*a*d^2*x*\sinh(d*x + c) - (a*d^3/b)^{1/3}*((\sqrt{-3}*b*x^2 + b*x^2)*\cosh(d*x + c)^2 - (\sqrt{-3}*b*x^2 + b*x^2)*\sinh(d*x + c)^2)*\text{Ei}(d*x - 1/2*(a*d^3/b)^{1/3}*(\sqrt{-3} + 1))*\cosh(1/2*(a*d^3/b)^{1/3}*(\sqrt{-3} + 1) + c) + (-a*d^3/b)^{1/3}*((\sqrt{-3}*b*x^2 + b*x^2)*\cosh(d*x + c)^2 - (\sqrt{-3}*b*x^2 + b*x^2)*\sinh(d*x + c)^2)*\text{Ei}(-d*x - 1/2*(-a*d^3/b)^{1/3}*(\sqrt{-3} + 1))*\cosh(1/2*(-a*d^3/b)^{1/3}*(\sqrt{-3} + 1) - c) + (a*d^3/b)^{1/3}*((\sqrt{-3})*b*x^2 - b*x^2)*\cosh(d*x + c)^2 - (\sqrt{-3})*b*x^2 - b*x^2)*\sinh(d*x + c)^2*\text{Ei}(d*x + 1/2*(a*d^3/b)^{1/3}*(\sqrt{-3} - 1))*\cosh(1/2*(a*d^3/b)^{1/3}*(\sqrt{-3} - 1) - c) - (-a*d^3/b)^{1/3}*((\sqrt{-3})*b*x^2 - b*x^2)*\cosh(d*x + c)^2 - (\sqrt{-3})*b*x^2 - b*x^2)*\sinh(d*x + c)^2*\text{Ei}(-d*x + 1/2*(-a*d^3/b)^{1/3}*(\sqrt{-3} - 1))*\cosh(1/2*(-a*d^3/b)^{1/3}*(\sqrt{-3} - 1) + c) - 2*(b*x^2*\cosh(d*x + c)^2 - b*x^2*\sinh(d*x + c)^2)*(-a*d^3/b)^{1/3}*\text{Ei}(-d*x + (-a*d^3/b)^{1/3})*\cosh(c + (-a*d^3/b)^{1/3}) + 2*(b*x^2*\cosh(d*x + c)^2 - b*x^2*\sinh(d*x + c)^2)*(a*d^3/b)^{1/3}*\text{Ei}(d*x + (a*d^3/b)^{1/3})*\cosh(-c + (a*d^3/b)^{1/3}) - (a*d^3/b)^{1/3}*((\sqrt{-3})*b*x^2 + b*x^2)*\cosh(d*x + c)^2 - (\sqrt{-3})*b*x^2 + b*x^2)*\sinh(d*x + c)^2*\text{Ei}(d*x - 1/2*(a*d^3/b)^{1/3}*(\sqrt{-3} + 1))*\sinh(1/2*(a*d^3/b)^{1/3}*(\sqrt{-3} + 1) + c) + (-a*d^3/b)^{1/3}*((\sqrt{-3})*b*x^2 + b*x^2)*\cosh(d*x + c)^2 - (\sqrt{-3})*b*x^2 + b*x^2)*\sinh(d*x + c)^2*\text{Ei}(-d*x - 1/2*(-a*d^3/b)^{1/3}*(\sqrt{-3} + 1))*\sinh(1/2*(-a*d^3/b)^{1/3}*(\sqrt{-3} + 1) - c) - (a*d^3/b)^{1/3}*((\sqrt{-3})*b*x^2 - b*x^2)*\cosh(d*x + c)^2 - (\sqrt{-3})*b*x^2 - b*x^2)*\sinh(d*x + c)^2*\text{Ei}(d*x + 1/2*(a*d^3/b)^{1/3}*(\sqrt{-3} - 1))*\sinh(1/2*(a*d^3/b)^{1/3}*(\sqrt{-3} - 1) - c) + (-a*d^3/b)^{1/3}*((\sqrt{-3})*b*x^2 - b*x^2)*\cosh(d*x + c)^2 - (\sqrt{-3})*b*x^2 - b*x^2)*\sinh(d*x + c)^2*\text{Ei}(-d*x + 1/2*(-a*d^3/b)^{1/3}*(\sqrt{-3} - 1))*\sinh(1/2*(-a*d^3/b)^{1/3}*(\sqrt{-3} - 1) + c) + 2*(b*x^2*\cosh(d*x + c)^2 - b*x^2*\sinh(d*x + c)^2)*(-a*d^3/b)^{1/3}*\text{Ei}(-d*x + (-a*d^3/b)^{1/3})*\sinh(c + (-a*d^3/b)^{1/3}) - 2*(b*x^2*\cosh(d*x + c)^2 - b*x^2*\sinh(d*x + c)^2)*(a*d^3/b)^{1/3}*\text{Ei}(d*x + (a*d^3/b)^{1/3})*\sinh(-c + (a*d^3/b)^{1/3}) + 6*a*d*\cosh(d*x + c) - 3*(a*d^3*x^2*\text{Ei}(d*x) + a*d^3*x^2*\text{Ei}(-d*x))*\cosh(c) - 3*(a*d^3*x^2*\text{Ei}(d*x) - a*d^3*x^2*\text{Ei}(-d*x))*\sinh(c))/(a^2*d*x^2*\cosh(d*x + c)^2 - a^2*d*x^2*\sinh(d*x + c)^2)$$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(dx + c)}{(bx^3 + a)x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/x^3/(b*x^3+a),x, algorithm="giac")

[Out] integrate(cosh(d*x + c)/((b*x^3 + a)*x^3), x)

maple [C] time = 0.14, size = 240, normalized size = 0.59

$$\frac{d e^{-dx-c}}{4ax} - \frac{e^{-dx-c}}{4a x^2} + \frac{d^2 \left(\sum_{R1=\text{RootOf}(b_Z^3-3cb_Z^2+3bc^2_Z+a d^3-b c^3)} \frac{e^{-R1} \text{Ei}(1, dx - R1+c)}{-R1^2-2_R1c+c^2} \right)}{6a} - \frac{d^2 e^{-c} \text{Ei}(1, dx)}{4a} - \frac{d e^{dx+c}}{4ax} - \frac{e^{dx+c}}{4a x^2} +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d*x+c)/x^3/(b*x^3+a), x)

[Out] 1/4*d*exp(-d*x-c)/a/x-1/4*exp(-d*x-c)/a/x^2+1/6*d^2/a*sum(1/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c), _R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/4*d^2/a*exp(-c)*Ei(1,d*x)-1/4*d*exp(d*x+c)/a/x-1/4*exp(d*x+c)/a/x^2+1/6*d^2/a*sum(1/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c), _R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/4*d^2/a*exp(c)*Ei(1,-d*x)

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/x^3/(b*x^3+a), x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(c + dx)}{x^3 (bx^3 + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(c + d*x)/(x^3*(a + b*x^3)), x)

[Out] int(cosh(c + d*x)/(x^3*(a + b*x^3)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/x**3/(b*x**3+a), x)

[Out] Timed out

$$3.102 \quad \int \frac{x^3 \cosh(c+dx)}{(a+bx^3)^2} dx$$

Optimal. Leaf size=718

$$\frac{\sqrt[3]{-1} \cosh\left(\frac{\sqrt[3]{-1} \sqrt[3]{ad}}{\sqrt[3]{b}} + c\right) \text{Chi}\left(\frac{\sqrt[3]{-1} \sqrt[3]{ad}}{\sqrt[3]{b}} - dx\right)}{9a^{2/3}b^{4/3}} + \frac{(-1)^{2/3} \cosh\left(c - \frac{(-1)^{2/3} \sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(-xd - \frac{(-1)^{2/3} \sqrt[3]{ad}}{\sqrt[3]{b}}\right)}{9a^{2/3}b^{4/3}} + \frac{\cosh\left(c - \frac{(-1)^{2/3} \sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(-xd - \frac{(-1)^{2/3} \sqrt[3]{ad}}{\sqrt[3]{b}}\right)}{9a^{2/3}b^{4/3}}$$

[Out] $1/9*\text{Chi}(a^{(1/3)}*d/b^{(1/3)}+d*x)*\cosh(c-a^{(1/3)}*d/b^{(1/3)})/a^{(2/3)}/b^{(4/3)}-1/9*(-1)^{(1/3)}*\text{Chi}((-1)^{(1/3)}*a^{(1/3)}*d/b^{(1/3)}-d*x)*\cosh(c+(-1)^{(1/3)}*a^{(1/3)}*d/b^{(1/3)})/a^{(2/3)}/b^{(4/3)}+1/9*(-1)^{(2/3)}*\text{Chi}(-(-1)^{(2/3)}*a^{(1/3)}*d/b^{(1/3)}-d*x)*\cosh(c-(-1)^{(2/3)}*a^{(1/3)}*d/b^{(1/3)})/a^{(2/3)}/b^{(4/3)}-1/3*x*\cosh(d*x+c)/b/(b*x^3+a)-1/9*(-1)^{(2/3)}*d*\cosh(c+(-1)^{(1/3)}*a^{(1/3)}*d/b^{(1/3)})*\text{Shi}(-(-1)^{(1/3)}*a^{(1/3)}*d/b^{(1/3)}+d*x)/a^{(1/3)}/b^{(5/3)}-1/9*d*\cosh(c-a^{(1/3)}*d/b^{(1/3)})*\text{Shi}(a^{(1/3)}*d/b^{(1/3)}+d*x)/a^{(1/3)}/b^{(5/3)}+1/9*(-1)^{(1/3)}*d*\cosh(c-(-1)^{(2/3)}*a^{(1/3)}*d/b^{(1/3)})*\text{Shi}((-1)^{(2/3)}*a^{(1/3)}*d/b^{(1/3)}+d*x)/a^{(1/3)}/b^{(5/3)}-1/9*d*\text{Chi}(a^{(1/3)}*d/b^{(1/3)}+d*x)*\sinh(c-a^{(1/3)}*d/b^{(1/3)})/a^{(1/3)}/b^{(5/3)}+1/9*\text{Shi}(a^{(1/3)}*d/b^{(1/3)}+d*x)*\sinh(c-a^{(1/3)}*d/b^{(1/3)})/a^{(2/3)}/b^{(4/3)}-1/9*(-1)^{(2/3)}*d*\text{Chi}((-1)^{(1/3)}*a^{(1/3)}*d/b^{(1/3)}-d*x)*\sinh(c+(-1)^{(1/3)}*a^{(1/3)}*d/b^{(1/3)})/a^{(1/3)}/b^{(5/3)}-1/9*(-1)^{(1/3)}*\text{Shi}(-(-1)^{(1/3)}*a^{(1/3)}*d/b^{(1/3)}+d*x)*\sinh(c+(-1)^{(1/3)}*a^{(1/3)}*d/b^{(1/3)})/a^{(2/3)}/b^{(4/3)}+1/9*(-1)^{(1/3)}*d*\text{Chi}(-(-1)^{(2/3)}*a^{(1/3)}*d/b^{(1/3)}-d*x)*\sinh(c-(-1)^{(2/3)}*a^{(1/3)}*d/b^{(1/3)})/a^{(1/3)}/b^{(5/3)}+1/9*(-1)^{(2/3)}*\text{Shi}((-1)^{(2/3)}*a^{(1/3)}*d/b^{(1/3)}+d*x)*\sinh(c-(-1)^{(2/3)}*a^{(1/3)}*d/b^{(1/3)})/a^{(2/3)}/b^{(4/3)}$

Rubi [A] time = 1.08, antiderivative size = 718, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {5291, 5281, 3303, 3298, 3301, 5292}

$$\frac{\sqrt[3]{-1} \cosh\left(\frac{\sqrt[3]{-1} \sqrt[3]{ad}}{\sqrt[3]{b}} + c\right) \text{Chi}\left(\frac{\sqrt[3]{-1} \sqrt[3]{ad}}{\sqrt[3]{b}} - dx\right)}{9a^{2/3}b^{4/3}} + \frac{(-1)^{2/3} \cosh\left(c - \frac{(-1)^{2/3} \sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(-xd - \frac{(-1)^{2/3} \sqrt[3]{ad}}{\sqrt[3]{b}}\right)}{9a^{2/3}b^{4/3}} + \frac{\cosh\left(c - \frac{(-1)^{2/3} \sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(-xd - \frac{(-1)^{2/3} \sqrt[3]{ad}}{\sqrt[3]{b}}\right)}{9a^{2/3}b^{4/3}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(x^3*\text{Cosh}[c + d*x])/(a + b*x^3)^2, x]$

[Out] $-(x*\text{Cosh}[c + d*x])/(3*b*(a + b*x^3)) - ((-1)^{(1/3)}*\text{Cosh}[c + ((-1)^{(1/3)}*a^{(1/3)}*d)/b^{(1/3)}]*\text{CoshIntegral}[((-1)^{(1/3)}*a^{(1/3)}*d)/b^{(1/3)} - d*x])/(9*a^{(2/3)}*b^{(4/3)}) + ((-1)^{(2/3)}*\text{Cosh}[c - ((-1)^{(2/3)}*a^{(1/3)}*d)/b^{(1/3)}]*\text{CoshIntegral}[-(((-1)^{(2/3)}*a^{(1/3)}*d)/b^{(1/3)}) - d*x])/(9*a^{(2/3)}*b^{(4/3)}) + (\text{Cosh}[c - (a^{(1/3)}*d)/b^{(1/3)}]*\text{CoshIntegral}[(a^{(1/3)}*d)/b^{(1/3)} + d*x])/(9*a^{(2/3)}*b^{(4/3)})$

$$\begin{aligned} & /3)*b^{(4/3)} - (d*\text{CoshIntegral}[(a^{(1/3)}*d)/b^{(1/3)} + d*x]*\text{Sinh}[c - (a^{(1/3)} \\ & *d)/b^{(1/3)}])/(9*a^{(1/3)}*b^{(5/3)}) - ((-1)^{(2/3)}*d*\text{CoshIntegral}[((-1)^{(1/3)}* \\ & a^{(1/3)}*d)/b^{(1/3)} - d*x]*\text{Sinh}[c + ((-1)^{(1/3)}*a^{(1/3)}*d)/b^{(1/3)}])/(9*a^{(1 \\ & /3)}*b^{(5/3)}) + ((-1)^{(1/3)}*d*\text{CoshIntegral}[-(((-1)^{(2/3)}*a^{(1/3)}*d)/b^{(1/3)}) \\ & - d*x]*\text{Sinh}[c - ((-1)^{(2/3)}*a^{(1/3)}*d)/b^{(1/3)}])/(9*a^{(1/3)}*b^{(5/3)}) + ((- \\ & 1)^{(2/3)}*d*\text{Cosh}[c + ((-1)^{(1/3)}*a^{(1/3)}*d)/b^{(1/3)}]*\text{SinhIntegral}[((-1)^{(1/3)} \\ &)*a^{(1/3)}*d)/b^{(1/3)} - d*x])/(9*a^{(1/3)}*b^{(5/3)}) + ((-1)^{(1/3)}*\text{Sinh}[c + ((- \\ & 1)^{(1/3)}*a^{(1/3)}*d)/b^{(1/3)}]*\text{SinhIntegral}[((-1)^{(1/3)}*a^{(1/3)}*d)/b^{(1/3)} - \\ & d*x])/(9*a^{(2/3)}*b^{(4/3)}) - (d*\text{Cosh}[c - (a^{(1/3)}*d)/b^{(1/3)}]*\text{SinhIntegral}[(\\ & a^{(1/3)}*d)/b^{(1/3)} + d*x])/(9*a^{(1/3)}*b^{(5/3)}) + (\text{Sinh}[c - (a^{(1/3)}*d)/b^{(1 \\ & /3)}]*\text{SinhIntegral}[(a^{(1/3)}*d)/b^{(1/3)} + d*x])/(9*a^{(2/3)}*b^{(4/3)}) + ((-1)^{(\\ & 1/3)}*d*\text{Cosh}[c - ((-1)^{(2/3)}*a^{(1/3)}*d)/b^{(1/3)}]*\text{SinhIntegral}[((-1)^{(2/3)}*a^{ \\ & (1/3)}*d)/b^{(1/3)} + d*x])/(9*a^{(1/3)}*b^{(5/3)}) + ((-1)^{(2/3)}*\text{Sinh}[c - ((-1)^{(\\ & 2/3)}*a^{(1/3)}*d)/b^{(1/3)}]*\text{SinhIntegral}[((-1)^{(2/3)}*a^{(1/3)}*d)/b^{(1/3)} + d*x] \\ &)/(9*a^{(2/3)}*b^{(4/3)}) \end{aligned}$$

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x]
&& EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x]
&& EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)
]/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

Rule 5281

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Int[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d}, x]
&& ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 5291

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[(x^(m - n + 1)*(a + b*x^n)^(p + 1)*Cosh[c + d*x])/(b*n*(p + 1)), x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1)
```

) * Cosh[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^n)^(p + 1)*Sinh[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1] && IGtQ[n, 0] && RationalQ[m] && (GtQ[m - n + 1, 0] || GtQ[n, 2])

Rule 5292

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*Sinh[(c_) + (d_)*(x_)], x_Symbol] :> Int[ExpandIntegrand[Sinh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])

Rubi steps

$$\begin{aligned} \int \frac{x^3 \cosh(c + dx)}{(a + bx^3)^2} dx &= -\frac{x \cosh(c + dx)}{3b(a + bx^3)} + \frac{\int \frac{\cosh(c+dx)}{a+bx^3} dx}{3b} + \frac{d \int \frac{x \sinh(c+dx)}{a+bx^3} dx}{3b} \\ &= -\frac{x \cosh(c + dx)}{3b(a + bx^3)} + \frac{\int \left(-\frac{\cosh(c+dx)}{3a^{2/3}(-\sqrt[3]{a}-\sqrt[3]{b}x)} - \frac{\cosh(c+dx)}{3a^{2/3}(-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x)} - \frac{\cosh(c+dx)}{3a^{2/3}(-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x)} \right) dx}{3b} \\ &= -\frac{x \cosh(c + dx)}{3b(a + bx^3)} - \frac{\int \frac{\cosh(c+dx)}{-\sqrt[3]{a}-\sqrt[3]{b}x} dx}{9a^{2/3}b} - \frac{\int \frac{\cosh(c+dx)}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x} dx}{9a^{2/3}b} - \frac{\int \frac{\cosh(c+dx)}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x} dx}{9a^{2/3}b} - \frac{d \int \frac{\sinh(c+dx)}{a+bx^3} dx}{9a^{2/3}b} \\ &= -\frac{x \cosh(c + dx)}{3b(a + bx^3)} - \frac{\cosh\left(c - \frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right) \int \frac{\cosh\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}} + dx\right)}{-\sqrt[3]{a}-\sqrt[3]{b}x} dx}{9a^{2/3}b} - \frac{\left(d \cosh\left(c - \frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right) \int \frac{\sinh\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{\sqrt[3]{a}+\sqrt[3]{b}x} dx}{9\sqrt[3]{a}b^{4/3}} \\ &= -\frac{x \cosh(c + dx)}{3b(a + bx^3)} - \frac{\sqrt[3]{-1} \cosh\left(c + \frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right) \text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}} - dx\right)}{9a^{2/3}b^{4/3}} + \frac{(-1)^{2/3} \cosh\left(c - \frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9\sqrt[3]{a}b^{4/3}} \end{aligned}$$

Mathematica [C] time = 0.37, size = 363, normalized size = 0.51

$$-\text{RootSum}\left[\#1^3b + a\&, \frac{\sinh(\#1d+c)\text{Chi}(d(x-\#1))-\#1d \sinh(\#1d+c)\text{Chi}(d(x-\#1))-\cosh(\#1d+c)\text{Chi}(d(x-\#1))+\#1d \cosh(\#1d+c)\text{Chi}(d(x-\#1))}{\#1^3b + a}\right]$$

Antiderivative was successfully verified.

[In] Integrate[(x^3*Cosh[c + d*x])/(a + b*x^3)^2, x]

```
[Out] ((-6*b*x*Cosh[c + d*x])/(a + b*x^3) - RootSum[a + b*#1^3 & , (-Cosh[c + d*
#1]*CoshIntegral[d*(x - #1)] + CoshIntegral[d*(x - #1)]*Sinh[c + d*#1] + C
osh[c + d*#1]*SinhIntegral[d*(x - #1)] - Sinh[c + d*#1]*SinhIntegral[d*(x -
#1)] + d*Cosh[c + d*#1]*CoshIntegral[d*(x - #1)]*#1 - d*CoshIntegral[d*(x
- #1)]*Sinh[c + d*#1]*#1 - d*Cosh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1 + d
*Sinh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1)/#1^2 & ] + RootSum[a + b*#1^3
& , (Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] + CoshIntegral[d*(x - #1)]*Sin
h[c + d*#1] + Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] + Sinh[c + d*#1]*Sinh
Integral[d*(x - #1)] + d*Cosh[c + d*#1]*CoshIntegral[d*(x - #1)]*#1 + d*Cos
hIntegral[d*(x - #1)]*Sinh[c + d*#1]*#1 + d*Cosh[c + d*#1]*SinhIntegral[d*(
x - #1)]*#1 + d*Sinh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1)/#1^2 & ])/(18*b
^2)
```

fricas [B] time = 0.60, size = 2050, normalized size = 2.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(d*x+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] -1/36*(12*a*d*x*cosh(d*x + c) - ((a*d^3/b)^(2/3)*((b*x^3 - sqrt(-3)*(b*x^3
+ a) + a)*cosh(d*x + c)^2 - (b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c
)^2) - (a*d^3/b)^(1/3)*((b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2
- (b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2))*Ei(d*x - 1/2*(a*d^3/
b)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1) + c) + ((-
a*d^3/b)^(2/3)*((b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x^3
- sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2) - (-a*d^3/b)^(1/3)*((b*x^3 +
sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x^3 + sqrt(-3)*(b*x^3 + a) +
a)*sinh(d*x + c)^2))*Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*cosh(1
/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1) - c) - ((a*d^3/b)^(2/3)*((b*x^3 + sqrt(-
3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*si
nh(d*x + c)^2) - (a*d^3/b)^(1/3)*((b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*cosh(d
*x + c)^2 - (b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2))*Ei(d*x + 1
/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1)
- c) + ((-a*d^3/b)^(2/3)*((b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^
2 - (b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2) - (-a*d^3/b)^(1/3)*
((b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x^3 - sqrt(-3)*(b*
x^3 + a) + a)*sinh(d*x + c)^2))*Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) -
1))*cosh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c) - 2*((-a*d^3/b)^(2/3)*((b
*x^3 + a)*cosh(d*x + c)^2 - (b*x^3 + a)*sinh(d*x + c)^2) - (-a*d^3/b)^(1/3)
*((b*x^3 + a)*cosh(d*x + c)^2 - (b*x^3 + a)*sinh(d*x + c)^2))*Ei(-d*x + (-a
*d^3/b)^(1/3))*cosh(c + (-a*d^3/b)^(1/3)) + 2*((a*d^3/b)^(2/3)*((b*x^3 + a)
*cosh(d*x + c)^2 - (b*x^3 + a)*sinh(d*x + c)^2) - (a*d^3/b)^(1/3)*((b*x^3 +
a)*cosh(d*x + c)^2 - (b*x^3 + a)*sinh(d*x + c)^2))*Ei(d*x + (a*d^3/b)^(1/3)
))*cosh(-c + (a*d^3/b)^(1/3)) - ((a*d^3/b)^(2/3)*((b*x^3 - sqrt(-3)*(b*x^3
```


+ a) + a)*cosh(d*x + c)^2 - (b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2) - (a*d^3/b)^(1/3)*((b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2))*Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1)) + ((-a*d^3/b)^(2/3)*((b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2) - (-a*d^3/b)^(1/3)*((b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2))*Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1) - c) + ((a*d^3/b)^(2/3)*((b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2) - (a*d^3/b)^(1/3)*((b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2))*Ei(d*x + 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1))*sinh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1) - c) - ((-a*d^3/b)^(2/3)*((b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2) - (-a*d^3/b)^(1/3)*((b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2))*Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1))*sinh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c) + 2*((-a*d^3/b)^(2/3)*((b*x^3 + a)*cosh(d*x + c)^2 - (b*x^3 + a)*sinh(d*x + c)^2) - (-a*d^3/b)^(1/3)*((b*x^3 + a)*cosh(d*x + c)^2 - (b*x^3 + a)*sinh(d*x + c)^2))*Ei(-d*x + (-a*d^3/b)^(1/3))*sinh(c + (-a*d^3/b)^(1/3)) - 2*((a*d^3/b)^(2/3)*((b*x^3 + a)*cosh(d*x + c)^2 - (b*x^3 + a)*sinh(d*x + c)^2) - (a*d^3/b)^(1/3)*((b*x^3 + a)*cosh(d*x + c)^2 - (b*x^3 + a)*sinh(d*x + c)^2))*Ei(d*x + (a*d^3/b)^(1/3))*sinh(-c + (a*d^3/b)^(1/3)))/((a*b^2*d*x^3 + a^2*b*d)*cosh(d*x + c)^2 - (a*b^2*d*x^3 + a^2*b*d)*sinh(d*x + c)^2)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \cosh(dx + c)}{(bx^3 + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*cosh(d*x+c)/(b*x^3+a)^2,x, algorithm="giac")

[Out] integrate(x^3*cosh(d*x + c)/(b*x^3 + a)^2, x)

maple [C] time = 0.25, size = 877, normalized size = 1.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*cosh(d*x+c)/(b*x^3+a)^2,x)

[Out] -1/6*d^3*exp(-d*x-c)/b/(b*d^3*x^3+a*d^3)*x-1/18/d/a/b^2*sum((3*_R1^2*b*c^2-_R1*a*d^3-5*_R1*b*c^3-2*a*c*d^3+2*b*c^4+3*_R1*b*c^2+a*d^3-b*c^3)/(_R1^2-2*_

$R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c), _R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/18/d*c^3/a/b*sum((_R1-c+2)/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c), _R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/6/d*c^2/a/b*sum((_R1^2-_R1*c+_R1+c)/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c), _R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/6/d*c/a/b^2*sum((2*_R1^2*b*c-3*_R1*b*c^2-a*d^3+b*c^3+2*_R1*b*c)/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c), _R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/6*d^3*exp(d*x+c)/b/(b*d^3*x^3+a*d^3)*x+1/18/d/a/b^2*sum((3*_R1^2*b*c^2-_R1*a*d^3-5*_R1*b*c^3-2*a*c*d^3+2*b*c^4-3*_R1*b*c^2-a*d^3+b*c^3)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c), _R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/18/d*c^3/a/b*sum((_R1-c-2)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c), _R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/6/d*c^2/a/b*sum((_R1^2-_R1*c-_R1-c)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c), _R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/6/d*c/a/b^2*sum((2*_R1^2*b*c-3*_R1*b*c^2-a*d^3+b*c^3+2*_R1*b*c)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c), _R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{(d^2x^3e^{2c} + 3dx^2e^{2c} + 12xe^{2c})e^{dx} - (d^2x^3 - 3dx^2 + 12x)e^{-dx}}{2(b^2d^3x^6e^c + 2abd^3x^3e^c + a^2d^3e^c)} + \frac{1}{2} \int -\frac{6(ad^2x^2e^c - 10bx^3e^c + 3adx e^c + 2a^2e^c)}{b^3d^3x^9 + 3ab^2d^3x^6 + 3a^2bd^3x^3 + a^3d^3e^c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*cosh(d*x+c)/(b*x^3+a)^2,x, algorithm="maxima")

[Out] 1/2*((d^2*x^3*e^(2*c) + 3*d*x^2*e^(2*c) + 12*x*e^(2*c))*e^(d*x) - (d^2*x^3 - 3*d*x^2 + 12*x)*e^(-d*x))/(b^2*d^3*x^6*e^c + 2*a*b*d^3*x^3*e^c + a^2*d^3*e^c) + 1/2*integrate(-6*(a*d^2*x^2*e^c - 10*b*x^3*e^c + 3*a*d*x*e^c + 2*a*e^c)*e^(d*x)/(b^3*d^3*x^9 + 3*a*b^2*d^3*x^6 + 3*a^2*b*d^3*x^3 + a^3*d^3), x) - 1/2*integrate(-6*(a*d^2*x^2 - 10*b*x^3 - 3*a*d*x + 2*a)*e^(-d*x)/(b^3*d^3*x^9*e^c + 3*a*b^2*d^3*x^6*e^c + 3*a^2*b*d^3*x^3*e^c + a^3*d^3*e^c), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3 \cosh(c + dx)}{(bx^3 + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3*cosh(c + d*x))/(a + b*x^3)^2,x)

[Out] int((x^3*cosh(c + d*x))/(a + b*x^3)^2, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cosh(d*x+c)/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

$$3.103 \quad \int \frac{x^2 \cosh(c+dx)}{(a+bx^3)^2} dx$$

Optimal. Leaf size=373

$$\frac{d \sinh\left(c - \frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right) \operatorname{Chi}\left(xd + \frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{2/3}b^{4/3}} - \frac{\sqrt[3]{-1} d \sinh\left(\frac{\sqrt[3]{-1} \sqrt[3]{a}d}{\sqrt[3]{b}} + c\right) \operatorname{Chi}\left(\frac{\sqrt[3]{-1} \sqrt[3]{a}d}{\sqrt[3]{b}} - dx\right)}{9a^{2/3}b^{4/3}} + \frac{(-1)^{2/3} d \sinh\left(c - \frac{(-1)^{2/3} \sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{2/3}b^{4/3}}$$

[Out] $-1/3 \cosh(dx+c)/b/(b*x^3+a) - 1/9 * (-1)^{(1/3)} * d * \cosh(c + (-1)^{(1/3)} * a^{(1/3)} * d/b^{(1/3)}) * \operatorname{Shi}(-(-1)^{(1/3)} * a^{(1/3)} * d/b^{(1/3)} + dx) / a^{(2/3)} / b^{(4/3)} + 1/9 * d * \cosh(c - a^{(1/3)} * d/b^{(1/3)}) * \operatorname{Shi}(a^{(1/3)} * d/b^{(1/3)} + dx) / a^{(2/3)} / b^{(4/3)} + 1/9 * (-1)^{(2/3)} * d * \cosh(c - (-1)^{(2/3)} * a^{(1/3)} * d/b^{(1/3)}) * \operatorname{Shi}((-1)^{(2/3)} * a^{(1/3)} * d/b^{(1/3)} + dx) / a^{(2/3)} / b^{(4/3)} + 1/9 * d * \operatorname{Chi}(a^{(1/3)} * d/b^{(1/3)} + dx) * \sinh(c - a^{(1/3)} * d/b^{(1/3)}) / a^{(2/3)} / b^{(4/3)} - 1/9 * (-1)^{(1/3)} * d * \operatorname{Chi}((-1)^{(1/3)} * a^{(1/3)} * d/b^{(1/3)} - dx) * \sinh(c + (-1)^{(1/3)} * a^{(1/3)} * d/b^{(1/3)}) / a^{(2/3)} / b^{(4/3)} + 1/9 * (-1)^{(2/3)} * d * \operatorname{Chi}(-(-1)^{(2/3)} * a^{(1/3)} * d/b^{(1/3)} - dx) * \sinh(c - (-1)^{(2/3)} * a^{(1/3)} * d/b^{(1/3)}) / a^{(2/3)} / b^{(4/3)}$

Rubi [A] time = 0.60, antiderivative size = 373, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {5289, 5280, 3303, 3298, 3301}

$$\frac{d \sinh\left(c - \frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right) \operatorname{Chi}\left(xd + \frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{2/3}b^{4/3}} - \frac{\sqrt[3]{-1} d \sinh\left(\frac{\sqrt[3]{-1} \sqrt[3]{a}d}{\sqrt[3]{b}} + c\right) \operatorname{Chi}\left(\frac{\sqrt[3]{-1} \sqrt[3]{a}d}{\sqrt[3]{b}} - dx\right)}{9a^{2/3}b^{4/3}} + \frac{(-1)^{2/3} d \sinh\left(c - \frac{(-1)^{2/3} \sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{2/3}b^{4/3}}$$

Antiderivative was successfully verified.

[In] Int[(x^2*Cosh[c + d*x])/(a + b*x^3)^2,x]

[Out] $-\operatorname{Cosh}[c + d*x]/(3*b*(a + b*x^3)) + (d*\operatorname{CoshIntegral}[(a^{(1/3)}*d)/b^{(1/3)} + d*x]*\operatorname{Sinh}[c - (a^{(1/3)}*d)/b^{(1/3)}])/(9*a^{(2/3)}*b^{(4/3)}) - ((-1)^{(1/3)}*d*\operatorname{CoshIntegral}[((-1)^{(1/3)}*a^{(1/3)}*d)/b^{(1/3)} - d*x]*\operatorname{Sinh}[c + ((-1)^{(1/3)}*a^{(1/3)}*d)/b^{(1/3)}])/(9*a^{(2/3)}*b^{(4/3)}) + ((-1)^{(2/3)}*d*\operatorname{CoshIntegral}[(-((-1)^{(2/3)}*a^{(1/3)}*d)/b^{(1/3)} - d*x]*\operatorname{Sinh}[c - ((-1)^{(2/3)}*a^{(1/3)}*d)/b^{(1/3)}])/(9*a^{(2/3)}*b^{(4/3)}) + ((-1)^{(1/3)}*d*\operatorname{Cosh}[c + ((-1)^{(1/3)}*a^{(1/3)}*d)/b^{(1/3)}]*\operatorname{SinhIntegral}[((-1)^{(1/3)}*a^{(1/3)}*d)/b^{(1/3)} - d*x])/(9*a^{(2/3)}*b^{(4/3)}) + (d*\operatorname{Cosh}[c - (a^{(1/3)}*d)/b^{(1/3)}]*\operatorname{SinhIntegral}[(a^{(1/3)}*d)/b^{(1/3)} + d*x])/(9*a^{(2/3)}*b^{(4/3)}) + ((-1)^{(2/3)}*d*\operatorname{Cosh}[c - ((-1)^{(2/3)}*a^{(1/3)}*d)/b^{(1/3)}]*\operatorname{SinhIntegral}[((-1)^{(2/3)}*a^{(1/3)}*d)/b^{(1/3)} + d*x])/(9*a^{(2/3)}*b^{(4/3)})$

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x]
&& EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x]
&& EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*e - c*f)/d],
Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d],
Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

Rule 5280

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Symbol] :> Int[ExpandIntegrand[
Sinh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d}, x]
&& ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 5289

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[(e^m*(a + b*x^n)^(p + 1)*Cosh[c + d*x]/(b*n*(p + 1)), x] - Dist[(d*e^m)/(b*n*(p + 1)),
Int[(a + b*x^n)^(p + 1)*Sinh[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
&& IntegerQ[p] && EqQ[m - n + 1, 0] && LtQ[p, -1] && (IntegerQ[n] || GtQ[e, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2 \cosh(c+dx)}{(a+bx^3)^2} dx &= -\frac{\cosh(c+dx)}{3b(a+bx^3)} + \frac{d \int \frac{\sinh(c+dx)}{a+bx^3} dx}{3b} \\
&= -\frac{\cosh(c+dx)}{3b(a+bx^3)} + \frac{d \int \left(-\frac{\sinh(c+dx)}{3a^{2/3}(-\sqrt[3]{a}-\sqrt[3]{b}x)} - \frac{\sinh(c+dx)}{3a^{2/3}(-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x)} - \frac{\sinh(c+dx)}{3a^{2/3}(-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x)} \right) dx}{3b} \\
&= -\frac{\cosh(c+dx)}{3b(a+bx^3)} - \frac{d \int \frac{\sinh(c+dx)}{-\sqrt[3]{a}-\sqrt[3]{b}x} dx}{9a^{2/3}b} - \frac{d \int \frac{\sinh(c+dx)}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x} dx}{9a^{2/3}b} - \frac{d \int \frac{\sinh(c+dx)}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x} dx}{9a^{2/3}b} \\
&= -\frac{\cosh(c+dx)}{3b(a+bx^3)} - \frac{\left(d \cosh\left(c - \frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right) \int \frac{\sinh\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}} + dx\right)}{-\sqrt[3]{a}-\sqrt[3]{b}x} dx \right)}{9a^{2/3}b} - \frac{\left(id \cosh\left(c + \frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right) \int \frac{\sinh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}} + dx\right)}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x} dx \right)}{9a^{2/3}b} \\
&= -\frac{\cosh(c+dx)}{3b(a+bx^3)} + \frac{d \operatorname{Chi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}} + dx\right) \sinh\left(c - \frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{2/3}b^{4/3}} - \frac{\sqrt[3]{-1} d \operatorname{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}} - dx\right) \sinh\left(c - \frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{2/3}b^{4/3}}
\end{aligned}$$

Mathematica [C] time = 0.18, size = 203, normalized size = 0.54

$$-d \operatorname{RootSum}\left[\#1^3 b + a \&, \frac{-\sinh(\#1 d + c) \operatorname{Chi}(d(x - \#1)) + \cosh(\#1 d + c) \operatorname{Chi}(d(x - \#1)) + \sinh(\#1 d + c) \operatorname{Shi}(d(x - \#1)) - \cosh(\#1 d + c) \operatorname{Shi}(d(x - \#1))}{\#1^2} \& \right]$$

Antiderivative was successfully verified.

[In] Integrate[(x^2*Cosh[c + d*x])/(a + b*x^3)^2,x]

[Out] ((-6*b*Cosh[c + d*x])/(a + b*x^3) - d*RootSum[a + b*#1^3 & , (Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] - CoshIntegral[d*(x - #1)]*Sinh[c + d*#1] - Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] + Sinh[c + d*#1]*SinhIntegral[d*(x - #1)]))/#1^2 &] + d*RootSum[a + b*#1^3 & , (Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] + CoshIntegral[d*(x - #1)]*Sinh[c + d*#1] + Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] + Sinh[c + d*#1]*SinhIntegral[d*(x - #1)])/#1^2 &])/(18*b^2)

fricas [B] time = 0.47, size = 1276, normalized size = 3.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*cosh(d*x+c)/(b*x^3+a)^2,x, algorithm="fricas")

[Out]
$$-1/36*((a*d^3/b)^{1/3}*((b*x^3 + \sqrt{-3}*(b*x^3 + a) + a)*\cosh(d*x + c)^2 - (b*x^3 + \sqrt{-3}*(b*x^3 + a) + a)*\sinh(d*x + c)^2)*\text{Ei}(d*x - 1/2*(a*d^3/b)^{1/3}*(\sqrt{-3} + 1))*\cosh(1/2*(a*d^3/b)^{1/3}*(\sqrt{-3} + 1) + c) + (-a*d^3/b)^{1/3}*((b*x^3 + \sqrt{-3}*(b*x^3 + a) + a)*\cosh(d*x + c)^2 - (b*x^3 + \sqrt{-3}*(b*x^3 + a) + a)*\sinh(d*x + c)^2)*\text{Ei}(-d*x - 1/2*(-a*d^3/b)^{1/3}*(\sqrt{-3} + 1))*\cosh(1/2*(-a*d^3/b)^{1/3}*(\sqrt{-3} + 1) - c) + (a*d^3/b)^{1/3}*((b*x^3 - \sqrt{-3}*(b*x^3 + a) + a)*\cosh(d*x + c)^2 - (b*x^3 - \sqrt{-3}*(b*x^3 + a) + a)*\sinh(d*x + c)^2)*\text{Ei}(d*x + 1/2*(a*d^3/b)^{1/3}*(\sqrt{-3} - 1))*\cosh(1/2*(a*d^3/b)^{1/3}*(\sqrt{-3} - 1) - c) + (-a*d^3/b)^{1/3}*((b*x^3 - \sqrt{-3}*(b*x^3 + a) + a)*\cosh(d*x + c)^2 - (b*x^3 - \sqrt{-3}*(b*x^3 + a) + a)*\sinh(d*x + c)^2)*\text{Ei}(-d*x + 1/2*(-a*d^3/b)^{1/3}*(\sqrt{-3} - 1))*\cosh(1/2*(-a*d^3/b)^{1/3}*(\sqrt{-3} - 1) + c) - 2*(-a*d^3/b)^{1/3}*((b*x^3 + a)*\cosh(d*x + c)^2 - (b*x^3 + a)*\sinh(d*x + c)^2)*\text{Ei}(-d*x + (-a*d^3/b)^{1/3})*\cosh(c + (-a*d^3/b)^{1/3}) - 2*(a*d^3/b)^{1/3}*((b*x^3 + a)*\cosh(d*x + c)^2 - (b*x^3 + a)*\sinh(d*x + c)^2)*\text{Ei}(d*x + (a*d^3/b)^{1/3})*\cosh(-c + (a*d^3/b)^{1/3}) + (a*d^3/b)^{1/3}*((b*x^3 + \sqrt{-3}*(b*x^3 + a) + a)*\cosh(d*x + c)^2 - (b*x^3 + \sqrt{-3}*(b*x^3 + a) + a)*\sinh(d*x + c)^2)*\text{Ei}(d*x - 1/2*(a*d^3/b)^{1/3}*(\sqrt{-3} + 1))*\sinh(1/2*(a*d^3/b)^{1/3}*(\sqrt{-3} + 1) + c) + (-a*d^3/b)^{1/3}*((b*x^3 + \sqrt{-3}*(b*x^3 + a) + a)*\cosh(d*x + c)^2 - (b*x^3 + \sqrt{-3}*(b*x^3 + a) + a)*\sinh(d*x + c)^2)*\text{Ei}(-d*x - 1/2*(-a*d^3/b)^{1/3}*(\sqrt{-3} + 1))*\sinh(1/2*(-a*d^3/b)^{1/3}*(\sqrt{-3} + 1) - c) - (a*d^3/b)^{1/3}*((b*x^3 - \sqrt{-3}*(b*x^3 + a) + a)*\cosh(d*x + c)^2 - (b*x^3 - \sqrt{-3}*(b*x^3 + a) + a)*\sinh(d*x + c)^2)*\text{Ei}(d*x + 1/2*(a*d^3/b)^{1/3}*(\sqrt{-3} - 1))*\sinh(1/2*(a*d^3/b)^{1/3}*(\sqrt{-3} - 1) - c) - (-a*d^3/b)^{1/3}*((b*x^3 - \sqrt{-3}*(b*x^3 + a) + a)*\cosh(d*x + c)^2 - (b*x^3 - \sqrt{-3}*(b*x^3 + a) + a)*\sinh(d*x + c)^2)*\text{Ei}(-d*x + 1/2*(-a*d^3/b)^{1/3}*(\sqrt{-3} - 1))*\sinh(1/2*(-a*d^3/b)^{1/3}*(\sqrt{-3} - 1) + c) + 2*(-a*d^3/b)^{1/3}*((b*x^3 + a)*\cosh(d*x + c)^2 - (b*x^3 + a)*\sinh(d*x + c)^2)*\text{Ei}(-d*x + (-a*d^3/b)^{1/3})*\sinh(c + (-a*d^3/b)^{1/3}) + 2*(a*d^3/b)^{1/3}*((b*x^3 + a)*\cosh(d*x + c)^2 - (b*x^3 + a)*\sinh(d*x + c)^2)*\text{Ei}(d*x + (a*d^3/b)^{1/3})*\sinh(-c + (a*d^3/b)^{1/3}) + 12*a*\cosh(d*x + c))/((a*b^2*x^3 + a^2*b)*\cosh(d*x + c)^2 - (a*b^2*x^3 + a^2*b)*\sinh(d*x + c)^2)$$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \cosh(dx + c)}{(bx^3 + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*cosh(d*x+c)/(b*x^3+a)^2,x, algorithm="giac")

[Out] integrate(x^2*cosh(d*x + c)/(b*x^3 + a)^2, x)

maple [C] time = 0.19, size = 594, normalized size = 1.59

$$\frac{d^3 e^{-dx-c}}{6b(bd^3x^3 + ad^3)} \sum_{R1=\text{RootOf}(b_Z^3-3cb_Z^2+3b^2_Z+a d^3-bc^3)} \frac{(2_{R1}^2bc-3_{R1}bc^2-ad^3+bc^3+2_{R1}bc)e^{-R1} \text{Ei}(1,dx-R1+c)}{_{R1}^2-2_{R1}c+c^2} c^2 \left(\dots \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*cosh(d*x+c)/(b*x^3+a)^2,x)`

[Out] $-1/6*d^3*\exp(-d*x-c)/b/(b*d^3*x^3+a*d^3)-1/18/a/b^2*\text{sum}((2*_R1^2*b*c-3*_R1*b*c^2-a*d^3+b*c^3+2*_R1*b*c)/(_R1^2-2*_R1*c+c^2)*\exp(-_R1)*\text{Ei}(1,d*x-_R1+c),$
 $_R1=\text{RootOf}(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/18*c^2/a/b*\text{sum}((_R1-c+2)/(_R1^2-2*_R1*c+c^2)*\exp(-_R1)*\text{Ei}(1,d*x-_R1+c),$
 $_R1=\text{RootOf}(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/9*c/a/b*\text{sum}((_R1^2-_R1*c+_R1+c)/(_R1^2-2*_R1*c+c^2)*\exp(-_R1)*\text{Ei}(1,d*x-_R1+c),$
 $_R1=\text{RootOf}(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/6*d^3*\exp(d*x+c)/b/(b*d^3*x^3+a*d^3)+1/18/a/b^2*\text{sum}((2*_R1^2*b*c-3*_R1*b*c^2-a*d^3+b*c^3-2*_R1*b*c)/(_R1^2-2*_R1*c+c^2)*\exp(_R1)*\text{Ei}(1,$
 $-d*x+_R1-c),$
 $_R1=\text{RootOf}(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/18*c^2/a/b*\text{sum}((_R1-c-2)/(_R1^2-2*_R1*c+c^2)*\exp(_R1)*\text{Ei}(1,-d*x+_R1-c),$
 $_R1=\text{RootOf}(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/9*c/a/b*\text{sum}((_R1^2-_R1*c-_R1-c)/(_R1^2-2*_R1*c+c^2)*\exp(_R1)*\text{Ei}(1,-d*x+_R1-c),$
 $_R1=\text{RootOf}(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{(dx^2e^{(2c)} + 4xe^{(2c)})e^{(dx)} - (dx^2 - 4x)e^{(-dx)}}{2(b^2d^2x^6e^c + 2abd^2x^3e^c + a^2d^2e^c)} + \frac{1}{2} \int \frac{2(10bx^3e^c - 3adxe^c - 2ae^c)e^{(dx)}}{b^3d^2x^9 + 3ab^2d^2x^6 + 3a^2bd^2x^3 + a^3d^2} dx + \frac{1}{2} \int \frac{2(10bx^3e^c - 3adxe^c - 2ae^c)e^{(-dx)}}{b^3d^2x^9e^c + 3ab^2d^2x^6e^c + 3a^2bd^2x^3e^c + a^3d^2e^c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*cosh(d*x+c)/(b*x^3+a)^2,x, algorithm="maxima")`

[Out] $1/2*((d*x^2*e^{(2*c)} + 4*x*e^{(2*c)})*e^{(d*x)} - (d*x^2 - 4*x)*e^{(-d*x)})/(b^2*d^2*x^6*e^c + 2*a*b*d^2*x^3*e^c + a^2*d^2*e^c) + 1/2*\text{integrate}(2*(10*b*x^3*e^c - 3*a*d*x*e^c - 2*a*e^c)*e^{(d*x)}/(b^3*d^2*x^9 + 3*a*b^2*d^2*x^6 + 3*a^2*b*d^2*x^3 + a^3*d^2), x) + 1/2*\text{integrate}(2*(10*b*x^3 + 3*a*d*x - 2*a)*e^{(-d*x)}/(b^3*d^2*x^9*e^c + 3*a*b^2*d^2*x^6*e^c + 3*a^2*b*d^2*x^3*e^c + a^3*d^2*e^c), x)$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2 \cosh(c + dx)}{(bx^3 + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*cosh(c + d*x))/(a + b*x^3)^2,x)
```

```
[Out] int((x^2*cosh(c + d*x))/(a + b*x^3)^2, x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cosh(d*x+c)/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

$$3.104 \quad \int \frac{x \cosh(c+dx)}{(a+bx^3)^2} dx$$

Optimal. Leaf size=695

$$\frac{(-1)^{2/3} \cosh\left(\frac{\sqrt[3]{-1} \sqrt[3]{ad}}{\sqrt[3]{b}} + c\right) \text{Chi}\left(\frac{\sqrt[3]{-1} \sqrt[3]{ad}}{\sqrt[3]{b}} - dx\right)}{9a^{4/3}b^{2/3}} + \frac{\sqrt[3]{-1} \cosh\left(c - \frac{(-1)^{2/3} \sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(-xd - \frac{(-1)^{2/3} \sqrt[3]{ad}}{\sqrt[3]{b}}\right) \cosh\left(c - \frac{(-1)^{2/3} \sqrt[3]{ad}}{\sqrt[3]{b}}\right)}{9a^{4/3}b^{2/3}}$$

[Out] $-1/9*\text{Chi}(a^{1/3}*d/b^{1/3}+d*x)*\cosh(c-a^{1/3}*d/b^{1/3})/a^{4/3}/b^{2/3}-1/9*(-1)^{(2/3)}*\text{Chi}((-1)^{(1/3)}*a^{1/3}*d/b^{1/3}-d*x)*\cosh(c+(-1)^{(1/3)}*a^{1/3}*d/b^{1/3})/a^{4/3}/b^{2/3}+1/9*(-1)^{(1/3)}*\text{Chi}((-1)^{(2/3)}*a^{1/3}*d/b^{1/3}-d*x)*\cosh(c-(-1)^{(2/3)}*a^{1/3}*d/b^{1/3})/a^{4/3}/b^{2/3}+1/3*\cosh(d*x+c)/a/b/x-1/3*\cosh(d*x+c)/b/x/(b*x^3+a)-1/9*d*\cosh(c+(-1)^{(1/3)}*a^{1/3}*d/b^{1/3})*\text{Shi}(-(-1)^{(1/3)}*a^{1/3}*d/b^{1/3}+d*x)/a/b-1/9*d*\cosh(c-a^{1/3}*d/b^{1/3})*\text{Shi}(a^{1/3}*d/b^{1/3}+d*x)/a/b-1/9*d*\cosh(c-(-1)^{(2/3)}*a^{1/3}*d/b^{1/3})*\text{Shi}((-1)^{(2/3)}*a^{1/3}*d/b^{1/3}+d*x)/a/b-1/9*d*\text{Chi}(a^{1/3}*d/b^{1/3}+d*x)*\sinh(c-a^{1/3}*d/b^{1/3})/a/b-1/9*\text{Shi}(a^{1/3}*d/b^{1/3}+d*x)*\sinh(c-a^{1/3}*d/b^{1/3})/a^{4/3}/b^{2/3}-1/9*d*\text{Chi}((-1)^{(1/3)}*a^{1/3}*d/b^{1/3}-d*x)*\sinh(c+(-1)^{(1/3)}*a^{1/3}*d/b^{1/3})/a/b-1/9*(-1)^{(2/3)}*\text{Shi}(-(-1)^{(1/3)}*a^{1/3}*d/b^{1/3}+d*x)*\sinh(c+(-1)^{(1/3)}*a^{1/3}*d/b^{1/3})/a^{4/3}/b^{2/3}-1/9*d*\text{Chi}(-(-1)^{(2/3)}*a^{1/3}*d/b^{1/3}-d*x)*\sinh(c-(-1)^{(2/3)}*a^{1/3}*d/b^{1/3})/a/b+1/9*(-1)^{(1/3)}*\text{Shi}((-1)^{(2/3)}*a^{1/3}*d/b^{1/3}+d*x)*\sinh(c-(-1)^{(2/3)}*a^{1/3}*d/b^{1/3})/a^{4/3}/b^{2/3}$

Rubi [A] time = 1.32, antiderivative size = 695, normalized size of antiderivative = 1.00, number of steps used = 34, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {5291, 5293, 3297, 3303, 3298, 3301, 5292}

$$\frac{(-1)^{2/3} \cosh\left(\frac{\sqrt[3]{-1} \sqrt[3]{ad}}{\sqrt[3]{b}} + c\right) \text{Chi}\left(\frac{\sqrt[3]{-1} \sqrt[3]{ad}}{\sqrt[3]{b}} - dx\right)}{9a^{4/3}b^{2/3}} + \frac{\sqrt[3]{-1} \cosh\left(c - \frac{(-1)^{2/3} \sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(-xd - \frac{(-1)^{2/3} \sqrt[3]{ad}}{\sqrt[3]{b}}\right) \cosh\left(c - \frac{(-1)^{2/3} \sqrt[3]{ad}}{\sqrt[3]{b}}\right)}{9a^{4/3}b^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[(x*Cosh[c + d*x])/(a + b*x^3)^2,x]

[Out] $\text{Cosh}[c + d*x]/(3*a*b*x) - \text{Cosh}[c + d*x]/(3*b*x*(a + b*x^3)) - ((-1)^{(2/3)}*\cosh[c + ((-1)^{(1/3)}*a^{1/3}*d)/b^{1/3}]*\text{CoshIntegral}[((-1)^{(1/3)}*a^{1/3}*d)/b^{1/3} - d*x]/(9*a^{4/3}*b^{2/3}) + ((-1)^{(1/3)}*\cosh[c - ((-1)^{(2/3)}*a^{1/3}*d)/b^{1/3}]*\text{CoshIntegral}[(-((-1)^{(2/3)}*a^{1/3}*d)/b^{1/3}) - d*x]/(9*a^{4/3}*b^{2/3}) - (\cosh[c - (a^{1/3}*d)/b^{1/3}]*\text{CoshIntegral}[(a^{1/3}*d)/b^{1/3} + d*x]/(9*a^{4/3}*b^{2/3}) - (d*\text{CoshIntegral}[(a^{1/3}*d)/b^{1/3} +$

$$\begin{aligned}
& d*x]*\text{Sinh}[c - (a^{(1/3)}*d)/b^{(1/3)}]/(9*a*b) - (d*\text{CoshIntegral}[((-1)^{(1/3)}* \\
& a^{(1/3)}*d)/b^{(1/3)} - d*x]*\text{Sinh}[c + ((-1)^{(1/3)}*a^{(1/3)}*d)/b^{(1/3)}]/(9*a*b) \\
& - (d*\text{CoshIntegral}[(-((-1)^{(2/3)}*a^{(1/3)}*d)/b^{(1/3)}) - d*x]*\text{Sinh}[c - ((-1)^{(2/3)}* \\
& a^{(1/3)}*d)/b^{(1/3)}]/(9*a*b) + (d*\text{Cosh}[c + ((-1)^{(1/3)}*a^{(1/3)}*d)/b^{(1/3)}] \\
& *\text{SinhIntegral}[((-1)^{(1/3)}*a^{(1/3)}*d)/b^{(1/3)} - d*x]/(9*a*b) + ((-1)^{(2/3)}* \\
& \text{Sinh}[c + ((-1)^{(1/3)}*a^{(1/3)}*d)/b^{(1/3)}]*\text{SinhIntegral}[((-1)^{(1/3)}*a^{(1/3)}* \\
& d)/b^{(1/3)} - d*x]/(9*a^{(4/3)}*b^{(2/3)}) - (d*\text{Cosh}[c - (a^{(1/3)}*d)/b^{(1/3)}] \\
&)*\text{SinhIntegral}[(a^{(1/3)}*d)/b^{(1/3)} + d*x]/(9*a*b) - (\text{Sinh}[c - (a^{(1/3)}*d) \\
& /b^{(1/3)}]*\text{SinhIntegral}[(a^{(1/3)}*d)/b^{(1/3)} + d*x]/(9*a^{(4/3)}*b^{(2/3)}) - (d \\
& *\text{Cosh}[c - ((-1)^{(2/3)}*a^{(1/3)}*d)/b^{(1/3)}]*\text{SinhIntegral}[((-1)^{(2/3)}*a^{(1/3)}* \\
& d)/b^{(1/3)} + d*x]/(9*a*b) + ((-1)^{(1/3)}*\text{Sinh}[c - ((-1)^{(2/3)}*a^{(1/3)}*d)/b^{(1/3)}] \\
& *\text{SinhIntegral}[((-1)^{(2/3)}*a^{(1/3)}*d)/b^{(1/3)} + d*x]/(9*a^{(4/3)}*b^{(2/3)}))
\end{aligned}$$

Rule 3297

```

Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
+ d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]

```

Rule 3298

```

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)])/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]

```

Rule 3301

```

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)])/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]

```

Rule 3303

```

Int[sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]

```

Rule 5291

```

Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x^(m - n + 1)*(a + b*x^n)^(p + 1)*Cosh[c + d*x])/(b*n*(p + 1)), x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1)*Cosh[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^n)^(p + 1)*Cosh[c + d*x], x], x]

```

$x)^{(p+1)} \sinh[c + dx], x], x) /;$ FreeQ[{a, b, c, d}, x] && ILtQ[p, -1] && IGtQ[n, 0] && RationalQ[m] && (GtQ[m - n + 1, 0] || GtQ[n, 2])

Rule 5292

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*Sinh[(c_) + (d_)*(x_)], x_Symbol] := Int[ExpandIntegrand[Sinh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])

Rule 5293

Int[Cosh[(c_) + (d_)*(x_)]*(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])

Rubi steps

$$\begin{aligned}
\int \frac{x \cosh(c+dx)}{(a+bx^3)^2} dx &= -\frac{\cosh(c+dx)}{3bx(a+bx^3)} - \frac{\int \frac{\cosh(c+dx)}{x^2(a+bx^3)} dx}{3b} + \frac{d \int \frac{\sinh(c+dx)}{x(a+bx^3)} dx}{3b} \\
&= -\frac{\cosh(c+dx)}{3bx(a+bx^3)} - \frac{\int \left(\frac{\cosh(c+dx)}{ax^2} - \frac{bx \cosh(c+dx)}{a(a+bx^3)} \right) dx}{3b} + \frac{d \int \left(\frac{\sinh(c+dx)}{ax} - \frac{bx^2 \sinh(c+dx)}{a(a+bx^3)} \right) dx}{3b} \\
&= -\frac{\cosh(c+dx)}{3bx(a+bx^3)} + \frac{\int \frac{x \cosh(c+dx)}{a+bx^3} dx}{3a} - \frac{\int \frac{\cosh(c+dx)}{x^2} dx}{3ab} - \frac{d \int \frac{x^2 \sinh(c+dx)}{a+bx^3} dx}{3a} + \frac{d \int \frac{\sinh(c+dx)}{x}}{3ab} \\
&= \frac{\cosh(c+dx)}{3abx} - \frac{\cosh(c+dx)}{3bx(a+bx^3)} + \frac{\int \left(-\frac{\cosh(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}(\sqrt[3]{a}+\sqrt[3]{b}x)} - \frac{(-1)^{2/3} \cosh(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x)} + \frac{\sqrt[3]{-1}}{3\sqrt[3]{a}\sqrt[3]{b}} \right) dx}{3a} \\
&= \frac{\cosh(c+dx)}{3abx} - \frac{\cosh(c+dx)}{3bx(a+bx^3)} + \frac{d \operatorname{Chi}(dx) \sinh(c)}{3ab} + \frac{d \cosh(c) \operatorname{Shi}(dx)}{3ab} - \frac{\int \frac{\cosh(c+dx)}{\sqrt[3]{a}+\sqrt[3]{b}x} dx}{9a^{4/3}\sqrt[3]{b}} \\
&= \frac{\cosh(c+dx)}{3abx} - \frac{\cosh(c+dx)}{3bx(a+bx^3)} - \frac{\cosh\left(c - \frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right) \int \frac{\cosh\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{\sqrt[3]{a}+\sqrt[3]{b}x} dx}{9a^{4/3}\sqrt[3]{b}} - \frac{\left(d \cosh\left(c - \frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)}{9a^{4/3}\sqrt[3]{b}} \\
&= \frac{\cosh(c+dx)}{3abx} - \frac{\cosh(c+dx)}{3bx(a+bx^3)} - \frac{(-1)^{2/3} \cosh\left(c + \frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right) \operatorname{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}} - dx\right)}{9a^{4/3}b^{2/3}} + \frac{\sqrt[3]{-1}}{9a^{4/3}\sqrt[3]{b}}
\end{aligned}$$

Mathematica [C] time = 0.20, size = 387, normalized size = 0.56

$$(a+bx^3) \operatorname{RootSum}\left[\#1^3b+a\&, \frac{-\sinh(\#1d+c)\operatorname{Chi}(d(x-\#1))-\#1d \sinh(\#1d+c)\operatorname{Chi}(d(x-\#1))+\cosh(\#1d+c)\operatorname{Chi}(d(x-\#1))+\#1d \cosh(\#1d+c)}{\dots}\right]$$

Warning: Unable to verify antiderivative.

[In] Integrate[(x*Cosh[c + d*x])/(a + b*x^3)^2,x]

[Out] (6*b*x^2*Cosh[c + d*x] + (a + b*x^3)*RootSum[a + b*#1^3 & , (Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] - CoshIntegral[d*(x - #1)]*Sinh[c + d*#1] - Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] + Sinh[c + d*#1]*SinhIntegral[d*(x - #1)] + d*Cosh[c + d*#1]*CoshIntegral[d*(x - #1)]*#1 - d*CoshIntegral[d*(x - #1)

```
) * Sinh[c + d*#1] * #1 - d * Cosh[c + d*#1] * SinhIntegral[d*(x - #1)] * #1 + d * Sinh[c + d*#1] * SinhIntegral[d*(x - #1)] * #1 / #1 & ] - (a + b*x^3) * RootSum[a + b*#1^3 & , (-Cosh[c + d*#1] * CoshIntegral[d*(x - #1)]) - CoshIntegral[d*(x - #1)] * Sinh[c + d*#1] - Cosh[c + d*#1] * SinhIntegral[d*(x - #1)] - Sinh[c + d*#1] * SinhIntegral[d*(x - #1)] + d * Cosh[c + d*#1] * CoshIntegral[d*(x - #1)] * #1 + d * CoshIntegral[d*(x - #1)] * Sinh[c + d*#1] * #1 + d * Cosh[c + d*#1] * SinhIntegral[d*(x - #1)] * #1 + d * Sinh[c + d*#1] * SinhIntegral[d*(x - #1)] * #1 / #1 & ] / (18*a*b*(a + b*x^3))
```

fricas [B] time = 0.94, size = 2135, normalized size = 3.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(d*x+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/36*(12*a*b*d^2*x^2*cosh(d*x + c) - (2*(a*b*d^3*x^3 + a^2*d^3)*cosh(d*x + c)^2 - 2*(a*b*d^3*x^3 + a^2*d^3)*sinh(d*x + c)^2 - (a*d^3/b)^(2/3)*((b^2*x^3 + a*b - sqrt(-3)*(b^2*x^3 + a*b))*cosh(d*x + c)^2 - (b^2*x^3 + a*b - sqrt(-3)*(b^2*x^3 + a*b))*sinh(d*x + c)^2))*Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1) + c) + (2*(a*b*d^3*x^3 + a^2*d^3)*cosh(d*x + c)^2 - 2*(a*b*d^3*x^3 + a^2*d^3)*sinh(d*x + c)^2 + (-a*d^3/b)^(2/3)*((b^2*x^3 + a*b - sqrt(-3)*(b^2*x^3 + a*b))*cosh(d*x + c)^2 - (b^2*x^3 + a*b - sqrt(-3)*(b^2*x^3 + a*b))*sinh(d*x + c)^2))*Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1) - c) - (2*(a*b*d^3*x^3 + a^2*d^3)*cosh(d*x + c)^2 - 2*(a*b*d^3*x^3 + a^2*d^3)*sinh(d*x + c)^2 - (a*d^3/b)^(2/3)*((b^2*x^3 + a*b + sqrt(-3)*(b^2*x^3 + a*b))*cosh(d*x + c)^2 - (b^2*x^3 + a*b + sqrt(-3)*(b^2*x^3 + a*b))*sinh(d*x + c)^2))*Ei(d*x + 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1) - c) + (2*(a*b*d^3*x^3 + a^2*d^3)*cosh(d*x + c)^2 - 2*(a*b*d^3*x^3 + a^2*d^3)*sinh(d*x + c)^2 + (-a*d^3/b)^(2/3)*((b^2*x^3 + a*b + sqrt(-3)*(b^2*x^3 + a*b))*cosh(d*x + c)^2 - (b^2*x^3 + a*b + sqrt(-3)*(b^2*x^3 + a*b))*sinh(d*x + c)^2))*Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c) + 2*((a*b*d^3*x^3 + a^2*d^3)*cosh(d*x + c)^2 - (a*b*d^3*x^3 + a^2*d^3)*sinh(d*x + c)^2 - (-a*d^3/b)^(2/3)*((b^2*x^3 + a*b)*cosh(d*x + c)^2 - (b^2*x^3 + a*b)*sinh(d*x + c)^2))*Ei(-d*x + (-a*d^3/b)^(1/3))*cosh(c + (-a*d^3/b)^(1/3)) - 2*((a*b*d^3*x^3 + a^2*d^3)*cosh(d*x + c)^2 - (a*b*d^3*x^3 + a^2*d^3)*sinh(d*x + c)^2 + (a*d^3/b)^(2/3)*((b^2*x^3 + a*b)*cosh(d*x + c)^2 - (b^2*x^3 + a*b)*sinh(d*x + c)^2))*Ei(d*x + (a*d^3/b)^(1/3))*cosh(-c + (a*d^3/b)^(1/3)) - (2*(a*b*d^3*x^3 + a^2*d^3)*cosh(d*x + c)^2 - 2*(a*b*d^3*x^3 + a^2*d^3)*sinh(d*x + c)^2 - (a*d^3/b)^(2/3)*((b^2*x^3 + a*b - sqrt(-3)*(b^2*x^3 + a*b))*cosh(d*x + c)^2 - (b^2*x^3 + a*b - sqrt(-3)*(b^2*x^3 + a*b))*sinh(d*x + c)^2))*Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1) + c) + (2*(a*b*d^3*x^3 + a^2*d^3)*cosh(d*x + c)^2 - 2*(a*b*d^3*x^3 + a^2*d^3)*si
```

$$\begin{aligned} & \text{nh}(d*x + c)^2 + (-a*d^3/b)^{(2/3)}*((b^2*x^3 + a*b - \sqrt{-3}*(b^2*x^3 + a*b)) \\ &)*\cosh(d*x + c)^2 - (b^2*x^3 + a*b - \sqrt{-3}*(b^2*x^3 + a*b))*\sinh(d*x + c \\ &)^2)*\text{Ei}(-d*x - 1/2*(-a*d^3/b)^{(1/3)}*(\sqrt{-3} + 1))*\sinh(1/2*(-a*d^3/b)^{(1/3)} \\ &)*(\sqrt{-3} + 1) - c) + (2*(a*b*d^3*x^3 + a^2*d^3)*\cosh(d*x + c)^2 - 2*(a \\ & *b*d^3*x^3 + a^2*d^3)*\sinh(d*x + c)^2 - (a*d^3/b)^{(2/3)}*((b^2*x^3 + a*b + \sqrt{-3} \\ &)*(b^2*x^3 + a*b))*\cosh(d*x + c)^2 - (b^2*x^3 + a*b + \sqrt{-3}*(b^2*x^3 + a*b)) \\ &)*\sinh(d*x + c)^2)*\text{Ei}(d*x + 1/2*(a*d^3/b)^{(1/3)}*(\sqrt{-3} - 1))*\sinh(1/2*(a*d^3/b)^{(1/3)} \\ &)*(\sqrt{-3} - 1) - c) - (2*(a*b*d^3*x^3 + a^2*d^3)*\cosh(d*x + c)^2 - 2*(a*b*d^3*x^3 + a^2*d^3) \\ &)*\sinh(d*x + c)^2 + (-a*d^3/b)^{(2/3)}*((b^2*x^3 + a*b + \sqrt{-3}*(b^2*x^3 + a*b)) \\ &)*\cosh(d*x + c)^2 - (b^2*x^3 + a*b + \sqrt{-3}*(b^2*x^3 + a*b))*\sinh(d*x + c)^2)*\text{Ei}(-d*x + 1/2*(-a*d^3/b)^{(1/3)} \\ &)*(\sqrt{-3} - 1))*\sinh(1/2*(-a*d^3/b)^{(1/3)}*(\sqrt{-3} - 1) + c) - 2*((a*b*d^3*x^3 + a^2*d^3) \\ &)*\cosh(d*x + c)^2 - (a*b*d^3*x^3 + a^2*d^3)*\sinh(d*x + c)^2 - (-a*d^3/b)^{(2/3)}*((b^2*x^3 + a*b) \\ &)*\cosh(d*x + c)^2 - (b^2*x^3 + a*b)*\sinh(d*x + c)^2)*\text{Ei}(-d*x + (-a*d^3/b)^{(1/3)})*\sinh(c + (-a*d^3/b)^{(1/3)}) + \\ & 2*((a*b*d^3*x^3 + a^2*d^3)*\cosh(d*x + c)^2 - (a*b*d^3*x^3 + a^2*d^3)*\sinh(d*x + c)^2 + (a*d^3/b)^{(2/3)} \\ &)*((b^2*x^3 + a*b)*\cosh(d*x + c)^2 - (b^2*x^3 + a*b)*\sinh(d*x + c)^2)*\text{Ei}(d*x + (a*d^3/b)^{(1/3)})*\sinh(-c + (a*d^3/b)^{(1/3)}) \\ &) / ((a^2*b^2*d^2*x^3 + a^3*b*d^2)*\cosh(d*x + c)^2 - (a^2*b^2*d^2*x^3 + a^3*b*d^2)*\sinh(d*x + c)^2) \end{aligned}$$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \cosh(dx + c)}{(bx^3 + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cosh(d*x+c)/(b*x^3+a)^2,x, algorithm="giac")

[Out] integrate(x*cosh(d*x + c)/(b*x^3 + a)^2, x)

maple [C] time = 0.16, size = 395, normalized size = 0.57

$$\frac{d^3 e^{-dx-c} x^2}{6a(b d^3 x^3 + a d^3)} - \frac{d \left(\sum_{R1=\text{RootOf}(b_Z^3-3cb_Z^2+3bc^2_Z+a d^3-bc^3)} \frac{(-R1^2-R1c+R1+c)e^{-R1} \text{Ei}(1,dx-R1+c)}{-R1^2-2R1c+c^2} \right)}{18ab} + \frac{dc}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*cosh(d*x+c)/(b*x^3+a)^2,x)

[Out] 1/6*d^3*exp(-d*x-c)*x^2/a/(b*d^3*x^3+a*d^3)-1/18*d/a/b*sum((R1^2-R1*c+R1+c)/(-R1^2-2*R1*c+c^2)*exp(-R1)*Ei(1,d*x-R1+c), R1=RootOf(_Z^3*b-3*_Z^2*

$b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/18*d*c/a/b*\text{sum}((_R1-c+2)/(_R1^2-2*_R1*c+c^2)$
 $*\text{exp}(-_R1)*\text{Ei}(1,d*x-_R1+c),_R1=\text{RootOf}(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*$
 $c^3))+1/6*d^3*\text{exp}(d*x+c)*x^2/a/(b*d^3*x^3+a*d^3)+1/18*d/a/b*\text{sum}((_R1^2-_R1*$
 $c-_R1-c)/(_R1^2-2*_R1*c+c^2)*\text{exp}(_R1)*\text{Ei}(1,-d*x+_R1-c),_R1=\text{RootOf}(_Z^3*b-3*$
 $_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/18*d*c/a/b*\text{sum}((_R1-c-2)/(_R1^2-2*_R1*c$
 $+c^2)*\text{exp}(_R1)*\text{Ei}(1,-d*x+_R1-c),_R1=\text{RootOf}(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d$
 $^3-b*c^3))$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{x e^{(dx+2c)} - x e^{-dx}}{2(b^2 dx^6 e^c + 2 ab dx^3 e^c + a^2 d e^c)} + \frac{1}{2} \int \frac{(5 b x^3 e^c - a e^c) e^{(dx)}}{b^3 dx^9 + 3 ab^2 dx^6 + 3 a^2 b dx^3 + a^3 d} dx - \frac{1}{2} \int \frac{(5 b x^3 - a) e^{(-dx)}}{b^3 dx^9 e^c + 3 ab^2 dx^6 e^c + 3 a^2 b dx^3 e^c + a^3 d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cosh(d*x+c)/(b*x^3+a)^2,x, algorithm="maxima")

[Out] 1/2*(x*e^(d*x + 2*c) - x*e^(-d*x))/(b^2*d*x^6*e^c + 2*a*b*d*x^3*e^c + a^2*d*e^c) + 1/2*integrate((5*b*x^3*e^c - a*e^c)*e^(d*x)/(b^3*d*x^9 + 3*a*b^2*d*x^6 + 3*a^2*b*d*x^3 + a^3*d), x) - 1/2*integrate((5*b*x^3 - a)*e^(-d*x)/(b^3*d*x^9*e^c + 3*a*b^2*d*x^6*e^c + 3*a^2*b*d*x^3*e^c + a^3*d*e^c), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x \cosh(c + dx)}{(bx^3 + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x*cosh(c + d*x))/(a + b*x^3)^2,x)

[Out] int((x*cosh(c + d*x))/(a + b*x^3)^2, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cosh(d*x+c)/(b*x**3+a)**2,x)

[Out] Timed out

$$3.105 \quad \int \frac{\cosh(c+dx)}{(a+bx^3)^2} dx$$

Optimal. Leaf size=739

$$\frac{d \sinh\left(c - \frac{\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(xd + \frac{\sqrt[3]{ad}}{\sqrt[3]{b}}\right)}{9a^{4/3}b^{2/3}} + \frac{(-1)^{2/3} d \sinh\left(\frac{\sqrt[3]{-1} \sqrt[3]{ad}}{\sqrt[3]{b}} + c\right) \text{Chi}\left(\frac{\sqrt[3]{-1} \sqrt[3]{ad}}{\sqrt[3]{b}} - dx\right)}{9a^{4/3}b^{2/3}} - \frac{\sqrt[3]{-1} d \sinh\left(c - \frac{(-1)^{2/3} \sqrt[3]{ad}}{\sqrt[3]{b}}\right)}{9a^{4/3}b^{2/3}}$$

[Out] $2/9 * \text{Chi}(a^{1/3} * d/b^{1/3} + d*x) * \cosh(c - a^{1/3} * d/b^{1/3}) / a^{5/3} / b^{1/3} - 2/9 * (-1)^{1/3} * \text{Chi}((-1)^{1/3} * a^{1/3} * d/b^{1/3} - d*x) * \cosh(c + (-1)^{1/3} * a^{1/3} * d/b^{1/3}) / a^{5/3} / b^{1/3} + 2/9 * (-1)^{2/3} * \text{Chi}(-(-1)^{2/3} * a^{1/3} * d/b^{1/3} - d*x) * \cosh(c - (-1)^{2/3} * a^{1/3} * d/b^{1/3}) / a^{5/3} / b^{1/3} + 1/3 * \cosh(d*x + c) / a/b/x^2 - 1/3 * \cosh(d*x + c) / b/x^2 / (b*x^3 + a) + 1/9 * (-1)^{2/3} * d * \cosh(c + (-1)^{1/3} * a^{1/3} * d/b^{1/3}) * \text{Shi}(-(-1)^{1/3} * a^{1/3} * d/b^{1/3} + d*x) / a^{4/3} / b^{2/3} + 1/9 * d * \cosh(c - a^{1/3} * d/b^{1/3}) * \text{Shi}(a^{1/3} * d/b^{1/3} + d*x) / a^{4/3} / b^{2/3} - 1/9 * (-1)^{1/3} * d * \cosh(c - (-1)^{2/3} * a^{1/3} * d/b^{1/3}) * \text{Shi}((-1)^{2/3} * a^{1/3} * d/b^{1/3} + d*x) / a^{4/3} / b^{2/3} + 1/9 * d * \text{Chi}(a^{1/3} * d/b^{1/3} + d*x) * \sinh(c - a^{1/3} * d/b^{1/3}) / a^{4/3} / b^{2/3} + 2/9 * \text{Shi}(a^{1/3} * d/b^{1/3} + d*x) * \sinh(c - a^{1/3} * d/b^{1/3}) / a^{5/3} / b^{1/3} + 1/9 * (-1)^{2/3} * d * \text{Chi}((-1)^{1/3} * a^{1/3} * d/b^{1/3} - d*x) * \sinh(c + (-1)^{1/3} * a^{1/3} * d/b^{1/3}) / a^{4/3} / b^{2/3} - 2/9 * (-1)^{1/3} * \text{Shi}(-(-1)^{1/3} * a^{1/3} * d/b^{1/3} + d*x) * \sinh(c + (-1)^{1/3} * a^{1/3} * d/b^{1/3}) / a^{5/3} / b^{1/3} - 1/9 * (-1)^{1/3} * d * \text{Chi}(-(-1)^{2/3} * a^{1/3} * d/b^{1/3} - d*x) * \sinh(c - (-1)^{2/3} * a^{1/3} * d/b^{1/3}) / a^{4/3} / b^{2/3} + 2/9 * (-1)^{2/3} * \text{Shi}((-1)^{2/3} * a^{1/3} * d/b^{1/3} + d*x) * \sinh(c - (-1)^{2/3} * a^{1/3} * d/b^{1/3}) / a^{5/3} / b^{1/3}$

Rubi [A] time = 1.32, antiderivative size = 739, normalized size of antiderivative = 1.00, number of steps used = 36, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5279, 5293, 3297, 3303, 3298, 3301, 5281, 5292}

$$\frac{d \sinh\left(c - \frac{\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(xd + \frac{\sqrt[3]{ad}}{\sqrt[3]{b}}\right)}{9a^{4/3}b^{2/3}} + \frac{(-1)^{2/3} d \sinh\left(\frac{\sqrt[3]{-1} \sqrt[3]{ad}}{\sqrt[3]{b}} + c\right) \text{Chi}\left(\frac{\sqrt[3]{-1} \sqrt[3]{ad}}{\sqrt[3]{b}} - dx\right)}{9a^{4/3}b^{2/3}} - \frac{\sqrt[3]{-1} d \sinh\left(c - \frac{(-1)^{2/3} \sqrt[3]{ad}}{\sqrt[3]{b}}\right)}{9a^{4/3}b^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]/(a + b*x^3)^2, x]

[Out] $\text{Cosh}[c + d*x] / (3*a*b*x^2) - \text{Cosh}[c + d*x] / (3*b*x^2*(a + b*x^3)) - (2*(-1)^{1/3} * \text{Cosh}[c + ((-1)^{1/3} * a^{1/3} * d) / b^{1/3}] * \text{CoshIntegral}[\frac{((-1)^{1/3} * a^{1/3} * d) / b^{1/3} - d*x}{9*a^{5/3} * b^{1/3}}]) / (9*a^{5/3} * b^{1/3}) + (2*(-1)^{2/3} * \text{Cosh}[c - ((-1)^{2/3} * a^{1/3} * d) / b^{1/3}] * \text{CoshIntegral}[-\frac{((-1)^{2/3} * a^{1/3} * d) / b^{1/3}}{9*a^{5/3} * b^{1/3}}]) / (9*a^{5/3} * b^{1/3}) - d$

$$\begin{aligned} & *x]/(9*a^{(5/3)*b^{(1/3)}}) + (2*Cosh[c - (a^{(1/3)*d}/b^{(1/3)})]*CoshIntegral[(a^{(1/3)*d}/b^{(1/3)} + d*x)]/(9*a^{(5/3)*b^{(1/3)}}) + (d*CoshIntegral[(a^{(1/3)*d}/b^{(1/3)} + d*x)*Sinh[c - (a^{(1/3)*d}/b^{(1/3)})]/(9*a^{(4/3)*b^{(2/3)}}) + ((-1)^{(2/3)}*d*CoshIntegral[((-1)^{(1/3)*a^{(1/3)*d}/b^{(1/3)} - d*x]*Sinh[c + ((-1)^{(1/3)*a^{(1/3)*d}/b^{(1/3)})]/(9*a^{(4/3)*b^{(2/3)}}) - ((-1)^{(1/3)*d*CoshIntegral[-(((-1)^{(2/3)*a^{(1/3)*d}/b^{(1/3)} - d*x)*Sinh[c - ((-1)^{(2/3)*a^{(1/3)*d}/b^{(1/3)})]/(9*a^{(4/3)*b^{(2/3)}}) - ((-1)^{(2/3)*d*Cosh[c + ((-1)^{(1/3)*a^{(1/3)*d}/b^{(1/3)}]*SinhIntegral[((-1)^{(1/3)*a^{(1/3)*d}/b^{(1/3)} - d*x)]/(9*a^{(4/3)*b^{(2/3)}}) + (2*(-1)^{(1/3)*Sinh[c + ((-1)^{(1/3)*a^{(1/3)*d}/b^{(1/3)}]*SinhIntegral[(((-1)^{(1/3)*a^{(1/3)*d}/b^{(1/3)} - d*x)]/(9*a^{(5/3)*b^{(1/3)}}) + (d*Cosh[c - (a^{(1/3)*d}/b^{(1/3)})]*SinhIntegral[(a^{(1/3)*d}/b^{(1/3)} + d*x)]/(9*a^{(4/3)*b^{(2/3)}}) + (2*Sinh[c - (a^{(1/3)*d}/b^{(1/3)})]*SinhIntegral[(a^{(1/3)*d}/b^{(1/3)} + d*x)]/(9*a^{(5/3)*b^{(1/3)}}) - ((-1)^{(1/3)*d*Cosh[c - ((-1)^{(2/3)*a^{(1/3)*d}/b^{(1/3)}]*SinhIntegral[((-1)^{(2/3)*a^{(1/3)*d}/b^{(1/3)} + d*x)]/(9*a^{(4/3)*b^{(2/3)}}) + (2*(-1)^{(2/3)*Sinh[c - ((-1)^{(2/3)*a^{(1/3)*d}/b^{(1/3)}]*SinhIntegral[(((-1)^{(2/3)*a^{(1/3)*d}/b^{(1/3)} + d*x)]/(9*a^{(5/3)*b^{(1/3)}}) \end{aligned}$$
Rule 3297

$$\text{Int}[(c_.) + (d_.)*(x_)^{(m_)}*\sin[(e_.) + (f_.)*(x_)], x_Symbol] \text{ :> } \text{Simp}[(c + d*x)^{(m + 1)}*\sin[e + f*x]/(d*(m + 1)), x] - \text{Dist}[f/(d*(m + 1)), \text{Int}[(c + d*x)^{(m + 1)}*\cos[e + f*x], x], x] \text{ /; } \text{FreeQ}\{c, d, e, f\}, x\} \ \&\& \ \text{LtQ}[m, -1]$$
Rule 3298

$$\text{Int}[\sin[(e_.) + (\text{Complex}[0, fz_])*(f_.)*(x_)]]/((c_.) + (d_.)*(x_)), x_Symbol] \text{ :> } \text{Simp}[(I*\text{SinhIntegral}[(c*f*fz)/d + f*fz*x])/d, x] \text{ /; } \text{FreeQ}\{c, d, e, f, fz\}, x\} \ \&\& \ \text{EqQ}[d*e - c*f*fz*I, 0]$$
Rule 3301

$$\text{Int}[\sin[(e_.) + (\text{Complex}[0, fz_])*(f_.)*(x_)]]/((c_.) + (d_.)*(x_)), x_Symbol] \text{ :> } \text{Simp}[\text{CoshIntegral}[(c*f*fz)/d + f*fz*x]/d, x] \text{ /; } \text{FreeQ}\{c, d, e, f, fz\}, x\} \ \&\& \ \text{EqQ}[d*(e - \text{Pi}/2) - c*f*fz*I, 0]$$
Rule 3303

$$\text{Int}[\sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*(x_)), x_Symbol] \text{ :> } \text{Dist}[\cos[(d*e - c*f)/d], \text{Int}[\sin[(c*f)/d + f*x]/(c + d*x), x], x] + \text{Dist}[\sin[(d*e - c*f)/d], \text{Int}[\cos[(c*f)/d + f*x]/(c + d*x), x], x] \text{ /; } \text{FreeQ}\{c, d, e, f\}, x\} \ \&\& \ \text{NeQ}[d*e - c*f, 0]$$
Rule 5279

$$\text{Int}[\text{Cosh}[(c_.) + (d_.)*(x_)]*((a_.) + (b_.)*(x_)^{(n_)})^{(p_)}, x_Symbol] \text{ :> } \text{Si}$$

```
mp[(x^(-n + 1)*(a + b*x^n)^(p + 1)*Cosh[c + d*x])/(b*n*(p + 1)), x] + (-Dist
t[(-n + 1)/(b*n*(p + 1)), Int[((a + b*x^n)^(p + 1)*Cosh[c + d*x])/x^n, x],
x] - Dist[d/(b*n*(p + 1)), Int[x^(-n + 1)*(a + b*x^n)^(p + 1)*Sinh[c + d*x]
, x], x]) /; FreeQ[{a, b, c, d}, x] && IntegerQ[p] && IGtQ[n, 0] && LtQ[p,
-1] && GtQ[n, 2]
```

Rule 5281

```
Int[Cosh[(c_.) + (d_.)*(x_.)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := In
t[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 5292

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_.)], x_Sy
mbol] := Int[ExpandIntegrand[Sinh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rule 5293

```
Int[Cosh[(c_.) + (d_.)*(x_.)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] := Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh(c+dx)}{(a+bx^3)^2} dx &= -\frac{\cosh(c+dx)}{3bx^2(a+bx^3)} - \frac{2 \int \frac{\cosh(c+dx)}{x^3(a+bx^3)} dx}{3b} + \frac{d \int \frac{\sinh(c+dx)}{x^2(a+bx^3)} dx}{3b} \\
&= -\frac{\cosh(c+dx)}{3bx^2(a+bx^3)} - \frac{2 \int \left(\frac{\cosh(c+dx)}{ax^3} - \frac{b \cosh(c+dx)}{a(a+bx^3)} \right) dx}{3b} + \frac{d \int \left(\frac{\sinh(c+dx)}{ax^2} - \frac{bx \sinh(c+dx)}{a(a+bx^3)} \right) dx}{3b} \\
&= -\frac{\cosh(c+dx)}{3bx^2(a+bx^3)} + \frac{2 \int \frac{\cosh(c+dx)}{a+bx^3} dx}{3a} - \frac{2 \int \frac{\cosh(c+dx)}{x^3} dx}{3ab} - \frac{d \int \frac{x \sinh(c+dx)}{a+bx^3} dx}{3a} + \frac{d \int \frac{\sinh(c+dx)}{x^2} dx}{3ab} \\
&= \frac{\cosh(c+dx)}{3abx^2} - \frac{\cosh(c+dx)}{3bx^2(a+bx^3)} - \frac{d \sinh(c+dx)}{3abx} + \frac{2 \int \left(-\frac{\cosh(c+dx)}{3a^{2/3}(-\sqrt[3]{a}-\sqrt[3]{b}x)} - \frac{\cosh(c+dx)}{3a^{2/3}(-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x)} \right) dx}{3a} \\
&= \frac{\cosh(c+dx)}{3abx^2} - \frac{\cosh(c+dx)}{3bx^2(a+bx^3)} - \frac{2 \int \frac{\cosh(c+dx)}{-\sqrt[3]{a}-\sqrt[3]{b}x} dx}{9a^{5/3}} - \frac{2 \int \frac{\cosh(c+dx)}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x} dx}{9a^{5/3}} - \frac{2 \int \frac{\cosh(c+dx)}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x} dx}{9a^{5/3}} \\
&= \frac{\cosh(c+dx)}{3abx^2} - \frac{\cosh(c+dx)}{3bx^2(a+bx^3)} + \frac{d^2 \cosh(c) \text{Chi}(dx)}{3ab} + \frac{d^2 \sinh(c) \text{Shi}(dx)}{3ab} - \frac{(d^2 \cosh(c)) \int}{3ab} \\
&= \frac{\cosh(c+dx)}{3abx^2} - \frac{\cosh(c+dx)}{3bx^2(a+bx^3)} - \frac{2\sqrt[3]{-1} \cosh\left(c + \frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}} - dx\right)}{9a^{5/3}\sqrt[3]{b}} + \frac{2(-1)^{2/3} \cosh\left(c + \frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}} - dx\right)}{9a^{5/3}\sqrt[3]{b}}
\end{aligned}$$

Mathematica [C] time = 0.21, size = 387, normalized size = 0.52

$$(a+bx^3) \text{RootSum}\left[\#1^3b+a\&, \frac{-2 \sinh(\#1d+c) \text{Chi}(d(x-\#1))-\#1d \sinh(\#1d+c) \text{Chi}(d(x-\#1))+2 \cosh(\#1d+c) \text{Chi}(d(x-\#1))+\#1d \cosh(\#1d+c)}{9a^{5/3}\sqrt[3]{b}}\right]$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d*x]/(a + b*x^3)^2, x]

[Out] (6*b*x*Cosh[c + d*x] + (a + b*x^3)*RootSum[a + b*#1^3 & , (2*Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] - 2*CoshIntegral[d*(x - #1)]*Sinh[c + d*#1] - 2*Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] + 2*Sinh[c + d*#1]*SinhIntegral[d*(x - #1)] + d*Cosh[c + d*#1]*CoshIntegral[d*(x - #1)]*#1 - d*CoshIntegral[d*(x - #1)]*Sinh[c + d*#1]*#1 - d*Cosh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1 +

$c)^2 - (b*x^3 - \sqrt{-3}*(b*x^3 + a) + a)*\sinh(d*x + c)^2 + 2*(-a*d^3/b)^{(1/3)}*((b*x^3 + \sqrt{-3}*(b*x^3 + a) + a)*\cosh(d*x + c)^2 - (b*x^3 + \sqrt{-3}*(b*x^3 + a) + a)*\sinh(d*x + c)^2)*\text{Ei}(-d*x - 1/2*(-a*d^3/b)^{(1/3)}*(\sqrt{-3} + 1))*\sinh(1/2*(-a*d^3/b)^{(1/3)}*(\sqrt{-3} + 1) - c) + ((a*d^3/b)^{(2/3)}*((b*x^3 + \sqrt{-3}*(b*x^3 + a) + a)*\cosh(d*x + c)^2 - (b*x^3 + \sqrt{-3}*(b*x^3 + a) + a)*\sinh(d*x + c)^2) + 2*(a*d^3/b)^{(1/3)}*((b*x^3 - \sqrt{-3}*(b*x^3 + a) + a)*\cosh(d*x + c)^2 - (b*x^3 - \sqrt{-3}*(b*x^3 + a) + a)*\sinh(d*x + c)^2))*\text{Ei}(d*x + 1/2*(a*d^3/b)^{(1/3)}*(\sqrt{-3} - 1))*\sinh(1/2*(a*d^3/b)^{(1/3)}*(\sqrt{-3} - 1) - c) - ((-a*d^3/b)^{(2/3)}*((b*x^3 + \sqrt{-3}*(b*x^3 + a) + a)*\cosh(d*x + c)^2 - (b*x^3 + \sqrt{-3}*(b*x^3 + a) + a)*\sinh(d*x + c)^2) + 2*(-a*d^3/b)^{(1/3)}*((b*x^3 - \sqrt{-3}*(b*x^3 + a) + a)*\cosh(d*x + c)^2 - (b*x^3 - \sqrt{-3}*(b*x^3 + a) + a)*\sinh(d*x + c)^2))*\text{Ei}(-d*x + 1/2*(-a*d^3/b)^{(1/3)}*(\sqrt{-3} - 1))*\sinh(1/2*(-a*d^3/b)^{(1/3)}*(\sqrt{-3} - 1) + c) + 2*((-a*d^3/b)^{(2/3)}*((b*x^3 + a)*\cosh(d*x + c)^2 - (b*x^3 + a)*\sinh(d*x + c)^2) + 2*(-a*d^3/b)^{(1/3)}*((b*x^3 + a)*\cosh(d*x + c)^2 - (b*x^3 + a)*\sinh(d*x + c)^2))*\text{Ei}(-d*x + (-a*d^3/b)^{(1/3)}*\sinh(c + (-a*d^3/b)^{(1/3)})) - 2*((a*d^3/b)^{(2/3)}*((b*x^3 + a)*\cosh(d*x + c)^2 - (b*x^3 + a)*\sinh(d*x + c)^2) + 2*(a*d^3/b)^{(1/3)}*((b*x^3 + a)*\cosh(d*x + c)^2 - (b*x^3 + a)*\sinh(d*x + c)^2))*\text{Ei}(d*x + (a*d^3/b)^{(1/3)}*\sinh(-c + (a*d^3/b)^{(1/3)})))/((a^2*b*d*x^3 + a^3*d)*\cosh(d*x + c)^2 - (a^2*b*d*x^3 + a^3*d)*\sinh(d*x + c)^2)$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(dx + c)}{(bx^3 + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/(b*x^3+a)^2,x, algorithm="giac")

[Out] integrate(cosh(d*x + c)/(b*x^3 + a)^2, x)

maple [C] time = 0.12, size = 226, normalized size = 0.31

$$\frac{d^3 e^{-dx-c} x}{6a (b d^3 x^3 + a d^3)} - \frac{d^2 \left(\sum_{_R1=\text{RootOf}(b_Z^3-3cb_Z^2+3bc^2_Z+a d^3-b c^3)} \frac{(-_R1-c+2)e^{-_R1} \text{Ei}(1,dx-_R1+c)}{_R1^2-2_R1c+c^2} \right)}{18ab} + \frac{d^3 e^{dx+c} x}{6a (b d^3 x^3 + a d^3)} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d*x+c)/(b*x^3+a)^2,x)

[Out] 1/6*d^3*exp(-d*x-c)*x/a/(b*d^3*x^3+a*d^3)-1/18*d^2/a/b*sum((_R1-c+2)/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b

$*c^2+a*d^3-b*c^3))+1/6*d^3*\exp(d*x+c)*x/a/(b*d^3*x^3+a*d^3)+1/18*d^2/a/b*\sum((_R1-c-2)/(_R1^2-2*_R1*c+c^2)*\exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(dx + c)}{(bx^3 + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/(b*x^3+a)^2,x, algorithm="maxima")

[Out] integrate(cosh(d*x + c)/(b*x^3 + a)^2, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(c + dx)}{(bx^3 + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(c + d*x)/(a + b*x^3)^2,x)

[Out] int(cosh(c + d*x)/(a + b*x^3)^2, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/(b*x**3+a)**2,x)

[Out] Timed out

$$3.106 \quad \int \frac{\cosh(c+dx)}{x(a+bx^3)^2} dx$$

Optimal. Leaf size=697

$$\frac{d \sinh\left(c - \frac{\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(xd + \frac{\sqrt[3]{ad}}{\sqrt[3]{b}}\right)}{9a^{5/3}\sqrt[3]{b}} + \frac{\sqrt[3]{-1} d \sinh\left(\frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}} + c\right) \text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}} - dx\right)}{9a^{5/3}\sqrt[3]{b}} - \frac{(-1)^{2/3} d \sinh\left(c - \frac{(-1)^{2/3}\sqrt[3]{ad}}{\sqrt[3]{b}}\right)}{9a^{5/3}\sqrt[3]{b}}$$

[Out] Chi(d*x)*cosh(c)/a^2-1/3*Chi(a^(1/3)*d/b^(1/3)+d*x)*cosh(c-a^(1/3)*d/b^(1/3))/a^2-1/3*Chi((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*cosh(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a^2-1/3*Chi(-(-1)^(2/3)*a^(1/3)*d/b^(1/3)-d*x)*cosh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^2+1/3*cosh(d*x+c)/a/b/x^3-1/3*cosh(d*x+c)/b/x^3/(b*x^3+a)+1/9*(-1)^(1/3)*d*cosh(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Shi(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(5/3)/b^(1/3)-1/9*d*cosh(c-a^(1/3)*d/b^(1/3))*Shi(a^(1/3)*d/b^(1/3)+d*x)/a^(5/3)/b^(1/3)-1/9*(-1)^(2/3)*d*cosh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))*Shi((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(5/3)/b^(1/3)+Shi(d*x)*sinh(c)/a^2-1/9*d*Chi(a^(1/3)*d/b^(1/3)+d*x)*sinh(c-a^(1/3)*d/b^(1/3))/a^(5/3)/b^(1/3)-1/3*Shi(a^(1/3)*d/b^(1/3)+d*x)*sinh(c-a^(1/3)*d/b^(1/3))/a^2+1/9*(-1)^(1/3)*d*Chi((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*sinh(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a^(5/3)/b^(1/3)-1/3*Shi(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)*sinh(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a^2-1/9*(-1)^(2/3)*d*Chi(-(-1)^(2/3)*a^(1/3)*d/b^(1/3)-d*x)*sinh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(5/3)/b^(1/3)-1/3*Shi((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)*sinh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^2

Rubi [A] time = 1.47, antiderivative size = 697, normalized size of antiderivative = 1.00, number of steps used = 41, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {5291, 5293, 3297, 3303, 3298, 3301, 5292, 5280}

$$\frac{d \sinh\left(c - \frac{\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(xd + \frac{\sqrt[3]{ad}}{\sqrt[3]{b}}\right)}{9a^{5/3}\sqrt[3]{b}} + \frac{\sqrt[3]{-1} d \sinh\left(\frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}} + c\right) \text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}} - dx\right)}{9a^{5/3}\sqrt[3]{b}} - \frac{(-1)^{2/3} d \sinh\left(c - \frac{(-1)^{2/3}\sqrt[3]{ad}}{\sqrt[3]{b}}\right)}{9a^{5/3}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]/(x*(a + b*x^3)^2), x]

[Out] Cosh[c + d*x]/(3*a*b*x^3) - Cosh[c + d*x]/(3*b*x^3*(a + b*x^3)) + (Cosh[c]*CoshIntegral[d*x])/a^2 - (Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[(((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*a^2) - (Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d*x])/(3*a^2) - (Cosh[c - (a^(1/3)*d)/b^(1/3)]*CoshIntegral[(a^(1/3)*d)/b^(1/3)])

$$\begin{aligned}
& + d*x]/(3*a^2) - (d*CoshIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sinh[c - (a^(1/3)*d)/b^(1/3)]/(9*a^(5/3)*b^(1/3)) + ((-1)^(1/3)*d*CoshIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]/(9*a^(5/3)*b^(1/3)) - ((-1)^(2/3)*d*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d*x]*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]/(9*a^(5/3)*b^(1/3)) + (Sinh[c]*SinhIntegral[d*x])/a^2 - ((-1)^(1/3)*d*Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]/(9*a^(5/3)*b^(1/3)) + (Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]/(3*a^2) - (d*Cosh[c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x]/(9*a^(5/3)*b^(1/3)) - (Sinh[c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x]/(3*a^2) - ((-1)^(2/3)*d*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]/(9*a^(5/3)*b^(1/3)) - (Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]/(3*a^2)
\end{aligned}$$

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]
```

Rule 5280

```
Int[((a_.) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Symbol] := Int[ExpandIntegrand[Sinh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
```

} , x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])

Rule 5291

Int[Cosh[(c_.) + (d_.)*(x_.)]*(x_.)^(m_.)*((a_.) + (b_.)*(x_.)^(n_.))^(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*(a + b*x^n)^(p + 1)*Cosh[c + d*x])/(b*n*(p + 1)), x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1)*Cosh[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^n)^(p + 1)*Sinh[c + d*x], x], x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1] && IGtQ[n, 0] && RationalQ[m] && (GtQ[m - n + 1, 0] || GtQ[n, 2])

Rule 5292

Int[(x_.)^(m_.)*((a_.) + (b_.)*(x_.)^(n_.))^(p_.)*Sinh[(c_.) + (d_.)*(x_.)], x_Symbol] :> Int[ExpandIntegrand[Sinh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])

Rule 5293

Int[Cosh[(c_.) + (d_.)*(x_.)]*(x_.)^(m_.)*((a_.) + (b_.)*(x_.)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])

Rubi steps

$$\begin{aligned}
\int \frac{\cosh(c+dx)}{x(a+bx^3)^2} dx &= -\frac{\cosh(c+dx)}{3bx^3(a+bx^3)} - \frac{\int \frac{\cosh(c+dx)}{x^4(a+bx^3)} dx}{b} + \frac{d \int \frac{\sinh(c+dx)}{x^3(a+bx^3)} dx}{3b} \\
&= -\frac{\cosh(c+dx)}{3bx^3(a+bx^3)} - \frac{\int \left(\frac{\cosh(c+dx)}{ax^4} - \frac{b \cosh(c+dx)}{a^2x} + \frac{b^2x^2 \cosh(c+dx)}{a^2(a+bx^3)} \right) dx}{b} + \frac{d \int \left(\frac{\sinh(c+dx)}{ax^3} - \frac{b \sinh(c+dx)}{a(a+bx^3)} \right) dx}{3b} \\
&= -\frac{\cosh(c+dx)}{3bx^3(a+bx^3)} + \frac{\int \frac{\cosh(c+dx)}{x} dx}{a^2} - \frac{\int \frac{\cosh(c+dx)}{x^4} dx}{ab} - \frac{b \int \frac{x^2 \cosh(c+dx)}{a+bx^3} dx}{a^2} - \frac{d \int \frac{\sinh(c+dx)}{a+bx^3} dx}{3a} \\
&= \frac{\cosh(c+dx)}{3abx^3} - \frac{\cosh(c+dx)}{3bx^3(a+bx^3)} - \frac{d \sinh(c+dx)}{6abx^2} - \frac{b \int \left(\frac{\cosh(c+dx)}{3b^{2/3}(\sqrt[3]{a} + \sqrt[3]{b}x)} + \frac{\cosh(c+dx)}{3b^{2/3}(-\sqrt[3]{-1} \sqrt[3]{a} + \sqrt[3]{b})} \right) dx}{a^2} \\
&= \frac{\cosh(c+dx)}{3abx^3} - \frac{d^2 \cosh(c+dx)}{6abx} - \frac{\cosh(c+dx)}{3bx^3(a+bx^3)} + \frac{\cosh(c)\text{Chi}(dx)}{a^2} + \frac{\sinh(c)\text{Shi}(dx)}{a^2} \\
&= \frac{\cosh(c+dx)}{3abx^3} - \frac{\cosh(c+dx)}{3bx^3(a+bx^3)} + \frac{\cosh(c)\text{Chi}(dx)}{a^2} + \frac{\sinh(c)\text{Shi}(dx)}{a^2} - \frac{d^3 \int \frac{\sinh(c+dx)}{x} dx}{6ab} \\
&= \frac{\cosh(c+dx)}{3abx^3} - \frac{\cosh(c+dx)}{3bx^3(a+bx^3)} + \frac{\cosh(c)\text{Chi}(dx)}{a^2} - \frac{\cosh\left(c + \frac{\sqrt[3]{-1} \sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(\frac{\sqrt[3]{-1} \sqrt[3]{ad}}{\sqrt[3]{b}}\right)}{3a^2} \\
&= \frac{\cosh(c+dx)}{3abx^3} - \frac{\cosh(c+dx)}{3bx^3(a+bx^3)} + \frac{\cosh(c)\text{Chi}(dx)}{a^2} - \frac{\cosh\left(c + \frac{\sqrt[3]{-1} \sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(\frac{\sqrt[3]{-1} \sqrt[3]{ad}}{\sqrt[3]{b}}\right)}{3a^2}
\end{aligned}$$

Mathematica [C] time = 0.83, size = 411, normalized size = 0.59

$$-3\text{RootSum}\left[\#1^3b + a\&, -\sinh(\#1d + c)\text{Chi}(d(x - \#1)) + \cosh(\#1d + c)\text{Chi}(d(x - \#1)) + \sinh(\#1d + c)\text{Shi}(d(x - \#1))\right]$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d*x]/(x*(a + b*x^3)^2), x]

[Out] ((6*a*Cosh[c]*Cosh[d*x])/(a + b*x^3) + 18*Cosh[c]*CoshIntegral[d*x] - 3*RootSum[a + b*#1^3 &, Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] - CoshIntegral[

$$d*(x - \#1)]*Sinh[c + d*\#1] - Cosh[c + d*\#1]*SinhIntegral[d*(x - \#1)] + Sinh[c + d*\#1]*SinhIntegral[d*(x - \#1)] &] - 3*RootSum[a + b*\#1^3 & , Cosh[c + d*\#1]*CoshIntegral[d*(x - \#1)] + CoshIntegral[d*(x - \#1)]*Sinh[c + d*\#1] + Cosh[c + d*\#1]*SinhIntegral[d*(x - \#1)] + Sinh[c + d*\#1]*SinhIntegral[d*(x - \#1)] &] + (a*d*RootSum[a + b*\#1^3 & , (Cosh[c + d*\#1]*CoshIntegral[d*(x - \#1)] - CoshIntegral[d*(x - \#1)]*Sinh[c + d*\#1] - Cosh[c + d*\#1]*SinhIntegral[d*(x - \#1)] + Sinh[c + d*\#1]*SinhIntegral[d*(x - \#1)])/\#1^2 &])/b - (a*d*RootSum[a + b*\#1^3 & , (Cosh[c + d*\#1]*CoshIntegral[d*(x - \#1)] + CoshIntegral[d*(x - \#1)]*Sinh[c + d*\#1] + Cosh[c + d*\#1]*SinhIntegral[d*(x - \#1)] + Sinh[c + d*\#1]*SinhIntegral[d*(x - \#1)])/\#1^2 &])/b + (6*a*Sinh[c]*Sinh[d*x])/(a + b*x^3) + 18*Sinh[c]*SinhIntegral[d*x])/(18*a^2)$$

fricas [B] time = 0.68, size = 1773, normalized size = 2.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/x/(b*x^3+a)^2,x, algorithm="fricas")

[Out]
$$-1/36*((6*(b*x^3 + a)*\cosh(d*x + c)^2 - 6*(b*x^3 + a)*\sinh(d*x + c)^2 - (a*d^3/b)^{(1/3)}*((b*x^3 + \sqrt{-3}*(b*x^3 + a) + a)*\cosh(d*x + c)^2 - (b*x^3 + \sqrt{-3}*(b*x^3 + a) + a)*\sinh(d*x + c)^2))*Ei(d*x - 1/2*(a*d^3/b)^{(1/3)}*(\sqrt{-3} + 1))*\cosh(1/2*(a*d^3/b)^{(1/3)}*(\sqrt{-3} + 1) + c) + (6*(b*x^3 + a)*\cosh(d*x + c)^2 - 6*(b*x^3 + a)*\sinh(d*x + c)^2 - (-a*d^3/b)^{(1/3)}*((b*x^3 + \sqrt{-3}*(b*x^3 + a) + a)*\cosh(d*x + c)^2 - (b*x^3 + \sqrt{-3}*(b*x^3 + a) + a)*\sinh(d*x + c)^2))*Ei(-d*x - 1/2*(-a*d^3/b)^{(1/3)}*(\sqrt{-3} + 1))*\cosh(1/2*(-a*d^3/b)^{(1/3)}*(\sqrt{-3} + 1) - c) + (6*(b*x^3 + a)*\cosh(d*x + c)^2 - 6*(b*x^3 + a)*\sinh(d*x + c)^2 - (a*d^3/b)^{(1/3)}*((b*x^3 - \sqrt{-3}*(b*x^3 + a) + a)*\cosh(d*x + c)^2 - (b*x^3 - \sqrt{-3}*(b*x^3 + a) + a)*\sinh(d*x + c)^2))*Ei(d*x + 1/2*(a*d^3/b)^{(1/3)}*(\sqrt{-3} - 1))*\cosh(1/2*(a*d^3/b)^{(1/3)}*(\sqrt{-3} - 1) - c) + (6*(b*x^3 + a)*\cosh(d*x + c)^2 - 6*(b*x^3 + a)*\sinh(d*x + c)^2 - (-a*d^3/b)^{(1/3)}*((b*x^3 - \sqrt{-3}*(b*x^3 + a) + a)*\cosh(d*x + c)^2 - (b*x^3 - \sqrt{-3}*(b*x^3 + a) + a)*\sinh(d*x + c)^2))*Ei(-d*x + 1/2*(-a*d^3/b)^{(1/3)}*(\sqrt{-3} - 1))*\cosh(1/2*(-a*d^3/b)^{(1/3)}*(\sqrt{-3} - 1) + c) + 2*(3*(b*x^3 + a)*\cosh(d*x + c)^2 - 3*(b*x^3 + a)*\sinh(d*x + c)^2 + (-a*d^3/b)^{(1/3)}*((b*x^3 + a)*\cosh(d*x + c)^2 - (b*x^3 + a)*\sinh(d*x + c)^2))*Ei(-d*x + (-a*d^3/b)^{(1/3)}*\cosh(c + (-a*d^3/b)^{(1/3)}) + 2*(3*(b*x^3 + a)*\cosh(d*x + c)^2 - 3*(b*x^3 + a)*\sinh(d*x + c)^2 + (a*d^3/b)^{(1/3)}*((b*x^3 + a)*\cosh(d*x + c)^2 - (b*x^3 + a)*\sinh(d*x + c)^2))*Ei(d*x + (a*d^3/b)^{(1/3)}*\cosh(-c + (a*d^3/b)^{(1/3)}) + (6*(b*x^3 + a)*\cosh(d*x + c)^2 - 6*(b*x^3 + a)*\sinh(d*x + c)^2 - (a*d^3/b)^{(1/3)}*((b*x^3 + \sqrt{-3}*(b*x^3 + a) + a)*\cosh(d*x + c)^2 - (b*x^3 + \sqrt{-3}*(b*x^3 + a) + a)*\sinh(d*x + c)^2))*Ei(d*x - 1/2*(a*d^3/b)^{(1/3)}*(\sqrt{-3} + 1))*\sinh(1/2*(a*d^3/b)^{(1/3)}*(\sqrt{-3} + 1) + c) + (6*(b*x^3 + a)*\cosh(d*x + c)^2 - 6*(b*x^3 + a)*\sinh(d*x + c)^2 - (-a*d^3/b)^{(1/3)}*((b*x^3 + \sqrt{-3}*(b*x^3 + a) + a)*\cosh(d*x + c)^2$$

$$\begin{aligned}
& - (b*x^3 + \sqrt{-3}*(b*x^3 + a) + a)*\sinh(d*x + c)^2)*\text{Ei}(-d*x - 1/2*(-a*d^3/b)^{1/3}*(\sqrt{-3} + 1))*\sinh(1/2*(-a*d^3/b)^{1/3}*(\sqrt{-3} + 1) - c) - \\
& (6*(b*x^3 + a)*\cosh(d*x + c)^2 - 6*(b*x^3 + a)*\sinh(d*x + c)^2 - (a*d^3/b)^{1/3}*((b*x^3 - \sqrt{-3}*(b*x^3 + a) + a)*\cosh(d*x + c)^2 - (b*x^3 - \sqrt{-3}*(b*x^3 + a) + a)*\sinh(d*x + c)^2))*\text{Ei}(d*x + 1/2*(a*d^3/b)^{1/3}*(\sqrt{-3} - 1))*\sinh(1/2*(a*d^3/b)^{1/3}*(\sqrt{-3} - 1) - c) - \\
& (6*(b*x^3 + a)*\cosh(d*x + c)^2 - 6*(b*x^3 + a)*\sinh(d*x + c)^2 - (-a*d^3/b)^{1/3}*((b*x^3 - \sqrt{-3}*(b*x^3 + a) + a)*\cosh(d*x + c)^2 - (b*x^3 - \sqrt{-3}*(b*x^3 + a) + a)*\sinh(d*x + c)^2))*\text{Ei}(-d*x + 1/2*(-a*d^3/b)^{1/3}*(\sqrt{-3} - 1))*\sinh(1/2*(-a*d^3/b)^{1/3}*(\sqrt{-3} - 1) + c) - \\
& 2*(3*(b*x^3 + a)*\cosh(d*x + c)^2 - 3*(b*x^3 + a)*\sinh(d*x + c)^2 + (-a*d^3/b)^{1/3}*((b*x^3 + a)*\cosh(d*x + c)^2 - (b*x^3 + a)*\sinh(d*x + c)^2))*\text{Ei}(-d*x + (-a*d^3/b)^{1/3})*\sinh(c + (-a*d^3/b)^{1/3}) - \\
& 2*(3*(b*x^3 + a)*\cosh(d*x + c)^2 - 3*(b*x^3 + a)*\sinh(d*x + c)^2 + (a*d^3/b)^{1/3}*((b*x^3 + a)*\cosh(d*x + c)^2 - (b*x^3 + a)*\sinh(d*x + c)^2))*\text{Ei}(d*x + (a*d^3/b)^{1/3})*\sinh(-c + (a*d^3/b)^{1/3}) - \\
& 12*a*\cosh(d*x + c) - 18*((b*x^3 + a)*\text{Ei}(d*x) + (b*x^3 + a)*\text{Ei}(-d*x))*\cosh(c) - 18*((b*x^3 + a)*\text{Ei}(d*x) - (b*x^3 + a)*\text{Ei}(-d*x))*\sinh(c))/((a^2*b*x^3 + a^3)*\cosh(d*x + c)^2 - (a^2*b*x^3 + a^3)*\sinh(d*x + c)^2)
\end{aligned}$$

giac [F] time = 0.00, size = 0, normalized size = 0.00

sage0x

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/x/(b*x^3+a)^2,x, algorithm="giac")

[Out] sage0*x

maple [C] time = 0.15, size = 338, normalized size = 0.48

$$\frac{e^{-dx-c}d^3}{6a((dx+c)^3b-3(dx+c)^2bc+3(dx+c)bc^2+ad^3-bc^3)} + \frac{\sum_{R1=\text{RootOf}(b_Z^3-3cb_Z^2+3bc^2_Z+a d^3-bc^3)} (-a d^3+3_R1^2)}{18a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d*x+c)/x/(b*x^3+a)^2,x)

[Out] $1/6*\exp(-d*x-c)/a*d^3/((d*x+c)^3*b-3*(d*x+c)^2*b*c+3*(d*x+c)*b*c^2+a*d^3-b*c^3)+1/18/a^2/b*\sum((-a*d^3+3*_R1^2*b-6*_R1*b*c+3*b*c^2)/(_R1^2-2*_R1*c+c^2))*\exp(-_R1)*\text{Ei}(1,d*x-_R1+c),_R1=\text{RootOf}(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/2/a^2*\exp(-c)*\text{Ei}(1,d*x)+1/6*\exp(d*x+c)/a*d^3/((d*x+c)^3*b-3*(d*x+c)^2*b*c+3*(d*x+c)*b*c^2+a*d^3-b*c^3)+1/18/a^2/b*\sum((a*d^3+3*_R1^2*b-6*_R1*b*c+3*b*c^2)/(_R1^2-2*_R1*c+c^2))*\exp(_R1)*\text{Ei}(1,-d*x+_R1-c),_R1=\text{RootOf}(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/2/a^2*\exp(c)*\text{Ei}(1,-d*x)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(dx + c)}{(bx^3 + a)^2 x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/x/(b*x^3+a)^2,x, algorithm="maxima")

[Out] integrate(cosh(d*x + c)/((b*x^3 + a)^2*x), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(c + dx)}{x(bx^3 + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(c + d*x)/(x*(a + b*x^3)^2),x)

[Out] int(cosh(c + d*x)/(x*(a + b*x^3)^2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/x/(b*x**3+a)**2,x)

[Out] Timed out

$$3.107 \quad \int \frac{x^5 \cosh(c+dx)}{(a+bx^3)^3} dx$$

Optimal. Leaf size=784

$$\frac{2d \sinh\left(c - \frac{\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(xd + \frac{\sqrt[3]{ad}}{\sqrt[3]{b}}\right)}{27a^{2/3}b^{7/3}} - \frac{2\sqrt[3]{-1} d \sinh\left(\frac{\sqrt[3]{-1} \sqrt[3]{ad}}{\sqrt[3]{b}} + c\right) \text{Chi}\left(\frac{\sqrt[3]{-1} \sqrt[3]{ad}}{\sqrt[3]{b}} - dx\right)}{27a^{2/3}b^{7/3}} + \frac{2(-1)^{2/3} d \sinh\left(c - \frac{(-1)^{2/3}}{\sqrt[3]{b}}\right)}{27a^{2/3}b^{7/3}}$$

[Out] $-1/54*d^2*\text{Chi}(a^{(1/3)}*d/b^{(1/3)}+d*x)*\cosh(c-a^{(1/3)}*d/b^{(1/3)})/a^{(1/3)}/b^{(8/3)} - 1/54*(-1)^{(2/3)}*d^2*\text{Chi}((-1)^{(1/3)}*a^{(1/3)}*d/b^{(1/3)}-d*x)*\cosh(c+(-1)^{(1/3)}*a^{(1/3)}*d/b^{(1/3)})/a^{(1/3)}/b^{(8/3)} + 1/54*(-1)^{(1/3)}*d^2*\text{Chi}(-(-1)^{(2/3)}*a^{(1/3)}*d/b^{(1/3)}-d*x)*\cosh(c-(-1)^{(2/3)}*a^{(1/3)}*d/b^{(1/3)})/a^{(1/3)}/b^{(8/3)} - 1/6*x^3*\cosh(d*x+c)/b/(b*x^3+a)^2 - 1/6*\cosh(d*x+c)/b^2/(b*x^3+a) - 2/27*(-1)^{(1/3)}*d*\cosh(c+(-1)^{(1/3)}*a^{(1/3)}*d/b^{(1/3)})*\text{Shi}(-(-1)^{(1/3)}*a^{(1/3)}*d/b^{(1/3)}+d*x)/a^{(2/3)}/b^{(7/3)} + 2/27*d*\cosh(c-a^{(1/3)}*d/b^{(1/3)})*\text{Shi}(a^{(1/3)}*d/b^{(1/3)}+d*x)/a^{(2/3)}/b^{(7/3)} + 2/27*(-1)^{(2/3)}*d*\cosh(c-(-1)^{(2/3)}*a^{(1/3)}*d/b^{(1/3)})*\text{Shi}((-1)^{(2/3)}*a^{(1/3)}*d/b^{(1/3)}+d*x)/a^{(2/3)}/b^{(7/3)} + 2/27*d*\text{Chi}(a^{(1/3)}*d/b^{(1/3)}+d*x)*\sinh(c-a^{(1/3)}*d/b^{(1/3)})/a^{(2/3)}/b^{(7/3)} - 1/54*d^2*\text{Shi}(a^{(1/3)}*d/b^{(1/3)}+d*x)*\sinh(c-a^{(1/3)}*d/b^{(1/3)})/a^{(1/3)}/b^{(8/3)} - 2/27*(-1)^{(1/3)}*d*\text{Chi}((-1)^{(1/3)}*a^{(1/3)}*d/b^{(1/3)}-d*x)*\sinh(c+(-1)^{(1/3)}*a^{(1/3)}*d/b^{(1/3)})/a^{(2/3)}/b^{(7/3)} - 1/54*(-1)^{(2/3)}*d^2*\text{Shi}(-(-1)^{(1/3)}*a^{(1/3)}*d/b^{(1/3)}+d*x)*\sinh(c+(-1)^{(1/3)}*a^{(1/3)}*d/b^{(1/3)})/a^{(1/3)}/b^{(8/3)} + 2/27*(-1)^{(2/3)}*d*\text{Chi}(-(-1)^{(2/3)}*a^{(1/3)}*d/b^{(1/3)}-d*x)*\sinh(c-(-1)^{(2/3)}*a^{(1/3)}*d/b^{(1/3)})/a^{(2/3)}/b^{(7/3)} + 1/54*(-1)^{(1/3)}*d^2*\text{Shi}((-1)^{(2/3)}*a^{(1/3)}*d/b^{(1/3)}+d*x)*\sinh(c-(-1)^{(2/3)}*a^{(1/3)}*d/b^{(1/3)})/a^{(1/3)}/b^{(8/3)} - 1/18*d*x*\sinh(d*x+c)/b^2/(b*x^3+a)$

Rubi [A] time = 1.63, antiderivative size = 784, normalized size of antiderivative = 1.00, number of steps used = 36, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {5291, 5289, 5280, 3303, 3298, 3301, 5290, 5293}

$$\frac{2d \sinh\left(c - \frac{\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(xd + \frac{\sqrt[3]{ad}}{\sqrt[3]{b}}\right)}{27a^{2/3}b^{7/3}} - \frac{2\sqrt[3]{-1} d \sinh\left(\frac{\sqrt[3]{-1} \sqrt[3]{ad}}{\sqrt[3]{b}} + c\right) \text{Chi}\left(\frac{\sqrt[3]{-1} \sqrt[3]{ad}}{\sqrt[3]{b}} - dx\right)}{27a^{2/3}b^{7/3}} + \frac{2(-1)^{2/3} d \sinh\left(c - \frac{(-1)^{2/3}}{\sqrt[3]{b}}\right)}{27a^{2/3}b^{7/3}}$$

Antiderivative was successfully verified.

[In] Int[(x^5*Cosh[c + d*x])/(a + b*x^3)^3,x]

[Out] $-(x^3*\text{Cosh}[c + d*x])/(6*b*(a + b*x^3)^2) - \text{Cosh}[c + d*x]/(6*b^2*(a + b*x^3)) - ((-1)^{(2/3)}*d^2*\text{Cosh}[c + ((-1)^{(1/3)}*a^{(1/3)}*d)/b^{(1/3)}]*\text{CoshIntegral}[((-1)^{(1/3)}*a^{(1/3)}*d)/b^{(1/3)} - d*x])/(54*a^{(1/3)}*b^{(8/3)}) + ((-1)^{(1/3)}*d^2*$

```

2*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d*x]/(54*a^(1/3)*b^(8/3)) - (d^2*Cosh[c - (a^(1/3)*d)/b^(1/3)]*CoshIntegral[(a^(1/3)*d)/b^(1/3) + d*x]/(54*a^(1/3)*b^(8/3)) + (2*d*CoshIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sinh[c - (a^(1/3)*d)/b^(1/3)])/(27*a^(2/3)*b^(7/3)) - (2*(-1)^(1/3)*d*CoshIntegral[(-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)])/(27*a^(2/3)*b^(7/3)) + (2*(-1)^(2/3)*d*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d*x]*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(27*a^(2/3)*b^(7/3)) - (d*x*Sinh[c + d*x])/(18*b^2*(a + b*x^3)) + (2*(-1)^(1/3)*d*Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[(-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]/(27*a^(2/3)*b^(7/3)) + ((-1)^(2/3)*d^2*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[(-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]/(54*a^(1/3)*b^(8/3)) + (2*d*Cosh[c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x]/(27*a^(2/3)*b^(7/3)) - (d^2*Sinh[c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x]/(54*a^(1/3)*b^(8/3)) + (2*(-1)^(2/3)*d*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[(-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]/(27*a^(2/3)*b^(7/3)) + ((-1)^(1/3)*d^2*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[(-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]/(54*a^(1/3)*b^(8/3)))

```

Rule 3298

```

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]

```

Rule 3301

```

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]

```

Rule 3303

```

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]

```

Rule 5280

```

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Symbol]
:> Int[ExpandIntegrand[Sinh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])

```


Rule 5289

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(e^m*(a + b*x^n)^(p + 1)*Cosh[c + d*x])/(b*n*(p + 1)), x] - Dist[(d*e^m)/(b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*Sinh[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IntegerQ[p] && EqQ[m - n + 1, 0] && LtQ[p, -1] && (IntegerQ[n] || GtQ[e, 0])
```

Rule 5290

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[(x^(m - n + 1)*(a + b*x^n)^(p + 1)*Sinh[c + d*x])/(b*n*(p + 1)), x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1)*Sinh[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^n)^(p + 1)*Cosh[c + d*x], x], x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1] && IGtQ[n, 0] && RationalQ[m] && (GtQ[m - n + 1, 0] || GtQ[n, 2])
```

Rule 5291

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x^(m - n + 1)*(a + b*x^n)^(p + 1)*Cosh[c + d*x])/(b*n*(p + 1)), x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1)*Cosh[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^n)^(p + 1)*Sinh[c + d*x], x], x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1] && IGtQ[n, 0] && RationalQ[m] && (GtQ[m - n + 1, 0] || GtQ[n, 2])
```

Rule 5293

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5 \cosh(c+dx)}{(a+bx^3)^3} dx &= -\frac{x^3 \cosh(c+dx)}{6b(a+bx^3)^2} + \frac{\int \frac{x^2 \cosh(c+dx)}{(a+bx^3)^2} dx}{2b} + \frac{d \int \frac{x^3 \sinh(c+dx)}{(a+bx^3)^2} dx}{6b} \\
&= -\frac{x^3 \cosh(c+dx)}{6b(a+bx^3)^2} - \frac{\cosh(c+dx)}{6b^2(a+bx^3)} - \frac{dx \sinh(c+dx)}{18b^2(a+bx^3)} + \frac{d \int \frac{\sinh(c+dx)}{a+bx^3} dx}{18b^2} + \frac{d \int \frac{\sinh(c+dx)}{a+bx^3} dx}{6b^2} \\
&= -\frac{x^3 \cosh(c+dx)}{6b(a+bx^3)^2} - \frac{\cosh(c+dx)}{6b^2(a+bx^3)} - \frac{dx \sinh(c+dx)}{18b^2(a+bx^3)} + \frac{d \int \left(\frac{\sinh(c+dx)}{3a^{2/3}(-\sqrt[3]{a}-\sqrt[3]{b}x)} - \frac{\sinh(c+dx)}{3a^{2/3}(-\sqrt[3]{a}+\sqrt[3]{b}x)} \right) dx}{6b^2} \\
&= -\frac{x^3 \cosh(c+dx)}{6b(a+bx^3)^2} - \frac{\cosh(c+dx)}{6b^2(a+bx^3)} - \frac{dx \sinh(c+dx)}{18b^2(a+bx^3)} - \frac{d \int \frac{\sinh(c+dx)}{-\sqrt[3]{a}-\sqrt[3]{b}x} dx}{54a^{2/3}b^2} - \frac{d \int \frac{\sinh(c+dx)}{-\sqrt[3]{a}+\sqrt[3]{b}x} dx}{54a^{2/3}b^2} \\
&= -\frac{x^3 \cosh(c+dx)}{6b(a+bx^3)^2} - \frac{\cosh(c+dx)}{6b^2(a+bx^3)} - \frac{dx \sinh(c+dx)}{18b^2(a+bx^3)} - \frac{\left(d \cosh\left(c - \frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right) \int \frac{\sinh\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}} + dx\right)}{-\sqrt[3]{a}-\sqrt[3]{b}x} dx \right)}{54a^{2/3}b^2} \\
&= -\frac{x^3 \cosh(c+dx)}{6b(a+bx^3)^2} - \frac{\cosh(c+dx)}{6b^2(a+bx^3)} - \frac{(-1)^{2/3}d^2 \cosh\left(c + \frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right) \text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}} - dx\right)}{54\sqrt[3]{a}b^{8/3}} + \dots
\end{aligned}$$

Mathematica [C] time = 0.68, size = 397, normalized size = 0.51

$$d\text{RootSum}\left[\#1^3b + a\&, \frac{4 \sinh(\#1d+c)\text{Chi}(d(x-\#1))-\#1d \sinh(\#1d+c)\text{Chi}(d(x-\#1))-4 \cosh(\#1d+c)\text{Chi}(d(x-\#1))+\#1d \cosh(\#1d+c)\text{Chi}(d(x-\#1))}{\dots}\right]$$

Antiderivative was successfully verified.

[In] Integrate[(x^5*Cosh[c + d*x])/(a + b*x^3)^3,x]

[Out] (d*RootSum[a + b*#1^3 & , (-4*Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] + 4*CoshIntegral[d*(x - #1)]*Sinh[c + d*#1] + 4*Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] - 4*Sinh[c + d*#1]*SinhIntegral[d*(x - #1)] + d*Cosh[c + d*#1]*CoshIntegral[d*(x - #1)]*#1 - d*CoshIntegral[d*(x - #1)]*Sinh[c + d*#1]*#1 - d*Cosh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1 + d*Sinh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1)/#1^2 &] + d*RootSum[a + b*#1^3 & , (4*Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] + 4*CoshIntegral[d*(x - #1)]*Sinh[c + d*#1] + 4*Cosh[c


```

2))*Ei(d*x + (a*d^3/b)^(1/3))*cosh(-c + (a*d^3/b)^(1/3)) + ((a*d^3/b)^(2/3)
*((b^2*x^6 + 2*a*b*x^3 + a^2 - sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*cosh(d
*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2 - sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^
2))*sinh(d*x + c)^2) - 4*(a*d^3/b)^(1/3)*((b^2*x^6 + 2*a*b*x^3 + a^2 + sqrt
(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 +
a^2 + sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*sinh(d*x + c)^2))*Ei(d*x - 1/2*
(a*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1) + c
) + ((-a*d^3/b)^(2/3)*((b^2*x^6 + 2*a*b*x^3 + a^2 - sqrt(-3)*(b^2*x^6 + 2*a
*b*x^3 + a^2))*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2 - sqrt(-3)*(b^2
*x^6 + 2*a*b*x^3 + a^2))*sinh(d*x + c)^2) - 4*(-a*d^3/b)^(1/3)*((b^2*x^6 +
2*a*b*x^3 + a^2 + sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*cosh(d*x + c)^2 - (
b^2*x^6 + 2*a*b*x^3 + a^2 + sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*sinh(d*x
+ c)^2))*Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(-a*d^3/b)
^(1/3)*(sqrt(-3) + 1) - c) - ((a*d^3/b)^(2/3)*((b^2*x^6 + 2*a*b*x^3 + a^2 +
sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^
3 + a^2 + sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*sinh(d*x + c)^2) - 4*(a*d^
3/b)^(1/3)*((b^2*x^6 + 2*a*b*x^3 + a^2 - sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^
2))*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2 - sqrt(-3)*(b^2*x^6 + 2*a*
b*x^3 + a^2))*sinh(d*x + c)^2))*Ei(d*x + 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1)
)*sinh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1) - c) - ((-a*d^3/b)^(2/3)*((b^2*x^
6 + 2*a*b*x^3 + a^2 + sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*cosh(d*x + c)^2
- (b^2*x^6 + 2*a*b*x^3 + a^2 + sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*sinh(
d*x + c)^2) - 4*(-a*d^3/b)^(1/3)*((b^2*x^6 + 2*a*b*x^3 + a^2 - sqrt(-3)*(b^
2*x^6 + 2*a*b*x^3 + a^2))*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2 - sq
rt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*sinh(d*x + c)^2))*Ei(-d*x + 1/2*(-a*d^3
/b)^(1/3)*(sqrt(-3) - 1))*sinh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c) + 2
*((-a*d^3/b)^(2/3)*((b^2*x^6 + 2*a*b*x^3 + a^2))*cosh(d*x + c)^2 - (b^2*x^6
+ 2*a*b*x^3 + a^2))*sinh(d*x + c)^2) - 4*(-a*d^3/b)^(1/3)*((b^2*x^6 + 2*a*b*
x^3 + a^2))*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2))*sinh(d*x + c)^2))*
Ei(-d*x + (-a*d^3/b)^(1/3))*sinh(c + (-a*d^3/b)^(1/3)) + 2*((a*d^3/b)^(2/3)
*((b^2*x^6 + 2*a*b*x^3 + a^2))*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2)
*sinh(d*x + c)^2) - 4*(a*d^3/b)^(1/3)*((b^2*x^6 + 2*a*b*x^3 + a^2))*cosh(d*x
+ c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2))*sinh(d*x + c)^2))*Ei(d*x + (a*d^3/b)^
(1/3))*sinh(-c + (a*d^3/b)^(1/3)) - 36*(2*a*b*x^3 + a^2)*cosh(d*x + c) - 12
*(a*b*d*x^4 + a^2*d*x)*sinh(d*x + c))/((a*b^4*x^6 + 2*a^2*b^3*x^3 + a^3*b^2
)*cosh(d*x + c)^2 - (a*b^4*x^6 + 2*a^2*b^3*x^3 + a^3*b^2)*sinh(d*x + c)^2)

```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5 \cosh(dx + c)}{(bx^3 + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*cosh(d*x+c)/(b*x^3+a)^3,x, algorithm="giac")

[Out] integrate(x^5*cosh(d*x + c)/(b*x^3 + a)^3, x)

maple [C] time = 0.68, size = 2448, normalized size = 3.12

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5*cosh(d*x+c)/(b*x^3+a)^3,x)

[Out]
$$\frac{1}{108} \frac{d^3 c^5}{a^2 b} \sum \left(\frac{{}_R1^2 - 2 {}_R1 c + c^2 + 6 {}_R1 - 6 c + 10}{({}_R1^2 - 2 {}_R1 c + c^2)} \right) \exp(-{}_R1) \operatorname{Ei}(1, d x - {}_R1 + c), {}_R1 = \operatorname{RootOf}(_Z^3 b - 3 _Z^2 b c + 3 _Z b c^2 + a d^3 - b c^3) - \frac{5}{108} \frac{d^3 c^4}{a^2 b^2} \sum \left(\frac{{}_R1^2 b c - 2 {}_R1 b c^2 - a d^3 + b c^3 + 4 {}_R1^2 b - 2 {}_R1 b c - 2 b c^2 + 4 {}_R1 b + 6 b^2 c}{({}_R1^2 - 2 {}_R1 c + c^2)} \right) \exp(-{}_R1) \operatorname{Ei}(1, d x - {}_R1 + c), {}_R1 = \operatorname{RootOf}(_Z^3 b - 3 _Z^2 b c + 3 _Z b c^2 + a d^3 - b c^3) + \frac{5}{54} \frac{d^3 c^3}{a^2 b^2} \sum \left(\frac{{}_R1^2 b c^2 - {}_R1 a d^3 - 2 {}_R1 b c^3 - a c d^3 + b c^4 + 8 {}_R1^2 b c - 10 {}_R1 b c^2 - 2 a d^3 + 2 b c^3 + 8 {}_R1 b c + 2 b c^2}{({}_R1^2 - 2 {}_R1 c + c^2)} \right) \exp(-{}_R1) \operatorname{Ei}(1, d x - {}_R1 + c), {}_R1 = \operatorname{RootOf}(_Z^3 b - 3 _Z^2 b c + 3 _Z b c^2 + a d^3 - b c^3) - \frac{1}{6} d^6 \exp(-d x - c) / b / (b^2 d^6 x^6 + 2 a b d^6 x^3 + a^2 d^6) x^3 + 1 / 108 \frac{d^3}{a^2 b^3} \sum \left(\frac{10 {}_R1^2 a b c^2 d^3 - {}_R1^2 b^2 c^5 - {}_R1 a^2 d^6 - 10 {}_R1 a b c^3 d^3 + 2 {}_R1 b^2 c^6 - 4 a^2 c d^6 + 5 a b c^4 d^3 - b^2 c^7 - 10 {}_R1^2 a b c d^3 - 20 {}_R1^2 b^2 c^4 + 20 {}_R1 a b c^2 d^3 + 34 {}_R1 b^2 c^5 + 4 a^2 d^6 + 10 a b c^3 d^3 - 14 b^2 c^6 - 10 {}_R1 a b c d^3 - 20 {}_R1 b^2 c^4 - 10 a b c^2 d^3 + 10 b^2 c^5}{({}_R1^2 - 2 {}_R1 c + c^2)} \right) \exp(-{}_R1) \operatorname{Ei}(1, d x - {}_R1 + c), {}_R1 = \operatorname{RootOf}(_Z^3 b - 3 _Z^2 b c + 3 _Z b c^2 + a d^3 - b c^3) + \frac{5}{54} \frac{d^3 c^2}{a^2 b^2} \sum \left(\frac{{}_R1^2 a d^3 - {}_R1^2 b c^3 + {}_R1 a c d^3 + 2 {}_R1 b c^4 + a c^2 d^3 - b c^5 - 12 {}_R1^2 b c^2 + 18 {}_R1 b c^3 + 6 a c d^3 - 6 b c^4 - 12 {}_R1 b c^2 - 2 a d^3 + 2 b c^3}{({}_R1^2 - 2 {}_R1 c + c^2)} \right) \exp(-{}_R1) \operatorname{Ei}(1, d x - {}_R1 + c), {}_R1 = \operatorname{RootOf}(_Z^3 b - 3 _Z^2 b c + 3 _Z b c^2 + a d^3 - b c^3) - \frac{1}{12} d^6 \exp(-d x - c) a / b^2 / (b^2 d^6 x^6 + 2 a b d^6 x^3 + a^2 d^6) - \frac{5}{108} \frac{d^3 c}{a^2 b^3} \sum \left(\frac{4 {}_R1^2 a b c d^3 - {}_R1^2 b^2 c^4 - 2 {}_R1 a b c^2 d^3 + 2 {}_R1 b^2 c^5 - a^2 d^6 + 2 a b c^3 d^3 - b^2 c^6 - 2 {}_R1^2 a b d^3 - 16 {}_R1^2 b^2 c^3 + 4 {}_R1 a b c d^3 + 26 {}_R1 b^2 c^4 + 10 a b c^2 d^3 - 10 b^2 c^5 - 2 {}_R1 a b d^3 - 16 {}_R1 b^2 c^3 - 6 a b c d^3 + 6 b^2 c^4}{({}_R1^2 - 2 {}_R1 c + c^2)} \right) \exp(-{}_R1) \operatorname{Ei}(1, d x - {}_R1 + c), {}_R1 = \operatorname{RootOf}(_Z^3 b - 3 _Z^2 b c + 3 _Z b c^2 + a d^3 - b c^3) + \frac{1}{36} d^7 \exp(-d x - c) / b / (b^2 d^6 x^6 + 2 a b d^6 x^3 + a^2 d^6) x^4 + \frac{1}{36} d^7 \exp(-d x - c) a / b^2 / (b^2 d^6 x^6 + 2 a b d^6 x^3 + a^2 d^6) x - \frac{1}{12} d^6 \exp(d x + c) a / b^2 / (b^2 d^6 x^6 + 2 a b d^6 x^3 + a^2 d^6) - \frac{5}{108} \frac{d^3 c}{a^2 b^3} \sum \left(\frac{4 {}_R1^2 a b c d^3 - {}_R1^2 b^2 c^4 - 2 {}_R1 a b c^2 d^3 + 2 {}_R1 b^2 c^5 - a^2 d^6 + 2 a b c^3 d^3 - b^2 c^6 + 2 {}_R1^2 a b d^3 + 16 {}_R1^2 b^2 c^3 - 4 {}_R1 a b c d^3 - 26 {}_R1 b^2 c^4 - 10 a b c^2 d^3 + 10 b^2 c^5 - 2 {}_R1 a b d^3 - 16 {}_R1 b^2 c^3 - 6 a b c d^3 + 6 b^2 c^4}{({}_R1^2 - 2 {}_R1 c + c^2)} \right) \exp({}_R1) \operatorname{Ei}(1, -d x + {}_R1 - c), {}_R1 = \operatorname{RootOf}(_Z^3 b - 3 _Z^2 b c + 3 _Z b c^2 + a d^3 - b c^3) + \frac{1}{108} \frac{d^3}{a^2 b^3} \sum \left(\frac{10 {}_R1^2 a b c^2 d^3 - {}_R1^2 b^2 c^5 - {}_R1 a^2 d^6 - 10 {}_R1 a b c^3 d^3 + 2 {}_R1 b^2 c^6 - 4 a^2 c d^6 + 5 a b c^4 d^3 - b^2 c^7 + 10 {}_R1^2 a b c d^3 + 20 {}_R1^2 b^2 c^4 - 20 {}_R1 a b c^2 d^3 - 34 {}_R1 b^2 c^5 - 4 a^2 d^6 - 10 a b c^3 d^3 + 14 b^2 c^6 - 10 {}_R1 a b c d^3 - 20 {}_R1 b^2 c^4 - 10 a b c^2 d^3 + 10 b^2 c^5}{({}_R1^2 - 2 {}_R1 c + c^2)} \right) \exp({}_R1) \operatorname{Ei}(1, -d x + {}_R1 - c), {}_R1 = \operatorname{RootOf}(_Z^3 b - 3 _Z^2 b c + 3 _Z b c^2 + a d^3 - b c^3)$$

```
) + 1/108/d^3*c^5/a^2/b*sum(( _R1^2-2*_R1*c+c^2-6*_R1+6*c+10)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c), _R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-5/108/d^3*c^4/a^2/b^2*sum(( _R1^2*b*c-2*_R1*b*c^2-a*d^3+b*c^3-4*_R1^2*b+2*_R1*b*c+2*b*c^2+4*_R1*b+6*b*c)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c), _R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+5/54/d^3*c^3/a^2/b^2*sum(( _R1^2*b*c^2-_R1*a*d^3-2*_R1*b*c^3-a*c*d^3+b*c^4-8*_R1^2*b*c+10*_R1*b*c^2+2*a*d^3-2*b*c^3+8*_R1*b*c+2*b*c^2)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c), _R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+5/54/d^3*c^2/a^2/b^2*sum(( _R1^2*a*d^3-_R1^2*b*c^3+_R1*a*c*d^3+2*_R1*b*c^4+a*c^2*d^3-b*c^5+12*_R1^2*b*c^2-18*_R1*b*c^3-6*a*c*d^3+6*b*c^4-12*_R1*b*c^2-2*a*d^3+2*b*c^3)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c), _R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/36*d^7*exp(d*x+c)/b/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)*x^4-1/36*d^7*exp(d*x+c)*a/b^2/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)*x-1/6*d^6*exp(d*x+c)/b/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)*x^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{(bd^4x^5e^{(2c)} + 4bd^3x^4e^{(2c)} + 20bd^2x^3e^{(2c)} + 120bdx^2e^{(2c)} - 3(3ad^3e^{(2c)} - 280be^{(2c)})x)e^{(dx)} - (bd^4x^5 - 4bd^3x^4 + 2(b^4d^5x^9e^c + 3ab^3d^5x^6e^c + 3a^2b^2d^5x^3e^c + a^3bd^5e^c))}{2(b^4d^5x^9e^c + 3ab^3d^5x^6e^c + 3a^2b^2d^5x^3e^c + a^3bd^5e^c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*cosh(d*x+c)/(b*x^3+a)^3,x, algorithm="maxima")

```
[Out] 1/2*((b*d^4*x^5*e^(2*c) + 4*b*d^3*x^4*e^(2*c) + 20*b*d^2*x^3*e^(2*c) + 120*b*d*x^2*e^(2*c) - 3*(3*a*d^3*e^(2*c) - 280*b*e^(2*c))*x)*e^(d*x) - (b*d^4*x^5 - 4*b*d^3*x^4 + 20*b*d^2*x^3 - 120*b*d*x^2 + 3*(3*a*d^3 + 280*b)*x)*e^(-d*x))/(b^4*d^5*x^9*e^c + 3*a*b^3*d^5*x^6*e^c + 3*a^2*b^2*d^5*x^3*e^c + a^3*b*d^5*e^c) - 1/2*integrate(3*(60*a*b*d^2*x^2*e^c - 3*a^2*d^3*e^c + 4*(9*a*b*d^3*e^c - 560*b^2*e^c)*x^3 + 280*a*b*e^c - 3*(a^2*d^4*e^c - 120*a*b*d*e^c)*x)*e^(d*x)/(b^5*d^5*x^12 + 4*a*b^4*d^5*x^9 + 6*a^2*b^3*d^5*x^6 + 4*a^3*b^2*d^5*x^3 + a^4*b*d^5), x) - 1/2*integrate(-3*(60*a*b*d^2*x^2 + 3*a^2*d^3 - 4*(9*a*b*d^3 + 560*b^2)*x^3 + 280*a*b - 3*(a^2*d^4 + 120*a*b*d)*x)*e^(-d*x)/(b^5*d^5*x^12*e^c + 4*a*b^4*d^5*x^9*e^c + 6*a^2*b^3*d^5*x^6*e^c + 4*a^3*b^2*d^5*x^3*e^c + a^4*b*d^5*e^c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5 \cosh(c + dx)}{(bx^3 + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^5*cosh(c + d*x))/(a + b*x^3)^3,x)

```
[Out] int((x^5*cosh(c + d*x))/(a + b*x^3)^3, x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*cosh(d*x+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

$$3.108 \quad \int \frac{x^4 \cosh(c+dx)}{(a+bx^3)^3} dx$$

Optimal. Leaf size=1105

$$\frac{\sqrt[3]{-1} \cosh\left(c + \frac{\sqrt[3]{-1} \sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(\frac{\sqrt[3]{-1} \sqrt[3]{ad}}{\sqrt[3]{b}} - dx\right) d^2}{54a^{2/3}b^{7/3}} + \frac{(-1)^{2/3} \cosh\left(c - \frac{(-1)^{2/3} \sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(-xd - \frac{(-1)^{2/3} \sqrt[3]{ad}}{\sqrt[3]{b}}\right) d^2}{54a^{2/3}b^{7/3}} + \frac{\cosh\left(c - \frac{(-1)^{2/3} \sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(-xd - \frac{(-1)^{2/3} \sqrt[3]{ad}}{\sqrt[3]{b}}\right) d^2}{54a^{2/3}b^{7/3}}$$

[Out] $-1/27*\text{Chi}(a^{1/3}*d/b^{1/3}+d*x)*\cosh(c-a^{1/3}*d/b^{1/3})/a^{4/3}/b^{5/3}+1/54*d^2*\text{Chi}(a^{1/3}*d/b^{1/3}+d*x)*\cosh(c-a^{1/3}*d/b^{1/3})/a^{2/3}/b^{7/3}-1/27*(-1)^{(2/3)}*\text{Chi}((-1)^{(1/3)}*a^{1/3}*d/b^{1/3}-d*x)*\cosh(c+(-1)^{(1/3)}*a^{1/3}*d/b^{1/3})/a^{4/3}/b^{5/3}-1/54*(-1)^{(1/3)}*d^2*\text{Chi}((-1)^{(1/3)}*a^{1/3}*d/b^{1/3}-d*x)*\cosh(c+(-1)^{(1/3)}*a^{1/3}*d/b^{1/3})/a^{2/3}/b^{7/3}+1/27*(-1)^{(1/3)}*\text{Chi}(-(-1)^{(2/3)}*a^{1/3}*d/b^{1/3}-d*x)*\cosh(c-(-1)^{(2/3)}*a^{1/3}*d/b^{1/3})/a^{4/3}/b^{5/3}+1/54*(-1)^{(2/3)}*d^2*\text{Chi}(-(-1)^{(2/3)}*a^{1/3}*d/b^{1/3}-d*x)*\cosh(c-(-1)^{(2/3)}*a^{1/3}*d/b^{1/3})/a^{2/3}/b^{7/3}+1/9*\cosh(d*x+c)/a/b^2/x-1/6*x^2*\cosh(d*x+c)/b/(b*x^3+a)^2-1/9*\cosh(d*x+c)/b^2/x/(b*x^3+a)-1/27*d*\cosh(c+(-1)^{(1/3)}*a^{1/3}*d/b^{1/3})*\text{Shi}(-(-1)^{(1/3)}*a^{1/3}*d/b^{1/3}+d*x)/a/b^2-1/27*d*\cosh(c-a^{1/3}*d/b^{1/3})*\text{Shi}(a^{1/3}*d/b^{1/3}+d*x)/a/b^2-1/27*d*\cosh(c-(-1)^{(2/3)}*a^{1/3}*d/b^{1/3})*\text{Shi}((-1)^{(2/3)}*a^{1/3}*d/b^{1/3}+d*x)/a/b^2-1/27*d*\text{Chi}(a^{1/3}*d/b^{1/3}+d*x)*\sinh(c-a^{1/3}*d/b^{1/3})/a/b^2-1/27*\text{Shi}(a^{1/3}*d/b^{1/3}+d*x)*\sinh(c-a^{1/3}*d/b^{1/3})/a^{4/3}/b^{5/3}+1/54*d^2*\text{Shi}(a^{1/3}*d/b^{1/3}+d*x)*\sinh(c-a^{1/3}*d/b^{1/3})/a^{2/3}/b^{7/3}-1/27*d*\text{Chi}((-1)^{(1/3)}*a^{1/3}*d/b^{1/3}-d*x)*\sinh(c+(-1)^{(1/3)}*a^{1/3}*d/b^{1/3})/a/b^2-1/27*(-1)^{(2/3)}*\text{Shi}(-(-1)^{(1/3)}*a^{1/3}*d/b^{1/3}+d*x)*\sinh(c+(-1)^{(1/3)}*a^{1/3}*d/b^{1/3})/a^{4/3}/b^{5/3}-1/54*(-1)^{(1/3)}*d^2*\text{Shi}(-(-1)^{(1/3)}*a^{1/3}*d/b^{1/3}+d*x)*\sinh(c+(-1)^{(1/3)}*a^{1/3}*d/b^{1/3})/a^{2/3}/b^{7/3}-1/27*d*\text{Chi}(-(-1)^{(2/3)}*a^{1/3}*d/b^{1/3}-d*x)*\sinh(c-(-1)^{(2/3)}*a^{1/3}*d/b^{1/3})/a/b^2+1/27*(-1)^{(1/3)}*\text{Shi}((-1)^{(2/3)}*a^{1/3}*d/b^{1/3}+d*x)*\sinh(c-(-1)^{(2/3)}*a^{1/3}*d/b^{1/3})/a^{4/3}/b^{5/3}+1/54*(-1)^{(2/3)}*d^2*\text{Shi}((-1)^{(2/3)}*a^{1/3}*d/b^{1/3}+d*x)*\sinh(c-(-1)^{(2/3)}*a^{1/3}*d/b^{1/3})/a^{2/3}/b^{7/3}-1/18*d*\sinh(d*x+c)/b^2/(b*x^3+a)$

Rubi [A] time = 1.85, antiderivative size = 1105, normalized size of antiderivative = 1.00, number of steps used = 47, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {5291, 5293, 3297, 3303, 3298, 3301, 5292, 5288, 5281}

result too large to display

Antiderivative was successfully verified.

[In] Int[(x^4*Cosh[c + d*x])/(a + b*x^3)^3,x]


```
[Out] Cosh[c + d*x]/(9*a*b^2*x) - (x^2*Cosh[c + d*x])/(6*b*(a + b*x^3)^2) - Cosh[
c + d*x]/(9*b^2*x*(a + b*x^3)) - ((-1)^(2/3)*Cosh[c + ((-1)^(1/3)*a^(1/3)*d
)/b^(1/3)]*CoshIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]/(27*a^(4/3)*
b^(5/3)) - ((-1)^(1/3)*d^2*Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CoshInt
egral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]/(54*a^(2/3)*b^(7/3)) + ((-1)^(
1/3)*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[-(((-1)^(2/3)*a^
(1/3)*d)/b^(1/3)) - d*x]/(27*a^(4/3)*b^(5/3)) + ((-1)^(2/3)*d^2*Cosh[c - (
(-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3
)) - d*x]/(54*a^(2/3)*b^(7/3)) - (Cosh[c - (a^(1/3)*d)/b^(1/3)]*CoshIntegr
al[(a^(1/3)*d)/b^(1/3) + d*x]/(27*a^(4/3)*b^(5/3)) + (d^2*Cosh[c - (a^(1/3
)*d)/b^(1/3)]*CoshIntegral[(a^(1/3)*d)/b^(1/3) + d*x]/(54*a^(2/3)*b^(7/3))
- (d*CoshIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sinh[c - (a^(1/3)*d)/b^(1/3)]
)/(27*a*b^2) - (d*CoshIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sinh[c
+ ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]/(27*a*b^2) - (d*CoshIntegral[-(((-1)^(2
/3)*a^(1/3)*d)/b^(1/3)) - d*x]*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]/(2
7*a*b^2) - (d*Sinh[c + d*x])/(18*b^2*(a + b*x^3)) + (d*Cosh[c + ((-1)^(1/3)
*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]/(2
7*a*b^2) + ((-1)^(2/3)*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegra
l[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]/(27*a^(4/3)*b^(5/3)) + ((-1)^(1/3)
*d^2*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(1/3)*a^(1
/3)*d)/b^(1/3) - d*x]/(54*a^(2/3)*b^(7/3)) - (d*Cosh[c - (a^(1/3)*d)/b^(1/
3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x]/(27*a*b^2) - (Sinh[c - (a^(1/3
)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x]/(27*a^(4/3)*b^(5/3))
+ (d^2*Sinh[c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*
x]/(54*a^(2/3)*b^(7/3)) - (d*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*Sinh
Integral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]/(27*a*b^2) + ((-1)^(1/3)*Si
nh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*d)/
b^(1/3) + d*x]/(27*a^(4/3)*b^(5/3)) + ((-1)^(2/3)*d^2*Sinh[c - ((-1)^(2/3)
*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]/(5
4*a^(2/3)*b^(7/3))
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
+ d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)])/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x]
&& EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x]
+ Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /;
FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]
```

Rule 5281

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Int[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /;
FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 5288

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)],
x_Symbol]
:> Simp[(e^m*(a + b*x^n)^(p + 1)*Sinh[c + d*x])/(b*n*(p + 1)), x]
- Dist[(d*e^m)/(b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*Cosh[c + d*x], x], x] /;
FreeQ[{a, b, c, d, e, m, n}, x] && IntegerQ[p] && EqQ[m - n + 1, 0]
&& LtQ[p, -1] && (IntegerQ[n] || GtQ[e, 0])
```

Rule 5291

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[(x^(m - n + 1)*(a + b*x^n)^(p + 1)*Cosh[c + d*x])/(b*n*(p + 1)), x]
+ (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1)*Cosh[c + d*x], x], x]
- Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^n)^(p + 1)*Sinh[c + d*x], x], x]) /;
FreeQ[{a, b, c, d}, x] && ILtQ[p, -1] && IGtQ[n, 0] && RationalQ[m] && (GtQ[m - n + 1, 0] || GtQ[n, 2])
```

Rule 5292

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Symbol]
:> Int[ExpandIntegrand[Sinh[c + d*x], x^m*(a + b*x^n)^p, x], x] /;
FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 5293

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
```

eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])

Rubi steps

$$\begin{aligned}
 \int \frac{x^4 \cosh(c+dx)}{(a+bx^3)^3} dx &= -\frac{x^2 \cosh(c+dx)}{6b(a+bx^3)^2} + \frac{\int \frac{x \cosh(c+dx)}{(a+bx^3)^2} dx}{3b} + \frac{d \int \frac{x^2 \sinh(c+dx)}{(a+bx^3)^2} dx}{6b} \\
 &= -\frac{x^2 \cosh(c+dx)}{6b(a+bx^3)^2} - \frac{\cosh(c+dx)}{9b^2 x(a+bx^3)} - \frac{d \sinh(c+dx)}{18b^2(a+bx^3)} - \frac{\int \frac{\cosh(c+dx)}{x^2(a+bx^3)} dx}{9b^2} + \frac{d \int \frac{\sinh(c+dx)}{x(a+bx^3)} dx}{9b^2} \\
 &= -\frac{x^2 \cosh(c+dx)}{6b(a+bx^3)^2} - \frac{\cosh(c+dx)}{9b^2 x(a+bx^3)} - \frac{d \sinh(c+dx)}{18b^2(a+bx^3)} - \frac{\int \left(\frac{\cosh(c+dx)}{ax^2} - \frac{bx \cosh(c+dx)}{a(a+bx^3)} \right) dx}{9b^2} \\
 &= -\frac{x^2 \cosh(c+dx)}{6b(a+bx^3)^2} - \frac{\cosh(c+dx)}{9b^2 x(a+bx^3)} - \frac{d \sinh(c+dx)}{18b^2(a+bx^3)} - \frac{\int \frac{\cosh(c+dx)}{x^2} dx}{9ab^2} + \frac{\int \frac{x \cosh(c+dx)}{a+bx^3} dx}{9ab} \\
 &= \frac{\cosh(c+dx)}{9ab^2 x} - \frac{x^2 \cosh(c+dx)}{6b(a+bx^3)^2} - \frac{\cosh(c+dx)}{9b^2 x(a+bx^3)} - \frac{d \sinh(c+dx)}{18b^2(a+bx^3)} + \frac{\int \left(-\frac{\cosh(c+dx)}{3 \sqrt[3]{a} \sqrt[3]{b} (\sqrt[3]{a} + \sqrt[3]{b} x)} \right) dx}{54a^{2/3} b^{7/3}} \\
 &= \frac{\cosh(c+dx)}{9ab^2 x} - \frac{x^2 \cosh(c+dx)}{6b(a+bx^3)^2} - \frac{\cosh(c+dx)}{9b^2 x(a+bx^3)} - \frac{\sqrt[3]{-1} d^2 \cosh \left(c + \frac{\sqrt[3]{-1} \sqrt[3]{ad}}{\sqrt[3]{b}} \right) \text{Chi} \left(\frac{\sqrt[3]{-1} \sqrt[3]{ad}}{\sqrt[3]{b}} \right)}{54a^{2/3} b^{7/3}} \\
 &= \frac{\cosh(c+dx)}{9ab^2 x} - \frac{x^2 \cosh(c+dx)}{6b(a+bx^3)^2} - \frac{\cosh(c+dx)}{9b^2 x(a+bx^3)} - \frac{\sqrt[3]{-1} d^2 \cosh \left(c + \frac{\sqrt[3]{-1} \sqrt[3]{ad}}{\sqrt[3]{b}} \right) \text{Chi} \left(\frac{\sqrt[3]{-1} \sqrt[3]{ad}}{\sqrt[3]{b}} \right)}{54a^{2/3} b^{7/3}} \\
 &= \frac{\cosh(c+dx)}{9ab^2 x} - \frac{x^2 \cosh(c+dx)}{6b(a+bx^3)^2} - \frac{\cosh(c+dx)}{9b^2 x(a+bx^3)} - \frac{(-1)^{2/3} \cosh \left(c + \frac{\sqrt[3]{-1} \sqrt[3]{ad}}{\sqrt[3]{b}} \right) \text{Chi} \left(\frac{\sqrt[3]{-1} \sqrt[3]{ad}}{\sqrt[3]{b}} \right)}{27a^{4/3} b^{5/3}}
 \end{aligned}$$

$$\begin{aligned}
& t(-3)*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3))*\sinh(d*x + c)^2))*Ei(d*x \\
& - 1/2*(a*d^3/b)^{(1/3)}*(\sqrt{-3} + 1))*\cosh(1/2*(a*d^3/b)^{(1/3)}*(\sqrt{-3} + \\
& 1) + c) - (4*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3))*\cosh(d*x + c)^2 - \\
& 4*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3))*\sinh(d*x + c)^2 + 2*(-a*d^3/b) \\
& ^{(2/3)}*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b - \sqrt{-3}*(b^3*x^6 + 2*a*b^2*x^3 \\
& + a^2*b))*\cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^2*b - \sqrt{-3}*(b^3*x \\
& ^6 + 2*a*b^2*x^3 + a^2*b))*\sinh(d*x + c)^2) + (-a*d^3/b)^{(1/3)}*((a*b^2*d^3 \\
& *x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3 + \sqrt{-3}*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^ \\
& 3 + a^3*d^3))*\cosh(d*x + c)^2 - (a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3 \\
& + \sqrt{-3}*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3))*\sinh(d*x + c)^2))*E \\
& i(-d*x - 1/2*(-a*d^3/b)^{(1/3)}*(\sqrt{-3} + 1))*\cosh(1/2*(-a*d^3/b)^{(1/3)}*(\sqrt{-3} \\
& + 1) - c) + (4*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3))*\cosh(d*x \\
& + c)^2 - 4*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3))*\sinh(d*x + c)^2 - 2* \\
& (a*d^3/b)^{(2/3)}*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b + \sqrt{-3}*(b^3*x^6 + 2*a*b \\
& ^2*x^3 + a^2*b))*\cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^2*b + \sqrt{-3} \\
&)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*\sinh(d*x + c)^2) + (a*d^3/b)^{(1/3)}*((a*b \\
& ^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3 - \sqrt{-3}*(a*b^2*d^3*x^6 + 2*a^2*b*d \\
& ^3*x^3 + a^3*d^3))*\cosh(d*x + c)^2 - (a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^ \\
& 3*d^3 - \sqrt{-3}*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3))*\sinh(d*x + c) \\
& ^2))*Ei(d*x + 1/2*(a*d^3/b)^{(1/3)}*(\sqrt{-3} - 1))*\cosh(1/2*(a*d^3/b)^{(1/3)}* \\
& (\sqrt{-3} - 1) - c) - (4*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3))*\cosh(d \\
& *x + c)^2 - 4*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3))*\sinh(d*x + c)^2 + \\
& 2*(-a*d^3/b)^{(2/3)}*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b + \sqrt{-3}*(b^3*x^6 + 2 \\
& *a*b^2*x^3 + a^2*b))*\cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^2*b + \sqrt{-3} \\
&)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*\sinh(d*x + c)^2) + (-a*d^3/b)^{(1/3)}* \\
& ((a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3 - \sqrt{-3}*(a*b^2*d^3*x^6 + 2*a \\
& ^2*b*d^3*x^3 + a^3*d^3))*\cosh(d*x + c)^2 - (a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 \\
& + a^3*d^3 - \sqrt{-3}*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3))*\sinh(d*x \\
& + c)^2))*Ei(-d*x + 1/2*(-a*d^3/b)^{(1/3)}*(\sqrt{-3} - 1))*\cosh(1/2*(-a*d^3/b) \\
&)^{(1/3)}*(\sqrt{-3} - 1) + c) - 2*(2*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d \\
& ^3))*\cosh(d*x + c)^2 - 2*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3))*\sinh(d* \\
& x + c)^2 - 2*(-a*d^3/b)^{(2/3)}*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b)*\cosh(d*x + c) \\
&)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*\sinh(d*x + c)^2) - (-a*d^3/b)^{(1/3)}*((\\
& (a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3))*\cosh(d*x + c)^2 - (a*b^2*d^3*x^ \\
& 6 + 2*a^2*b*d^3*x^3 + a^3*d^3))*\sinh(d*x + c)^2))*Ei(-d*x + (-a*d^3/b)^{(1/3)} \\
&)*\cosh(c + (-a*d^3/b)^{(1/3)}) + 2*(2*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3* \\
& d^3))*\cosh(d*x + c)^2 - 2*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3))*\sinh(d \\
& *x + c)^2 + 2*(a*d^3/b)^{(2/3)}*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b)*\cosh(d*x + c) \\
&)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*\sinh(d*x + c)^2) - (a*d^3/b)^{(1/3)}*((\\
& a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3))*\cosh(d*x + c)^2 - (a*b^2*d^3*x^6 \\
& + 2*a^2*b*d^3*x^3 + a^3*d^3))*\sinh(d*x + c)^2))*Ei(d*x + (a*d^3/b)^{(1/3)})*\c \\
& osh(-c + (a*d^3/b)^{(1/3)}) + (4*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3))* \\
& \cosh(d*x + c)^2 - 4*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3))*\sinh(d*x + \\
& c)^2 - 2*(a*d^3/b)^{(2/3)}*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b - \sqrt{-3}*(b^3*x^ \\
& 6 + 2*a*b^2*x^3 + a^2*b))*\cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^2*b
\end{aligned}$$

$$\begin{aligned}
& - \sqrt{-3} * (b^3 * x^6 + 2 * a * b^2 * x^3 + a^2 * b) * \sinh(d * x + c)^2 + (a * d^3 / b)^{(1/3)} * \\
& ((a * b^2 * d^3 * x^6 + 2 * a^2 * b * d^3 * x^3 + a^3 * d^3 + \sqrt{-3} * (a * b^2 * d^3 * x^6 + \\
& 2 * a^2 * b * d^3 * x^3 + a^3 * d^3)) * \cosh(d * x + c)^2 - (a * b^2 * d^3 * x^6 + 2 * a^2 * b * d^3 \\
& * x^3 + a^3 * d^3 + \sqrt{-3} * (a * b^2 * d^3 * x^6 + 2 * a^2 * b * d^3 * x^3 + a^3 * d^3)) * \sinh \\
& (d * x + c)^2) * \operatorname{Ei}(d * x - 1/2 * (a * d^3 / b)^{(1/3)} * (\sqrt{-3} + 1)) * \sinh(1/2 * (a * d^3 / \\
& b)^{(1/3)} * (\sqrt{-3} + 1) + c) - (4 * (a * b^2 * d^3 * x^6 + 2 * a^2 * b * d^3 * x^3 + a^3 * d^3) * \\
& \cosh(d * x + c)^2 - 4 * (a * b^2 * d^3 * x^6 + 2 * a^2 * b * d^3 * x^3 + a^3 * d^3) * \sinh(d * x \\
& + c)^2 + 2 * (-a * d^3 / b)^{(2/3)} * ((b^3 * x^6 + 2 * a * b^2 * x^3 + a^2 * b - \sqrt{-3} * (b^3 * x^6 \\
& + 2 * a * b^2 * x^3 + a^2 * b)) * \cosh(d * x + c)^2 - (b^3 * x^6 + 2 * a * b^2 * x^3 + a^2 * b - \\
& \sqrt{-3} * (b^3 * x^6 + 2 * a * b^2 * x^3 + a^2 * b)) * \sinh(d * x + c)^2) + (-a * d^3 / \\
& b)^{(1/3)} * ((a * b^2 * d^3 * x^6 + 2 * a^2 * b * d^3 * x^3 + a^3 * d^3 + \sqrt{-3} * (a * b^2 * d^3 * \\
& x^6 + 2 * a^2 * b * d^3 * x^3 + a^3 * d^3)) * \cosh(d * x + c)^2 - (a * b^2 * d^3 * x^6 + 2 * a^2 * \\
& b * d^3 * x^3 + a^3 * d^3 + \sqrt{-3} * (a * b^2 * d^3 * x^6 + 2 * a^2 * b * d^3 * x^3 + a^3 * d^3)) \\
& * \sinh(d * x + c)^2) * \operatorname{Ei}(-d * x - 1/2 * (-a * d^3 / b)^{(1/3)} * (\sqrt{-3} + 1)) * \sinh(1/2 * \\
& (-a * d^3 / b)^{(1/3)} * (\sqrt{-3} + 1) - c) - (4 * (a * b^2 * d^3 * x^6 + 2 * a^2 * b * d^3 * x^3 \\
& + a^3 * d^3) * \cosh(d * x + c)^2 - 4 * (a * b^2 * d^3 * x^6 + 2 * a^2 * b * d^3 * x^3 + a^3 * d^3) * \\
& \sinh(d * x + c)^2 - 2 * (a * d^3 / b)^{(2/3)} * ((b^3 * x^6 + 2 * a * b^2 * x^3 + a^2 * b + \sqrt{-3} * \\
& (-3) * (b^3 * x^6 + 2 * a * b^2 * x^3 + a^2 * b)) * \cosh(d * x + c)^2 - (b^3 * x^6 + 2 * a * b^2 * x^3 \\
& + a^2 * b + \sqrt{-3} * (b^3 * x^6 + 2 * a * b^2 * x^3 + a^2 * b)) * \sinh(d * x + c)^2) + (\\
& a * d^3 / b)^{(1/3)} * ((a * b^2 * d^3 * x^6 + 2 * a^2 * b * d^3 * x^3 + a^3 * d^3 - \sqrt{-3} * (a * b^2 * \\
& d^3 * x^6 + 2 * a^2 * b * d^3 * x^3 + a^3 * d^3)) * \cosh(d * x + c)^2 - (a * b^2 * d^3 * x^6 + \\
& 2 * a^2 * b * d^3 * x^3 + a^3 * d^3 - \sqrt{-3} * (a * b^2 * d^3 * x^6 + 2 * a^2 * b * d^3 * x^3 + a^3 \\
& * d^3)) * \sinh(d * x + c)^2) * \operatorname{Ei}(d * x + 1/2 * (a * d^3 / b)^{(1/3)} * (\sqrt{-3} - 1)) * \sinh(\\
& 1/2 * (a * d^3 / b)^{(1/3)} * (\sqrt{-3} - 1) - c) + (4 * (a * b^2 * d^3 * x^6 + 2 * a^2 * b * d^3 * x^3 \\
& + a^3 * d^3) * \cosh(d * x + c)^2 - 4 * (a * b^2 * d^3 * x^6 + 2 * a^2 * b * d^3 * x^3 + a^3 * d^3) * \\
& \sinh(d * x + c)^2 + 2 * (-a * d^3 / b)^{(2/3)} * ((b^3 * x^6 + 2 * a * b^2 * x^3 + a^2 * b + \sqrt{-3} * \\
& (-3) * (b^3 * x^6 + 2 * a * b^2 * x^3 + a^2 * b)) * \cosh(d * x + c)^2 - (b^3 * x^6 + 2 * a * b^2 * x^3 \\
& + a^2 * b + \sqrt{-3} * (b^3 * x^6 + 2 * a * b^2 * x^3 + a^2 * b)) * \sinh(d * x + c)^2) \\
& + (-a * d^3 / b)^{(1/3)} * ((a * b^2 * d^3 * x^6 + 2 * a^2 * b * d^3 * x^3 + a^3 * d^3 - \sqrt{-3} * \\
& (a * b^2 * d^3 * x^6 + 2 * a^2 * b * d^3 * x^3 + a^3 * d^3)) * \cosh(d * x + c)^2 - (a * b^2 * d^3 * x^6 \\
& + 2 * a^2 * b * d^3 * x^3 + a^3 * d^3 - \sqrt{-3} * (a * b^2 * d^3 * x^6 + 2 * a^2 * b * d^3 * x^3 \\
& + a^3 * d^3)) * \sinh(d * x + c)^2) * \operatorname{Ei}(-d * x + 1/2 * (-a * d^3 / b)^{(1/3)} * (\sqrt{-3} - 1) \\
&) * \sinh(1/2 * (-a * d^3 / b)^{(1/3)} * (\sqrt{-3} - 1) + c) + 2 * (2 * (a * b^2 * d^3 * x^6 + 2 * a \\
& ^2 * b * d^3 * x^3 + a^3 * d^3) * \cosh(d * x + c)^2 - 2 * (a * b^2 * d^3 * x^6 + 2 * a^2 * b * d^3 * x^3 \\
& + a^3 * d^3) * \sinh(d * x + c)^2 - 2 * (-a * d^3 / b)^{(2/3)} * ((b^3 * x^6 + 2 * a * b^2 * x^3 + \\
& a^2 * b) * \cosh(d * x + c)^2 - (b^3 * x^6 + 2 * a * b^2 * x^3 + a^2 * b) * \sinh(d * x + c)^2) \\
& - (-a * d^3 / b)^{(1/3)} * ((a * b^2 * d^3 * x^6 + 2 * a^2 * b * d^3 * x^3 + a^3 * d^3) * \cosh(d * x + \\
& c)^2 - (a * b^2 * d^3 * x^6 + 2 * a^2 * b * d^3 * x^3 + a^3 * d^3) * \sinh(d * x + c)^2) * \operatorname{Ei}(-d * \\
& x + (-a * d^3 / b)^{(1/3)}) * \sinh(c + (-a * d^3 / b)^{(1/3)}) - 2 * (2 * (a * b^2 * d^3 * x^6 + 2 * \\
& a^2 * b * d^3 * x^3 + a^3 * d^3) * \cosh(d * x + c)^2 - 2 * (a * b^2 * d^3 * x^6 + 2 * a^2 * b * d^3 * x^3 \\
& + a^3 * d^3) * \sinh(d * x + c)^2 + 2 * (a * d^3 / b)^{(2/3)} * ((b^3 * x^6 + 2 * a * b^2 * x^3 + \\
& a^2 * b) * \cosh(d * x + c)^2 - (b^3 * x^6 + 2 * a * b^2 * x^3 + a^2 * b) * \sinh(d * x + c)^2) \\
& - (a * d^3 / b)^{(1/3)} * ((a * b^2 * d^3 * x^6 + 2 * a^2 * b * d^3 * x^3 + a^3 * d^3) * \cosh(d * x + \\
& c)^2 - (a * b^2 * d^3 * x^6 + 2 * a^2 * b * d^3 * x^3 + a^3 * d^3) * \sinh(d * x + c)^2) * \operatorname{Ei}(d * x \\
& + (a * d^3 / b)^{(1/3)}) * \sinh(-c + (a * d^3 / b)^{(1/3)}) - 12 * (2 * a * b^2 * d^2 * x^5 - a^2 * b
\end{aligned}$$

$*d^2*x^2)*\cosh(d*x + c) + 12*(a^2*b*d^3*x^3 + a^3*d^3)*\sinh(d*x + c))/((a^2*b^4*d^2*x^6 + 2*a^3*b^3*d^2*x^3 + a^4*b^2*d^2)*\cosh(d*x + c)^2 - (a^2*b^4*d^2*x^6 + 2*a^3*b^3*d^2*x^3 + a^4*b^2*d^2)*\sinh(d*x + c)^2)$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 \cosh(dx + c)}{(bx^3 + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*cosh(d*x+c)/(b*x^3+a)^3,x, algorithm="giac")

[Out] integrate(x^4*cosh(d*x + c)/(b*x^3 + a)^3, x)

maple [C] time = 0.48, size = 1927, normalized size = 1.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4*cosh(d*x+c)/(b*x^3+a)^3,x)

[Out] $-1/27/d^2*c/a^2/b^2*\sum((_R1^2*a*d^3-_R1^2*b*c^3+_R1*a*c*d^3+2*_R1*b*c^4+a*c^2*d^3-b*c^5-12*_R1^2*b*c^2+18*_R1*b*c^3+6*a*c*d^3-6*b*c^4-12*_R1*b*c^2-2*a*d^3+2*b*c^3)/(_R1^2-2*_R1*c+c^2)*\exp(-_R1)*\text{Ei}(1,d*x-_R1+c), _R1=\text{RootOf}(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/108/d^2/a^2/b^3*\sum((4*_R1^2*a*b*c*d^3-_R1^2*b^2*c^4-2*_R1*a*b*c^2*d^3+2*_R1*b^2*c^5-a^2*d^6+2*a*b*c^3*d^3-b^2*c^6-2*_R1^2*a*b*d^3-16*_R1^2*b^2*c^3+4*_R1*a*b*c*d^3+26*_R1*b^2*c^4+10*a*b*c^2*d^3-10*b^2*c^5-2*_R1*a*b*d^3-16*_R1*b^2*c^3-6*a*b*c*d^3+6*b^2*c^4)/(_R1^2-2*_R1*c+c^2)*\exp(-_R1)*\text{Ei}(1,d*x-_R1+c), _R1=\text{RootOf}(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/36*d^7*\exp(-d*x-c)*a/b^2/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)+1/36*d^7*\exp(-d*x-c)/b/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)*x^3+1/18*d^6*\exp(-d*x-c)/a/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)*x^5-1/36*d^6*\exp(-d*x-c)/b/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)*x^2-1/108/d^2*c^4/a^2/b*\sum((_R1^2-2*_R1*c+c^2+6*_R1-6*c+10)/(_R1^2-2*_R1*c+c^2)*\exp(-_R1)*\text{Ei}(1,d*x-_R1+c), _R1=\text{RootOf}(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/27/d^2*c^3/a^2/b^2*\sum((_R1^2*b*c-2*_R1*b*c^2-a*d^3+b*c^3+4*_R1^2*b-2*_R1*b*c-2*b*c^2+4*_R1*b+6*b*c)/(_R1^2-2*_R1*c+c^2)*\exp(-_R1)*\text{Ei}(1,d*x-_R1+c), _R1=\text{RootOf}(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/18/d^2*c^2/a^2/b^2*\sum((_R1^2*b*c^2-_R1*a*d^3-2*_R1*b*c^3-a*c*d^3+b*c^4+8*_R1^2*b*c-10*_R1*b*c^2-2*a*d^3+2*b*c^3+8*_R1*b*c+2*b*c^2)/(_R1^2-2*_R1*c+c^2)*\exp(-_R1)*\text{Ei}(1,d*x-_R1+c), _R1=\text{RootOf}(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/18/d^2*c^2/a^2/b^2*\sum((_R1^2*b*c^2-_R1*a*d^3-2*_R1*b*c^3-a*c*d^3+b*c^4-8*_R1^2*b*c+10*_R1*b*c^2+2*a*d^3-2*b*c^3+8*_R1*b*c+2*b*c^2)/(_R1^2-2*_R1*c+c^2)*\exp(_R1)*\text{Ei}(1,-d*x+_R1-c), _R1=\text{RootOf}(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/27/d^2*c/a^2/b^2*su$

m(($_R1^2*a*d^3 - _R1^2*b*c^3 + _R1*a*c*d^3 + 2*_R1*b*c^4 + a*c^2*d^3 - b*c^5 + 12*_R1^2*b*c^2 - 18*_R1*b*c^3 - 6*a*c*d^3 + 6*b*c^4 - 12*_R1*b*c^2 - 2*a*d^3 + 2*b*c^3$)/($_R1^2 - 2*_R1*c + c^2$))*exp($_R1$)*Ei(1, -d*x + $_R1 - c$), $_R1 = \text{RootOf}(_Z^3*b - 3*_Z^2*b*c + 3*_Z*b*c^2 + a*d^3 - b*c^3)$)-1/36*d^7*exp(d*x+c)*a/b^2/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)+1/108/d^2/a^2/b^3*sum((4*_R1^2*a*b*c*d^3 - _R1^2*b^2*c^4 - 2*_R1*a*b*c^2*d^3 + 2*_R1*b^2*c^5 - a^2*d^6 + 2*a*b*c^3*d^3 - b^2*c^6 + 2*_R1^2*a*b*d^3 + 16*_R1^2*b^2*c^3 - 4*_R1*a*b*c*d^3 - 26*_R1*b^2*c^4 - 10*a*b*c^2*d^3 + 10*b^2*c^5 - 2*_R1*a*b*d^3 - 16*_R1*b^2*c^3 - 6*a*b*c*d^3 + 6*b^2*c^4)/($_R1^2 - 2*_R1*c + c^2$))*exp($_R1$)*Ei(1, -d*x + $_R1 - c$), $_R1 = \text{RootOf}(_Z^3*b - 3*_Z^2*b*c + 3*_Z*b*c^2 + a*d^3 - b*c^3)$)-1/36*d^7*exp(d*x+c)/b/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)*x^3-1/108/d^2*c^4/a^2/b*sum(($_R1^2 - 2*_R1*c + c^2 - 6*_R1 + 6*c + 10$)/($_R1^2 - 2*_R1*c + c^2$))*exp($_R1$)*Ei(1, -d*x + $_R1 - c$), $_R1 = \text{RootOf}(_Z^3*b - 3*_Z^2*b*c + 3*_Z*b*c^2 + a*d^3 - b*c^3)$))+1/27/d^2*c^3/a^2/b^2*sum(($_R1^2*b*c - 2*_R1*b*c^2 - a*d^3 + b*c^3 - 4*_R1^2*b + 2*_R1*b*c + 2*b*c^2 + 4*_R1*b + 6*b*c$)/($_R1^2 - 2*_R1*c + c^2$))*exp($_R1$)*Ei(1, -d*x + $_R1 - c$), $_R1 = \text{RootOf}(_Z^3*b - 3*_Z^2*b*c + 3*_Z*b*c^2 + a*d^3 - b*c^3)$))+1/18*d^6*exp(d*x+c)/a/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)*x^2

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{(d^3 x^4 e^{2c} + 5 d^2 x^3 e^{2c} + 30 dx^2 e^{2c} + 210 x e^{2c}) e^{dx} - (d^3 x^4 - 5 d^2 x^3 + 30 dx^2 - 210 x) e^{-dx}}{2(b^3 d^4 x^9 e^c + 3 ab^2 d^4 x^6 e^c + 3 a^2 b d^4 x^3 e^c + a^3 d^4 e^c)} - \frac{1}{2} \int \frac{3(15 ad^2 x^2)}{b^4 d^4 x^1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*cosh(d*x+c)/(b*x^3+a)^3,x, algorithm="maxima")

[Out] 1/2*((d^3*x^4*e^(2*c) + 5*d^2*x^3*e^(2*c) + 30*d*x^2*e^(2*c) + 210*x*e^(2*c))*e^(d*x) - (d^3*x^4 - 5*d^2*x^3 + 30*d*x^2 - 210*x)*e^(-d*x))/(b^3*d^4*x^9*e^c + 3*a*b^2*d^4*x^6*e^c + 3*a^2*b*d^4*x^3*e^c + a^3*d^4*e^c) - 1/2*integrate(3*(15*a*d^2*x^2*e^c + (3*a*d^3*e^c - 560*b*e^c)*x^3 + 90*a*d*x*e^c + 70*a*e^c)*e^(d*x)/(b^4*d^4*x^12 + 4*a*b^3*d^4*x^9 + 6*a^2*b^2*d^4*x^6 + 4*a^3*b*d^4*x^3 + a^4*d^4), x) + 1/2*integrate(-3*(15*a*d^2*x^2 - (3*a*d^3 + 560*b)*x^3 - 90*a*d*x + 70*a)*e^(-d*x)/(b^4*d^4*x^12*e^c + 4*a*b^3*d^4*x^9*e^c + 6*a^2*b^2*d^4*x^6*e^c + 4*a^3*b*d^4*x^3*e^c + a^4*d^4*e^c), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4 \cosh(c + dx)}{(bx^3 + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4*cosh(c + d*x))/(a + b*x^3)^3,x)


```
[Out] int((x^4*cosh(c + d*x))/(a + b*x^3)^3, x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*cosh(d*x+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

$$3.109 \quad \int \frac{x^3 \cosh(c+dx)}{(a+bx^3)^3} dx$$

Optimal. Leaf size=776

$$\frac{\sqrt[3]{-1} \cosh\left(\frac{\sqrt[3]{-1} \sqrt[3]{ad}}{\sqrt[3]{b}} + c\right) \text{Chi}\left(\frac{\sqrt[3]{-1} \sqrt[3]{ad}}{\sqrt[3]{b}} - dx\right)}{27a^{5/3}b^{4/3}} + \frac{(-1)^{2/3} \cosh\left(c - \frac{(-1)^{2/3} \sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(-xd - \frac{(-1)^{2/3} \sqrt[3]{ad}}{\sqrt[3]{b}}\right)}{27a^{5/3}b^{4/3}} + \frac{\cosh\left(c - \frac{(-1)^{2/3} \sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(-xd - \frac{(-1)^{2/3} \sqrt[3]{ad}}{\sqrt[3]{b}}\right)}{27a^{5/3}b^{4/3}}$$

[Out] $\frac{1}{27} \text{Chi}(a^{1/3}d/b^{1/3}+dx) \cosh(c-a^{1/3}d/b^{1/3})/a^{5/3}/b^{4/3} - 1/54 d^2 \text{Chi}(a^{1/3}d/b^{1/3}+dx) \cosh(c-a^{1/3}d/b^{1/3})/a/b^2 - 1/27 (-1)^{1/3} \text{Chi}((-1)^{1/3}a^{1/3}d/b^{1/3}-dx) \cosh(c+(-1)^{1/3}a^{1/3}d/b^{1/3})/a^{5/3}/b^{4/3} - 1/54 d^2 \text{Chi}((-1)^{1/3}a^{1/3}d/b^{1/3}-dx) \cosh(c+(-1)^{1/3}a^{1/3}d/b^{1/3})/a/b^2 + 1/27 (-1)^{2/3} \text{Chi}(-(-1)^{2/3}a^{1/3}d/b^{1/3}-dx) \cosh(c-(-1)^{2/3}a^{1/3}d/b^{1/3})/a^{5/3}/b^{4/3} - 1/54 d^2 \text{Chi}(-(-1)^{2/3}a^{1/3}d/b^{1/3}-dx) \cosh(c-(-1)^{2/3}a^{1/3}d/b^{1/3})/a/b^2 + 1/18 \cosh(dx+c)/a/b^2/x^2 - 1/6 x \cosh(dx+c)/b/(bx^3+a)^2 - 1/18 \cosh(dx+c)/b^2/x^2/(bx^3+a) + 1/27 \text{Shi}(a^{1/3}d/b^{1/3}+dx) \sinh(c-a^{1/3}d/b^{1/3})/a^{5/3}/b^{4/3} - 1/54 d^2 \text{Shi}(a^{1/3}d/b^{1/3}+dx) \sinh(c-a^{1/3}d/b^{1/3})/a/b^2 - 1/27 (-1)^{1/3} \text{Shi}(-(-1)^{1/3}a^{1/3}d/b^{1/3}+dx) \sinh(c+(-1)^{1/3}a^{1/3}d/b^{1/3})/a^{5/3}/b^{4/3} - 1/54 d^2 \text{Shi}(-(-1)^{1/3}a^{1/3}d/b^{1/3}+dx) \sinh(c+(-1)^{1/3}a^{1/3}d/b^{1/3})/a/b^2 + 1/27 (-1)^{2/3} \text{Shi}((-1)^{2/3}a^{1/3}d/b^{1/3}+dx) \sinh(c-(-1)^{2/3}a^{1/3}d/b^{1/3})/a^{5/3}/b^{4/3} - 1/54 d^2 \text{Shi}((-1)^{2/3}a^{1/3}d/b^{1/3}+dx) \sinh(c-(-1)^{2/3}a^{1/3}d/b^{1/3})/a/b^2 + 1/18 d \sinh(dx+c)/a/b^2/x - 1/18 d \sinh(dx+c)/b^2/x/(bx^3+a)$

Rubi [A] time = 2.66, antiderivative size = 776, normalized size of antiderivative = 1.00, number of steps used = 71, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {5291, 5279, 5293, 3297, 3303, 3298, 3301, 5281, 5292, 5290}

$$\frac{\sqrt[3]{-1} \cosh\left(\frac{\sqrt[3]{-1} \sqrt[3]{ad}}{\sqrt[3]{b}} + c\right) \text{Chi}\left(\frac{\sqrt[3]{-1} \sqrt[3]{ad}}{\sqrt[3]{b}} - dx\right)}{27a^{5/3}b^{4/3}} + \frac{(-1)^{2/3} \cosh\left(c - \frac{(-1)^{2/3} \sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(-xd - \frac{(-1)^{2/3} \sqrt[3]{ad}}{\sqrt[3]{b}}\right)}{27a^{5/3}b^{4/3}} + \frac{\cosh\left(c - \frac{(-1)^{2/3} \sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(-xd - \frac{(-1)^{2/3} \sqrt[3]{ad}}{\sqrt[3]{b}}\right)}{27a^{5/3}b^{4/3}}$$

Antiderivative was successfully verified.

[In] Int[(x^3*Cosh[c + d*x])/(a + b*x^3)^3,x]

[Out] $\frac{\text{Cosh}[c + d*x]}{(18*a*b^2*x^2) - (x*\text{Cosh}[c + d*x])/(6*b*(a + b*x^3)^2) - \text{Cosh}[c + d*x]/(18*b^2*x^2*(a + b*x^3)) - ((-1)^{1/3}*\text{Cosh}[c + ((-1)^{1/3}a^{1/3}d/b^{1/3})*d]/b^{1/3})*\text{CoshIntegral}[((-1)^{1/3}a^{1/3}d)/b^{1/3} - d*x]}{(27*a^{5/3}b^{4/3})}$

$$\begin{aligned} & /3)*b^{(4/3)} - (d^2*\text{Cosh}[c + ((-1)^{(1/3)}*a^{(1/3)}*d)/b^{(1/3)}]*\text{CoshIntegral}[(\\ & (-1)^{(1/3)}*a^{(1/3)}*d)/b^{(1/3)} - d*x])/(54*a*b^2) + ((-1)^{(2/3)}*\text{Cosh}[c - ((- \\ & 1)^{(2/3)}*a^{(1/3)}*d)/b^{(1/3)}]*\text{CoshIntegral}[-(((-1)^{(2/3)}*a^{(1/3)}*d)/b^{(1/3)} \\ & - d*x])/(27*a^{(5/3)}*b^{(4/3)} - (d^2*\text{Cosh}[c - ((-1)^{(2/3)}*a^{(1/3)}*d)/b^{(1/3)} \\ &])*\text{CoshIntegral}[-(((-1)^{(2/3)}*a^{(1/3)}*d)/b^{(1/3)} - d*x])/(54*a*b^2) + (\text{Cos} \\ & h[c - (a^{(1/3)}*d)/b^{(1/3)}]*\text{CoshIntegral}[(a^{(1/3)}*d)/b^{(1/3)} + d*x])/(27*a^{(\\ & 5/3)}*b^{(4/3)} - (d^2*\text{Cosh}[c - (a^{(1/3)}*d)/b^{(1/3)}]*\text{CoshIntegral}[(a^{(1/3)}*d) \\ & /b^{(1/3)} + d*x])/(54*a*b^2) + (d*\text{Sinh}[c + d*x])/(18*a*b^2*x) - (d*\text{Sinh}[c + \\ & d*x])/(18*b^2*x*(a + b*x^3)) + ((-1)^{(1/3)}*\text{Sinh}[c + ((-1)^{(1/3)}*a^{(1/3)}*d)/ \\ & b^{(1/3)}]*\text{SinhIntegral}[((-1)^{(1/3)}*a^{(1/3)}*d)/b^{(1/3)} - d*x])/(27*a^{(5/3)}*b^{(\\ & 4/3)} + (d^2*\text{Sinh}[c + ((-1)^{(1/3)}*a^{(1/3)}*d)/b^{(1/3)}]*\text{SinhIntegral}[((-1)^{(\\ & 1/3)}*a^{(1/3)}*d)/b^{(1/3)} - d*x])/(54*a*b^2) + (\text{Sinh}[c - (a^{(1/3)}*d)/b^{(1/3)}] \\ & *\text{SinhIntegral}[(a^{(1/3)}*d)/b^{(1/3)} + d*x])/(27*a^{(5/3)}*b^{(4/3)} - (d^2*\text{Sinh}[\\ & c - (a^{(1/3)}*d)/b^{(1/3)}]*\text{SinhIntegral}[(a^{(1/3)}*d)/b^{(1/3)} + d*x])/(54*a*b^2 \\ &) + ((-1)^{(2/3)}*\text{Sinh}[c - ((-1)^{(2/3)}*a^{(1/3)}*d)/b^{(1/3)}]*\text{SinhIntegral}[((-1) \\ & ^{(2/3)}*a^{(1/3)}*d)/b^{(1/3)} + d*x])/(27*a^{(5/3)}*b^{(4/3)} - (d^2*\text{Sinh}[c - ((-1) \\ &)^{(2/3)}*a^{(1/3)}*d)/b^{(1/3)}]*\text{SinhIntegral}[((-1)^{(2/3)}*a^{(1/3)}*d)/b^{(1/3)} + d \\ & *x])/(54*a*b^2) \end{aligned}$$

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
+ d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)])/((c_.) + (d_.)*(x_)), x_Symbol]
:= Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)])/((c_.) + (d_.)*(x_)), x_Symbol]
:= Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

Rule 5279

```
Int[Cosh[(c_.) + (d_.)*(x_.)]*((a_.) + (b_.)*(x_.)^(n_.))^(p_), x_Symbol] := Simp[(x^(-n + 1)*(a + b*x^n)^(p + 1)*Cosh[c + d*x])/(b*n*(p + 1)), x] + (-Dist[(-n + 1)/(b*n*(p + 1)), Int[((a + b*x^n)^(p + 1)*Cosh[c + d*x])/x^n, x], x] - Dist[d/(b*n*(p + 1)), Int[x^(-n + 1)*(a + b*x^n)^(p + 1)*Sinh[c + d*x], x], x]) /; FreeQ[{a, b, c, d}, x] && IntegerQ[p] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[n, 2]
```

Rule 5281

```
Int[Cosh[(c_.) + (d_.)*(x_.)]*((a_.) + (b_.)*(x_.)^(n_.))^(p_), x_Symbol] := Int[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 5290

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_.)^(n_.))^(p_)*Sinh[(c_.) + (d_.)*(x_.)], x_Symbol] := Simp[(x^(m - n + 1)*(a + b*x^n)^(p + 1)*Sinh[c + d*x])/(b*n*(p + 1)), x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1)*Sinh[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^n)^(p + 1)*Cosh[c + d*x], x], x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1] && IGtQ[n, 0] && RationalQ[m] && (GtQ[m - n + 1, 0] || GtQ[n, 2])
```

Rule 5291

```
Int[Cosh[(c_.) + (d_.)*(x_.)]*(x_)^(m_.)*((a_.) + (b_.)*(x_.)^(n_.))^(p_), x_Symbol] := Simp[(x^(m - n + 1)*(a + b*x^n)^(p + 1)*Cosh[c + d*x])/(b*n*(p + 1)), x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1)*Cosh[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^n)^(p + 1)*Sinh[c + d*x], x], x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1] && IGtQ[n, 0] && RationalQ[m] && (GtQ[m - n + 1, 0] || GtQ[n, 2])
```

Rule 5292

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_.)^(n_.))^(p_)*Sinh[(c_.) + (d_.)*(x_.)], x_Symbol] := Int[ExpandIntegrand[Sinh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 5293

```
Int[Cosh[(c_.) + (d_.)*(x_.)]*(x_)^(m_.)*((a_.) + (b_.)*(x_.)^(n_.))^(p_), x_Symbol] := Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
```

2] || EqQ[p, -1])

Rubi steps

$$\begin{aligned}
\int \frac{x^3 \cosh(c+dx)}{(a+bx^3)^3} dx &= -\frac{x \cosh(c+dx)}{6b(a+bx^3)^2} + \frac{\int \frac{\cosh(c+dx)}{(a+bx^3)^2} dx}{6b} + \frac{d \int \frac{x \sinh(c+dx)}{(a+bx^3)^2} dx}{6b} \\
&= -\frac{x \cosh(c+dx)}{6b(a+bx^3)^2} - \frac{\cosh(c+dx)}{18b^2x^2(a+bx^3)} - \frac{d \sinh(c+dx)}{18b^2x(a+bx^3)} - \frac{\int \frac{\cosh(c+dx)}{x^3(a+bx^3)} dx}{9b^2} + \frac{d^2 \int \frac{\cosh(c+dx)}{x(a+bx^3)} dx}{18b^2} \\
&= -\frac{x \cosh(c+dx)}{6b(a+bx^3)^2} - \frac{\cosh(c+dx)}{18b^2x^2(a+bx^3)} - \frac{d \sinh(c+dx)}{18b^2x(a+bx^3)} - \frac{\int \left(\frac{\cosh(c+dx)}{ax^3} - \frac{b \cosh(c+dx)}{a(a+bx^3)} \right) dx}{9b^2} \\
&= -\frac{x \cosh(c+dx)}{6b(a+bx^3)^2} - \frac{\cosh(c+dx)}{18b^2x^2(a+bx^3)} - \frac{d \sinh(c+dx)}{18b^2x(a+bx^3)} - \frac{\int \frac{\cosh(c+dx)}{x^3} dx}{9ab^2} + \frac{\int \frac{\cosh(c+dx)}{a+bx^3} dx}{9ab} \\
&= \frac{\cosh(c+dx)}{18ab^2x^2} - \frac{x \cosh(c+dx)}{6b(a+bx^3)^2} - \frac{\cosh(c+dx)}{18b^2x^2(a+bx^3)} - \frac{d \sinh(c+dx)}{18b^2x(a+bx^3)} + \frac{\int \left(-\frac{\cosh(c+dx)}{3a^2/3(-\sqrt[3]{a})} \right) dx}{18ab^2} \\
&= \frac{\cosh(c+dx)}{18ab^2x^2} - \frac{x \cosh(c+dx)}{6b(a+bx^3)^2} - \frac{\cosh(c+dx)}{18b^2x^2(a+bx^3)} + \frac{d^2 \cosh(c) \text{Chi}(dx)}{18ab^2} + \frac{d \sinh(c+dx)}{18ab^2x} \\
&= \frac{\cosh(c+dx)}{18ab^2x^2} - \frac{x \cosh(c+dx)}{6b(a+bx^3)^2} - \frac{\cosh(c+dx)}{18b^2x^2(a+bx^3)} + \frac{d^2 \cosh(c) \text{Chi}(dx)}{18ab^2} + \frac{d \sinh(c+dx)}{18ab^2x} \\
&= \frac{\cosh(c+dx)}{18ab^2x^2} - \frac{x \cosh(c+dx)}{6b(a+bx^3)^2} - \frac{\cosh(c+dx)}{18b^2x^2(a+bx^3)} - \frac{\sqrt[3]{-1} \cosh\left(c + \frac{\sqrt[3]{-1} \sqrt[3]{a} d}{\sqrt[3]{b}}\right) \text{Chi}\left(\frac{\sqrt[3]{-1} \sqrt[3]{a} d}{\sqrt[3]{b}}\right)}{27a^{5/3}b^{4/3}}
\end{aligned}$$

Mathematica [C] time = 0.71, size = 429, normalized size = 0.55

$$\text{RootSum}\left[\#1^3b + a\&, \frac{-\#1^2d^2 \sinh(\#1d+c)\text{Chi}(d(x-\#1))+\#1^2d^2 \cosh(\#1d+c)\text{Chi}(d(x-\#1))+\#1^2d^2 \sinh(\#1d+c)\text{Shi}(d(x-\#1))-\#1^2d^2 \cosh(\#1d+c)\text{Shi}(d(x-\#1))}{27a^{5/3}b^{4/3}}\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Cosh[c + d*x])/(a + b*x^3)^3,x]
```

```
[Out] -1/108*(RootSum[a + b*#1^3 & , (-2*Cosh[c + d*#1]*CoshIntegral[d*(x - #1)]
+ 2*CoshIntegral[d*(x - #1)]*Sinh[c + d*#1] + 2*Cosh[c + d*#1]*SinhIntegral
[d*(x - #1)] - 2*Sinh[c + d*#1]*SinhIntegral[d*(x - #1)] + d^2*Cosh[c + d*#
1]*CoshIntegral[d*(x - #1)]*#1^2 - d^2*CoshIntegral[d*(x - #1)]*Sinh[c + d*
#1]*#1^2 - d^2*Cosh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1^2 + d^2*Sinh[c +
d*#1]*SinhIntegral[d*(x - #1)]*#1^2)/#1^2 & ] + RootSum[a + b*#1^3 & , (-2*
Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] - 2*CoshIntegral[d*(x - #1)]*Sinh[c
+ d*#1] - 2*Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] - 2*Sinh[c + d*#1]*Sin
hIntegral[d*(x - #1)] + d^2*Cosh[c + d*#1]*CoshIntegral[d*(x - #1)]*#1^2 +
d^2*CoshIntegral[d*(x - #1)]*Sinh[c + d*#1]*#1^2 + d^2*Cosh[c + d*#1]*SinhI
ntegral[d*(x - #1)]*#1^2 + d^2*Sinh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1^2
)/#1^2 & ] - (6*b*x*((-2*a + b*x^3)*Cosh[c + d*x] + d*x*(a + b*x^3)*Sinh[c
+ d*x]))/(a + b*x^3)^2/(a*b^2)
```

fricas [B] time = 0.65, size = 2962, normalized size = 3.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(d*x+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] -1/108*(((a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*cosh(d*x + c)^2 - (a*b
^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*sinh(d*x + c)^2 + (a*d^3/b)^(1/3)*(
(b^3*x^6 + 2*a*b^2*x^3 + a^2*b + sqrt(-3)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*
cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^2*b + sqrt(-3)*(b^3*x^6 + 2*a*
b^2*x^3 + a^2*b))*sinh(d*x + c)^2))*Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt(-3)
+ 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1) + c) + ((a*b^2*d^3*x^6 + 2*a^
2*b*d^3*x^3 + a^3*d^3)*cosh(d*x + c)^2 - (a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 +
a^3*d^3)*sinh(d*x + c)^2 - (-a*d^3/b)^(1/3)*((b^3*x^6 + 2*a*b^2*x^3 + a^2*
b + sqrt(-3)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*cosh(d*x + c)^2 - (b^3*x^6 +
2*a*b^2*x^3 + a^2*b + sqrt(-3)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*sinh(d*x +
c)^2))*Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(-a*d^3/b)^(
1/3)*(sqrt(-3) + 1) - c) + ((a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*cos
h(d*x + c)^2 - (a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*sinh(d*x + c)^2
+ (a*d^3/b)^(1/3)*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b - sqrt(-3)*(b^3*x^6 + 2*a
*b^2*x^3 + a^2*b))*cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^2*b - sqrt(
-3)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*sinh(d*x + c)^2))*Ei(d*x + 1/2*(a*d^3/
b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1) - c) + ((a
*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*cosh(d*x + c)^2 - (a*b^2*d^3*x^6
+ 2*a^2*b*d^3*x^3 + a^3*d^3)*sinh(d*x + c)^2 - (-a*d^3/b)^(1/3)*((b^3*x^6 +
2*a*b^2*x^3 + a^2*b - sqrt(-3)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*cosh(d*x +
```

$$\begin{aligned}
& c)^2 - (b^3x^6 + 2ab^2x^3 + a^2b - \sqrt{-3})(b^3x^6 + 2ab^2x^3 + \\
& a^2b))\sinh(dx + c)^2) * Ei(-dx + 1/2*(-a^3d^3/b)^{1/3} * (\sqrt{-3} - 1)) * \co \\
& sh(1/2*(-a^3d^3/b)^{1/3} * (\sqrt{-3} - 1) + c) + ((ab^2d^3x^6 + 2a^2b^3d^3 \\
& *x^3 + a^3d^3) * \cosh(dx + c)^2 - (ab^2d^3x^6 + 2a^2b^3d^3x^3 + a^3d^3 \\
&) * \sinh(dx + c)^2 + 2*(-a^3d^3/b)^{1/3} * ((b^3x^6 + 2ab^2x^3 + a^2b) * \co \\
& sh(dx + c)^2 - (b^3x^6 + 2ab^2x^3 + a^2b) * \sinh(dx + c)^2)) * Ei(-dx + \\
& (-a^3d^3/b)^{1/3}) * \cosh(c + (-a^3d^3/b)^{1/3}) + ((ab^2d^3x^6 + 2a^2b^3d^3 \\
& *x^3 + a^3d^3) * \cosh(dx + c)^2 - (ab^2d^3x^6 + 2a^2b^3d^3x^3 + a^3d^3 \\
&) * \sinh(dx + c)^2 - 2*(a^3d^3/b)^{1/3} * ((b^3x^6 + 2ab^2x^3 + a^2b) * \co \\
& sh(dx + c)^2 - (b^3x^6 + 2ab^2x^3 + a^2b) * \sinh(dx + c)^2)) * Ei(dx + \\
& (a^3d^3/b)^{1/3}) * \cosh(-c + (a^3d^3/b)^{1/3}) + ((ab^2d^3x^6 + 2a^2b^3d^3 \\
& *x^3 + a^3d^3) * \cosh(dx + c)^2 - (ab^2d^3x^6 + 2a^2b^3d^3x^3 + a^3d^3 \\
&) * \sinh(dx + c)^2 + (a^3d^3/b)^{1/3} * ((b^3x^6 + 2ab^2x^3 + a^2b + \sqrt{-3} \\
&) * \cosh(dx + c)^2 - (b^3x^6 + 2ab^2x^3 + a^2b) * \sinh(dx + c)^2)) * \\
& Ei(dx - 1/2*(a^3d^3/b)^{1/3} * (\sqrt{-3} + 1)) * \sinh(1/2*(a^3d^3/b)^{1/3} * (\sqrt{-3} \\
& + 1) + c) + ((ab^2d^3x^6 + 2a^2b^3d^3x^3 + a^3d^3) * \cosh(dx + c) \\
& ^2 - (ab^2d^3x^6 + 2a^2b^3d^3x^3 + a^3d^3) * \sinh(dx + c)^2 - (-a^3d^3/ \\
& b)^{1/3} * ((b^3x^6 + 2ab^2x^3 + a^2b + \sqrt{-3})(b^3x^6 + 2ab^2x^3 \\
& + a^2b)) * \cosh(dx + c)^2 - (b^3x^6 + 2ab^2x^3 + a^2b + \sqrt{-3})(b^3x^6 \\
& + 2ab^2x^3 + a^2b) * \sinh(dx + c)^2)) * Ei(-dx - 1/2*(-a^3d^3/b)^{1/3} \\
&) * (\sqrt{-3} + 1)) * \sinh(1/2*(-a^3d^3/b)^{1/3} * (\sqrt{-3} + 1) - c) - ((ab^2d^3 \\
& *x^6 + 2a^2b^3d^3x^3 + a^3d^3) * \cosh(dx + c)^2 - (ab^2d^3x^6 + 2a^2b^3d^3 \\
& *x^3 + a^3d^3) * \sinh(dx + c)^2 + (a^3d^3/b)^{1/3} * ((b^3x^6 + 2ab^2x^3 \\
& + a^2b - \sqrt{-3})(b^3x^6 + 2ab^2x^3 + a^2b)) * \cosh(dx + c)^2 - \\
& (b^3x^6 + 2ab^2x^3 + a^2b - \sqrt{-3})(b^3x^6 + 2ab^2x^3 + a^2b)) \\
&) * \sinh(dx + c)^2) * Ei(dx + 1/2*(a^3d^3/b)^{1/3} * (\sqrt{-3} - 1)) * \sinh(1/2*(a \\
& ^3d^3/b)^{1/3} * (\sqrt{-3} - 1) - c) - ((ab^2d^3x^6 + 2a^2b^3d^3x^3 + a^3d^3 \\
&) * \cosh(dx + c)^2 - (ab^2d^3x^6 + 2a^2b^3d^3x^3 + a^3d^3) * \sinh(dx \\
& + c)^2 - (-a^3d^3/b)^{1/3} * ((b^3x^6 + 2ab^2x^3 + a^2b - \sqrt{-3})(b^3 \\
& *x^6 + 2ab^2x^3 + a^2b)) * \cosh(dx + c)^2 - (b^3x^6 + 2ab^2x^3 + a^2 \\
& *b - \sqrt{-3})(b^3x^6 + 2ab^2x^3 + a^2b)) * \sinh(dx + c)^2)) * Ei(-dx + \\
& 1/2*(-a^3d^3/b)^{1/3} * (\sqrt{-3} - 1)) * \sinh(1/2*(-a^3d^3/b)^{1/3} * (\sqrt{-3} - \\
& 1) + c) - ((ab^2d^3x^6 + 2a^2b^3d^3x^3 + a^3d^3) * \cosh(dx + c)^2 - (a \\
& *b^2d^3x^6 + 2a^2b^3d^3x^3 + a^3d^3) * \sinh(dx + c)^2 + 2*(-a^3d^3/b)^{1 \\
& /3} * ((b^3x^6 + 2ab^2x^3 + a^2b) * \cosh(dx + c)^2 - (b^3x^6 + 2ab^2x^3 \\
& ^3 + a^2b) * \sinh(dx + c)^2)) * Ei(-dx + (-a^3d^3/b)^{1/3}) * \sinh(c + (-a^3d^3/ \\
& b)^{1/3}) - ((ab^2d^3x^6 + 2a^2b^3d^3x^3 + a^3d^3) * \cosh(dx + c)^2 - \\
& (ab^2d^3x^6 + 2a^2b^3d^3x^3 + a^3d^3) * \sinh(dx + c)^2 - 2*(a^3d^3/b)^{1 \\
& /3} * ((b^3x^6 + 2ab^2x^3 + a^2b) * \cosh(dx + c)^2 - (b^3x^6 + 2ab^2x^3 \\
& + a^2b) * \sinh(dx + c)^2)) * Ei(dx + (a^3d^3/b)^{1/3}) * \sinh(-c + (a^3d^3/b) \\
&)^{1/3}) - 6*(ab^2d^3x^4 - 2a^2b^3d^3x) * \cosh(dx + c) - 6*(ab^2d^2x^5 + \\
& a^2b^3d^2x^2) * \sinh(dx + c) / ((a^2b^4d^3x^6 + 2a^3b^3d^3x^3 + a^4b^2d^3 \\
&) * \cosh(dx + c)^2 - (a^2b^4d^3x^6 + 2a^3b^3d^3x^3 + a^4b^2d^3) * \sinh(dx \\
& + c)^2)
\end{aligned}$$

$\wedge 2*d^6)*x+1/36/d*c/a^2/b^2*\text{sum}((_R1^2*b*c^2-_R1*a*d^3-2*_R1*b*c^3-a*c*d^3+b*c^4-8*_R1^2*b*c+10*_R1*b*c^2+2*a*d^3-2*b*c^3+8*_R1*b*c+2*b*c^2)/(_R1^2-2*_R1*c+c^2)*\text{exp}(_R1)*\text{Ei}(1,-d*x+_R1-c), _R1=\text{RootOf}(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{(d^2x^3e^{(2c)} + 6dx^2e^{(2c)} + 42xe^{(2c)})e^{(dx)} - (d^2x^3 - 6dx^2 + 42x)e^{(-dx)}}{2(b^3d^3x^9e^c + 3ab^2d^3x^6e^c + 3a^2bd^3x^3e^c + a^3d^3e^c)} + \frac{1}{2} \int -\frac{3(3ad^2x^2e^c - 112bx^3e^c + 18adx^3e^c + 18a^2d^3x^9e^c + 3a^2b^2d^3x^6e^c + 6a^2b^2d^3x^3e^c + a^3d^3e^c)}{b^4d^3x^{12} + 4ab^3d^3x^9 + 6a^2b^2d^3x^6 + 4a^3bd^3x^3 + a^4d^3e^c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*cosh(d*x+c)/(b*x^3+a)^3,x, algorithm="maxima")

[Out] $\frac{1}{2}((d^2*x^3*e^{(2*c)} + 6*d*x^2*e^{(2*c)} + 42*x*e^{(2*c)})*e^{(d*x)} - (d^2*x^3 - 6*d*x^2 + 42*x)*e^{(-d*x)})/(b^3*d^3*x^9*e^c + 3*a*b^2*d^3*x^6*e^c + 3*a^2*b*d^3*x^3*e^c + a^3*d^3*e^c) + \frac{1}{2}*\text{integrate}(-3*(3*a*d^2*x^2*e^c - 112*b*x^3*e^c + 18*a*d*x*e^c + 14*a*e^c)*e^{(d*x)})/(b^4*d^3*x^{12} + 4*a*b^3*d^3*x^9 + 6*a^2*b^2*d^3*x^6 + 4*a^3*b*d^3*x^3 + a^4*d^3), x) - \frac{1}{2}*\text{integrate}(-3*(3*a*d^2*x^2 - 112*b*x^3 - 18*a*d*x + 14*a)*e^{(-d*x)})/(b^4*d^3*x^{12}*e^c + 4*a*b^3*d^3*x^9*e^c + 6*a^2*b^2*d^3*x^6*e^c + 4*a^3*b*d^3*x^3*e^c + a^4*d^3*e^c), x)$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3 \cosh(c + dx)}{(bx^3 + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3*cosh(c + d*x))/(a + b*x^3)^3,x)

[Out] int((x^3*cosh(c + d*x))/(a + b*x^3)^3, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*cosh(d*x+c)/(b*x**3+a)**3,x)

[Out] Timed out

$$3.110 \quad \int \frac{x^2 \cosh(c+dx)}{(a+bx^3)^3} dx$$

Optimal. Leaf size=781

$$\frac{(-1)^{2/3} d^2 \cosh\left(\frac{\sqrt[3]{-1} \sqrt[3]{ad}}{\sqrt[3]{b}} + c\right) \text{Chi}\left(\frac{\sqrt[3]{-1} \sqrt[3]{ad}}{\sqrt[3]{b}} - dx\right)}{54a^{4/3} b^{5/3}} - \frac{\sqrt[3]{-1} d^2 \cosh\left(c - \frac{(-1)^{2/3} \sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(-xd - \frac{(-1)^{2/3} \sqrt[3]{ad}}{\sqrt[3]{b}}\right)}{54a^{4/3} b^{5/3}} + \frac{d^2 \cosh\left(\frac{\sqrt[3]{-1} \sqrt[3]{ad}}{\sqrt[3]{b}} + c\right)}{54a^{4/3} b^{5/3}}$$

[Out] $\frac{1}{54} d^2 \text{Chi}(a^{1/3} d/b^{1/3} + dx) \cosh(c - a^{1/3} d/b^{1/3}) / a^{4/3} / b^{5/3} + \frac{1}{54} (-1)^{2/3} d^2 \text{Chi}((-1)^{1/3} a^{1/3} d/b^{1/3} - dx) \cosh(c + (-1)^{1/3} a^{1/3} d/b^{1/3}) / a^{4/3} / b^{5/3} - \frac{1}{54} (-1)^{1/3} d^2 \text{Chi}((-1)^{2/3} a^{1/3} d/b^{1/3} - dx) \cosh(c - (-1)^{2/3} a^{1/3} d/b^{1/3}) / a^{4/3} / b^{5/3} - \frac{1}{6} \cosh(dx+c) / b / (b x^3 + a)^2 - \frac{1}{27} (-1)^{1/3} d \cosh(c + (-1)^{1/3} a^{1/3} d/b^{1/3}) \text{Shi}((-1)^{1/3} a^{1/3} d/b^{1/3} + dx) / a^{5/3} / b^{4/3} + \frac{1}{27} d \cosh(c - a^{1/3} d/b^{1/3}) \text{Shi}(a^{1/3} d/b^{1/3} + dx) / a^{5/3} / b^{4/3} + \frac{1}{27} (-1)^{2/3} d \cosh(c - (-1)^{2/3} a^{1/3} d/b^{1/3}) \text{Shi}((-1)^{2/3} a^{1/3} d/b^{1/3} + dx) / a^{5/3} / b^{4/3} + \frac{1}{27} d \text{Chi}(a^{1/3} d/b^{1/3} + dx) \sinh(c - a^{1/3} d/b^{1/3}) / a^{5/3} / b^{4/3} + \frac{1}{54} d^2 \text{Shi}(a^{1/3} d/b^{1/3} + dx) \sinh(c - a^{1/3} d/b^{1/3}) / a^{4/3} / b^{5/3} - \frac{1}{27} (-1)^{1/3} d \text{Chi}((-1)^{1/3} a^{1/3} d/b^{1/3} - dx) \sinh(c + (-1)^{1/3} a^{1/3} d/b^{1/3}) / a^{5/3} / b^{4/3} + \frac{1}{54} (-1)^{2/3} d^2 \text{Shi}((-1)^{1/3} a^{1/3} d/b^{1/3} + dx) \sinh(c + (-1)^{1/3} a^{1/3} d/b^{1/3}) / a^{4/3} / b^{5/3} + \frac{1}{27} (-1)^{2/3} d \text{Chi}((-1)^{2/3} a^{1/3} d/b^{1/3} - dx) \sinh(c - (-1)^{2/3} a^{1/3} d/b^{1/3}) / a^{5/3} / b^{4/3} - \frac{1}{54} (-1)^{1/3} d^2 \text{Shi}((-1)^{2/3} a^{1/3} d/b^{1/3} + dx) \sinh(c - (-1)^{2/3} a^{1/3} d/b^{1/3}) / a^{4/3} / b^{5/3} + \frac{1}{18} d \sinh(dx+c) / a / b^2 / x^2 - \frac{1}{18} d \sinh(dx+c) / b^2 / x^2 / (b x^3 + a)$

Rubi [A] time = 1.45, antiderivative size = 781, normalized size of antiderivative = 1.00, number of steps used = 37, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {5289, 5278, 5292, 3297, 3303, 3298, 3301, 5280, 5293}

$$\frac{(-1)^{2/3} d^2 \cosh\left(\frac{\sqrt[3]{-1} \sqrt[3]{ad}}{\sqrt[3]{b}} + c\right) \text{Chi}\left(\frac{\sqrt[3]{-1} \sqrt[3]{ad}}{\sqrt[3]{b}} - dx\right)}{54a^{4/3} b^{5/3}} - \frac{\sqrt[3]{-1} d^2 \cosh\left(c - \frac{(-1)^{2/3} \sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(-xd - \frac{(-1)^{2/3} \sqrt[3]{ad}}{\sqrt[3]{b}}\right)}{54a^{4/3} b^{5/3}} + \frac{d^2 \cosh\left(\frac{\sqrt[3]{-1} \sqrt[3]{ad}}{\sqrt[3]{b}} + c\right)}{54a^{4/3} b^{5/3}}$$

Antiderivative was successfully verified.

[In] Int[(x^2*Cosh[c + d*x])/(a + b*x^3)^3, x]

[Out] $-\frac{\text{Cosh}[c + d*x]}{(6*b*(a + b*x^3)^2)} + \frac{((-1)^{2/3} d^2 \text{Cosh}[c + ((-1)^{1/3} a^{1/3} d/b^{1/3} - dx)] \text{CoshIntegral}[((-1)^{1/3} a^{1/3} d/b^{1/3} - dx)]}{(54 a^{4/3} b^{5/3})} - \frac{((-1)^{1/3} d^2 \text{Cosh}[c - ((-1)^{2/3} a^{1/3} d/b^{1/3} - dx)] \text{CoshIntegral}[((-1)^{2/3} a^{1/3} d/b^{1/3} - dx)]}{(54 a^{4/3} b^{5/3})} + \frac{d^2 \text{Cosh}[c + ((-1)^{2/3} a^{1/3} d/b^{1/3} - dx)]}{(54 a^{4/3} b^{5/3})}$

```

*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d*x]/(54*a^(4/3)*b^(5/3)
) + (d^2*Cosh[c - (a^(1/3)*d)/b^(1/3)]*CoshIntegral[(a^(1/3)*d)/b^(1/3) + d
*x])/ (54*a^(4/3)*b^(5/3)) + (d*CoshIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sinh
[c - (a^(1/3)*d)/b^(1/3)])/(27*a^(5/3)*b^(4/3)) - ((-1)^(1/3)*d*CoshIntegra
l[(-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(
1/3)])/(27*a^(5/3)*b^(4/3)) + ((-1)^(2/3)*d*CoshIntegral[-(((-1)^(2/3)*a^(1
/3)*d)/b^(1/3)) - d*x]*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(27*a^(5/3
)*b^(4/3)) + (d*Sinh[c + d*x])/ (18*a*b^2*x^2) - (d*Sinh[c + d*x])/ (18*b^2*x
^2*(a + b*x^3)) + ((-1)^(1/3)*d*Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*Si
nhIntegral[(-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/ (27*a^(5/3)*b^(4/3)) - ((
-1)^(2/3)*d^2*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[(-1)^(
1/3)*a^(1/3)*d)/b^(1/3) - d*x])/ (54*a^(4/3)*b^(5/3)) + (d*Cosh[c - (a^(1/3)
*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/ (27*a^(5/3)*b^(4/3))
+ (d^2*Sinh[c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x
])/ (54*a^(4/3)*b^(5/3)) + ((-1)^(2/3)*d*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(
1/3)]*SinhIntegral[(-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/ (27*a^(5/3)*b^(4/
3)) - ((-1)^(1/3)*d^2*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral
[(-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/ (54*a^(4/3)*b^(5/3))

```

Rule 3297

```

Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/ (d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
+ d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]

```

Rule 3298

```

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)])/((c_.) + (d_.)*(x_)), x_Symbol]
:= Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]

```

Rule 3301

```

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)])/((c_.) + (d_.)*(x_)), x_Symbol]
:= Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]

```

Rule 3303

```

Int[sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]

```

Rule 5278

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Symbol] := Si
mp[(x^(-n + 1)*(a + b*x^n)^(p + 1)*Sinh[c + d*x])/(b*n*(p + 1)), x] + (-Dis
t[(-n + 1)/(b*n*(p + 1)), Int[((a + b*x^n)^(p + 1)*Sinh[c + d*x])/x^n, x],
x] - Dist[d/(b*n*(p + 1)), Int[x^(-n + 1)*(a + b*x^n)^(p + 1)*Cosh[c + d*x]
, x], x]) /; FreeQ[{a, b, c, d}, x] && IntegerQ[p] && IGtQ[n, 0] && LtQ[p,
-1] && GtQ[n, 2]
```

Rule 5280

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Symbol] := In
t[ExpandIntegrand[Sinh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 5289

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_), x_Symbol] := Simp[(e^m*(a + b*x^n)^(p + 1)*Cosh[c + d*x])/(b*n*(p + 1))
, x] - Dist[(d*e^m)/(b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*Sinh[c + d*x], x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IntegerQ[p] && EqQ[m - n + 1, 0
] && LtQ[p, -1] && (IntegerQ[n] || GtQ[e, 0])
```

Rule 5292

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Sy
mbol] := Int[ExpandIntegrand[Sinh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rule 5293

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] := Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2 \cosh(c + dx)}{(a + bx^3)^3} dx &= -\frac{\cosh(c + dx)}{6b(a + bx^3)^2} + \frac{d \int \frac{\sinh(c+dx)}{(a+bx^3)^2} dx}{6b} \\
&= -\frac{\cosh(c + dx)}{6b(a + bx^3)^2} - \frac{d \sinh(c + dx)}{18b^2 x^2 (a + bx^3)} - \frac{d \int \frac{\sinh(c+dx)}{x^3(a+bx^3)} dx}{9b^2} + \frac{d^2 \int \frac{\cosh(c+dx)}{x^2(a+bx^3)} dx}{18b^2} \\
&= -\frac{\cosh(c + dx)}{6b(a + bx^3)^2} - \frac{d \sinh(c + dx)}{18b^2 x^2 (a + bx^3)} - \frac{d \int \left(\frac{\sinh(c+dx)}{ax^3} - \frac{b \sinh(c+dx)}{a(a+bx^3)} \right) dx}{9b^2} + \frac{d^2 \int \left(\frac{\cosh(c+dx)}{ax^2} \right) dx}{9b^2} \\
&= -\frac{\cosh(c + dx)}{6b(a + bx^3)^2} - \frac{d \sinh(c + dx)}{18b^2 x^2 (a + bx^3)} - \frac{d \int \frac{\sinh(c+dx)}{x^3} dx}{9ab^2} + \frac{d \int \frac{\sinh(c+dx)}{a+bx^3} dx}{9ab} + \frac{d^2 \int \frac{\cosh(c+dx)}{x^2} dx}{18ab^2} \\
&= -\frac{d^2 \cosh(c + dx)}{18ab^2 x} - \frac{\cosh(c + dx)}{6b(a + bx^3)^2} + \frac{d \sinh(c + dx)}{18ab^2 x^2} - \frac{d \sinh(c + dx)}{18b^2 x^2 (a + bx^3)} + \frac{d \int \left(-\frac{\sinh(c+dx)}{3a^{2/3}} \right) dx}{3a^{2/3}} \\
&= -\frac{\cosh(c + dx)}{6b(a + bx^3)^2} + \frac{d \sinh(c + dx)}{18ab^2 x^2} - \frac{d \sinh(c + dx)}{18b^2 x^2 (a + bx^3)} - \frac{d \int \frac{\sinh(c+dx)}{-\sqrt[3]{a} - \sqrt[3]{b} x} dx}{27a^{5/3} b} - \frac{d \int \frac{\sinh(c+dx)}{-\sqrt[3]{a} + \sqrt[3]{b} x} dx}{27a^{5/3} b} \\
&= -\frac{\cosh(c + dx)}{6b(a + bx^3)^2} + \frac{d^3 \text{Chi}(dx) \sinh(c)}{18ab^2} + \frac{d \sinh(c + dx)}{18ab^2 x^2} - \frac{d \sinh(c + dx)}{18b^2 x^2 (a + bx^3)} + \frac{d^3 \cosh(c)}{18ab^2} \\
&= -\frac{\cosh(c + dx)}{6b(a + bx^3)^2} + \frac{(-1)^{2/3} d^2 \cosh\left(c + \frac{\sqrt[3]{-1} \sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(\frac{\sqrt[3]{-1} \sqrt[3]{ad}}{\sqrt[3]{b}} - dx\right)}{54a^{4/3} b^{5/3}} - \frac{\sqrt[3]{-1} d^2 \cosh\left(c + \frac{\sqrt[3]{-1} \sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(\frac{\sqrt[3]{-1} \sqrt[3]{ad}}{\sqrt[3]{b}} + dx\right)}{54a^{4/3} b^{5/3}}
\end{aligned}$$

Mathematica [C] time = 0.46, size = 423, normalized size = 0.54

$$d\text{RootSum}\left[\#1^3 b + a \&, \frac{-2 \sinh(\#1 d + c) \text{Chi}(d(x - \#1)) - \#1 d \sinh(\#1 d + c) \text{Chi}(d(x - \#1)) + 2 \cosh(\#1 d + c) \text{Chi}(d(x - \#1)) + \#1 d \cosh(\#1 d + c) \text{Chi}(d(x - \#1))}{54 a^{4/3} b^{5/3}}\right]$$

Antiderivative was successfully verified.

[In] Integrate[(x^2*Cosh[c + d*x])/(a + b*x^3)^3,x]

```
[Out] -1/108*(d*RootSum[a + b*#1^3 & , (2*Cosh[c + d*#1]*CoshIntegral[d*(x - #1)]
- 2*CoshIntegral[d*(x - #1)]*Sinh[c + d*#1] - 2*Cosh[c + d*#1]*SinhIntegra
l[d*(x - #1)] + 2*Sinh[c + d*#1]*SinhIntegral[d*(x - #1)] + d*Cosh[c + d*#1
]*CoshIntegral[d*(x - #1)]*#1 - d*CoshIntegral[d*(x - #1)]*Sinh[c + d*#1]*#
1 - d*Cosh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1 + d*Sinh[c + d*#1]*SinhInt
egral[d*(x - #1)]*#1)/#1^2 & ] + d*RootSum[a + b*#1^3 & , (-2*Cosh[c + d*#1
]*CoshIntegral[d*(x - #1)] - 2*CoshIntegral[d*(x - #1)]*Sinh[c + d*#1] - 2*
Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] - 2*Sinh[c + d*#1]*SinhIntegral[d*(
x - #1)] + d*Cosh[c + d*#1]*CoshIntegral[d*(x - #1)]*#1 + d*CoshIntegral[d*
(x - #1)]*Sinh[c + d*#1]*#1 + d*Cosh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1
+ d*Sinh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1)/#1^2 & ] - (6*b*Cosh[d*x]*(-
3*a*Cosh[c] + d*x*(a + b*x^3)*Sinh[c]))/(a + b*x^3)^2 - (6*b*(d*x*(a + b*x
^3)*Cosh[c] - 3*a*Sinh[c])*Sinh[d*x))/(a + b*x^3)^2)/(a*b^2)
```

fricas [B] time = 0.69, size = 2972, normalized size = 3.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(d*x+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] -1/216*(36*a^2*cosh(d*x + c) + ((a*d^3/b)^(2/3)*((b^2*x^6 + 2*a*b*x^3 + a^2
- sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b
*x^3 + a^2 - sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*sinh(d*x + c)^2) + 2*(a*
d^3/b)^(1/3)*((b^2*x^6 + 2*a*b*x^3 + a^2 + sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 +
a^2))*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2 + sqrt(-3)*(b^2*x^6 + 2*
a*b*x^3 + a^2))*sinh(d*x + c)^2))*Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) +
1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1) + c) + ((-a*d^3/b)^(2/3)*((b^2*
x^6 + 2*a*b*x^3 + a^2 - sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*cosh(d*x + c)
^2 - (b^2*x^6 + 2*a*b*x^3 + a^2 - sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*sin
h(d*x + c)^2) + 2*(-a*d^3/b)^(1/3)*((b^2*x^6 + 2*a*b*x^3 + a^2 + sqrt(-3)*(
b^2*x^6 + 2*a*b*x^3 + a^2))*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2 +
sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*sinh(d*x + c)^2))*Ei(-d*x - 1/2*(-a*d
^3/b)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1) - c) +
((a*d^3/b)^(2/3)*((b^2*x^6 + 2*a*b*x^3 + a^2 + sqrt(-3)*(b^2*x^6 + 2*a*b*x
^3 + a^2))*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2 + sqrt(-3)*(b^2*x^6
+ 2*a*b*x^3 + a^2))*sinh(d*x + c)^2) + 2*(a*d^3/b)^(1/3)*((b^2*x^6 + 2*a*b
*x^3 + a^2 - sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*cosh(d*x + c)^2 - (b^2*x
^6 + 2*a*b*x^3 + a^2 - sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*sinh(d*x + c)^
2))*Ei(d*x + 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(a*d^3/b)^(1/3)*(
sqrt(-3) - 1) - c) + ((-a*d^3/b)^(2/3)*((b^2*x^6 + 2*a*b*x^3 + a^2 + sqrt(-
3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^
2 + sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*sinh(d*x + c)^2) + 2*(-a*d^3/b)^(
1/3)*((b^2*x^6 + 2*a*b*x^3 + a^2 - sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*co
sh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2 - sqrt(-3)*(b^2*x^6 + 2*a*b*x^3
```

$$\begin{aligned}
& + a^2)) * \sinh(dx + c)^2) * \operatorname{Ei}(-dx + 1/2 * (-a*d^3/b)^{1/3} * (\sqrt{-3} - 1)) * \operatorname{cosh}(1/2 * (-a*d^3/b)^{1/3} * (\sqrt{-3} - 1) + c) - 2 * ((-a*d^3/b)^{2/3} * ((b^2*x^6 \\
& + 2*a*b*x^3 + a^2) * \operatorname{cosh}(dx + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2) * \sinh(dx \\
& + c)^2) + 2 * (-a*d^3/b)^{1/3} * ((b^2*x^6 + 2*a*b*x^3 + a^2) * \operatorname{cosh}(dx + c)^2 - \\
& (b^2*x^6 + 2*a*b*x^3 + a^2) * \sinh(dx + c)^2)) * \operatorname{Ei}(-dx + (-a*d^3/b)^{1/3}) * \\
& \operatorname{cosh}(c + (-a*d^3/b)^{1/3}) - 2 * ((a*d^3/b)^{2/3} * ((b^2*x^6 + 2*a*b*x^3 + a^2) \\
&) * \operatorname{cosh}(dx + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2) * \sinh(dx + c)^2) + 2 * (a*d^3 \\
& /b)^{1/3} * ((b^2*x^6 + 2*a*b*x^3 + a^2) * \operatorname{cosh}(dx + c)^2 - (b^2*x^6 + 2*a*b*x \\
& ^3 + a^2) * \sinh(dx + c)^2)) * \operatorname{Ei}(dx + (a*d^3/b)^{1/3}) * \operatorname{cosh}(-c + (a*d^3/b)^{1/3}) \\
& + ((a*d^3/b)^{2/3} * ((b^2*x^6 + 2*a*b*x^3 + a^2 - \sqrt{-3}) * (b^2*x^6 + \\
& 2*a*b*x^3 + a^2)) * \operatorname{cosh}(dx + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2 - \sqrt{-3}) * (\\
& b^2*x^6 + 2*a*b*x^3 + a^2)) * \sinh(dx + c)^2) + 2 * (a*d^3/b)^{1/3} * ((b^2*x^6 \\
& + 2*a*b*x^3 + a^2 + \sqrt{-3}) * (b^2*x^6 + 2*a*b*x^3 + a^2)) * \operatorname{cosh}(dx + c)^2 - \\
& (b^2*x^6 + 2*a*b*x^3 + a^2 + \sqrt{-3}) * (b^2*x^6 + 2*a*b*x^3 + a^2)) * \sinh(dx \\
& + c)^2) * \operatorname{Ei}(dx - 1/2 * (a*d^3/b)^{1/3} * (\sqrt{-3} + 1)) * \sinh(1/2 * (a*d^3/b)^{1/3} \\
& * (\sqrt{-3} + 1) + c) + ((-a*d^3/b)^{2/3} * ((b^2*x^6 + 2*a*b*x^3 + a^2 - \\
& \sqrt{-3}) * (b^2*x^6 + 2*a*b*x^3 + a^2)) * \operatorname{cosh}(dx + c)^2 - (b^2*x^6 + 2*a*b*x \\
& ^3 + a^2 - \sqrt{-3}) * (b^2*x^6 + 2*a*b*x^3 + a^2)) * \sinh(dx + c)^2) + 2 * (-a*d \\
& ^3/b)^{1/3} * ((b^2*x^6 + 2*a*b*x^3 + a^2 + \sqrt{-3}) * (b^2*x^6 + 2*a*b*x^3 + a \\
& ^2)) * \operatorname{cosh}(dx + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2 + \sqrt{-3}) * (b^2*x^6 + 2*a \\
& *b*x^3 + a^2)) * \sinh(dx + c)^2) * \operatorname{Ei}(-dx - 1/2 * (-a*d^3/b)^{1/3} * (\sqrt{-3} + \\
& 1)) * \sinh(1/2 * (-a*d^3/b)^{1/3} * (\sqrt{-3} + 1) - c) - ((a*d^3/b)^{2/3} * ((b^2 \\
& *x^6 + 2*a*b*x^3 + a^2 + \sqrt{-3}) * (b^2*x^6 + 2*a*b*x^3 + a^2)) * \operatorname{cosh}(dx + c \\
&)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2 + \sqrt{-3}) * (b^2*x^6 + 2*a*b*x^3 + a^2)) * \operatorname{si} \\
& nh(dx + c)^2) + 2 * (a*d^3/b)^{1/3} * ((b^2*x^6 + 2*a*b*x^3 + a^2 - \sqrt{-3}) * (\\
& b^2*x^6 + 2*a*b*x^3 + a^2)) * \operatorname{cosh}(dx + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2 - \\
& \sqrt{-3}) * (b^2*x^6 + 2*a*b*x^3 + a^2)) * \sinh(dx + c)^2) * \operatorname{Ei}(dx + 1/2 * (a*d^3 \\
& /b)^{1/3} * (\sqrt{-3} - 1)) * \sinh(1/2 * (a*d^3/b)^{1/3} * (\sqrt{-3} - 1) - c) - ((\\
& -a*d^3/b)^{2/3} * ((b^2*x^6 + 2*a*b*x^3 + a^2 + \sqrt{-3}) * (b^2*x^6 + 2*a*b*x^3 \\
& + a^2)) * \operatorname{cosh}(dx + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2 + \sqrt{-3}) * (b^2*x^6 + \\
& 2*a*b*x^3 + a^2)) * \sinh(dx + c)^2) + 2 * (-a*d^3/b)^{1/3} * ((b^2*x^6 + 2*a*b* \\
& x^3 + a^2 - \sqrt{-3}) * (b^2*x^6 + 2*a*b*x^3 + a^2)) * \operatorname{cosh}(dx + c)^2 - (b^2*x^ \\
& 6 + 2*a*b*x^3 + a^2 - \sqrt{-3}) * (b^2*x^6 + 2*a*b*x^3 + a^2)) * \sinh(dx + c)^2 \\
&)) * \operatorname{Ei}(-dx + 1/2 * (-a*d^3/b)^{1/3} * (\sqrt{-3} - 1)) * \sinh(1/2 * (-a*d^3/b)^{1/3} \\
& * (\sqrt{-3} - 1) + c) + 2 * ((-a*d^3/b)^{2/3} * ((b^2*x^6 + 2*a*b*x^3 + a^2) * \operatorname{cos} \\
& h(dx + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2) * \sinh(dx + c)^2) + 2 * (-a*d^3/b)^{1/3} \\
& * ((b^2*x^6 + 2*a*b*x^3 + a^2) * \operatorname{cosh}(dx + c)^2 - (b^2*x^6 + 2*a*b*x^3 + \\
& a^2) * \sinh(dx + c)^2)) * \operatorname{Ei}(-dx + (-a*d^3/b)^{1/3}) * \sinh(c + (-a*d^3/b)^{1/3} \\
&) + 2 * ((a*d^3/b)^{2/3} * ((b^2*x^6 + 2*a*b*x^3 + a^2) * \operatorname{cosh}(dx + c)^2 - (b^ \\
& 2*x^6 + 2*a*b*x^3 + a^2) * \sinh(dx + c)^2) + 2 * (a*d^3/b)^{1/3} * ((b^2*x^6 + 2 \\
& *a*b*x^3 + a^2) * \operatorname{cosh}(dx + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2) * \sinh(dx + c) \\
& ^2)) * \operatorname{Ei}(dx + (a*d^3/b)^{1/3}) * \sinh(-c + (a*d^3/b)^{1/3}) - 12 * (a*b*d*x^4 + \\
& a^2*d*x) * \sinh(dx + c) / ((a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b) * \operatorname{cosh}(dx + \\
& c)^2 - (a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b) * \sinh(dx + c)^2)
\end{aligned}$$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \cosh(dx + c)}{(bx^3 + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*cosh(d*x+c)/(b*x^3+a)^3,x, algorithm="giac")

[Out] integrate(x^2*cosh(d*x + c)/(b*x^3 + a)^3, x)

maple [C] time = 0.35, size = 994, normalized size = 1.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*cosh(d*x+c)/(b*x^3+a)^3,x)

[Out]
$$\begin{aligned} & -1/36*d^7*exp(-d*x-c)/a/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)*x^4-1/36*d^7*exp(-d*x-c)/b/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)*x-1/12*d^6*exp(-d*x-c)/b/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)-1/108/a^2/b^2*sum((_R1^2*b*c^2-_R1*a*d^3-2*_R1*b*c^3-a*c*d^3+b*c^4+8*_R1^2*b*c-10*_R1*b*c^2-2*a*d^3+2*b*c^3+8*_R1*b*c+2*b*c^2)/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c), _R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/108*c^2/a^2/b*sum((_R1^2-2*_R1*c+c^2+6*_R1-6*c+10)/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c), _R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/54*c/a^2/b^2*sum((_R1^2*b*c-2*_R1*b*c^2-a*d^3+b*c^3+4*_R1^2*b-2*_R1*b*c-2*b*c^2+4*_R1*b+6*b*c)/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c), _R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/36*d^7*exp(d*x+c)/a/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)*x^4+1/36*d^7*exp(d*x+c)/b/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)*x-1/12*d^6*exp(d*x+c)/b/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)-1/108/a^2/b^2*sum((_R1^2*b*c^2-_R1*a*d^3-2*_R1*b*c^3-a*c*d^3+b*c^4-8*_R1^2*b*c+10*_R1*b*c^2+2*a*d^3-2*b*c^3+8*_R1*b*c+2*b*c^2)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c), _R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/108*c^2/a^2/b*sum((_R1^2-2*_R1*c+c^2-6*_R1+6*c+10)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c), _R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/54*c/a^2/b^2*sum((_R1^2*b*c-2*_R1*b*c^2-a*d^3+b*c^3-4*_R1^2*b+2*_R1*b*c+2*b*c^2+4*_R1*b+6*b*c)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c), _R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3)) \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{(dx^2e^{(2c)} + 7xe^{(2c)})e^{(dx)} - (dx^2 - 7x)e^{(-dx)}}{2(b^3d^2x^9e^c + 3ab^2d^2x^6e^c + 3a^2bd^2x^3e^c + a^3d^2e^c)} + \frac{1}{2} \int \frac{(56bx^3e^c - 9adx^2e^c - 7ae^c)e^{(dx)}}{b^4d^2x^{12} + 4ab^3d^2x^9 + 6a^2b^2d^2x^6 + 4a^3bd^2x^3 + a^4d^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*cosh(d*x+c)/(b*x^3+a)^3,x, algorithm="maxima")

[Out] 1/2*((d*x^2*e^(2*c) + 7*x*e^(2*c))*e^(d*x) - (d*x^2 - 7*x)*e^(-d*x))/(b^3*d^2*x^9*e^c + 3*a*b^2*d^2*x^6*e^c + 3*a^2*b*d^2*x^3*e^c + a^3*d^2*e^c) + 1/2*integrate((56*b*x^3*e^c - 9*a*d*x*e^c - 7*a*e^c)*e^(d*x)/(b^4*d^2*x^12 + 4*a*b^3*d^2*x^9 + 6*a^2*b^2*d^2*x^6 + 4*a^3*b*d^2*x^3 + a^4*d^2), x) + 1/2*integrate((56*b*x^3 + 9*a*d*x - 7*a)*e^(-d*x)/(b^4*d^2*x^12*e^c + 4*a*b^3*d^2*x^9*e^c + 6*a^2*b^2*d^2*x^6*e^c + 4*a^3*b*d^2*x^3*e^c + a^4*d^2*e^c), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2 \cosh(c + d x)}{(b x^3 + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2*cosh(c + d*x))/(a + b*x^3)^3,x)

[Out] int((x^2*cosh(c + d*x))/(a + b*x^3)^3, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*cosh(d*x+c)/(b*x**3+a)**3,x)

[Out] Timed out

$$3.111 \quad \int \frac{x \cosh(c+dx)}{(a+bx^3)^3} dx$$

Optimal. Leaf size=1147

$$\frac{\sqrt[3]{-1} \cosh\left(c + \frac{\sqrt[3]{-1} \sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(\frac{\sqrt[3]{-1} \sqrt[3]{ad}}{\sqrt[3]{b}} - dx\right) d^2}{54a^{5/3}b^{4/3}} - \frac{(-1)^{2/3} \cosh\left(c - \frac{(-1)^{2/3} \sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(-xd - \frac{(-1)^{2/3} \sqrt[3]{ad}}{\sqrt[3]{b}}\right) d^2}{54a^{5/3}b^{4/3}} - \cosh\left(c + \frac{\sqrt[3]{-1} \sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(\frac{\sqrt[3]{-1} \sqrt[3]{ad}}{\sqrt[3]{b}} - dx\right) d^2$$

[Out] $-2/27*\text{Chi}(a^{1/3}*d/b^{1/3}+d*x)*\cosh(c-a^{1/3}*d/b^{1/3})/a^{7/3}/b^{2/3}-1/54*d^2*\text{Chi}(a^{1/3}*d/b^{1/3}+d*x)*\cosh(c-a^{1/3}*d/b^{1/3})/a^{5/3}/b^{4/3}-2/27*(-1)^{2/3}*\text{Chi}((-1)^{1/3}*a^{1/3}*d/b^{1/3}-d*x)*\cosh(c+(-1)^{1/3}*a^{1/3}*d/b^{1/3})/a^{7/3}/b^{2/3}+1/54*(-1)^{1/3}*d^2*\text{Chi}((-1)^{1/3}*a^{1/3}*d/b^{1/3}-d*x)*\cosh(c+(-1)^{1/3}*a^{1/3}*d/b^{1/3})/a^{5/3}/b^{4/3}+2/27*(-1)^{1/3}*\text{Chi}(-(-1)^{2/3}*a^{1/3}*d/b^{1/3}-d*x)*\cosh(c-(-1)^{2/3}*a^{1/3}*d/b^{1/3})/a^{7/3}/b^{2/3}-1/54*(-1)^{2/3}*d^2*\text{Chi}(-(-1)^{2/3}*a^{1/3}*d/b^{1/3}-d*x)*\cosh(c-(-1)^{2/3}*a^{1/3}*d/b^{1/3})/a^{5/3}/b^{4/3}-1/18*\cosh(d*x+c)/a/b^2/x^4+2/9*\cosh(d*x+c)/a^2/b/x-1/6*\cosh(d*x+c)/b/x/(b*x^3+a)^2+1/18*\cosh(d*x+c)/b^2/x^4/(b*x^3+a)-2/27*d*\cosh(c+(-1)^{1/3}*a^{1/3}*d/b^{1/3})*\text{Shi}(-(-1)^{1/3}*a^{1/3}*d/b^{1/3}+d*x)/a^2/b-2/27*d*\cosh(c-a^{1/3}*d/b^{1/3})*\text{Shi}(a^{1/3}*d/b^{1/3}+d*x)/a^2/b-2/27*d*\cosh(c-(-1)^{2/3}*a^{1/3}*d/b^{1/3})*\text{Shi}((-1)^{2/3}*a^{1/3}*d/b^{1/3}+d*x)/a^2/b-2/27*d*\text{Chi}(a^{1/3}*d/b^{1/3}+d*x)*\sinh(c-a^{1/3}*d/b^{1/3})/a^2/b-2/27*\text{Shi}(a^{1/3}*d/b^{1/3}+d*x)*\sinh(c-a^{1/3}*d/b^{1/3})/a^{7/3}/b^{2/3}-1/54*d^2*\text{Shi}(a^{1/3}*d/b^{1/3}+d*x)*\sinh(c-a^{1/3}*d/b^{1/3})/a^{5/3}/b^{4/3}-2/27*d*\text{Chi}((-1)^{1/3}*a^{1/3}*d/b^{1/3}-d*x)*\sinh(c+(-1)^{1/3}*a^{1/3}*d/b^{1/3})/a^2/b-2/27*(-1)^{2/3}*\text{Shi}(-(-1)^{1/3}*a^{1/3}*d/b^{1/3}+d*x)*\sinh(c+(-1)^{1/3}*a^{1/3}*d/b^{1/3})/a^{5/3}/b^{4/3}-2/27*d*\text{Chi}(-(-1)^{2/3}*a^{1/3}*d/b^{1/3}-d*x)*\sinh(c-(-1)^{2/3}*a^{1/3}*d/b^{1/3})/a^2/b+2/27*(-1)^{1/3}*\text{Shi}((-1)^{2/3}*a^{1/3}*d/b^{1/3}+d*x)*\sinh(c-(-1)^{2/3}*a^{1/3}*d/b^{1/3})/a^{7/3}/b^{2/3}-1/54*(-1)^{2/3}*d^2*\text{Shi}((-1)^{2/3}*a^{1/3}*d/b^{1/3}+d*x)*\sinh(c-(-1)^{2/3}*a^{1/3}*d/b^{1/3})/a^{5/3}/b^{4/3}+1/18*d*\sinh(d*x+c)/a/b^2/x^3-1/18*d*\sinh(d*x+c)/b^2/x^3/(b*x^3+a)$

Rubi [A] time = 3.20, antiderivative size = 1147, normalized size of antiderivative = 1.00, number of steps used = 89, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {5291, 5293, 3297, 3303, 3298, 3301, 5292, 5290, 5281}

result too large to display

Antiderivative was successfully verified.

[In] Int[(x*Cosh[c + d*x])/(a + b*x^3)^3,x]

```
[Out] -Cosh[c + d*x]/(18*a*b^2*x^4) + (2*Cosh[c + d*x])/(9*a^2*b*x) - Cosh[c + d*
x]/(6*b*x*(a + b*x^3)^2) + Cosh[c + d*x]/(18*b^2*x^4*(a + b*x^3)) - (2*(-1)
^(2/3)*Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[((-1)^(1/3)*a^(
1/3)*d)/b^(1/3) - d*x])/(27*a^(7/3)*b^(2/3)) + ((-1)^(1/3)*d^2*Cosh[c + ((
-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) -
d*x])/(54*a^(5/3)*b^(4/3)) + (2*(-1)^(1/3)*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)
/b^(1/3)]*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d*x])/(27*a^(7/3
)*b^(2/3)) - ((-1)^(2/3)*d^2*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CoshI
ntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d*x])/(54*a^(5/3)*b^(4/3)) - (2
*Cosh[c - (a^(1/3)*d)/b^(1/3)]*CoshIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(27
*a^(7/3)*b^(2/3)) - (d^2*Cosh[c - (a^(1/3)*d)/b^(1/3)]*CoshIntegral[(a^(1/3
)*d)/b^(1/3) + d*x])/(54*a^(5/3)*b^(4/3)) - (2*d*CoshIntegral[(a^(1/3)*d)/b
^(1/3) + d*x]*Sinh[c - (a^(1/3)*d)/b^(1/3)])/(27*a^2*b) - (2*d*CoshIntegral
[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1
/3)])/(27*a^2*b) - (2*d*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d*
x]*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(27*a^2*b) + (d*Sinh[c + d*x])
/(18*a*b^2*x^3) - (d*Sinh[c + d*x])/(18*b^2*x^3*(a + b*x^3)) + (2*d*Cosh[c
+ ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/
3) - d*x])/(27*a^2*b) + (2*(-1)^(2/3)*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/
3)]*SinhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(27*a^(7/3)*b^(2/3)
) - ((-1)^(1/3)*d^2*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[(
(-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(54*a^(5/3)*b^(4/3)) - (2*d*Cosh[c -
(a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(27*a^2*b) -
(2*Sinh[c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(
27*a^(7/3)*b^(2/3)) - (d^2*Sinh[c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1
/3)*d)/b^(1/3) + d*x])/(54*a^(5/3)*b^(4/3)) - (2*d*Cosh[c - ((-1)^(2/3)*a^(
1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(27*a^
2*b) + (2*(-1)^(1/3)*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[
((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(27*a^(7/3)*b^(2/3)) - ((-1)^(2/3)*d
^2*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3
)*d)/b^(1/3) + d*x])/(54*a^(5/3)*b^(4/3))
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
+ d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)])/((c_.) + (d_.)*(x_)), x_Symbol]
:= Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:= Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x]
&& EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

Rule 5281

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := In
t[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d}, x]
&& ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 5290

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Sy
mbol] := Simp[(x^(m - n + 1)*(a + b*x^n)^(p + 1)*Sinh[c + d*x])/(b*n*(p + 1
)), x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1
)*Sinh[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^
n)^(p + 1)*Cosh[c + d*x], x], x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1]
&& IGtQ[n, 0] && RationalQ[m] && (GtQ[m - n + 1, 0] || GtQ[n, 2])
```

Rule 5291

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] := Simp[(x^(m - n + 1)*(a + b*x^n)^(p + 1)*Cosh[c + d*x])/(b*n*(p + 1
)), x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1
)*Cosh[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^
n)^(p + 1)*Sinh[c + d*x], x], x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1]
&& IGtQ[n, 0] && RationalQ[m] && (GtQ[m - n + 1, 0] || GtQ[n, 2])
```

Rule 5292

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Sy
mbol] := Int[ExpandIntegrand[Sinh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rule 5293

```

Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] := Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])

```

Rubi steps

$$\begin{aligned}
\int \frac{x \cosh(c+dx)}{(a+bx^3)^3} dx &= -\frac{\cosh(c+dx)}{6bx(a+bx^3)^2} - \frac{\int \frac{\cosh(c+dx)}{x^2(a+bx^3)^2} dx}{6b} + \frac{d \int \frac{\sinh(c+dx)}{x(a+bx^3)^2} dx}{6b} \\
&= -\frac{\cosh(c+dx)}{6bx(a+bx^3)^2} + \frac{\cosh(c+dx)}{18b^2x^4(a+bx^3)} - \frac{d \sinh(c+dx)}{18b^2x^3(a+bx^3)} + \frac{2 \int \frac{\cosh(c+dx)}{x^5(a+bx^3)} dx}{9b^2} - \frac{d \int \frac{\sinh(c+dx)}{x^4(a+bx^3)} dx}{18b^2} \\
&= -\frac{\cosh(c+dx)}{6bx(a+bx^3)^2} + \frac{\cosh(c+dx)}{18b^2x^4(a+bx^3)} - \frac{d \sinh(c+dx)}{18b^2x^3(a+bx^3)} + \frac{2 \int \left(\frac{\cosh(c+dx)}{ax^5} - \frac{b \cosh(c+dx)}{a^2x^2} \right) dx}{9b^2} \\
&= -\frac{\cosh(c+dx)}{6bx(a+bx^3)^2} + \frac{\cosh(c+dx)}{18b^2x^4(a+bx^3)} - \frac{d \sinh(c+dx)}{18b^2x^3(a+bx^3)} + \frac{2 \int \frac{x \cosh(c+dx)}{a+bx^3} dx}{9a^2} + \frac{2 \int \frac{\cosh(c+dx)}{x^5} dx}{9ab^2} \\
&= -\frac{\cosh(c+dx)}{18ab^2x^4} - \frac{d^2 \cosh(c+dx)}{36ab^2x^2} + \frac{2 \cosh(c+dx)}{9a^2bx} - \frac{\cosh(c+dx)}{6bx(a+bx^3)^2} + \frac{\cosh(c+dx)}{18b^2x^4(a+bx^3)} \\
&= -\frac{\cosh(c+dx)}{18ab^2x^4} + \frac{d^2 \cosh(c+dx)}{108ab^2x^2} + \frac{2 \cosh(c+dx)}{9a^2bx} - \frac{\cosh(c+dx)}{6bx(a+bx^3)^2} + \frac{\cosh(c+dx)}{18b^2x^4(a+bx^3)} \\
&= -\frac{\cosh(c+dx)}{18ab^2x^4} + \frac{2 \cosh(c+dx)}{9a^2bx} - \frac{\cosh(c+dx)}{6bx(a+bx^3)^2} + \frac{\cosh(c+dx)}{18b^2x^4(a+bx^3)} + \frac{d \sinh(c+dx)}{18ab^2x^3} \\
&= -\frac{\cosh(c+dx)}{18ab^2x^4} + \frac{2 \cosh(c+dx)}{9a^2bx} - \frac{\cosh(c+dx)}{6bx(a+bx^3)^2} + \frac{\cosh(c+dx)}{18b^2x^4(a+bx^3)} + \frac{d^4 \cosh(c) \text{Chi}(c)}{36ab^2} \\
&= -\frac{\cosh(c+dx)}{18ab^2x^4} + \frac{2 \cosh(c+dx)}{9a^2bx} - \frac{\cosh(c+dx)}{6bx(a+bx^3)^2} + \frac{\cosh(c+dx)}{18b^2x^4(a+bx^3)} - \frac{d^4 \cosh(c) \text{Chi}(c)}{108ab^2} \\
&= -\frac{\cosh(c+dx)}{18ab^2x^4} + \frac{2 \cosh(c+dx)}{9a^2bx} - \frac{\cosh(c+dx)}{6bx(a+bx^3)^2} + \frac{\cosh(c+dx)}{18b^2x^4(a+bx^3)} - \frac{2(-1)^{2/3} \cosh(c)}{108ab^2}
\end{aligned}$$

Mathematica [C] time = 0.50, size = 669, normalized size = 0.58

RootSum $\left[\#1^3 b + a \&, \frac{-4\#1^2 b d \sinh(\#1 d + c) \text{Chi}(d(x - \#1)) + 4\#1^2 b d \cosh(\#1 d + c) \text{Chi}(d(x - \#1)) + 4\#1^2 b d \sinh(\#1 d + c) \text{Shi}(d(x - \#1)) - 4\#1^2 b d \cosh(\#1 d + c) \text{Shi}(d(x - \#1))}{(a + b\#1^3)^2} \right]$

Antiderivative was successfully verified.

[In] Integrate[(x*Cosh[c + d*x])/(a + b*x^3)^3,x]

[Out] (RootSum[a + b*#1^3 & , (- (a*d^2*Cosh[c + d*#1]*CoshIntegral[d*(x - #1)]) + a*d^2*CoshIntegral[d*(x - #1)]*Sinh[c + d*#1] + a*d^2*Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] - a*d^2*Sinh[c + d*#1]*SinhIntegral[d*(x - #1)] + 4*b*Cosh[c + d*#1]*CoshIntegral[d*(x - #1)]*#1 - 4*b*CoshIntegral[d*(x - #1)]*Sinh[c + d*#1]*#1 - 4*b*Cosh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1 + 4*b*Sinh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1 + 4*b*d*Cosh[c + d*#1]*CoshIntegral[d*(x - #1)]*#1^2 - 4*b*d*CoshIntegral[d*(x - #1)]*Sinh[c + d*#1]*#1^2 - 4*b*d*Cosh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1^2 + 4*b*d*Sinh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1^2)/#1^2 &] - RootSum[a + b*#1^3 & , (a*d^2*Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] + a*d^2*CoshIntegral[d*(x - #1)]*Sinh[c + d*#1] + a*d^2*Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] + a*d^2*Sinh[c + d*#1]*SinhIntegral[d*(x - #1)] - 4*b*Cosh[c + d*#1]*CoshIntegral[d*(x - #1)]*#1 - 4*b*CoshIntegral[d*(x - #1)]*Sinh[c + d*#1]*#1 - 4*b*Cosh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1 - 4*b*Sinh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1 + 4*b*d*Cosh[c + d*#1]*CoshIntegral[d*(x - #1)]*#1^2 + 4*b*d*CoshIntegral[d*(x - #1)]*Sinh[c + d*#1]*#1^2 + 4*b*d*Cosh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1^2 + 4*b*d*Sinh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1^2)/#1^2 &] + (6*b*Cosh[d*x]*(b*x^2*(7*a + 4*b*x^3)*Cosh[c] + a*d*(a + b*x^3)*Sinh[c]))/(a + b*x^3)^2 + (6*b*(a*d*(a + b*x^3)*Cosh[c] + b*x^2*(7*a + 4*b*x^3)*Sinh[c])*Sinh[d*x])/(a + b*x^3)^2)/(108*a^2*b^2)

fricas [B] time = 0.68, size = 4691, normalized size = 4.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cosh(d*x+c)/(b*x^3+a)^3,x, algorithm="fricas")

[Out] -1/216*((8*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*cosh(d*x + c)^2 - 8*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*sinh(d*x + c)^2 - 4*(a*d^3/b)^(2/3)*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b - sqrt(-3)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^2*b - sqrt(-3)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*sinh(d*x + c)^2) - (a*d^3/b)^(1/3)*((a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3 + sqrt(-3)*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3))*cosh(d*x + c)^2 - (a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3 + sqrt(-3)*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3))*sinh(d*x + c)^2) - (a*d^3/b)^(1/3)*((a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3 + sqrt(-3)*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3))*cosh(d*x + c)^2 - (a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3 + sqrt(-3)*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3))*sinh(d*x + c)^2))

$$\begin{aligned}
& t(-3)*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3))*\sinh(d*x + c)^2))*Ei(d*x \\
& - 1/2*(a*d^3/b)^{(1/3)}*(\sqrt{-3} + 1))*\cosh(1/2*(a*d^3/b)^{(1/3)}*(\sqrt{-3} + \\
& 1) + c) - (8*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3))*\cosh(d*x + c)^2 - \\
& 8*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3))*\sinh(d*x + c)^2 + 4*(-a*d^3/ \\
& b)^{(2/3)}*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b - \sqrt{-3})*(b^3*x^6 + 2*a*b^2*x^3 \\
& + a^2*b))*\cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^2*b - \sqrt{-3})*(b^3*x \\
& x^6 + 2*a*b^2*x^3 + a^2*b))*\sinh(d*x + c)^2) - (-a*d^3/b)^{(1/3)}*((a*b^2*d^3 \\
& *x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3 + \sqrt{-3})*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^ \\
& 3 + a^3*d^3))*\cosh(d*x + c)^2 - (a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3 \\
& + \sqrt{-3})*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3))*\sinh(d*x + c)^2))*E \\
& i(-d*x - 1/2*(-a*d^3/b)^{(1/3)}*(\sqrt{-3} + 1))*\cosh(1/2*(-a*d^3/b)^{(1/3)}*(\sqrt{-3} \\
& + 1) - c) + (8*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3))*\cosh(d*x \\
& + c)^2 - 8*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3))*\sinh(d*x + c)^2 - 4* \\
& (a*d^3/b)^{(2/3)}*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b + \sqrt{-3})*(b^3*x^6 + 2*a*b \\
& ^2*x^3 + a^2*b))*\cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^2*b + \sqrt{-3} \\
&)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*\sinh(d*x + c)^2) - (a*d^3/b)^{(1/3)}*((a*b \\
& ^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3 - \sqrt{-3})*(a*b^2*d^3*x^6 + 2*a^2*b* \\
& d^3*x^3 + a^3*d^3))*\cosh(d*x + c)^2 - (a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^ \\
& 3*d^3 - \sqrt{-3})*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3))*\sinh(d*x + c) \\
& ^2))*Ei(d*x + 1/2*(a*d^3/b)^{(1/3)}*(\sqrt{-3} - 1))*\cosh(1/2*(a*d^3/b)^{(1/3)}* \\
& (\sqrt{-3} - 1) - c) - (8*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3))*\cosh(d \\
& *x + c)^2 - 8*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3))*\sinh(d*x + c)^2 + \\
& 4*(-a*d^3/b)^{(2/3)}*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b + \sqrt{-3})*(b^3*x^6 + 2 \\
& *a*b^2*x^3 + a^2*b))*\cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^2*b + \sqrt{-3} \\
& t(-3)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*\sinh(d*x + c)^2) - (-a*d^3/b)^{(1/3)}* \\
& ((a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3 - \sqrt{-3})*(a*b^2*d^3*x^6 + 2*a \\
& ^2*b*d^3*x^3 + a^3*d^3))*\cosh(d*x + c)^2 - (a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 \\
& + a^3*d^3 - \sqrt{-3})*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3))*\sinh(d*x \\
& + c)^2))*Ei(-d*x + 1/2*(-a*d^3/b)^{(1/3)}*(\sqrt{-3} - 1))*\cosh(1/2*(-a*d^3/b \\
&)^{(1/3)}*(\sqrt{-3} - 1) + c) - 2*(4*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d \\
& ^3))*\cosh(d*x + c)^2 - 4*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3))*\sinh(d* \\
& x + c)^2 - 4*(-a*d^3/b)^{(2/3)}*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b)*\cosh(d*x + c \\
&)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*\sinh(d*x + c)^2) + (-a*d^3/b)^{(1/3)}* \\
& ((a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3))*\cosh(d*x + c)^2 - (a*b^2*d^3*x^ \\
& 6 + 2*a^2*b*d^3*x^3 + a^3*d^3))*\sinh(d*x + c)^2))*Ei(-d*x + (-a*d^3/b)^{(1/3} \\
&)*\cosh(c + (-a*d^3/b)^{(1/3})) + 2*(4*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3* \\
& d^3))*\cosh(d*x + c)^2 - 4*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3))*\sinh(d \\
& *x + c)^2 + 4*(a*d^3/b)^{(2/3)}*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b)*\cosh(d*x + c \\
&)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*\sinh(d*x + c)^2) + (a*d^3/b)^{(1/3)}*((\\
& a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3))*\cosh(d*x + c)^2 - (a*b^2*d^3*x^6 \\
& + 2*a^2*b*d^3*x^3 + a^3*d^3))*\sinh(d*x + c)^2))*Ei(d*x + (a*d^3/b)^{(1/3)})*c \\
& osh(-c + (a*d^3/b)^{(1/3})) + (8*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3))* \\
& \cosh(d*x + c)^2 - 8*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3))*\sinh(d*x + \\
& c)^2 - 4*(a*d^3/b)^{(2/3)}*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b - \sqrt{-3})*(b^3*x^ \\
& 6 + 2*a*b^2*x^3 + a^2*b))*\cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^2*b
\end{aligned}$$

$*b*d^2*x^2)*\cosh(d*x + c) - 12*(a^2*b*d^3*x^3 + a^3*d^3)*\sinh(d*x + c))/((a^3*b^3*d^2*x^6 + 2*a^4*b^2*d^2*x^3 + a^5*b*d^2)*\cosh(d*x + c)^2 - (a^3*b^3*d^2*x^6 + 2*a^4*b^2*d^2*x^3 + a^5*b*d^2)*\sinh(d*x + c)^2)$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \cosh(dx + c)}{(bx^3 + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cosh(d*x+c)/(b*x^3+a)^3,x, algorithm="giac")

[Out] integrate(x*cosh(d*x + c)/(b*x^3 + a)^3, x)

maple [C] time = 0.26, size = 810, normalized size = 0.71

$$\frac{d^6 e^{-dx-c} b x^5}{9a^2 (b^2 d^6 x^6 + 2ab d^6 x^3 + a^2 d^6)} - \frac{d^7 e^{-dx-c} x^3}{36a (b^2 d^6 x^6 + 2ab d^6 x^3 + a^2 d^6)} + \frac{7d^6 e^{-dx-c} x^2}{36a (b^2 d^6 x^6 + 2ab d^6 x^3 + a^2 d^6)} - \frac{d^5 e^{-dx-c} x}{36a (b^2 d^6 x^6 + 2ab d^6 x^3 + a^2 d^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*cosh(d*x+c)/(b*x^3+a)^3,x)

[Out] $\frac{1}{9}d^6 \exp(-d*x-c)/a^2/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)*b*x^5-1/36*d^7*\exp(-d*x-c)/a/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)*x^3+7/36*d^6*\exp(-d*x-c)/a/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)*x^2-1/36*d^7*\exp(-d*x-c)/b/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)-1/108*d/a^2/b^2*\sum((_R1^2*b*c-2*_R1*b*c^2-a*d^3+b*c^3+4*_R1^2*b-2*_R1*b*c-2*b*c^2+4*_R1*b+6*b*c)/(_R1^2-2*_R1*c+c^2)*\exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/108*d*c/a^2/b*\sum((_R1^2-2*_R1*c+c^2+6*_R1-6*c+10)/(_R1^2-2*_R1*c+c^2)*\exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/9*d^6*\exp(d*x+c)/a^2/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)*b*x^5+1/36*d^7*\exp(d*x+c)/a/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)*x^3+7/36*d^6*\exp(d*x+c)/a/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)*x^2+1/36*d^7*\exp(d*x+c)/b/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)-1/108*d/a^2/b^2*\sum((_R1^2*b*c-2*_R1*b*c^2-a*d^3+b*c^3-4*_R1^2*b+2*_R1*b*c+2*b*c^2+4*_R1*b+6*b*c)/(_R1^2-2*_R1*c+c^2)*\exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/108*d*c/a^2/b*\sum((_R1^2-2*_R1*c+c^2-6*_R1+6*c+10)/(_R1^2-2*_R1*c+c^2)*\exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{x e^{(dx+2c)} - x e^{(-dx)}}{2(b^3 dx^9 e^c + 3 ab^2 dx^6 e^c + 3 a^2 b dx^3 e^c + a^3 d e^c)} + \frac{1}{2} \int \frac{(8 b x^3 e^c - a e^c) e^{(dx)}}{b^4 dx^{12} + 4 ab^3 dx^9 + 6 a^2 b^2 dx^6 + 4 a^3 b dx^3 + a^4 d} dx - \frac{1}{2} \int \frac{1}{b^4 dx^{12} + 4 ab^3 dx^9 + 6 a^2 b^2 dx^6 + 4 a^3 b dx^3 + a^4 d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cosh(d*x+c)/(b*x^3+a)^3,x, algorithm="maxima")

[Out] $\frac{1}{2}*(x*e^{(d*x + 2*c)} - x*e^{(-d*x)})/(b^3*d*x^9*e^c + 3*a*b^2*d*x^6*e^c + 3*a^2*b*d*x^3*e^c + a^3*d*e^c) + \frac{1}{2}*integrate((8*b*x^3*e^c - a*e^c)*e^{(d*x)})/(b^4*d*x^{12} + 4*a*b^3*d*x^9 + 6*a^2*b^2*d*x^6 + 4*a^3*b*d*x^3 + a^4*d), x) - \frac{1}{2}*integrate((8*b*x^3 - a)*e^{(-d*x)})/(b^4*d*x^{12}*e^c + 4*a*b^3*d*x^9*e^c + 6*a^2*b^2*d*x^6*e^c + 4*a^3*b*d*x^3*e^c + a^4*d*e^c), x)$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x \cosh(c + dx)}{(bx^3 + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x*cosh(c + d*x))/(a + b*x^3)^3,x)

[Out] int((x*cosh(c + d*x))/(a + b*x^3)^3, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cosh(d*x+c)/(b*x**3+a)**3,x)

[Out] Timed out

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)

(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*      GradeAntiderivative[result,optimal] returns*)
(* "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(* "C" if result involves higher level functions than necessary*)
(* "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(* "A" if result can be considered optimal*)

GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
```

```

If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
  If[LeafCount[result]<=2*LeafCount[optimal],
    "A",
    "B"],
  "C"],
If[FreeQ[result,Integrate] && FreeQ[result,Int],
  "C",
"F"]]

```

```
(* ::Text:: *)
```

```
(*The following summarizes the type number assigned an *)
```

```
(*expression based on the functions it involves*)
```

```
(*1 = rational function*)
```

```
(*2 = algebraic function*)
```

```
(*3 = elementary function*)
```

```
(*4 = special function*)
```

```
(*5 = hyperpergeometric function*)
```

```
(*6 = appell function*)
```

```
(*7 = rootsum function*)
```

```
(*8 = integrate function*)
```

```
(*9 = unknown function*)
```

```
ExpnType[expn_] :=
```

```
  If[AtomQ[expn],
```

```
    1,
```

```
  If[ListQ[expn],
```

```
    Max[Map[ExpnType,expn]],
```

```
  If[Head[expn]===Power,
```

```
    If[IntegerQ[expn[[2]]],
```

```
      ExpnType[expn[[1]],
```

```
    If[Head[expn[[2]]]===Rational,
```

```
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
```

```
        1,
```

```
        Max[ExpnType[expn[[1]],2]],
```

```
      Max[ExpnType[expn[[1]],ExpnType[expn[[2]],3]],
```

```
  If[Head[expn]===Plus || Head[expn]===Times,
```

```
    Max[ExpnType[First[expn],ExpnType[Rest[expn]]],
```

```
  If[ElementaryFunctionQ[Head[expn]],
```

```
    Max[3,ExpnType[expn[[1]]],
```

```
  If[SpecialFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
```

```
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
```

```
  If[AppellFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
```

```

If[Head[expn]===RootSum,
  Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
If[Head[expn]===Integrate || Head[expn]===Int,
  Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
9]]]]]]]]]]

ElementaryFunctionQ[func_] :=
MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]

SpecialFunctionQ[func_] :=
MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]

HypergeometricFunctionQ[func_] :=
MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]

AppellFunctionQ[func_] :=
MemberQ[{AppellF1},func]

```

4.0.2 Maple grading function

```

# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017 Use Maple leaf count instead since buildin
#Nasser 03/23/2017 missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017 corrected the check for complex result
#Nasser 10/27/2017 check for leafsize and do not call ExpnType()
#
# if leaf size is "too large". Set at 500,000

```

```

#Nasser 12/22/2019 Added debug flag, added 'dilog' to special functions
# see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

    leaf_count_result:=leafcount(result);
    #do NOT call ExpnType() if leaf size is too large. Recursion problem
    if leaf_count_result > 500000 then
        return "B";
    fi;

    leaf_count_optimal:=leafcount(optimal);

    ExpnType_result:=ExpnType(result);
    ExpnType_optimal:=ExpnType(optimal);

    if debug then
        print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
ExpnType_optimal);
    fi;

# If result and optimal are mathematical expressions,
# GradeAntiderivative[result,optimal] returns
# "F" if the result fails to integrate an expression that
# is integrable
# "C" if result involves higher level functions than necessary
# "B" if result is more than twice the size of the optimal
# antiderivative
# "A" if result can be considered optimal

#This check below actually is not needed, since I only
#call this grading only for passed integrals. i.e. I check
#for "F" before calling this. But no harm of keeping it here.
#just in case.

if not type(result,freeof('int')) then
    return "F";
end if;

if ExpnType_result<=ExpnType_optimal then
    if debug then
        print("ExpnType_result<=ExpnType_optimal");
    fi;

```



```

if is_contains_complex(result) then
  if is_contains_complex(optimal) then
    if debug then
      print("both result and optimal complex");
    fi;
    #both result and optimal complex
    if leaf_count_result<=2*leaf_count_optimal then
      return "A";
    else
      return "B";
    end if
  else #result contains complex but optimal is not
    if debug then
      print("result contains complex but optimal is not");
    fi;
    return "C";
  end if
else # result do not contain complex
  # this assumes optimal do not as well
  if debug then
    print("result do not contain complex, this assumes optimal do not
as well");
  fi;
  if leaf_count_result<=2*leaf_count_optimal then
    if debug then
      print("leaf_count_result<=2*leaf_count_optimal");
    fi;
    return "A";
  else
    if debug then
      print("leaf_count_result>2*leaf_count_optimal");
    fi;
    return "B";
  end if
end if
else #ExpnType(result) > ExpnType(optimal)
  if debug then
    print("ExpnType(result) > ExpnType(optimal)");
  fi;
  return "C";
end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false

```

```

#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1
    else
      max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'^^') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1
      else
        max(2,ExpnType(op(1,expn)))
      end if
    else
      max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  end if
end proc:

```

```

elif HypergeometricFunctionQ(op(0,expn)) then
  max(5,apply(max,map(ExpnType,[op(expn)])))
elif AppellFunctionQ(op(0,expn)) then
  max(6,apply(max,map(ExpnType,[op(expn)])))
elif op(0,expn)='int' then
  max(8,apply(max,map(ExpnType,[op(expn)]))) else
9
end if
end proc:

ElementaryFunctionQ := proc(func)
  member(func,[
    exp,log,ln,
    sin,cos,tan,cot,sec,csc,
    arcsin,arccos,arctan,arccot,arcsec,arccsc,
    sinh,cosh,tanh,coth,sech,csch,
    arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
    erf,erfc,erfi,
    FresnelS,FresnelC,
    Ei,Ei,Li,Si,Ci,Shi,Chi,
    GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
    EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product. rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
    op(2,u)
  else
    apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

```

```
#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17 Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
  MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#           Port of original Maple grading function by
#           Albert Rich to use with Sympy/Python
#Dec 27, 2019 Nasser. Added `RootSum`. See problem 177, Timofeev file
#           added 'exp_polar'
from sympy import *

def leaf_count(expr):
  #sympy do not have leaf count function. This is approximation
  return round(1.7*count_ops(expr))

def is_sqrt(expr):
  if isinstance(expr,Pow):
    if expr.args[1] == Rational(1,2):
      return True
    else:
      return False
  else:
    return False

def is_elementary_function(func):
  return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
                 asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
                 asinh,acosh,atanh,acoth,asech,acsch
                 ]

def is_special_function(func):
  return func in [ erf,erfc,erfi,
                 fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
                 gamma,loggamma,digamma,zeta,polylog,LambertW,
                 elliptic_f,elliptic_e,elliptic_pi,exp_polar
                 ]

def is_hypergeometric_function(func):
  return func in [hyper]

def is_appell_function(func):
  return func in [appellf1]
```

```

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow): #type(expn,'`^`')
        if isinstance(expn.args[1],Integer): #type(op(2,expn),'integer')
            return expnType(expn.args[0]) #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational): #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)
))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type
(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0])) #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1) #max(4,apply(max,map(ExpnType,[op(expn)])))

```

```

elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
expn))
    m1 = max(map(expnType, list(expn.args)))
    return max(5,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
elif is_appell_function(expn.func):
    m1 = max(map(expnType, list(expn.args)))
    return max(6,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
elif isinstance(expn,RootSum):
    m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,
Apply[List,expn]],7]],
    return max(7,m1)
elif str(expn).find("Integral") != -1:
    m1 = max(map(expnType, list(expn.args)))
    return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"

```

4.0.4 SageMath grading function

```

#Dec 24, 2019. Nasser: Ported original Maple grading function by
#           Albert Rich to use with Sagemath. This is used to
#           grade Fracas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#           'arctan2', 'floor', 'abs', 'log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow: #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
        'sin','cos','tan','cot','sec','csc',
        'arcsin','arccos','arctan','arccot','arcsec','arccsc',
        'sinh','cosh','tanh','coth','sech','csch',
        'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
    ]
    if debug:
        if m:

```

```

        print ("func ", func , " is elementary_function")
    else:
        print ("func ", func , " is NOT elementary_function")

    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
        'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
sinh_integral'
        'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
        'polylog','lambert_w','elliptic_f','elliptic_e',
        'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")

    return m

def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in
sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:

```



```

        return expn.operator() is None
    if expn.parent() in (ZZ, QQ, AA, QQbar):
        return expn in expn.parent() # Should always return True
    if hasattr(expn.parent(), "base_ring") and hasattr(expn.parent(), "gens"):
        return expn in expn.parent().base_ring() or expn in expn.parent().
gens()
    return False

except AttributeError as error:
    return False

def expnType(expn):

    if debug:
        print(">>>>Enter expnType, expn=", expn)
        print(">>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list: #instance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(instance(expn.args[0],
Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.
args[0]))
    elif expn.operator() == operator.pow: #instance(expn,Pow)
        if type(expn.operands()[1])==Integer: #instance(expn.args[1],Integer)
            return expnType(expn.operands()[0]) #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational: #instance(expn.args[1],
Rational)
            if type(expn.operands()[0])==Rational: #instance(expn.args[0],
Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.
args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
instance(expn,Add) or instance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))

```

```

    return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()): #is_elementary_function(expn.
func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
    return max(4,m1) #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #
is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
    return max(5,m1) #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
    return max(6,m1) #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
#is checked before calling the grading function that is passed.
#but kept it here.
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
    return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

```

```
#main function
```

```
def grade_antiderivative(result,optimal):
```

```

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
leaf_count_optimal=",leaf_count_optimal)

```

```

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

```

```

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
expnType_optimal)

```

```

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex

```

```
        if leaf_count_result <= 2*leaf_count_optimal:
            return "A"
        else:
            return "B"
    else: #result contains complex but optimal is not
        return "C"
else: # result do not contain complex, this assumes optimal do not as
well
    if leaf_count_result <= 2*leaf_count_optimal:
        return "A"
    else:
        return "B"
else:
    return "C"
```